Exercise 1.1

1. Prove that the product of two consecutive positive integers is divisible by 2.

Sol:

Let, (n – 1) and n be two consecutive positive integers

∴ Their product = n(n – 1)

$= n^2 - n$

We know that any positive integer is of the form 2q or $2q + 1$, for some integer q

When n =2q, we have

$n^2 - n = (2q)^2 - 2q$

$= 4q^2 - 2q$

$2q(2q - 1)$

Then $n^2 - n$ is divisible by 2.

When n = 2q + 1, we have

$n^2 - n = (2q + 1)^2 - (2q + 1)$

$= 4q^2 + 4q + 1 - 2q - 1$

$= 4q^2 + 2q$

$= 2q(2q + 1)$

Then $n^2 - n$ is divisible by 2.

Hence the product of two consecutive positive integers is divisible by 2.

2. If a and b are two odd positive integers such that a > b, then prove that one of the two numbers $\frac{a+b}{2}$ and $\frac{a-b}{2}$ is odd and the other is even.

Sol:

Let a = 2q + 3 and b = 2q + 1 be two positive odd integers such that a > b

Now, $\frac{a+b}{2} = \frac{2q+3+2q+1}{2} = \frac{4q+4}{2} = 2q + 2 = an\ even\ number$

and $\frac{a-b}{2} = \frac{(2q+3)-(2q+1)}{2} = \frac{2q+3-2q-1}{2} = \frac{2}{2} = 1 = an\ odd\ number$

Hence one of the two numbers $\frac{a+b}{2}\ and\ \frac{a-b}{2}\ \ is\ odd\ and\ the\ other\ is\ even\ for\ any\ two\ positive\ odd\ integer$

3. Show that the square of an odd positive integer is of the form 8q + 1, for some integer q.

Sol:

By Euclid's division algorithm

a = bq + r, where 0 ≤ r ≤ b

Put b = 4

a = 4q + r, where 0 ≤ r ≤ 4

If r = 0, then a = 4q even

If r = 1, then a = 4q + 1 odd

If r = 2, then a = 4q + 2 even
If r = 3, then a = 4q + 3 odd
Now, $(4q+1)^2 = (4q)^2 + 2(4q)(1) + (1)^2$
= $16q^2 + 8q + 1$
= $8(2q^2 + q) + 1$
= 8m + 1 where m is some integer
Hence the square of an odd integer is of the form 8q + 1, for some integer q

4. Show that any positive odd integer is of the form 6q + 1 or, 6q + 3 or, 6q + 5, where q is some integer.

Sol:

Let a be any odd positive integer we need to prove that a is of the form 6q + 1, or 6q +3, 6q + 5, where q is some integer
Since a is an integer consider b = 6 another integer applying Euclid's division lemma we get
a = 6q + r for some integer q ≥ 0, and r = 0, 1, 2, 3, 4, 5 since 0 ≤ r < 6.
Therefore, a = 6q or 6q + 1 or 6q + 2 or 6q + 3 or 6q + 4 or 6q + 5
However since a is odd so a cannot take the values 6q, 6q + 2 and 6q + 4
(since all these are divisible by 2)
Also, 6q + 1 = 2 × 3q + 1 = 2k 1 + 1, where k1 is a positive integer
6q + 3 = (6q + 2) + 1 = 2 (3q + 1) + 1 = 2k2 + 1, where k2 is an integer
6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k3 + 1, where k3 is an integer
Clearly, 6q +1, 6q +3, 6q + 5 are of the form 2k + 1, where k is an integer
Therefore, 6q + 1, 6q + 3, 6q + 5 are odd numbers.
Therefore, any odd integer can be expressed is of the form
6q + 1, or 6q + 3, 6q + 5 where q is some integer
Concept insight: In order to solve such problems Euclid's division lemma is applied to two integers a and b the integer b must be taken in accordance with what is to be proved, for example here the integer b was taken 6 because a must be of the form 6q + 1, 6q +3, 6q + 5
Basic definition of even (divisible by 2) and odd numbers (not divisible by 2) and the fact that addiction and multiplication of integers is always an integer are applicable here.

5. Prove that the square of any positive integer is of the form 3m or, 3m + 1 but not of the form 3m +2.

Sol:

By Euclid's division algorithm
a = bq + r, where 0 ≤ r ≤ b
Put b = 3
a = 3q + r, where 0 ≤ r ≤ 3
If r = 0, then a = 3q

If r = 1, then a = 3q + 1
If r = 2, then a = 3q + 2
Now, $(3q)^2 = 9q^2$
$= 3 \times 3q^2$
$= 3m, where\ m\ is\ some\ integer$
$(3q+1)^2 = (3q)^2 + 2(3q)(1) + (1)^2$
$= 9q^2 + 6q + 1$
$= 3(3q^2 + 2q) + 1$
$= 3m + 1, where\ m\ is\ some\ integer$
$(3q+2)^2 = (3q)^2 + 2(3q)(2) + (2)^2$
$= 9q^2 + 12q + 4$
$= 9q^2 + 12q + 4$
$= 3(3q^2 + 4q + 1) + 1$
$= 3m + 1, where\ m\ is\ some\ integer$
Hence the square of any positive integer is of the form 3m, or 3m +1
But not of the form 3m + 2

6. Prove that the square of any positive integer is of the form 4q or 4q + 1 for some integer q.
Sol:
By Euclid's division Algorithm
a = bm + r, where $0 \le r \le b$
Put b = 4
a = 4m + r, where $0 \le r \le 4$
If r = 0, then a = 4m
If r = 1, then a = 4m + 1
If r = 2, then a = 4m + 2
If r = 3, then a = 4m + 3
Now, $(4m)^2 = 16m^2$
$= 4 \times 4m^2$
= 4q where q is some integer
$(4m+1)^2 = (4m)^2 + 2(4m)(1) + (1)^2$
$= 16m^2 + 8m + 1$
$= 4(4m^2 + 2m) + 1$
$= 4q + 1$ where q is some integer
$(4m+2)^2 = (4m)^2 + 2(4m)(2) + (2)^2$
$= 16m^2 + 24m + 9$
$= 16m^2 + 24m + 8 + 1$
$= 4(4m^2 + 6m + 2) + 1$
$= 4q + 1, where\ q\ is\ some\ integer$

Hence, the square of any positive integer is of the form 4q or 4q + 1 for some integer m

7. Prove that the square of any positive integer is of the form 5q, 5q + 1, 5q + 4 for some integer q.

Sol:

By Euclid's division algorithm
a = bm + r, where $0 \le r \le b$
Put b = 5
a = 5m + r, where $0 \le r \le 4$
If r = 0, then a = 5m
If r = 1, then a = 5m + 1
If r = 2, then a = 5m + 2
If r = 3, then a = 5m + 3
If r = 4, then a = 5m + 4
Now, $(5m)^2 = 25m^2$
= $5(5m^2)$
= 5q where q is some integer
$(5m+1)^2 = (5m)^2 + 2(5m)(1) + (1)^2$
$= 25m^2 + 10m + 1$
$= 5(5m^2 + 2m) + 1$
$= 5q + 1\ where\ q\ is\ some\ integer$
$(5m+1)^2 = (5m)^2 + 2(5m)(1)(1)^2$
$= 25m^2 + 10m + 1$
$= 5(5m^2 + 2m) + 1$
$= 5q + 1$ where q is some integer
$= (5m+2)^2 = (5m)^2 + 2(5m)(2) + (2)^2$
$= 25m^2 + 20m + 4$
$= 5(5m^2 + 4m) + 4$
$= 5q + 4, where\ q\ is\ some\ integer$
$= (5m+3)^2 = (5m)^2 + 2(5m)(3) + (3)^2$
$= 25m^2 + 30m + 9$
$= 25m^2 + 30m + 5 + 4$
$= 5(5m^2 + 6m + 1) + 4$
$= 5q + 1, where\ q\ is\ some\ integer$
$= (5m+4)^2 = (5m)^2 + 2(5m)(4) + (4)^2$
$= 25m^2 + 40m + 16$
$= 25m^2 + 40m + 15 + 1$
$= 5(5m^2) + 2(5m)(4) + (4)^2$
$= 5q + 1, where\ q\ is\ some\ integer$

Hence, the square of any positive integer is of the form 5q or 5q + 1, 5q + 4 for some integer q.

8. Prove that if a positive integer is of the form 6q + 5, then it is of the form 3q + 2 for some integer q, but not conversely.

Sol:

Let, n = 6q + 5, when q is a positive integer

We know that any positive integer is of the form 3k, or 3k + 1, or 3k + 2

∴ q = 3k or 3k + 1, or 3k + 2

If q = 3k, then

n = 6q + 5

= 6(3k) + 5

= 18k + 5

= 18k + 3 + 2

= 3(6k + 1) + 2

= 3m + 2, where m is some integer

If q = 3k + 1, then

n = 6q + 5

= 6(3k + 1) + 5

= 18k + 6 + 5

= 18k + 11

= 3(6k + 3) + 2

= 3m + 2, where m is some integer

If q = 3k + 2, then

n = 6q + 5

= 6(3k + 2) + 5

= 18k + 12 + 5

= 18k + 17

= 3(6k + 5) + 2

= 3m + 2, where m is some integer

Hence, if a positive integer is of the form 6q + 5, then it is of the form 3q + 2 for some integer q.

Conversely

Let n = 3q + 2

We know that a positive integer can be of the form 6k + 1, 6k + 2, 6k + 3, 6k + 4 or 6k + 5

So, now if q = 6k + 1 then

n = 3(6k + 1) + 2

= 18k + 5

= 6(3k) + 5

= 6m + 5, where m is some integer
So, now if q = 6k + 2 then
n = 3(6k + 2) + 2
= 18k + 8
= 6 (3k + 1) + 2
= 6m + 2, where m is some integer
Now, this is not of the form 6m + 5
Hence, if n is of the form 3q + 2, then it necessarily won't be of the form 6q + 5 always.

9. Prove that the square of any positive integer of the form 5q + 1 is of the same form.
Sol:
Let n = 5q + 1 where q is a positive integer
$\therefore n^2 = (5q+1)^2$
= $25q^2 + 10q + 1$
= $5(5q^2 + 2q) + 1$
= 5m + 1, where m is some integer
Hence, the square of any positive integer of the form 5q + 1 is of the same form.

10. Prove that the product of three consecutive positive integer is divisible by 6.
Sol:
Let, n be any positive integer. Since any positive integer is of the form 6q or 6q + 1 or 6q + 2 or, 6q + 3 or 6q + 4 or 6q + 5.
If n = 6q, then
n(n + 1)(n + 2) = (6q + 1)(6q + 2)(6q + 3)
= 6[(6q + 1)(3q + 1)(2q + 1)]
= 6m, which is divisible by 6?
If n = 6q + 1, then
n(n + 1)(n + 2) = (6q + 2)(6q + 3)(6q + 4)
= 6[(6q + 1)(3q + 1)(2q + 1)]
= 6m, which is divisible by 6
If n = 6q + 2, then
n(n + 1)(n + 2) = (6q + 2)(6q + 3)(6q + 4)
= 6[(3q + 1)(2q + 1)(6q + 4)]
= 6m, which is divisible by 6.
If n = 6q + 3, then
n(n + 1)(n + 2) = (6q + 3)(6q + 4)(6q + 5)
= 6[(6q + 1)(3q + 2)(2q + 5)]
= 6m, which is divisible by 6.
If n = 6q + 4, then
n(n + 1)(n + 2) = (6q + 4)(6q + 5)(6q + 6)

= 6[(6q + 4)(3q + 5)(2q + 1)]
= 6m, which is divisible by 6.
If n = 6q + 5, then
n(n + 1)(n + 2) = (6q + 5)(6q + 6)(6q + 7)
= 6[(6q + 5)(q + 1)(6q + 7)]
= 6m, which is divisible by 6.
Hero, the product of three consecutive positive integer is divisible by 6.

11. For any positive integer n , prove that $n^3 - n$ divisible by 6.
Sol:
We have $n^3 - n = n(n^2 - 1) = (n - 1)\,(n)\,(n + 1)$
Let, n be any positive integer. Since any positive integer is of the form 6q or 6q + 1 or, 6q + 2 or, 6q + 3 or, 6q + 4 or, 6q + 5.
If n = 6q, then
$(n - 1)(n)(n + 1) = (6q - 1)(6q)(6q + 1)$
= $6[(6q - 1)(q)(6q + 1)]$
= 6m, which is divisible by 6
If n = 6q + 1, then
$(n - 1)(n + 1) = (6q)(6q + 1)(6q + 2)$
= $6[(q)(6q + 1)(6q + 2)]$
= 6m, which is divisible by 6
If n = 6q + 2, then
$(n - 1)(n)(n + 1) = (6q + 1)(6q + 2)(6q + 3)$
= $6[(6q + 1)(3q + 1)(2q + 1)]$
= 6m, which is divisible by 6
If n = 6q + 3, then
$(n - 1)(n)(n + 1) = (6q + 3)(6q + 4)(6q + 5)$
= $6[(3q + 1)(2q + 1)(6q + 4)]$
= 6m, which is divisible by 6
If n = 6q + 4, then
$(n - 1)(n)(n + 1) = (6q + 3)(6q + 4)(6q + 5)$
= $6[(2q + 1)(3q + 2)(6q + 5)]$
= 6m, which is divisible by 6
If n = 6q + 5, then
$(n - 1)(n)(n + 1) = (6q + 4)(6q + 5)(6q + 6)$
= $6[(6q + 4)(6q + 5)(q + 1)]$
= 6m, which is divisible by 6
Hence, for any positive integer n, $n^3 – n$ is divisible by 6.

Exercise 1.2

1. Define HOE of two positive integers and find the HCF of the following pairs of numbers:
(i) 32 and 54 (ii) 18 and 24 (iii) 70 and 30 (iv) 56 and 88
(v) 475 and 495 (vi) 75 and 243 (vii) 240 and 6552 (viii) 155 and 1385
(ix) 100 and 190 (x) 105 and 120

Sol:

By applying Euclid's division lemma

(i) 5y = 32 × 1 + 22
Since remainder ≠ 0, apply division lemma on division of 32 and remainder 22.
32 = 22 × 1 + 10
Since remainder ≠ 0, apply division lemma on division of 22 and remainder 10.
22 = 10 × 2 + 2
Since remainder ≠ 0, apply division lemma on division of 10 and remainder 2.
10 = 2 × 5 [remainder 0]
Hence, HCF of 32 and 54 10 2

(ii) By applying division lemma
24 = 18 × 1 + 6
Since remainder = 6, apply division lemma on divisor of 18 and remainder 6.
18 = 6 × 3 + 0
∴ Hence, HCF of 18 and 24 = 6

(iii) By applying Euclid's division lemma
70 = 30 × 2 + 10
Since remainder ≠ 0, apply division lemma on divisor of 30 and remainder 10.
30 = 10 × 3 + 0
∴ Hence HCF of 70 and 30 is = 10.

(iv) By applying Euclid's division lemma
88 = 56 × 1 + 32
Since remainder ≠ 0, apply division lemma on divisor of 56 and remainder 32.
56 = 32 × 1 + 24
Since remainder ≠ 0, apply division lemma on divisor of 32 and remainder 24.
$32 = 24 \times 1 + 8$
Since remainder ≠ 0, apply division lemma on divisor of 24 and remainder 8.
$24 = 8 \times 3 + 0$
∴ HCF of 56 and 88 is = 8.

(v) By applying Euclid's division lemma
495 = 475 × 1 +20
Since remainder ≠ 0, apply division lemma on divisor of 475 and remainder 20.
475 = 20 × 23 + 15
Since remainder ≠ 0, apply division lemma on divisor of 20 and remainder 15.

$20 = 15 \times 1 + 5$
Since remainder $\neq 0$, apply division lemma on divisor of 15 and remainder 5.
$15 = 5 \times 3 + 0$
$\therefore$ HCF of 475 and 495 is = 5.

(vi) By applying Euclid's division lemma
$243 = 75 \times 3 + 18$
Since remainder $\neq 0$, apply division lemma on divisor of 75 and remainder 18.
$75 = 18 \times 4 + 3$
Since remainder $\neq 0$, apply division lemma on divisor of 18 and remainder 3.
$18 = 3 \times 6 + 0$
$\therefore$ HCF of 243 and 75 is = 3.

(vii) By applying Euclid's division lemma
$6552 = 240 \times 27 + 72$
Since remainder $\neq 0$, apply division lemma on divisor of 240 and remainder 72.
$210 = 72 \times 3 + 24$
Since remainder $\neq 0$, apply division lemma on divisor of 72 and remainder 24.
$72 = 24 \times 3 + 0$
$\therefore$ HCF of 6552 and 240 is = 24.

(viii) By applying Euclid's division lemma
$1385 = 155 \times 8 + 145$
Since remainder $\neq 0$, applying division lemma on divisor 155 and remainder 145
$155 = 145 \times 1 + 10$
Since remainder $\neq 0$, applying division lemma on divisor 10 and remainder 5
$10 = 5 \times 2 + 0$
$\therefore$ Hence HCF of 1385 and 155 = 5.

(ix) By applying Euclid's division lemma
$190 = 100 \times 1 + 90$
Since remainder $\neq 0$, applying division lemma on divisor 100 and remainder 90.
$90 = 10 \times 9 + 0$
$\therefore$ HCF of 100 and 190 = 10

(x) By applying Euclid's division lemma
$120 = 105 \times 1 + 15$
Since remainder $\neq 0$, applying division lemma on divisor 105 and remainder 15.
$105 = 15 \times 7 + 0$
$\therefore$ HCF of 105 and 120 = 15

2. Use Euclid's division algorithm to find the HCF of
(i) 135 and 225 (ii) 196 and 38220

Sol:

(i) 135 and 225

Step 1: Since 225 > 135. Apply Euclid's division lemma to a = 225 and b = 135 to find q and r such that 225 = 135q + r, 0 ≤ r < 135
On dividing 225 by 135 we get quotient as 1 and remainder as '90'
i.e., 225 = 135r 1 + 90
Step 2: Remainder 5 which is 90 7, we apply Euclid's division lemma to a = 135 and b = 90 to find whole numbers q and r such that 135 = 90 × q + r 0 ≤ r < 90 on dividing 135 by 90 we get quotient as 1 and remainder as 45
i.e., 135 = 90 × 1 + 45
Step3: Again remainder r = 45 to so we apply division lemma to a = 90 and b = 45 to find q and r such that 90 = 45 × q × r. 0 ≤ r < 45. On dividing 90 by 45we get quotient as 2 and remainder as 0 i.e., 90 = 2 × 45 + 0
Step 4: Since the remainder = 0, the divisor at this stage will be HCF of (135, 225)
Since the divisor at this stage is 45. Therefore the HCF of 135 and 225 is 45.

(ii) 867 and 255:
Step 1: Since 867 > 255, apply Euclid's division
Lemma a to a = 867 = 255 q + r, 0 < r < 255
On dividing 867 by 255 we get quotient as 3 and the remainder as low
Step 2: Since the remainder 102 to, we apply the division lemma to a = 255 and b = 102 to find 255 = 102q + 51 = 102r – 151
Step 3: Again remainder 0 is non-zero, so we apply the division lemma to a = 102 and b = 51 to find whole numbers q and r such that 102 = q = r when 0 ≤ r < 51
On dividing 102 by 51 quotient = 2 and remainder is '0'
i,e., = 102 = 51 × 2 + 0
Since the remainder is zero, the divisional this stage is the HCF.
Since the divisor at this stage is 51, ∴ HCF of 867 and 255 is '51'.

3. Find the HCF of the following pairs of integers and express it as a linear combination of them.
(i) 963 and 657 (ii) 592 and 252 (iii) 506 and 1155 (iv)1288 and 575

Sol:

(i) 963 and 6567
By applying Euclid's division lemma 963 = 657 × 1 + 306 …(i)
Since remainder ≠ 0, apply division lemma on divisor 657 and remainder 306
657 = 306 × 2 + 45 ….. (ii)
Since remainder ≠ 0, apply division lemma on divisor 306 and remainder 4
306 = 45 × 6 + 36 …..(iii)
Since remainder ≠ 0, apply division lemma on divisor 45 and remainder 36
45 = 36 × 1 + 9 …… (iv)
Since remainder ≠ 0, apply division lemma on divisor 36 and remainder 9
36 = 9 × 4 + 0

∴ HCF = 9

Now 9 = 45 – 36 × 1 [from (iv)]

= 45 – [306 – 45 × 6] × 1 [from (iii)]

= 45 – 306 × 1 + 45 × 6

= 45 × 7 – 306 × 1

= 657 × 7 – 306 × 14 – 306 × 1 [from (ii)]

= 657 × 7 – 306 × 15

= 657 × 7 – [963 – 657 × 1] × 15 [from (i)]

= 657 × 22 – 963 × 15

(ii) 595 and 252

By applying Euclid's division lemma

595 = 252 × 2 + 91 ….. (i)

Since remainder ≠ 0, apply division lemma on divisor 252 and remainder 91

252 = 91 × 2 + 70 …. (ii)

Since remainder ≠ 0, apply division lemma on divisor 91 and remainder 70

91 = 70 × 1 + 21 ….(iii)

Since remainder ≠ 0, apply division lemma on divisor 70 and remainder 20

70 = 21 × 3 + 7 …..(iv)

Since remainder ≠ 0, apply division lemma on divisor 21 and remainder 7

21 = 7 × 3 + 0

H.C.F = 7

Now, 7 = 70 – 21 × 3 [from (iv)]

= 70 – [90 – 70 × 1] × 3 [from (iii)]

= 70 – 91 × 3 + 70 × 3

= 70 × 4 – 91 × 3

= [252 – 91 × 2] × 4 – 91 × 3 [from (ii)]

= 252 × 4 – 91 × 8 – 91 × 3

= 252 × 4 – 91 × 11

= 252 × 4 – [595 – 252 × 2] × 11 [from (i)]

= 252 × 4 – 595 × 11 + 252 × 22

= 252 × 6 – 595 × 11

(iii) 506 and 1155

By applying Euclid's division lemma

1155 = 506 × 2 + 143 …. (i)

Since remainder ≠ 0, apply division lemma on division 506 and remainder 143

506 = 143 × 3 + 77 ….(ii)

Since remainder ≠ 0, apply division lemma on division 143 and remainder 77

143 = 77 × 1 + 56 ….(iii)

Since remainder ≠ 0, apply division lemma on division 77 and remainder 66

77 = 66 × 1 + 11 …(iv)

Since remainder ≠ 0, apply division lemma on divisor 36 and remainder 9
$66 = 11 \times 6 + 0$
∴ HCF = 11
Now, $11 = 77 – 6 \times 11$ [from (iv)]
$= 77 – [143 – 77 \times 1] \times 1$ [from (iii)]
$= 77 – 143 \times 1 – 77 \times 1$
$= 77 \times 2 – 143 \times 1$
$= [506 – 143 \times 3] \times 2 – 143 \times 1$ [from (ii)]
$= 506 \times 2 – 143 \times 6 – 143 \times 1$
$= 506 \times 2 – 143 \times 7$
$= 506 \times 2 – [1155 – 506 \times 27 \times 7]$ [from (i)]
$= 506 \times 2 – 1155 \times 7 + 506 \times 14$
$= 506 \times 16 – 115 \times 7$

(iv) 1288 and 575
By applying Euclid's division lemma
$1288 = 575 \times 2 + 138$ …(i)
Since remainder ≠ 0, apply division lemma on division 575 and remainder 138
$575 = 138 \times 1 + 23$ …(ii)
Since remainder ≠ 0, apply division lemma on division 138 and remainder 23 …(iii)
∴ HCF = 23
Now, $23 = 575 – 138 \times 4$ [from (ii)]
$= 575 – [1288 – 572 \times 2] \times 4$ [from (i)]
$= 575 – 1288 \times 4 + 575 \times 8$
$= 575 \times 9 – 1288 \times 4$

4. Express the HCF of 468 and 222 as 468x + 222y where x, y are integers in two different ways.
Sol:
Given integers are 468 and 222 where 468 > 222.
By applying Euclid's division lemma, we get $468 = 222 \times 2 + 24$ …(i)
Since remainder ≠ 0, apply division lemma on division 222 and remainder 24
$222 = 24 \times 9 + 6$ …(ii)
Since remainder ≠ 0, apply division lemma on division 24 and remainder 6
$24 = 6 \times 4 + 0$ …(iii)
We observe that the remainder = 0, so the last divisor 6 is the HCF of the 468 and 222
From (ii) we have
$6 = 222 – 24 \times 9$
⇒ $6 = 222 – [468 – 222 \times 2] \times 9$ [Substituting $24 = 468 – 222 \times 2$ from (i)]
⇒ $6 = 222 – 468 \times 9 – 222 \times 18$
⇒ $6 = 222 \times 19 – 468 \times 9$

⇒ 6 = 222y + 468x, where x = −9 and y = 19

5. If the HCF of 408 and 1032 is expressible in the form 1032 m − 408 × 5, find m.
Sol:
General integers are 408 and 1032 where 408 < 1032
By applying Euclid's division lemma, we get
1032 = 408 × 2 + 216
Since remainder ≠ 0, apply division lemma on division 408 and remainder 216
408 = 216 × 1 + 192
Since remainder ≠ 0, apply division lemma on division 216 and remainder 192
216 = 192 × 1 + 24
Since remainder ≠ 0, apply division lemma on division 192 and remainder 24
192 = 24 × 8 + 32
We observe that 32m under in 0. So the last divisor 24 is the H.C.F of 408 and 1032
∴ 216 = 1032m – 408 × 5
⇒ 1032 m = 24 + 408 × 5
$\Rightarrow 1032m = 24 + 2040$
$\Rightarrow 1032m = 2064$
$\Rightarrow m = \frac{2064}{1032} = 2$

6. If the HCF of 657 and 963 is expressible in the form 657 x + 963 x − 15, find x.
Sol:
657 and 963
By applying Euclid's division lemma
963 = 657 × 1 + 306
Since remainder ≠ 0, apply division lemma on division 657 and remainder 306
657 = 306 × 2 + 45
Since remainder ≠ 0, apply division lemma on division 306 and remainder 45
306 = 45 × 6 + 36
Since remainder ≠ 0, apply division lemma on division 45 and remainder 36
45 = 36 × 1 + 19
Since remainder ≠ 0, apply division lemma on division 36 and remainder 19
36 = 19 × 4 + 0
∴ HCF = 657
Given HCF = 657 + 963 × (-15)
$\Rightarrow 9 = 657 \times -1445$
$\Rightarrow 9 + 14445 = 657\ x$
$\Rightarrow 657x = 1445y$
$\Rightarrow x = \frac{1445y}{657}$

$\Rightarrow x = 22$

7. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?
Sol:
Members in arms = 616
Members in Band = 32
∴ Maximum numbers of columns
= HCF of 616 and 32
By applying Euclid's division lemma
$616 = 32 \times 19 + 8$
$32 = 8 \times 4 + 0$
∴ HCF = 8
Hence the maximum remainder number of columns in which they can each is 8

8. Find the largest number which divides 615 and 963 leaving remainder 6 in each case.
Sol:
The required number when the divides 615 and 963
Leaves remainder 616 is means 615 – 6 = 609 and 963 – 957 are completely divisible by the number
∴ the required number
= HCF of 609 and 957
By applying Euclid's division lemma
$957 = 609 \times 1 + 348$
$609 = 348 \times 1 + 261$
$348 = 261 \times 1 + 87$
$261 = 87 \times 370$
HCF = 87
Hence the required number is '87'

9. Find the greatest number which divides 285 and 1249 leaving remainders 9 and 7 respectively.
Sol:
The require number when divides 285 and 1249, leaves remainder 9 and 7, this means 285 – 9 = 276 and 1249 – 7 = 1242 are completely divisible by the number
∴ The required number = HCF of 276 and 1242
By applying Euclid's division lemma
$1242 = 276 \times 4 + 138$
$276 = 138 \times 2 + 0$

∴ HCF = 138
Hence remainder is = 0
Hence required number is 138

10. Find the largest number which exactly divides 280 and 1245 leaving remainders 4 and 3, respectively.
Sol:
The required number when divides 280 and 1245 leaves the remainder 4 and 3, this means 280 4 – 216 and 1245 – 3 = 1245 – 3 = 1242 are completely divisible by the number
∴ The required number = HCF of 276 and 1242
By applying Euclid's division lemma
1242 = 276 × 4 + 138
276 = 138 × 2 + 0
∴ HCF = 138
Hence the required numbers is 138

11. What is the largest number that divides 626, 3127 and 15628 and leaves remainders of 1, 2 and 3 respectively.
Sol:
The required number when divides 626, 3127 and 15628, leaves remainder 1, 2 and 3. This means 626 – 1 = 625, 3127 – 2 = 3125 and
15628 – 3 = 15625 are completely divisible by the number
∴ The required number = HCF of 625, 3125 and 15625
First consider 625 and 3125
By applying Euclid's division lemma
3125 = 625 × 5 + 0
HCF of 625 and 3125 = 625
Now consider 625 and 15625
By applying Euclid's division lemma
15625 = 625 × 25 + 0
∴ HCF of 625, 3125 and 15625 = 625
Hence required number is 625

12. Find the greatest number that will divide 445, 572 and 699 leaving remainders 4, 5 and 6 respectively.
Sol:
The required number when divides 445, 572 and 699 leaves remainders 4, 5 and 6
This means 445 – 4 = 441, 572 – 5 = 561 and
699 – 6 = 693 are completely divisible by the number
∴ The required number = HCF of 441, 567 and 693

First consider 441 and 567
By applying Euclid’s division lemma
$567 = 441 \times 1 + 126$
$441 = 126 \times 3 + 63$
$126 = 63 \times 2 + 0$
$\therefore$ HCF of 441 and 567 = 63
Now consider 63 and 693
By applying Euclid’s division lemma
$693 = 63 \times 11 + 0$
$\therefore$ HCF of 441, 567 and 693 = 63
Hence required number is 63

13. Find the greatest number which divides 2011 and 2623 leaving remainders 9 and 5 respectively.

Sol:
The required number when divides 2011 and 2623
Leaves remainders 9 and the means
$2011 - 9 = 2002$ and $2623 - 5 = 2618$ are completely divisible by the number
$\therefore$ The required number = HCF of 2002 and 2618
By applying Euclid’s division lemma
$2618 = 2002 \times 1 + 616$
$2002 = 616 \times 3 + 154$
$616 = 754 \times 4 + 0$
$\therefore$ HCF of 2002 and 2618 = 154
Hence required number is 154

14. The length, breadth and height of a room are 8 m 25 cm, 6 m 75 cm and 4 m 50 cm, respectively. Determine the longest rod which can measure the three dimensions of the room exactly.

Sol:
Length of room = 8m 25cm = 825 cm
Breadth of room = 6m 75m = 675 cm
Height of room = 4m 50m = 450 cm
$\therefore$ The required longest rod
= HCF of 825, 675 and 450
First consider 675 and 450
By applying Euclid’s division lemma
$675 = 450 \times 1 + 225$
$450 = 225 \times 2 + 0$
$\therefore$ HCF of 675 and 450 = 825

Now consider 625 and 825
By applying Euclid's division lemma
825 = 225 × 3 + 150
225 = 150 × 1 + 75
150 = 75 × 2 + 0
HCF of 825, 675 and 450 = 75

15. 105 goats, 140 donkeys and 175 cows have to be taken across a river. There is only one boat which will have to make many trips in order to do so. The lazy boatman has his own conditions for transporting them. He insists that he will take the same number of animals in every trip and they have to be of the same kind. He will naturally like to take the largest possible number each time. Can you tell how many animals went in each trip?

Sol:
Number of goats = 205
Number of donkey = 140
Number of cows = 175
∴ The largest number of animals in one trip = HCF of 105, 140 and 175
First consider 105 and 140
By applying Euclid's division lemma
140 = 105 × 1 + 35
105 = 35 × 3 + 0
∴ HCF of 105 and 140 = 35
Now consider 35 and 175
By applying Euclid's division lemma
175 = 35 × 5 + 0
HCF of 105, 140 and 175 = 35

16. 15 pastries and 12 biscuit packets have been donated for a school fete. These are to be packed in several smaller identical boxes with the same number of pastries and biscuit packets in each. How many biscuit packets and how many pastries will each box contain?

Sol:
Number of pastries = 15
Number of biscuit packets = 12
∴ The required no of boxes to contain equal number = HCF of 15 and 13
By applying Euclid's division lemma
15 = 12 × 13
12 = 2 × 9 = 0
∴ No. of boxes required = 3
Hence each box will contain $\frac{15}{3} = 5$ pastries and $\frac{2}{3}$ biscuit packets

17. A mason has to fit a bathroom with square marble tiles of the largest possible size. The size of the bathroom is 10 ft. by 8 ft. What would be the size in inches of the tile required that has to be cut and how many such tiles are required?

Sol:

Size of bathroom = 10ft by 8ft

= (10 × 12) inch by (8 × 12) inch

= 120 inch by 96 inch

The largest size of tile required = HCF of 120 and 96

By applying Euclid's division lemma

120 = 96 × 1 + 24

96 = 24 × 4 + 0

∴ HCF = 24

∴ Largest size of tile required = 24 inches

∴ No. of tiles required = $\frac{Area\ of\ bathroom}{area\ of\ 2\ tile}$

= $\frac{120 \times 96}{24 \times 24}$

= 5 × 4

= 20 tiles

18. Two brands of chocolates are available in packs of 24 and 15 respectively. If I need to buy an equal number of chocolates of both kinds, what is the least number of boxes of each kind I would need to buy?

Sol:

Number of chocolates of 1^{st} brand in one pack = 24

Number of chocolates of 2^{nd} b and in one pack = 15

∴ The least number of chocolates 1 need to purchase

= LCM of 24 and 15

= 2 × 24 × 2 × 2 × 3 × 5

= 120

∴ The number of packet of 1^{st} brand = $\frac{120}{24} = 5$

And the number of packet of 2^{nd} brand = $\frac{120}{15} = 8$

∴ Largest size of tile required = 24 inches

∴ No of tiles required = $\frac{area\ of\ bath\ room}{area\ of\ 1\ tile} = \frac{120 \times 96}{24 \times 24} = 5 \times 4 = 20 tiles$

No of chocolates of 1^{st} brand in one pack = 24

No of chocolate of 2^{nd} brand in one pack = 15

∴ The least number of chocolates I need to purchase

= LCM of 24 and 15

= $2 \times 2 \times 2 \times 3 \times 5$

= 120

∴ The number of packet of 1st brand = $\frac{120}{24} = 5$

All the number of packet of 2nd brand = $\frac{120}{15} = 8$

19. 144 cartons of Coke Cans and 90 cartons of Pepsi Cans are to be stacked in a Canteen. If each stack is of the same height and is to contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

Sol:

Number of cartons of coke cans = 144

Number of cartons of pepsi cans = 90

∴ The greatest number of cartons in one stock = HCF of 144 and 90

By applying Euclid's division lemma

144 = 90 × 1 + 54

90 = 54 × 1 + 36

54 = 36 × 1 + 18

36 = 18 × 2 + 0

∴ HCF = 18

Hence the greatest number cartons in one stock = 18

20. During a sale, colour pencils were being sold in packs of 24 each and crayons in packs of 32 each. If you want full packs of both and the same number of pencils and crayons, how many of each would you need to buy?

Sol:

Number of color pencils in one pack = 24

No of crayons in pack = 32

∴ The least number of both colors to be purchased

= LCM of 24 and 32

= $2 \times 2 \times 2 \times 2 \times 3$

= 96

∴ Number of packs of pencils to be bought = $\frac{96}{24} = 1$

And number of packs of crayon to be bought = $\frac{96}{32} = 3$

21. A merchant has 120 liters of oil of one kind, 180 liters of another kind and 240 liters of third kind. He wants to sell the oil by filling the three kinds of oil in tins of equal capacity. What should be the greatest capacity of such a tin?

Sol:

Quantity of oil A = 120 liters

Quantity of oil B = 180 liters

Quantity of oil C = 240 liters

We want to fill oils A, B and C in tins of the same capacity

∴ The greatest capacity of the tin chat can hold oil. A, B and C = HCF of 120, 180 and 240
By fundamental theorem of arithmetic
120 = $2^3 \times 3 \times 5$
180 = $2^2 \times 3^2 \times 5$
240 = $2^4 \times 3 \times 5$
HCF = $2^2 \times 3 \times 5 = 4 \times 3 \times 5 = 60\ litres$
The greatest capacity of tin = 60 liters

Exercise 2.1

1. Find the zeroes of each of the following quadratic polynomials and verify the relationship between the zeroes and their co efficient:

(i) f(x) = $x^2 - 2x - 8$

(ii) g(s) = $4s^2 - 4x + 1$

(iii) h(t) = $t^2 - 15$

(iv) p(x) = $x^2 + 2\sqrt{2}x + 6$

(v) q(x) = $\sqrt{3}x^2 + 10x + 7\sqrt{3}$

(vi) f(x) = $x^2 - (\sqrt{3} + 1)x + \sqrt{3}$

(vii) g(x) = $a(x^2 + 1) - x(a^2 + 1)$

(viii) $6x^2 - 3 - 7x$

Sol:

(i) f(x) = $x^2 - 2x - 8$

$f(x) = x^2 - 2x - 8 = x^2 - 4x + 2x - 8$

= $x(x - 4) + 2(x - 4)$

= $(x + 2)(x - 4)$

Zeroes of the polynomials are -2 and 4

Sum of the zeroes = $\frac{-\, co\ efficient\ of\ x}{co\ efficient\ of\ x}$

-2 + 4 = $\frac{-(-2)}{1}$

2 = 2

Product of the zeroes = $\frac{constant\ term}{co\ efficient\ of\ x^2}$

= 24 = $\frac{-8}{1}$

$-\,8 = -8$

∴ Hence the relationship verified

(ii) $9(5) = 45 - 45 + 1 = 45^2 - 25 - 25 + 1 = 25(25 - 1) - 1(25 - 1)$

= $(25 - 1)(25 - 1)$

Zeroes of the polynomials are $\frac{1}{2}\, and\, \frac{1}{2}$

Sum of zeroes = $\frac{-\, co\ efficient\ of\ s}{co\ efficient\ of\ s^2}$

$\frac{1}{2} + \frac{1}{2} = \frac{-(-4)}{4}$

1 = 1

Product of the zeroes = $\frac{constant\ term}{co\ efficient\ os\ s^2}$

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \Rightarrow \frac{1}{4} = \frac{1}{4}$

∴ Hence the relationship verified.

(iii) h(t) = $t^2 - 15 = (t^2) - (\sqrt{15})^2 = (t + \sqrt{15})(t - \sqrt{15})$

zeroes of the polynomials are $-\sqrt{15}\ and\ \sqrt{15}$

sum of zeroes = 0

$-\sqrt{15} + \sqrt{15} = 0$

$0 = 0$

Product of zeroes = $\frac{-15}{1}$

$-\sqrt{15}\times\sqrt{15}=-15$

-15 = -15

∴ Hence the relationship verified.

(iv) p(x) = $x^2+2\sqrt{2}x-6=x^2+3\sqrt{2}x+\sqrt{2}\times3\sqrt{2}$

= $x(x+3\sqrt{2})-\sqrt{2}(2+3\sqrt{2})=(x-\sqrt{2})(x+3\sqrt{2})$

Zeroes of the polynomial are 3$\sqrt{2}$ and -3$\sqrt{2}$

Sum of the zeroes = $\frac{-3\sqrt{2}}{1}$

$\sqrt{2}-3\sqrt{2}=-2\sqrt{2}$

$-2\sqrt{2}=-2\sqrt{2}$

$Product\ of\ zeroes\ \Rightarrow\ \sqrt{2}\times-3\sqrt{2}=-\frac{6}{1}$

-6 = -6

$Hence\ the\ relatioship\ varified$

(v) 2(x) = $\sqrt{3}x^2+10x+7\sqrt{3}=\sqrt{3}x^2+7x+3x+7\sqrt{3}$

= $\sqrt{3}x(x+\sqrt{3})+7(x+\sqrt{3})$

= $(\sqrt{3}x+7)(x+\sqrt{3})$

Zeroes of the polynomials are $-\sqrt{3},\frac{-7}{\sqrt{3}}$

Sum of zeroes = $\frac{-10}{\sqrt{3}}$

$\Rightarrow-\sqrt{3}-\frac{7}{\sqrt{3}}=\frac{-10}{\sqrt{3}}\Rightarrow\frac{-10}{\sqrt{3}}=\frac{-10}{\sqrt{3}}$

Product of zeroes = $\frac{7\sqrt{3}}{3}\Rightarrow\frac{\sqrt{3}x-7}{\sqrt{30}}=7$

⇒ 7 = 7

Hence, relationship verified.

(vi) f(x) = $x^2-(\sqrt{3}+1)x+\sqrt{3}=x^2-\sqrt{3}x-x+\sqrt{3}$

= x (x $-\sqrt{3}$) – 1 (x - $\sqrt{3}$)

= (x – 1) (x $-\sqrt{3}$)

Zeroes of the polynomials are 1 and $\sqrt{3}$

Sum of zeroes = $\frac{-\{coefficient\ of\ x\}}{co\ efficient\ of\ r^2}=\frac{-[-\sqrt{3}-1]}{1}$

1 + $\sqrt{3}=\sqrt{3}+1$

Product of zeroes = $\frac{constant\ term}{co\ efficient\ of\ x^2}=\frac{\sqrt{3}}{1}$

$1\times\sqrt{3}=\sqrt{3}=\sqrt{3}=\sqrt{3}$

∴ Hence, relationship verified

(vii) g(x) = $a[(x^2+1)-x(a^2+1)]^2=ax^2+a-a^2x-x$

= $ax^2-[(a^2+1)-x]+0=ax^2-a^2x-x+a$

$= ax(x-a) - 1(x-a) = (x-a)(ax-1)$

Zeroes of the polynomials = $\frac{1}{a}$ $and\ a$

Sum of the zeroes = $\frac{-[-a^2-1]}{a}$

$\Rightarrow \frac{1}{a} + a = \frac{a^2+1}{a} \Rightarrow \frac{a^2+1}{a} = \frac{a^2+1}{a}$

Product of zeroes = $\frac{a}{a}$

$\Rightarrow \frac{1}{a} \times a = \frac{a}{a} \Rightarrow \frac{a^2+1}{a} = \frac{a^2+1}{a}$

Product of zeroes = $\frac{a}{a} \Rightarrow 1 = 1$

Hence relationship verified

(viii) $6x^2 - 3 - 7x = 6x^2 - 7x - 3 = (3x+11)(2x-3)$

Zeroes of polynomials are $+\frac{3}{2}\ and\ \frac{-1}{3}$

Sum of zeroes = $\frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(co\ efficient\ of\ x)}{co\ efficient\ of\ x^2}$

Product of zeroes = $\frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{constant\ term}{co\ efficient\ of\ x^2}$

∴ Hence, relationship verified.

2. If α and β are the zeros of the quadratic polynomial f(x) = ax^2 + bx + c, then evaluate:

(i) $\alpha - \beta$

(ii) $\frac{1}{\alpha} - \frac{1}{\beta}$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

(iv) $\alpha^2\beta + \alpha\beta^2$

(v) $\alpha^4 + \beta^4$

(vi) $\frac{1}{a\alpha+b} + \frac{1}{a\beta+b}$

(vii) $\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b}$

(viii) $a\left[\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right] + b\left[\frac{\alpha}{a} + \frac{\beta}{\alpha}\right]$

Sol:

f(x) = $ax^2 + bx + c$

$\alpha + \beta = \frac{-b}{\boldsymbol{a}}$

$\alpha\beta = \frac{c}{a}$

$since\ \alpha +\ \beta\ are\ the\ roots\ (or) zeroes\ of\ the\ given\ polynomials$

(i) $\alpha - \beta$

The two zeroes of the polynomials are

$\frac{-b+\sqrt{b^2-4ac}}{2a} - \left(b\frac{-\sqrt{b^2-4ac}}{2a}\right) = -b + \frac{\sqrt{b^2-4ac}+b+\sqrt{b^2-4ac}}{2a} = \frac{2\sqrt{b^2-4ac}}{2a} = \frac{\sqrt{b^2-4ac}}{2a}$

(ii) $\frac{1}{\alpha} - \frac{1}{\beta} = \frac{\beta-\alpha}{\alpha\beta} = \frac{-(\alpha-\beta)}{\alpha\beta} \ldots (i)$

From (i) we know that $\alpha - \beta = \frac{\sqrt{b^2-4ac}}{2a}\ [from\ (i)]\alpha\beta = \frac{c}{a}$

Putting the values in the (a) = $-\left(\frac{\sqrt{b^2-4ac \times a}}{a \times c}\right) = \frac{-\sqrt{b^2-4ac}}{c}$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

$\Rightarrow \left[\frac{\alpha+\beta}{\alpha\beta}\right] - 2\alpha\beta$

$\Rightarrow \frac{-b}{a} \times \frac{a}{c} - 2\frac{c}{a} = -2\frac{c}{a} - \frac{b}{c} = \frac{-ab-2c^2}{ac} - \left[\frac{b}{c} + \frac{2c}{a}\right]$

(iv) $\alpha^2\,\beta + \alpha\,\beta^2$

$\alpha\,\beta(\alpha + \beta)$

$= \frac{c}{a}\left(\frac{-b}{a}\right)$

$= \frac{-bc}{a^2}$

(v) $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2 + \beta^2$

$= ((\alpha + \beta)^2 - 2\alpha\beta)^2 - 2(\alpha\beta)^2$

$= \left[\left(-\frac{b}{a}\right)^2 - 2\frac{c}{a}\right]^2 - \left[2\left(\frac{c}{a}\right)^2\right]$

$= \left[\frac{b^2-2ac}{a^2}\right]^2 - \frac{2c^2}{a^2}$

$= \frac{(b^2 2ac)^2 - 2a^2c^2}{a^4}$

(vi) $\frac{1}{a\alpha+b} + \frac{1}{\alpha\beta+b}$

$\Rightarrow \frac{a\beta+b+a\alpha+b}{(3\alpha+b)(\alpha\beta+b)}$

$= \frac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab\,\alpha+ab\beta+b^2}$

$= \frac{a(\alpha+\beta)+b}{a^2\alpha\beta+\alpha\beta(\alpha^2\beta)+b^2}$

$= \frac{a\times\frac{a+2b}{a}}{a\times\frac{c}{a}+\frac{abc(-b)+b^2}{a}} = \frac{b}{ac-b^2+b^2} = \frac{b}{ac}$

(vii) $\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b}$

$= \frac{\beta(a\beta+b)+\alpha(a\alpha+b)}{(a\alpha+b)(\alpha\beta+b)}$

$= \frac{a\beta^2+b\beta+a\alpha^2+b\alpha}{a^2\alpha\beta+ab\alpha+ab\beta+b^2}$

$= \frac{a\alpha^2+a\beta^2+b\beta^2+b\alpha}{a\times\frac{c}{a}+ab(\alpha+\beta)+b^2}$

$= \frac{a\left[(\alpha^2+\beta^2)+b(\alpha+\beta)\right]}{ac+ab+x\left(\frac{-b}{a}\right)+b^2}$

$= \frac{a\left[(\alpha+\beta)^2-2\alpha\beta\right]+bx-\frac{b}{a}}{ac}$

$= \frac{a\left[\frac{b^2}{a}-\frac{2c}{a}\right]-\frac{b^2}{a}}{ac} = \frac{a\times\left[\frac{b^2-2c}{a}\right]-b^2}{ac} = \frac{-2}{a}$

(viii) $a\left[\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right] + b\left[\frac{\alpha}{a} + \frac{\beta}{\alpha}\right]$

$= a\left[\frac{\alpha^3+\beta^3}{\alpha\beta}\right] + b\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)$

$= \frac{\alpha[(\alpha+\beta)^3 - 3\alpha\beta\,(\alpha+\beta)]}{\alpha\,\beta} + b(\alpha+\beta)^2 - 2\alpha\,\beta$

$= \frac{\alpha\left[\left(\frac{-b^3}{a^3}\right) + \frac{3b}{a}.\frac{c}{a} + b\left(\frac{b^2}{a^2} - \frac{2c}{a}\right)\right]}{\frac{c}{a}}$

$= \frac{a^2}{c}\left[\frac{-b^3}{a^3} + \frac{3bc}{a^2} + \frac{b^3}{a^2} - \frac{2bc}{a}\right]$

$= \frac{-a^2b^3}{ca^3} + \frac{3a^2bc}{ca^2} + \frac{b^3a^2}{a^2c} - \frac{2bca^2}{ac}$

$= \frac{-b^3}{ac} + 3b + \frac{b^3}{ac} - 2b$

= b

3. If α and β are the zeros of the quadratic polynomial f(x) = 6x^2 + x − 2, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Sol:

f(x) = $6x^2 - x - 2$

Since α and β are the zeroes of the given polynomial

∴ Sum of zeroes $[\alpha + \beta] = \frac{-1}{6}$

Product of zeroes $(\alpha\beta) = \frac{-1}{3}$

$= \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{1}{6}\right)^2 - 2\times\left(\frac{-1}{3}\right)}{-\frac{1}{3}} = \frac{\frac{1}{6} - \frac{2}{3}}{\frac{-1}{3}} = \frac{\frac{1+24}{36}}{\frac{-1}{3}}$

$= \frac{\frac{2}{36}}{\frac{1}{3}} = \frac{-25}{12}$

4. If a and are the zeros of the quadratic polynomial f(x) = $x^2 - x - 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$

Sol:

Since $\alpha + \beta$ are the zeroes of the polynomial: $x^2 - x - 4$

Sum of the roots $(\alpha + \beta)$ = 1

Product of the roots $(\alpha\beta)$ = −4

$\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{\alpha+\beta}{\alpha\beta} - \alpha\beta$

$= \frac{1}{-4} + 4 = \frac{-1}{4} + 4 = \frac{-1+16}{4} = \frac{15}{4}$

5. If α and β are the zeros of the quadratic polynomial p(x) = 4x^2 − 5x −1, find the value of $\alpha^2\beta + \alpha\beta^2$.

Sol:

Since α *and* β are the roots of the polynomial: $4x^2 - 5x - 1$

∴ Sum of the roots $\alpha + \beta = \frac{5}{4}$

Product of the roots $\alpha\beta = \frac{-1}{4}$

Hence $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta) = \frac{5}{4}\left(\frac{-1}{4}\right) = \frac{-5}{16}$

6. If a and 3 are the zeros of the quadratic polynomial f(x) = $x^2 + x - 2$, find the value of $\frac{1}{\alpha} - \frac{1}{\beta}$.

Sol:

Since α and β are the roots of the polynomial x + x – 2

∴ Sum of roots $\alpha + \beta = 1$

Product of roots $\alpha\beta\ 2 \Rightarrow -\frac{1}{\beta}$

$= \frac{\beta-\alpha}{\alpha\beta} \cdot \frac{(\alpha-\beta)}{\alpha\beta}$

$= \frac{\sqrt{(\alpha+\beta)^2-4\alpha\beta}}{\alpha\beta}$

$= \frac{\sqrt{1+8}}{+2} = \frac{3}{2}$

7. If α and β are the zeros of the quadratic polynomial f(x) = $x^2 - 5x + 4$, find the value of $\frac{1}{\alpha} - \frac{1}{\beta} - 2\alpha\beta$

Sol:

Since $\alpha\ and\ \beta$ are the roots of the quadratic polynomial

f(x) = $x^2 - 5x + 4$

Sum of roots = $\alpha + \beta = 5$

Product of roots = $\alpha\beta = 4$

$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\beta+\alpha}{\alpha\beta} - 2\alpha\beta = \frac{5}{4} - 2 \times 4 = \frac{5}{4} - 8 = \frac{-27}{4}$

8. If α and β are the zeros of the quadratic polynomial f(t) = $t^2 - 4t + 3$, find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$

Sol:

Since $\alpha\ and\ \beta$ are the zeroes of the polynomial f(t) = $t^2 - 4t + 3$

Since α + β = 4

Product of zeroes $\alpha\beta = 3$

$Hence\ \alpha^4\beta^3 + \alpha^3\beta^4 = \alpha^3\beta^3(\alpha + \beta) = [3]^3[4] = 108$

9. If α and β are the zeros of the quadratic polynomial p(y) = $5y^2 - 7y + 1$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$

Sol:

Since α *and* β are the zeroes of the polynomials
p(y) = $54^2 = 5y^2 - 7y + 1$
Sum of the zeroes $\alpha\ \beta = \frac{1}{6}$
Product of zeroes = $\alpha\ \beta = \frac{1}{6}$
$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{7\times5}{5\times1} = 7$

10. If α and β are the zeros of the quadratic polynomial p(s) = $3s^2 - 6s + 4$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$

Sol:

Since α *and* β are the zeroes of the polynomials
Sum of the zeroes $\alpha + \beta = \frac{6}{3}$
Product of the zeroes $\alpha\beta = \frac{4}{3}$
$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$
$\Rightarrow \frac{\alpha^2+\beta^2}{\alpha\beta} + 2\left[\frac{\alpha+\beta}{\alpha\beta}\right] + 3\alpha\beta$
$\Rightarrow \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} + 2\left[\frac{\alpha+\beta}{\alpha\beta}\right] + 3\alpha\beta$
$= \frac{[2]^2-2\times\frac{4}{3}+2\left[\frac{2\times3}{4}\right]+3\left[\frac{4}{3}\right]}{\frac{4}{3}}$
$= \frac{4-\frac{8}{3}}{\frac{4}{3}} + 7 \Rightarrow \frac{4}{3}\times\frac{3}{4}(1+7) \Rightarrow 8$

11. If α and β are the zeros of the quadratic polynomial f(x) = $x^2 - px + q$, prove that $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$

Sol:

Since α and β are the roots of the polynomials
f(x) = $x^2 - px + 2$
sum of zeroes = p = $\alpha + \beta$
Product of zeroes = q = $\alpha\beta$
LHS $= \frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$
$= \frac{\alpha^2+\beta^2}{\alpha\beta^2} = \frac{\left(\alpha^2 + \beta^2\right)^2-2(\alpha\beta)^2}{(\alpha\beta)^2}$
$= \frac{\left[(\alpha+\beta)^2-2\alpha\beta\right]^2-2(\alpha\beta)^2}{(\alpha\beta)^2}$
$= \frac{\left[(p)^2-2q\right]^2-2q^2}{q}$

$= \frac{p^4+4q^2-2p^2.2q-2q^2}{q^2}$

$= \frac{p^4+2q^2-4p^2q}{q^2} = \frac{p^4}{q^2} + 2 - \frac{4p^2}{q}$

$= \frac{p^4}{q^2} - \frac{4p^2}{q^2} = \frac{p^4}{q^2} + 2 - \frac{4p^2}{q}$

$= \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$

12. If the squared difference of the zeros of the quadratic polynomial f(x) = x^2 + px + 45 is equal to 144, find the value of p.

Sol:

Let the two zeroes of the polynomial be α and β

f(x) = $x^2 + px + 45$

sum of the zeroes = $-p$

Product of zeroes = 45

$\Rightarrow (\alpha - \beta)^2 - 4\alpha\beta = 144$

$\Rightarrow p^2 - 4 \times 45 = 144$

$\Rightarrow p^2 = 144 + 180$

$\Rightarrow p^2 = 324$

$p = \pm 1$

13. If the sum of the zeros of the quadratic polynomial f(t) = kt^2 + 2t + 3k is equal to their product, find the value of k.

Sol:

Let the two zeroes of the f(t) = $kt^2 + 2t + 3k\ be$ α and β

Sum of the zeroes $(\alpha + \beta)$

Product of the zeroes $\alpha\beta$

$\frac{-2}{k} = \frac{3k}{k}$

$-2k = 3k^2$

$2k + 3k^2 = 0$

k(3k + 2) = 0

k = 0

k = $\frac{-2}{3}$

14. If one zero of the quadratic polynomial f(x) = $4x^2 - 8kx - 9$ is negative of the other, find the value of k.

Sol:

Let the two zeroes of one polynomial

$f(x) = 4x^2 - 5k - 9\ be\ \alpha, -\alpha$

$\alpha \times \alpha = \frac{-9}{4}$

$t\alpha^2 = \frac{+9}{4}$

$\alpha = \frac{+3}{2}$

Sum of zeroes = $\frac{8k}{4} = 0$

Hence 8k = 0

Or k = 0

15. If α and β are the zeros of the quadratic polynomial f(x) = $x^2 - 1$, find a quadratic polynomial whose zeroes are $\frac{2\alpha}{\beta}$ *and* $\frac{2\beta}{\alpha}$

Sol:

f(x) = $x^2 - 1$

sum of zeroes $\alpha + \beta = 0$

Product of zeroes $\alpha\beta = -1$

Sum of zeroes = $\frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} = \frac{2\alpha^2+2\beta^2}{\alpha\beta}$

$= \frac{2((\alpha+\beta)^2-2\alpha\beta)}{\alpha\beta}$

$= \frac{2[(0)^2-2\times -1]}{-1}$

$= \frac{2(2)1}{-1}$

$= -4$

Product of zeroes = $\frac{2\alpha\times2\beta}{\alpha\beta} = \frac{4\alpha\beta}{\alpha\beta}$

Hence the quadratic equation is $x^2 - (sum\ of\ zeroes)x + product\ of\ zeroes$

$= k(x^2 + 4x + 14)$

16. If α and β are the zeros of the quadratic polynomial f(x) = $x^2 - 3x - 2$, find a quadratic polynomial whose zeroes are $\frac{1}{2\alpha+\beta} + \frac{1}{2\beta+\alpha}$.

Sol:

$f(x) = x^2 - 3x - 2$

Sum of zeroes $[\alpha + \beta] = 3$

Product of zeroes $[\alpha\beta] = -2$

Sum of zeroes = $\frac{1}{2\alpha+\beta} + \frac{1}{2\beta+\alpha}$

$= \frac{2\beta+\alpha+2\alpha+A}{(2\alpha+\beta)(2\beta+\alpha)}$

$= \frac{3\alpha+3\beta}{2(\alpha^2+\beta^2)+5\alpha\beta}$

$= \frac{3\times3}{2[2(\alpha+\beta)^2-2\alpha\beta+5\times(-2)]}$

$= \frac{9}{2[9]-10} = \frac{9}{16}$

Product of zeroes = $\frac{1}{\alpha+\beta} \times \frac{1}{2\beta+\alpha} = \frac{1}{4\alpha\beta+\alpha\beta+2\alpha^2+2\beta^2}$

$= \frac{1}{5\times-2+2[(\alpha+\beta)^2-2\alpha\beta]}$

$= \frac{1}{-10+2[9+4]}$

$= \frac{1}{10+26}$

$= \frac{1}{16}$

Quadratic equation = $x^2 - [sum\ of\ zeroes]x + product\ of\ zeroes$

$= x^2 - \frac{9x}{16} + \frac{1}{16}$

$= k\left[x^2 - \frac{9x}{16} + \frac{1}{16}\right]$

17. If α and β are the zeros of a quadratic polynomial such that a + 13 = 24 and a $-\beta$ = 8, find a quadratic polynomial having α and β as its zeros.

Sol:

$\alpha + \beta = 24$

$\alpha\ \beta = 8$

…………

$2\ \alpha = 32$

$\alpha = 16$

$\beta = 8$

$\alpha\ \beta = 16 \times 8 = 128$

Quadratic equation

$\Rightarrow x^2 - (sum\ of\ zeroes) + product\ of\ zeroes$

$\Rightarrow k[x^2 - 24x + 128]$

18. If α and β are the zeros of the quadratic polynomial f(x) = $x^2 -$ p (x + 1) — c, show that (α + 1)(β +1) = 1$-$ c.

Sol:

$f(x) = x^2 - p(x+1)c = x - px = -p - c$

Sum of zeroes = $\alpha + \beta$ = p

Product of zeroes = $-$ p – c = $\alpha\ \beta$

(α + 1+ β+) = $\alpha\ \beta$+ α + β + 1 = $-$p –c + p + 1

= 1 – c = R.H.S

∴ Hence proved

19. If If α and β are the zeros of the quadratic polynomial f(x) = x^2 – 2x + 3, find a polynomial whose roots are (i) α + 2, β + 2 (ii) $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$

Sol:

$f(x) = x^2 - 2x + 3$

Sum of zeroes = 2 = $(\alpha + \beta)$

Product of zeroes = 3 = $(\alpha\ \beta)$

$(i)\ sum\ of\ zeroes = (\alpha + 2) + (\beta + 2) = \alpha + \beta + 4 = 2 + 4 = 6$

$Product\ of\ zeroes = (\alpha + 2)(\beta + 2)$

$= \alpha\,\beta + 2\alpha + 2\beta + 4 = 3 + 2(2) + 4 = 11$

$Quadratic\ equation = x^2 - 6x + 11 = k[x^2 - 6x + 11]$

(ii) sum of zeroes = $\frac{\alpha-1}{\alpha+1} + \frac{\beta-1}{\beta+1}$

= $\frac{(\alpha-1)(\beta+1)+(\beta-1)(\alpha+1)}{(\alpha+1)(\beta+1)}$

= $\frac{\alpha\,\beta+\alpha-\beta-1+\alpha\,\beta+\beta+\beta-\alpha-1}{3+2+1}$

= $\frac{3-1+3-1}{3+2+1} = 4 = \frac{2}{3}$

Product of zeroes = $\frac{\alpha-1}{\beta\alpha+1} \times \frac{\beta-1}{\alpha+1} - \frac{\alpha\ 1-\alpha-\alpha\beta+1}{\alpha\,\beta+\alpha+\beta+1}$

= $\frac{3-(\alpha+\beta)+1}{3+2+1} = \frac{2}{6} = \frac{1}{3}$

Quadratic equation on $x^2 - \frac{2}{3} \times \frac{+1}{3} = 1\left[\frac{x^2-2x}{3} + \frac{1}{3}\right]$

20. If α and β are the zeroes of the polynomial f(x) = $x^2 + px + q$, form a polynomial whose zeroes are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$.

Sol:

$f(x) = x^2 + p + q$

Sum of zeroes = p = $\alpha + \beta$

Product of zeroes = q = $\alpha\ \beta$

Sum of the new polynomial = $(\alpha\ + \beta)^2 + (\alpha - \beta)^2$

$= (-p)^2 + \alpha^2 + \beta^2 - 2\alpha\,\beta$

$= p^2 + (\alpha\ +\ \beta)^2 - 2\alpha\,\beta - 2\alpha\,\beta$

$= p^2 + p^2 - 4q$

$= 2p^2 - 4q$

Product of zeroes = $(\alpha\ +\ \beta)^2 \times (\alpha - \beta)^2 = [-p]^2 \times (p^2 - 4q) = (p^2 - 4q)p^2$

Quadratic equation = $x^2 - [2p^2 - 4q] + p^2[-4q + p]$

$$f(x) = k\{x^2 - 2(p^2 - 28)x + p^2(q^2 - 4q)\}$$

Exercise 2.2

1. Verify that the numbers given alongside of the cubic polynomials below are their zeros. Also, verify the relationship between the zeros and coefficients in each case:

$(i)\ f(x) = 2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

(ii) $g(x) = x^3 - 4x^2 + 5x - 2; 2, 1, 1$

Sol:

$(i)\ f(x) = 2x^3 + x^2 - 5x + 2$

$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$

$= \frac{2}{8} + \frac{1}{4} - \frac{5}{2} + 2 = \frac{-4}{2} + 2 = 0$

$f(1) = 2(1)^3 + (1)^2 - 5(1) + 2 = 2 + 1 - 5 + 2 = 0$

$f(-2) = q(-2)^3 + (-2)^2 - 5(-2) + 2$

$= -16 + 4 + 10 + 2$

$= -16 + 16 = 0$

$= \propto + \beta + \gamma = \frac{-b}{a}$

$\frac{1}{2} + 1 - 2 = \frac{-1}{2}$

$\frac{1}{2} - 1 = \frac{-1}{2}$

$\frac{1}{2} = \frac{-1}{2}$

$\alpha\beta . \beta\gamma + r\alpha = \frac{c}{a}$

$\frac{1}{2} \times 1 + 1 \times -2 + -2 \times \frac{1}{2} = \frac{-5}{2}$

$\frac{1}{2} - 2 - 1 = \frac{-5}{2}$

$\frac{-5}{2} = \frac{-5}{2}$

$(ii)\ g(x) = x^3 - 4x^2 + 5 \times -2$

$g(2) = (2)^3 - 4(2)^2 + 5(2) - 2 = 8 - 16 + 10 - 2 = 18 - 18 = 0$

$g(1) = [1]^3 - 4[1]^2 + 5[1] - 2 = 1 - 4 + 5 - 2 = 6 - 6 = 0$

$\alpha + \beta + \gamma = \frac{-b}{a} (2) + 1 + 1 = -(-4) = 4 = 4$

$\alpha\,\beta + \beta\,\gamma + \gamma\alpha = \frac{c}{a}$

$2 \times 1 + 1\ \alpha\ 1 + 1 \times 2 = 5$

$2 + 1 + 2 = 5$

$5 = 5$

$\alpha\beta\gamma = -(-2)$

$2 \times 1 \times 1 = 2$

$2 = 2$

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and product of its zeros as 3, −1 and −3 respectively.

Sol:

Any cubic polynomial is of the form $ax^3 + bx^2 + cx + d = x^3 - sum\ of\ zeroes\ (x^2)$[product of zeroes] + sum of the products of its zeroes × - product of zeroes

= $x^3 - 2x^2 + (3 - x) + 3$

= k $[x^3 - 3x^2 - x - 3]$

k is any non-zero real numbers

3. If the zeros of the polynomial f(x) = $2x^3 - 15x^2 + 37x - 30$ are in A.P., find them.

Sol:

Let α = a – d , β = a and γ = a + d be the zeroes of polynomial.

$f(x) = 2x^3 - 15x^2 + 37x - 30$

$\alpha + \beta + \gamma = -\left(\frac{-15}{2}\right) = \frac{15}{2}$

$\alpha\beta\gamma = -\left(\frac{-30}{2}\right) = 15$

a – d + a + a + d = $\frac{15}{2}$ and $a(a-d)(a+a) = 15$

3a = $\frac{15}{2}$, a = $\frac{5}{2}$

$a(a^2 - d^2) = 15$

$a^2 - a^2 = \frac{15\times2}{5} \Rightarrow \left(\frac{5}{2}\right)^2 - d^2 = 6 \Rightarrow\Rightarrow \frac{25-6}{4} = d^2$

$d^2 = \frac{1}{4} \Rightarrow d = \frac{1}{2}$

$\therefore \alpha = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$

$\beta = \frac{5}{2} = \frac{5}{2}$

$\gamma = \frac{5}{2} + \frac{1}{2} = 3$

4. Find the condition that the zeros of the polynomial f(x) = $x^3 + 3px^2 + 3qx + r$ may be in A.P.

Sol:

$f(x) = x^3 + 3px^2 + 3qx + q$

$Let\ a - d, a, a + d$ be the zeroes of the polynomial

$The\ sum\ of\ zeroes = \frac{-b}{a}$

$a + a - d + a + d = \frac{b}{a}$

$3a = -3p$

a = $-p$

Since a is the zero of the polynomial f(x) therefore f(a) = 0 ⇒ $[a]^2 + 3pa^2 + 3qa + r = 0$

$\therefore f(a) = 0 \Rightarrow [a]^2 + 3pa^2 + 3qa + r = 0$
$\Rightarrow p^3 + 3p(-p)^2 + 3q(-p) + r = 0$
$\Rightarrow -p^3 + 3p^2 - pq + r = 0$
$\Rightarrow 2p^3 - pq + r = 0$

5. If the zeroes of the polynomial $f(x) = ax^3 + 3bx^2 + 3cx + d$ are in A.P., prove that $2b^3 - 3abc + a^2d = 0$

Sol:

Let a – d, a, a + d be the zeroes of the polynomial f(x)

The sum of zeroes ⇒ a – d + a + a + d = $\frac{-3b}{a}$

⇒ +3a $= -\frac{3b}{a} \Rightarrow a = \frac{-3b}{a \times 3} a = \frac{-b}{a}$

f(a) = 0 ⇒ $a(a)^2 + 3b(a)^2 + 3\,c(a) + d = 0$

$= a\left(\frac{-b}{a}\right)^3 + \frac{3b^2}{a^2} - \frac{3bc}{a} + d = 0$

$\Rightarrow \frac{2b^3}{a^2} - \frac{3bc}{a} + d = 0$

$\Rightarrow \frac{2b^3 - 3abc + a^2d}{a^2} = 0$

$\Rightarrow 2b^3 - 3abc + a^2d = 0$

6. If the zeroes of the polynomial $f(x) = x^3 - 12x^2 + 39x + k$ are in A.P., find the value of k.

Sol:

$f(x) = x^3 - 12x^2 + 39x - k$

Let $a - d, a, a + d$ be the zeroes of the polynomial f(x)

The sum of the zeroes = 12

3a = 12

a = 4

$f(a), -a(x)^3 - l^2(4)^2 + 39(4) + k = 0$

64 – 192 + 156 + k = 0

$= -28 = k$

k = -28

Exercise 2.3

1. Apply division algorithm to find the quotient q(x) and remainder r(x) on dividing f(x) by g(x) in each of the following:

(i) $f(x) = x^3 - 6x^2 + 11x - 6$, g(x) = $x^2 + x + 1$

(ii) $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 105(x) = 2x^2 + 7x + 1$

(iii) $f(x) = 4x^3 + 8x^2 + 8x + 7: 9(x) = 2x^2 - x + 1$

(iv) $f(x) = 15x^3 - 20x^2 + 13x - 12; g(x) = x^2 - 2x + 2$

Sol:

(i) $f(x) = x^3 - 6x^2 + 11x - 6$

g(x) = $x^2 + x + 1$

$$\begin{array}{r|l} & x - 7 \\ \hline x^2 + x + 1 & x^3 - 6x^2 + 11x - 6 \\ & x^3 + x^2 + x \\ \hline & \quad -7x^2 - 7x - 7 \\ & \quad -7x^2 - 7x - 7 \\ \hline & \qquad\qquad 17x - 1 \end{array}$$

(ii) $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 105(x) = 2x^2 + 7x + 1$

$$\begin{array}{r|l} & 5x^2 - 9x - 2 \\ \hline 2x^2 + 7x + 1 & 10x^4 + 17x^3 - 62x^2 + 30x - 3 \\ & 10x^4 + 35x^3 + 5x^2 \\ \hline & \qquad -18x^3 - 67x^2 + 30x \\ & \qquad -18x^3 \pm 63x^2 + 9x \\ \hline & \qquad\qquad -4x^2 + 39x - 3 \\ & \qquad\qquad \pm 4x^2 \pm 14x \pm 2 \\ \hline & \qquad\qquad\qquad\quad 53x - 1 \end{array}$$

(iii) $f(x) = 4x^3 + 8x^2 + 8x + 7: 9(x) = 2x^2 - x + 1$

$$\begin{array}{r|l} & \qquad 2x - 5 \\ \hline 2x^2 - 2 + 1 & 4x^3 + 8x^2 + 8^2 + 7 \\ & 4x^3 \mp 2x^2 \pm 2x \\ \hline & \qquad 10x^2 + 6x + 7 \\ & \qquad 10x^2 \pm 5x \pm 5 \\ \hline & \qquad\qquad 11x - 2 \end{array}$$

(iv) $f(x) = 15x^3 - 20x^2 + 13x - 12; g(x) = x^2 - 2x + 2$

$$\begin{array}{r|l} & \qquad 15x + 10 \\ \hline x^2 - 2x + 2 & 15x^3 - 20x^2 + 13x - 12 \\ & 15x^3 \mp 30x^2 \pm 30x \\ \hline & \qquad 10x^2 - 17x - 12 \\ & \qquad 10x^2 \pm 20x + 20 \\ \hline & \qquad\qquad 3x - 32 \end{array}$$

2. Check whether the first polynomial is a factor of the second polynomial by applying the division algorithm:

(i) $g(t) = t^2 - 3; f(t) = 2t^4 + 3t^3 - 2t^2 - 9t$

(ii) $g(x) = x^2 - 3x + 1, f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

(iii) $g(x) = 2x^2 - x + 3, f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

Sol:

(i) $g(t) = t^2 - 3; f(t) = 2t^4 + 3t^3 - 2t^2 - 9t$

	$2t^2 + 3t + 4$
$t^2 - 3$	$2t^4 + 3t^3 - 2t^2 - 9t$
	$2t^2 - 6t^2$
	$3t^3 + 4t - 9t$
	$3t^3 + 4t - 9t$
	$4t^2 - 12$
	$4t^2 \mp 12$

(ii) $g(x) = x^2 - 3x + 1, f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

	$x^2 - 1$
$x^3 - 3x + 1$	$x^5 - 4x^3 + x^2 + 3x + 1$
	$x^5 - 3x^3 + x^2$
	$-x^3 + 3x + 1$
	$-x^3 + 3x - 1$
	2

(iii) $g(x) = 2x^2 - x + 3, f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

	$3x^3 + x^2 - 2x - 5$
$2x^2 - x + 3$	$6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$
	$6x^5 - 3x^4 + 9x^3$
	$2x^4 - 5x^3 - 5x^2$
	$2x^4 \mp x^3 \pm 3x^2$
	$-4x^3 - 8x^2 - x$
	$\mp 4x^3 \pm 2x^2 - 6x$
	$-10x^2 - 5x - 15$
	$\mp 10x \pm 15x \mp 15$
	0

3. Obtain all zeros of the polynomial f(x) = $2x^4 + x^3 - 14x^2 - 19x - 6$, if two of its zeros are -2 and -1.

Sol:

$f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$

If the two zeroes of the polynomial are -2 and -1, then its factors are (x + 2) and (x + 1)

$(x + 2)(x + 1) = x^2 + x + 2x = x^2 + 3x + 2$

$$\begin{array}{r|l} & 2x^2 - 5x - 3 \\ \hline x^2 + 3x + 2 & 2x^4 + x^3 - 14x^2 - 19x - 6 \\ & 2x^4 + 6x^3 + 4x^2 \\ \hline & \quad -5x^3 - 18x^2 - 19x \\ & \quad -5x^3 \mp 15x^2 \mp 10x \\ \hline & \qquad -3x^2 - 9x - 6 \\ & \qquad -3x^2 - 9x - 6 \end{array}$$

∴ $2x^4 + x^3 - 14x^2 - 19x - 6$

= $(2x^2 - 5x - 3)[x^2 + 3x + 2] = [2x + 1][x - 3][x + 2][x + 1]$

∴ zero all x = $\frac{-1}{2}$ $, 3, -2, -1$

4. Obtain all zeros of f(x) = $x^3 + 13x^2 + 32x + 20$, if one of its zeros is −2.

Sol:

$f(x) = x^3 + 13x^2 + 32x + 20$

$$\begin{array}{r|l} & x^2 + 11x + 10 \\ \hline x + 2 & x^3 + 13x^2 + 32x + 20 \\ & x^3 \pm 2x^2 \\ \hline & \quad 11x^2 + 32x + 20 \\ & \quad 11x^2 \pm 22x \\ \hline & \qquad 10x + 20 \\ & \qquad 10x + 20 \\ \hline & \qquad\quad 0 \end{array}$$

$(x^2 + 11x + 10) = x^2 + 10x + x + 20(x + 10) + 1(x + 10) = (x + 1)(x + 10)$

∴ The zeroes of the polynomial are -1, -10, -2.

5. Obtain all zeros of the polynomial $f(x) = x^4 - 3x^2 = x^2 + 9x - 6$ if two of its zeros are $-\sqrt{3}, and\ \sqrt{3}$.

Sol:

$f(x) = (x^2 - 3x + 2) = \left(x + \sqrt{3}\right) \& \left(x - \sqrt{3}\right) = x^2 - 3$

$$\begin{array}{r|l} & x^2 - 3x + 2 \\ \hline x^2 - 3 & x^4 - 3x^2 = x^2 + 9x - 6 \\ & x^4 - 3x^2 \\ \hline & \quad -3x^2 + 2x^2 + 9x \\ & \quad -3x^2 \qquad \pm 9x \\ \hline & \qquad 2x^2 - 6 \\ & \qquad 2x^2 - 6 \end{array}$$

$(x^2 - 3)(x^2 - 3x + 2) = \left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)(x^2 - 2x - x + 2)$

$= \left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)(x - 2)(x - 2)$

Zeroes are $-\sqrt{3}, \sqrt{3}, 1, 2$

6. Find all zeros of the polynomial $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$, if its two zeroes are $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$

Sol:

If the zeroes of the polynomial are $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$

Its factors are $\left(x + \frac{\sqrt{3}}{2}\right)\left(x - \sqrt{\frac{3}{2}}\right) = \frac{x^2-3}{2}$

x = $-1, 2, \sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}$

= $[2x^2 - 2x - 4]\left(x^2 - \frac{3}{2}\right)$

$= (2x^2 - 4x + 2x - 4)\left(x + \sqrt{\frac{3}{2}}\right)$

$= [2[x(x+2) + 2(x-2)]]$

$= \left[x + \frac{\sqrt{3}}{2}\right]\left[x - \sqrt{\frac{3}{2}}\right]$

$= (x+2)(x-2)\left[x + \sqrt{\frac{3}{2}}\right]\left[x - \sqrt{\frac{3}{2}}\right]$

$x = -1, 2, \sqrt{\frac{3}{2}} - \sqrt{\frac{3}{2}}$

7. What must be added to the polynomial $f(x) = x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$?

Sol:

	$x^2 - 1$
$x^2 + 2x - 3$	$x^4 + 2x^3 - 2x^2 + x - 1$
	$x^4 + 2x^3 - 3x^2$
	$x^2 + x - 1$
	$x^2 + 2x - 3$
	$-x + 2$

we must add x -2 in order to get the resulting polynomial exactly divisible by $x^2 + 2x - 3$

8. What must be subtracted from the polynomial $x^4 + 2x^3 - 13x^2 - 12x + 21$, so that the resulting polynomial is exactly divisible by $x^2 - 4x + 3$?

Sol:

$$\begin{array}{r|l} & x^2+6x+8 \\ \hline x^2-4x+3 & x^4+2x^3-13x^2-12x+21 \\ & x^4-4x^3+3x^2 \\ \hline & \quad 6x^3-16x^2-12x \\ & \quad 6x^3-24x^2-18x \\ \hline & \qquad 8x^2-30x+21 \\ & \qquad 8x^2-32x+21 \\ \hline & \qquad\qquad 2x-2 \end{array}$$

We must subtract $[2x-2]+10m$ the given polynomial so as to get the resulting polynomial exactly divisible by x^2-x+3

9. Find all the zeroes of the polynomial $x^4+x^3-34x^2-4x+120$, if two of its zeroes are 2 and -2.

Sol:

$\Rightarrow f(x)=x^4+x^3-34x^2-4x+120$

⇒ x = -2 is a solution

x = -2 is a factor

x = -2 is a solution

x = +2 is a factor

here,

$(x-2)(x+2) is\ a\ factor\ of\ f(x)$

$x^2-4\ is\ a\ factor$

$$\begin{array}{r|l} & x^2+x-30 \\ \hline x^2-4 & x^4+x^3-34x^2-4x+120 \\ & -x^4 \quad -4x^2 \\ \hline & \quad x^3-30x^2-4x+120 \\ & \quad x^3 \qquad -4x \\ \hline & \qquad -30x^2 \quad +120 \\ & \qquad -30x^2 \quad +120 \\ \hline & \qquad\qquad 0 \end{array}$$

Hence, $x^4+x^3-34x^2-4x+120=(x^2-4)(x^2+x-30)$

$x^4+x^3-34x^2-4x+120=(x^2-4)(x^2+6x-5x-30)$

$x^4+x^3-34x^2-4x+120=(x^2-4)\left[\left(x(x+6)-5(x+6)\right)\right]$

$x^4+x^3-34x^2-4x+120=(x^2-4)(x+6)(x-5)$

Other zeroes are

x + 6 = 0 ⇒ x – 5 = 0

x = $-$ 6 x = 5

Set of zeroes for f(x) $[2,-2,-6,5]$

10. Find all zeros of the polynomial $2x^4 + 7x^3 - 19x^2 - 14x + 30$, if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.

Sol:

$f(x) = 2x^4 + 7x^3 - 19x^2 - 14x + 30$

$x = \sqrt{2}\ is\ a\ solution$

$x - \sqrt{2}\ is\ a\ solution$

$x - \sqrt{2}\ is\ a\ solution$

$x + \sqrt{2}\ is\ a\ factor$

Here, $(x + \sqrt{2})(x - \sqrt{2}) is\ a\ factor\ of\ f(x)$

$x^2 - 2\ is\ a\ factor\ of\ f(x)$

$$
\begin{array}{r|l}
 & 2x^2 + 7x - 15 \\
\hline
x^2 - 2 & 2x^4 + 7x^3 - 19^2 - 14x + 30 \\
 & 2x^4 \qquad\quad - 4x^2 \\
\cline{2-2}
 & \qquad\qquad 7x^3 - 15x^2 - 14x \\
 & \qquad\qquad 7x^3 - \qquad\ -14x \\
\cline{2-2}
 & \qquad\qquad\qquad -15x^2 \qquad + 30 \\
 & \qquad\qquad\qquad -15x^2 \qquad + 30 \\
\cline{2-2}
 & \qquad\qquad\qquad\qquad\qquad\quad 0
\end{array}
$$

Hence, $2x^4 + 7x^3 - 19x^2 - 14x + 30 = (x^2 - 2)(2x^2 + 7x - 15)$

$= (x^2 - 2)\ (2x^2 + 10x - 3x - 15)$

$= (x^2 - 2)\big(2x(x + 5) - 3(x + 5)\big)$

$= (x^2 - 2)(x + 5)(x - 3)$

Other zeroes are:

$x + 5 = 0$	$2x - 3 = 0$
$x = -5$	$2x = 3$
	$x = \frac{3}{2}$

Hence the set of zeroes for $f(x)\ \left\{-5, \frac{3}{2}, \sqrt{2}, -\sqrt{2}\right\}$

11. Find all the zeros of the polynomial $2x^3 + x^2 - 6x - 3$, if two of its zeros are $-\sqrt{3}$ and $\sqrt{3}$.

Sol:

$f(x) = 2x^3 + x^2 - 6x - 3$

$x = -\sqrt{3}$ is a solution

$x + \sqrt{3}\ is\ a\ factor$

$x = \sqrt{3}\ is\ a\ solution$

$x - \sqrt{3}\ is\ a\ factor$

Here, $(x + \sqrt{3})(x - \sqrt{3}) is\ a\ factor\ of\ f(x)$

$x^2 - 3\ is\ a\ factor\ of\ f(x)$

	2x + 1
x^2-3	$2x^3+x^2-6x-3$
	$2x^3 \quad -6x$
	$x^2 \quad -3$
	$x^2 \quad -3$
	0

Hence, $2x^3+x^2-6x-3=(x^2-3)(2x+1)$

Other zeroes of f(x) is $2\times+1=0$

$x=-\frac{1}{2}$

$Set\ of\ zeroes\ \left\{\sqrt{3},-\sqrt{3},\frac{-1}{2}\right\}$

12. Find all the zeros of the polynomial x^3+3x^2-2x-6, if two of its zeros are $-\sqrt{2}$ and $\sqrt{2}$.

Sol:

Since $-\sqrt{2}\ and\ \sqrt{2}$ are zeroes of polynomial $f(x)=x^3+3x^2-2x-6$

$\left(x+\sqrt{2}\right)\left(x-\sqrt{2}\right)=x^2-2\ is\ a\ factor\ of\ f(x)$

Now we divide $f(x)=x^3+3x^2-2x-6$ by

$g(x)=x^2-2\ to\ b\ find\ the\ other\ zeroes\ of\ f(x)$

	$x+3$
x^2-2	x^3+3x^2-2x-6
	$x^3 \quad -2x$
	$3x^2 \quad -6$
	$3x^2 \quad -6$
	0

By division algorithm, we have

$\Rightarrow x^3+3x^2-2-6=(x^2-2)(x+3)$

$\Rightarrow x^3+3x^2-2x-6=\left(x+\sqrt{2}\right)\left(x-\sqrt{2}\right)(x+3)$

Here the zeroes of the given polynomials are $-\sqrt{2},\sqrt{2}\ and-3$

Exercise 3.1

1. Akhila went to a fair in her village. She wanted to enjoy rides on the Giant Wheel and play Hoopla (a game in which you throw a rig on the items kept in the stall, and if the ring covers any object completely you get it). The number of times she played Hoopla is half the number of rides she had on the Giant Wheel. Each ride costs Rs 3, and a game of Hoopla costs Rs 4. If she spent Rs 20 in the fair, represent this situation algebraically and graphically.

Sol:

The pair of equations formed is:

$y - \frac{1}{2}x$

i.e., $x - 2y = 0$ ……..(1)

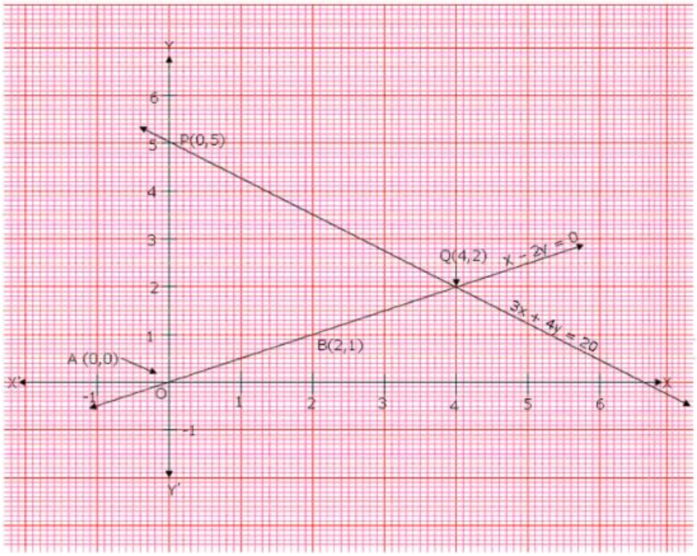
$3x + 4y = 20$ ………(2)

Let us represent these equations graphically. For this, we need at least two solutions for each equation. We give these solutions in Table

x	0	2
$y - \frac{x}{2}$	0	1

x	0	2	4
$y = \frac{20-3x}{4}$	5	0	2

Recall from Class IX that there are infinitely many solutions of each linear equation. So each of you choose any two values, which may not be the ones we have chosen. Can you guess why we have chosen $x = O$ in the first equation and in the second equation? When one of the variables is zero, the equation reduces to a linear equation is one variable, which can be solved easily. For instance, putting $x = O$ in Equation (2), we get 4y = 20 i.e., $y = 5$. Similarly, putting $y = O$ in Equation (2), we get $3x = 20$ *i.e.*, $x = \frac{20}{3}$. But as $\frac{20}{3}$ is not an integer, it will not be easy to plot exactly on the graph paper. So, we choose y = 2 which gives x = 4, an integral value.



Plot the points $A(O,O), B(2,1)$ and $P(O,5)$, $Q(412)$, corresponding to the draw the lines AB and PQ, representing the equations $x-2y=O$ and $3x+4y=20$, as shown in figure

In fig., observe that the two lines representing the two equations are intersecting at the point (4,2),

2. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." Is not this interesting? Represent this situation algebraically and graphically.

Sol:

Let the present age of Aftab and his daughter be x and y respectively. Seven years ago.

Age of Ahab $=x-7$

Age of his daughter $y-7$

According to the given condition.

$(x-7)=7(y-7)$

$\Rightarrow x-7=7y-49$

$\Rightarrow x-7y=-42$

Three years hence

Age of Aftab $=x+3$

Age of his daughter $=y+3$

According to the given condition,

$(x+3)=3(y+3)$

$\Rightarrow x+3=3y+9$

$\Rightarrow x-3y=6$

Thus, the given condition can be algebraically represented as

$x-7y=-42$

$x-3y=6$

$x-7y=-42 \Rightarrow x=-42+7y$

Three solution of this equation can be written in a table as follows:

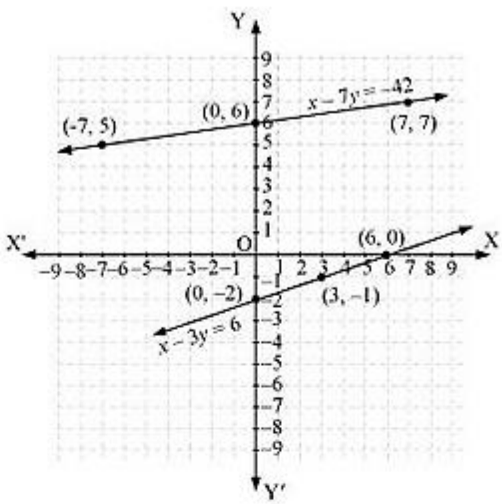
x	–7	0	7
y	5	6	7

$x-3y=6 \Rightarrow x=6+3y$

Three solution of this equation can be written in a table as follows:

x	6	3	0
y	0	-1	-2

The graphical representation is as follows:



Concept insight In order to represent a given situation mathematically, first see what we need to find out in the problem. Here. Aftab and his daughters present age needs to be found so, so the ages will be represented by variables z and y. The problem talks about their ages seven years ago and three years from now. Here, the words ’seven years ago’ means we have to subtract 7 from their present ages. and ‘three years from now’ or three years hence means we have to add 3 to their present ages. Remember in order to represent the algebraic equations graphically the solution set of equations must be taken as whole numbers only for the accuracy. Graph of the two linear equations will be represented by a straight line.

3. The path of a train A is given by the equation 3x + 4y – 12 = 0 and the path of another train B is given by the equation 6x + 8y – 48 = 0. Represent this situation graphically.

Sol:
The paths of two trains are giver by the following pair of linear equations.

$3x+4y-12=0 \qquad ...(1)$

$6x+8y-48=0 \qquad ...(2)$

In order to represent the above pair of linear equations graphically. We need two points on the line representing each equation. That is, we find two solutions of each equation as given below:
We have,

$3x+4y-12=0$

Putting $y=0$, we get

$3x+4\times 0-12=0$

$\Rightarrow \quad 3x=12$

$\Rightarrow \quad x=\frac{12}{3}=4$

Putting $x=0$, we get

$3\times 0+4y-12=0$

$\Rightarrow \quad 4y=12$

$\Rightarrow \quad y=\frac{12}{4}=3$

Thus, two solution of equation $3x+4y-12=0$ are $(0,3)$ *and* $(4,0)$

We have,

$6x+8y-48=0$

Putting $x=0$, we get

$6\times 0+8y-48=0$

$\Rightarrow \quad 8y=48$

$\Rightarrow \quad y=\frac{48}{8}$

$\Rightarrow \quad y=6$

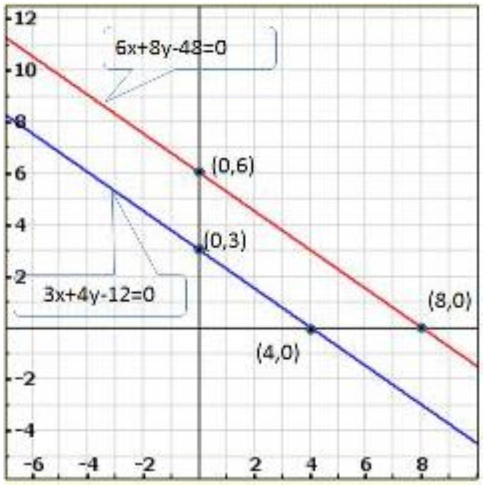
Putting $y=0$, we get

$6x+8\times 0=48=0$

$\Rightarrow \quad 6x=48$

$\Rightarrow \quad x=\frac{48}{6}=8$

Thus, two solution of equation $6x+8y-48=0$ are $(0,6)$ *and* $(8,0)$



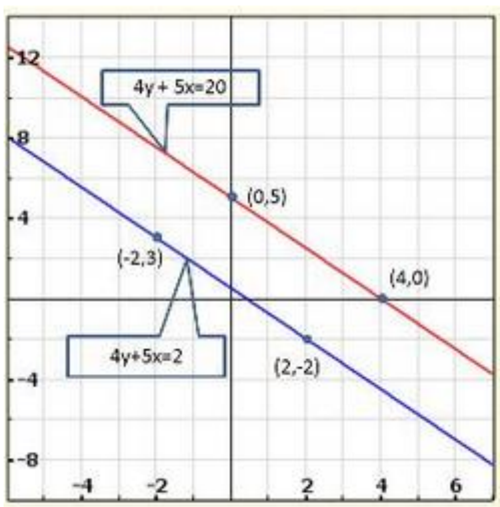
Clearly, two lines intersect at $(-1, 2)$

Hence, $x = -1, y = 2$ is the solution of the given system of equations.

4. Gloria is walking along the path joining (− 2, 3) and (2, − 2), while Suresh is walking along the path joining (0, 5) and (4, 0). Represent this situation graphically.

Sol:

It is given that Gloria is walking along the path Joining $(-2, 3)$ *and* $(2, -2)$, while Suresh is walking along the path joining $(0, 5)$ *and* $(4, 0)$.



We observe that the lines are parallel and they do not intersect anywhere.

5. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ *and* $\frac{c_1}{c_2}$ and and without drawing them, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincide:

(i) 5x− 4y + 8 = 0
7x + 6y − 9 = 0

(ii) 9x + 3y + 12 = 0
18x + 6y + 24 = 0

(iii) 6x − 3y + 10 = 0
2x – y + 9 = 0

Sol:

We have,

$$5x-4y+8=0$$
$$7x+6y-9=0$$

Here,

$$a_1=5, b_1=-4, c_1=8$$
$$a_2=7, b_2=6, c_2=-9$$

We have,

$$\frac{a_1}{a_2}=\frac{5}{7}, \frac{b_1}{b_2}=\frac{-4}{6}=\frac{-2}{3} \text{ and } \frac{c_1}{c_2}=\frac{8}{-9}=\frac{-8}{9}$$

$\therefore$ $$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

$\therefore$ Two lines are intersecting with each other at a point.

We have,

$$9x+3y+12=0$$
$$18+6y+24=0$$

Here,

$$a_1=9, b_1=3, c_1=12$$
$$a_2=18, b_2=6, c_2=24$$

Now,

$$\frac{a_1}{a_2}=\frac{9}{18}=\frac{1}{2},$$
$$\frac{b_1}{b_2}=\frac{3}{6}=\frac{1}{2}$$

And $$\frac{c_1}{c_2}=\frac{12}{24}=\frac{1}{2}$$

$\therefore$ $$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$\therefore$ Both the lines coincide.

We have,

$$6x-3y+10=0$$
$$2x-y+9=0$$

Here,

$$a_1=6, b_1=-3, c_1=10$$
$$a_2=2, b_2=-1, c_2=9$$

Now,

$$\frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1},$$

$$\frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1},$$

And $$\frac{c_1}{c_2} = \frac{10}{9}$$

$\therefore$ $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\therefore$ The lines are parallel

6. Given the linear equation 2x + 3y - 8 = 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is:
(i) intersecting lines (ii) parallel lines (iii) coincident lines.

Sol:

We have,

$$2x + 3y - 8 = 0$$

Let another equation of line is:

$$4x + 9y - 4 = 0$$

Here,

$$a_1 = 2, b_1 = 3, c_1 = -8$$
$$a_2 = 4, b_2 = 9, c_2 = -4$$

Now,

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2},$$

$$\frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3},$$

And $$\frac{c_1}{c_2} = \frac{-8}{-4} = \frac{2}{1}$$

$\therefore$ $$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\therefore$ $2x + 3y - 8 = 0$ *and* $4x + 9y - 4 = 0$ intersect each other at one point.

Hence, required equation of line is $4x + 9y - 4 = 0$

We have,

$$2x + 3y - 8 = 0$$

Let another equation of line is:

$$4x + 6y - 4 = 0$$

Here,

$$a_1 = 2, b_1 = 3, c_1 = -8$$
$$a_2 = 4, b_2 = 6, c_2 = -4$$

Now,

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2},$$
$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2},$$

And $$\frac{c_1}{c_2} = \frac{-8}{-4} = \frac{2}{1}$$

$\therefore$ $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\therefore$ Lines are parallel to each other.

Hence, required equation of line is $4x + 6y - 4 = 0.$

7. The cost of 2kg of apples and 1 kg of grapes on a day was found to be Rs 160. After a month, the cost of 4kg of apples and 2kg of grapes is Rs 300. Represent the situation algebraically and geometrically.

Sol:

Let the cost of 1 kg of apples and 1 kg grapes be Rs x and Rs y.

The given conditions can be algebraically represented as:

$$2x + y = 160$$
$$4x + 2y = 300$$
$$2x + y = 160 \Rightarrow y = 160 - 2x$$

Three solutions of this equation cab be written in a table as follows:

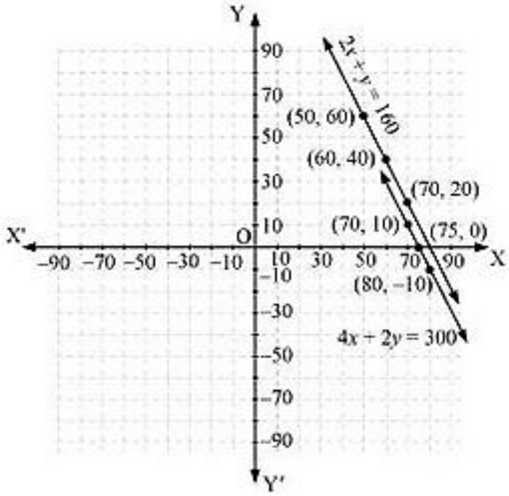
x	50	60	70
y	60	40	20

$$4x + 2y = 300 \Rightarrow y = \frac{300 - 4x}{2}$$

Three solutions of this equation cab be written in a table as follows:

x	70	80	75
y	10	-10	0

The graphical representation is as follows:



Concept insight: cost of apples and grapes needs to be found so the cost of 1 kg apples and 1kg grapes will be taken as the variables from the given condition of collective cost of apples and grapes, a pair of linear equations in two variables will be obtained. Then In order to represent the obtained equations graphically, take the values of variables as whole numbers only. Since these values are Large so take the suitable scale.

Exercise 3.2

Solve the following systems of equations graphically:

1. $x+y=3$

   $2x+5y=12$

Sol:

We have

$x+y=3$

$2x+5y=12$

Now,

$x+y=3$

When $y=0$, we have

$x=3$

When $x=0$, we have

$y=3$

Thus, we have the following table giving points on the line $x+y=3$

x	0	3
y	3	0

Now,

$2+5y=12$

$\Rightarrow y=\frac{12-2x}{5}$

When $x=1,$ we have

$y=\frac{12-1(1)}{5}=2$

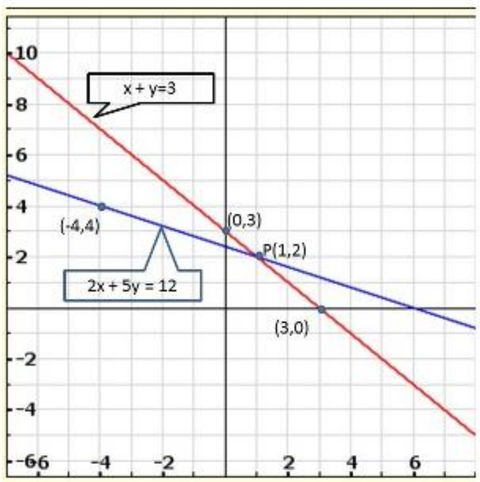
When $x=-4,$ we have

$y=\frac{12-1(4)}{5}=4$

Thus, we have the following table giving points on the line $2x+5y=12$

x	1	-4
y	2	4

Graph of the equation $x+y=3$ and $2x+5y=12$:



Clearly, two lines intersect at $P(1,2)$.

Hence, $x=1, y=2$ is the solution of the given system of equations.

2. $x-2y=5$

$2x+3y=10$

Sol:

We have

$x-2y=5$

$2x+3y=10$

Now,

$x-2y=5$

$\Rightarrow x=5+2y$

When $y = 0$, we have

$x = 5 + 2 \times 0 = 5$

When $y = -2$, we have

$x = 5 + 2 \times (-2) = 1$

Thus, we have the following table giving points on the line $x - 2y = 5$

x	5	1
y	0	-2

Now,

$$2x + 3y = 10$$

$$\Rightarrow \quad 2x = 10 - 3y$$

$$\Rightarrow \quad x = \frac{10 - 3y}{2}$$

When $y = 0$, we have

$x = \frac{10}{2} = 5$

When $y = 0$, we have

$x = \frac{10}{2} = 5$

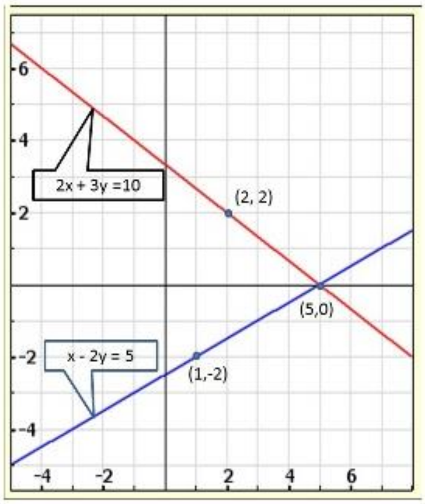
When $y = 2$, we have

$x = \frac{10 - 3 \times 2}{2} = 2$

Thus, we have the following table giving points on the line $2x + 3y = 10$

x	5	2
y	0	2

Graph of the equation $x - 2y = 5$ *and* $2x + 3y = 10$:



Clearly, two lines intersect at (5,0).
Hence, $x=5,\ y=0$ is the solution of the given system of equations.

3. $$3x+y+1=0$$ $$2x-3y+8=0$$

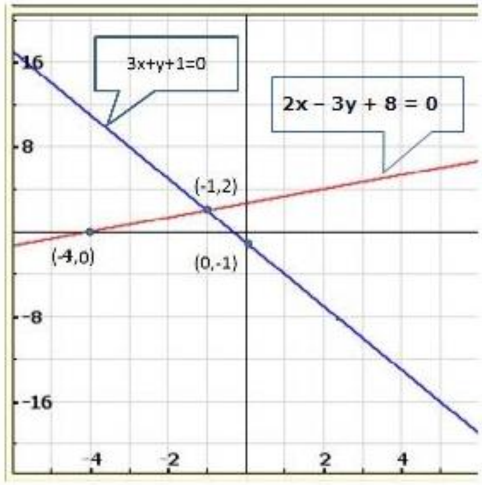
Sol:
We have,
$$3x+y+1=0$$
$$2x-3y+8=0$$
Now,
$$3x+y+1=0$$
$$\Rightarrow \quad y=-1-3x$$
When $x=0$, we have
$y=-1$
When $x=-1$, we have
$y=-1-3\times(-1)=2$
Thus, we have the following table giving points on the line $3x+y+1=0$

x	-1	0
y	2	-1

Now,
$$2x-3y+8=0$$
$$\Rightarrow \quad 2x=3y-8$$
$$\Rightarrow \quad x=\frac{3y-8}{2}$$
When $y=0$, we have
$$x=\frac{3\times0-8}{2}=-4$$
When $y=2$, we have
$$x=\frac{3\times2-8}{2}=-1$$
Thus, we have the following table giving points on the line $2x-3y+8=0$

x	-4	-1
y	0	-2

Graph of the equation are:



Clearly, two lines intersect at $(-1, 2)$.

Hence, $x = -1, y = 2$ is the solution of the given system of equations.

4. $$2x + y - 3 = 0$$
$$2x - 3y - 7 = 0$$

Sol:

We have

$$2x + y - 3 = 0$$
$$2x - 3y - 7 = 0$$

Now,

$$2x + y - 3 = 0$$
$$\Rightarrow \quad y = 3 - 2x$$

When $x = 0$, we have

$y = 3$

When $x = 1$, we have

$y = 1$

Thus, we have the following table giving points on the line $2x + y - 3 = 0$

x	0	1
y	3	1

Now,

$$2x - 3y - 7 = 0$$
$$\Rightarrow \quad 3y = 2x - 7$$
$$\Rightarrow \quad y = \frac{2 \times 5 - 7}{3} = 1$$

When $x = 5$, we have

$$y = \frac{2 \times 5 - 7}{3} = 1$$

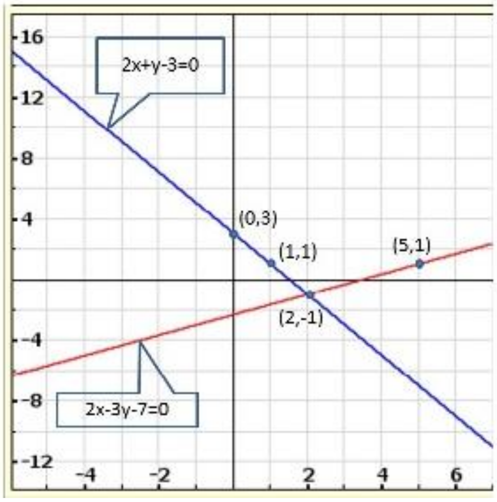
When $x = 2$, we have

$$y = \frac{2 \times 2 - 7}{3} = -1$$

Thus, we have the following table giving points on the line $2x - 3y - 7 = 0$

x	2	5
y	-1	1

Graph of the given equation are



Clearly, two lines intersect at $(2, -1)$.

Hence, $x = 2, y = -1$ is the solution of the given system of equations.

5. $$x + y = 6$$
$$x - y = 2$$

Sol:

We have.

$$x + y = 6$$
$$x - y = 2$$

Now,

$$x + y = 6$$

$\Rightarrow$ $y = 6 - x$

When $x = 2$, we have

$y = 4$

When $x = 3$, we have

$y = 3$

Thus, we have the following table giving points on the line $x + y = 6$

x	2	3
y	4	3

Now,

$$x - y = 2$$

$$\Rightarrow \quad y = x - 2$$

When $x = 0$, we have

$y = -2$

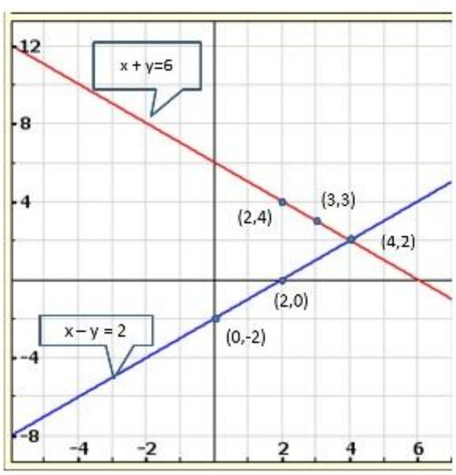
When $x = 2$, we have

$y = 0$

Thus, we have the following table giving points on the line $x - y = 6$

x	0	2
y	-2	0

Graph of the given equation are



Clearly, two lines intersect at (4,2).

Hence, $x = 4$, $y = 2$ is the solution of the given system of equations.

6. $$x - 2y = 6$$
$$3x - 6y = 0$$

Sol:

We have.

$$x - 2y = 6$$

$$3x - 6y = 0$$

Now,

$$x-2y=6$$

$$\Rightarrow \quad x=6+2y$$

When $y=-2$, we have

$x=6+2\times-2=2$

When $y=-3$, we have

$x=6+2\times-3=0$

Thus, we have the following table giving points on the line $x-2y=6$

x	2	0
y	-2	-3

Now,

$$3x-6y=0$$

$$\Rightarrow \quad 3x=6y$$

$$\Rightarrow \quad x=2y$$

When $y=0$, we have

$x=0$

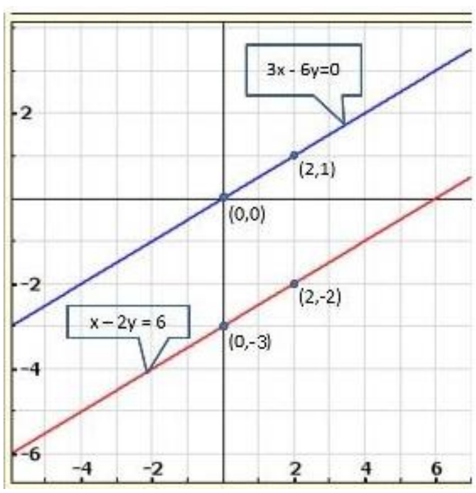
When $y=1$, we have

$x=2$

Thus, we have the following table giving points on the line $3x-6y=0$

x	0	2
y	0	1

Graph of the given equation are



Clearly, two lines are parallel to each other. So, the two lines have no common point Hence, the given system of equations has no solution.

7. $x+y=4$
$2x-3y=3$

Sol:

We have.

$x+y=4$

$2x-3y=3$

Now,

$x+y=4$

$\Rightarrow \quad x=4-y$

When $y=0,$ we have

$x=4$

When $y=2,$ we have

$x=2$

Thus, we have the following table giving points on the line $x+y=4$

x	4	2
y	0	2

Now,

$2x-3y=3$

$\Rightarrow \quad 2x=3y+3$

$\Rightarrow \quad x=\frac{3y+3}{2}$

When $y=1,$ we have

$x=3$

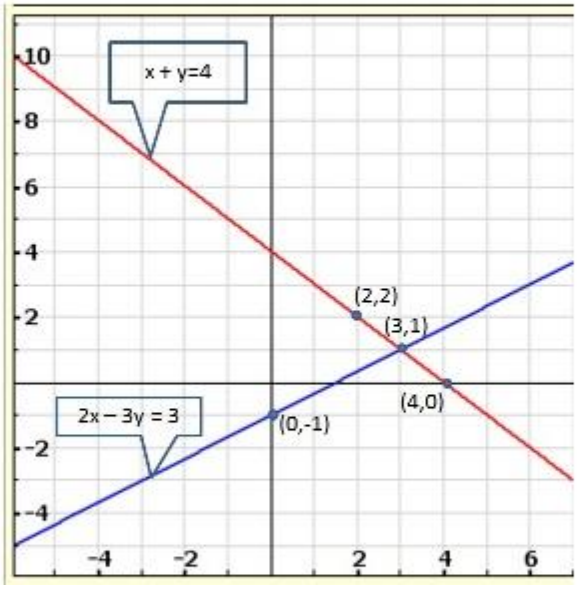
When $y=-1,$ we have

$x=0$

Thus, we have the following table giving points on the line $2x-3y=3$

x	3	0
y	1	-1

Graph of the given equation are



Clearly, two lines intersect at (3, 1).

Hence, $x=3,\ y=1$ is the solution of the given system of equations.

8. $2x+3y=4$

$x-y+3=0$

Sol:

We have.

$2x+3y=4$

$x-y+3=0$

Now,

$2x+3y=4$

$\Rightarrow \quad 2x=4-3y$

$\Rightarrow \quad x=\dfrac{4-3y}{2}$

When $y=0,$ we have

$x=\dfrac{4-3\times 2}{2}=-1$

When $y=2,$ we have

$x=\dfrac{4-3\times 2}{2}=-1$

Thus, we have the following table giving points on the line $2x+3y=4$

x	-1	2
y	2	0

Now,

$$x - y + 3 = 0$$

$$\Rightarrow \quad x = y - 3$$

When $y = 3$, we have

$x = 0$

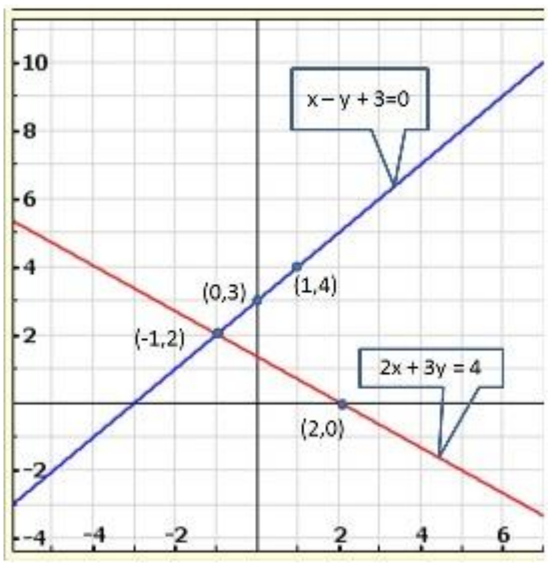
When $y = 4$, we have

$x = 1$

Thus, we have the following table giving points on the line $x - y + 3 = 0$

x	0	1
y	3	4

Graph of the given equation are



Clearly, two lines intersect at $(-1, 2)$.

Hence, $x = -1, y = 2$ is the solution of the given system of equations.

9. $$2x - 3y + 13 = 0$$
$$3x - 2y + 12 = 0$$

Sol:

We have,

$$2x - 3y + 13 = 0$$

$$3x - 2y + 12 = 0$$

Now,

$$2x - 3y + 13 = 0$$
$$\Rightarrow \quad 2x = 3y - 13$$
$$\Rightarrow \quad x = \frac{3y - 13}{2}$$

When $y = 1$, we have

$$x = \frac{3 \times 1 - 13}{2} = -5$$

When $y = 3$, we have

$$x = \frac{3 \times 3 - 13}{2} = -2$$

Thus, we have the following table giving points on the line $2x - 3y + 13 = 0$

x	-5	-2
y	1	3

Now,

$$3x - 2y + 12 = 0$$
$$\Rightarrow \quad 3x = 2y - 12$$
$$\Rightarrow \quad x = \frac{2y - 12}{3}$$

When $y = 0$, we have

$$x = \frac{2 \times 0 - 12}{3} = -14$$

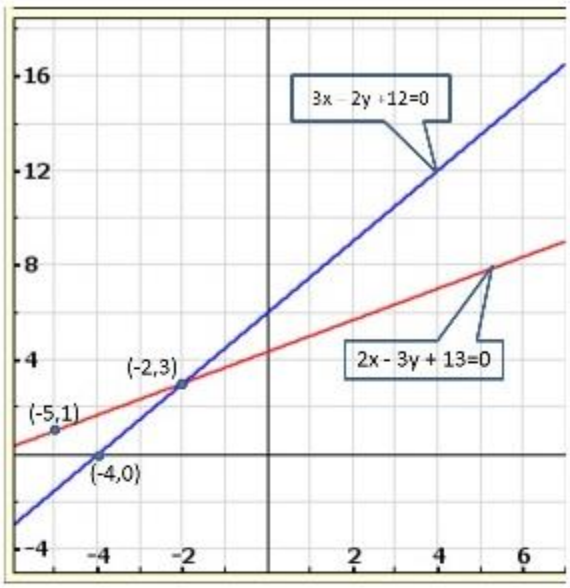
When $y = 3$, we have

$$x = \frac{2 \times 3 - 12}{3} = -2$$

Thus, we have the following table giving points on the line $3y - 2y + 12 = 0$

x	-4	-2
y	0	3

Graph of the given equations are:



Clearly, two lines intersect at (-2, 3)

Hence, $x=-2,\ y=3$ is the solution of the given system of equations.

10. $$2x+3y+5=0$$
$$3x+2y-12=0$$

Sol:

We have,

$$2x+3y+5=0$$
$$3x+2y-12=0$$

Now,

$$2x+3y+5=0$$

$\Rightarrow$ $2x=-3y-5$

$\Rightarrow$ $x=\dfrac{-3y-5}{2}$

When $y=1$, we have

$$x=\frac{-3\times1-5}{2}=-4$$

When $y=-1$, we have

$$x=\frac{-3\times(-1)-5}{2}=-1$$

Thus, we have the following table giving points on the line $2x+3y+5=0$

x	-4	-1
y	1	-1

Now,

$$3x-2y-12=0$$

$$\Rightarrow \quad 3x=2y+12$$

$$\Rightarrow \quad x=\frac{2y+12}{3}$$

When $y=0$, we have

$$x=\frac{2\times 0+12}{3}=4$$

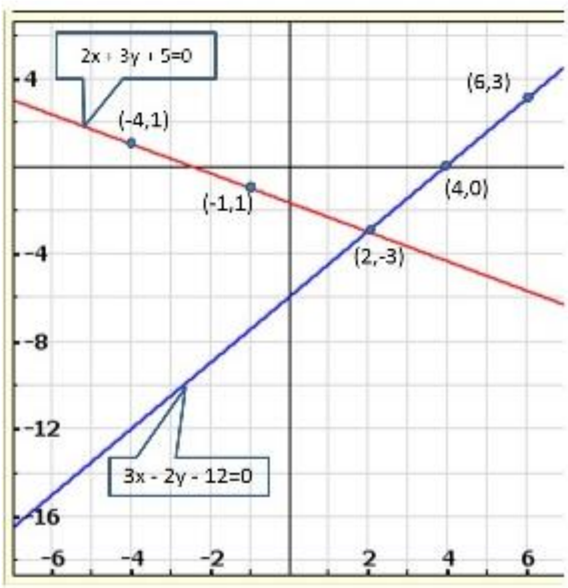
When $y=3$, we have

$$x=\frac{2\times 3+12}{3}=6$$

Thus we have the following table giving points on the line $3x-2y-12=0$

x	4	6
y	0	3

Graph of the given equations are:



Clearly, two lines intersect at $(2,-3)$.

Hence, $x=2,\ y=-3$ is the solution of the given system of equations.

Show graphically that each one of the following systems of equations has infinitely many solutions:

11. $2x+3y=6$

$4x+6y=12$

Sol:

We have,

$$2x+3y=6$$
$$4x+6y=12$$

Now,

$$2x+3y=6$$
$$\Rightarrow \quad 2x=6-3y$$
$$\Rightarrow \quad x=\frac{6-3y}{2}$$

When $y=0,$ we have

$x=3$

When $y=2,$ we have

$$x=\frac{6-3\times 2}{2}=0$$

Thus, we have the following table giving points on the line $2x+3y=6$

x	0	3
y	2	0

Now,

$$4x+6y=12$$
$$\Rightarrow \quad 4x=12-6y$$
$$\Rightarrow \quad x=\frac{12-6y}{4}$$

When $y=0,$ we have

$x=3$

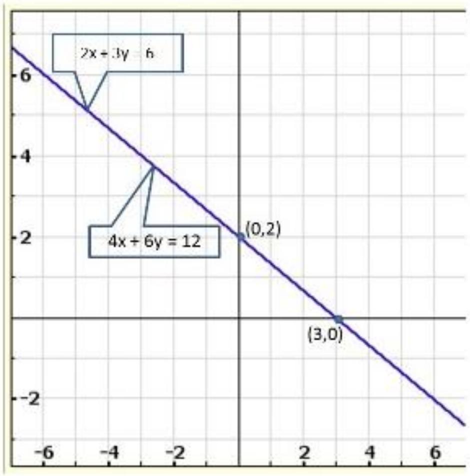
When $y=2,$ we have

$$x=\frac{12-6\times 2}{3}=0$$

Thus, we have the following table giving points on the line $4x+6y=12$

x	0	3
y	2	0

Graph of the given equations:



Thus, the graphs of the two equations are coincident.
Hence, the system of equations has infinitely many solutions.

12. $x-2y=5$
$3x-6y=15$

Sol:
We have,
$$x-2y=5$$
$$3x-6y=15$$
Now,
$$x-2y=5$$
$$\Rightarrow \quad x=2y+5$$
When $y=-1,$ we have
$$x=2(-1)+5=3$$
When $y=0,$ we have
$$x=2\times 0+5=5$$
Thus, we have the following table giving points on the line $x-2y=5$

x	3	5
y	1	0

Now,
$$3x-6y=15$$
$$\Rightarrow \quad 3x=15+6y$$
$$\Rightarrow \quad x=\frac{15+6y}{3}$$
When $y=-2,$ we have

$$x = \frac{15+6(-2)}{3} = 1$$

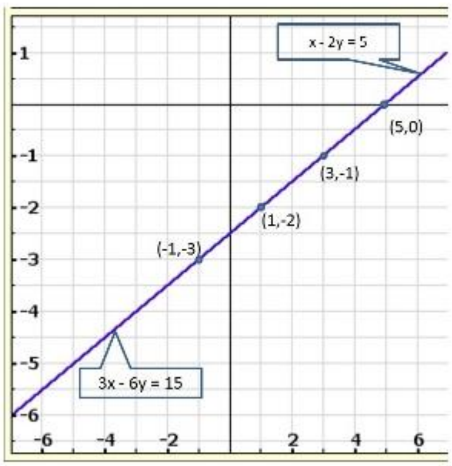
When $y = -3$, we have

$$x = \frac{15+6(-3)}{3} = -1$$

Thus, we have the following table giving points on the line $3x - 6y = 15$

x	1	-1
y	-2	-3

Graph of the given equations:



13. $3x + y = 8$
$6x + 2y = 16$

Sol:

We have,

$$3x + y = 8$$
$$6x + 2y = 16$$

Now,

$$3x + y = 8$$
$$\Rightarrow \quad y = 8 - 3x$$

When $x = 2$, we have

$y = 8, -3 \times 2 = 2$

When $x = 3$, we have

$y = 8, -3 \times 3 = -1$

Thus we have the following table giving points on the line $3x + y = 8$

x	2	3
y	2	-1

Now,

$$6x+2y=16$$

$$\Rightarrow \quad 2y=16-6x$$

$$\Rightarrow \quad y=\frac{16-6x}{2}$$

When $x=1,$ we have

$$y=\frac{16-6\times1}{2}=5$$

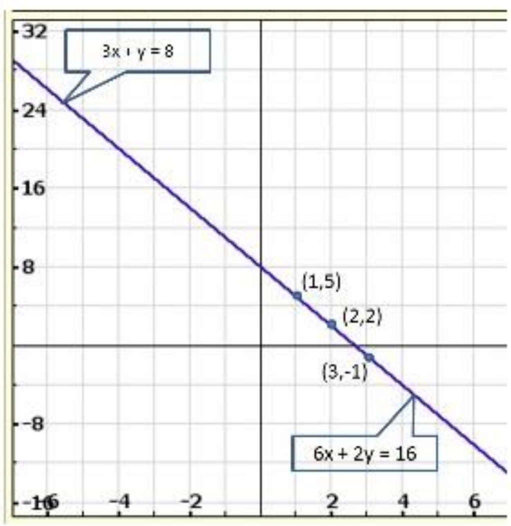
When $x=3,$ we have

$$y=\frac{16-6\times3}{2}=-1$$

Thus we have the following table giving points on the line $6x+2y=16$

x	1	3
y	5	-1

Graph of the given equations:



Thus, the graphs of the two equations are coincident.

Hence, the system of equations has infinitely many solutions,

14. $$x+2y+11=0$$
$$3x+6y+33=0$$

Sol:

We have,

$$x+2y+11=0$$

$$3x+6y+33=0$$

Now,

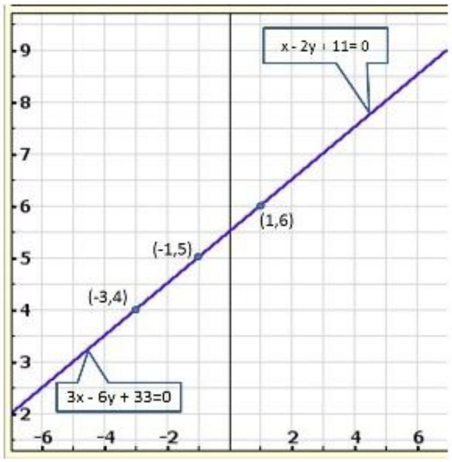
$$x-2y+11=0$$
$$\Rightarrow \quad x=2y-11$$
When $y=5$, we have
$$x=2\times5-11=-1$$
When $x=4$, we have
$$x=2\times4-11=-3$$
Thus we have the following table giving points on the line $x-2y+11=0$

x	-1	-3
y	5	4

Now,
$$3x-6y+33=0$$
$$\Rightarrow \quad 3x=6y-33$$
$$\Rightarrow \quad x=\frac{6y-33}{3}=1$$
When $y=6$, we have
$$x=\frac{6\times6-33}{3}=-1$$
When $y=5$, we have
$$x=\frac{6\times5\times-33}{2}=-1$$
Thus we have the following table giving points on the line $3x+6y+33=0$

x	1	-1
y	6	5

Graph of the given equations:



Thus, the graphs of the two equations are coincident,
Hence, the system of equations has infinitely many solutions,

Show graphically that each one of the following systems of equations is in-consistent (i.e., has no solution)

15. $3x-5y=20$
$6x-10y=-40$

Sol:

We have,

$3x-5y=20$

$6x-10y=-40$

Now

$\Rightarrow \quad 3x-5y=20$

$\Rightarrow \quad x=\frac{5y+20}{3}$

When $y=-1,$ we have

$x=\frac{5(-1)+20}{3}=5$

When $y=-4,$ we have

$x=\frac{5(-4)+20}{3}=0$

Thus we have the following table giving points on the line $3x-5y=20$

x	5	0
y	-1	-4

Now

$6x-10y=-40$

$\Rightarrow \quad 6x=-40+10y$

$\Rightarrow \quad x=\frac{-40+10y}{6}$

When $y=4,$ we have

$x=\frac{-40+10\times4}{6}=0$

When $y=1,$ we have

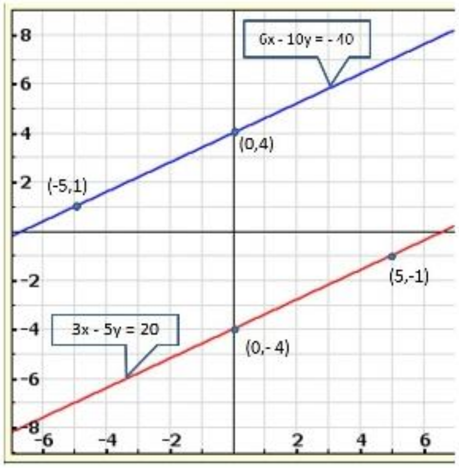
$x=\frac{-40+10\times1}{6}=-5$

Thus we have the following table giving points on the line $6x-10y=-40$

x	0	-5

y	4	1

Graph of the given equations:



Clearly, there is no common point between these two lines
Hence, given system of equations is in-consistent.

16. $x - 2y = 6$
$3x - 6y = 0$

Sol:

We have

$$x - 2y = 6$$
$$3x - 6y = 0$$

Now,

$$x - 2y = 6$$
$\Rightarrow$ $x = 6 + 2y$

When $y = 0$, we have

$x = 6 + 2 \times 0 = 6$

When $y = -2$, we have

$x = 6 + 2 \times (-2) = 2$

Thus, we have the following table giving points on the line $x - 2y = 6$

x	6	2
y	0	-2

Now,

$$3x - 6y = 0$$

$\Rightarrow \quad 3x = 6y$

$\Rightarrow \quad x = \frac{6y}{3}$

$\Rightarrow \quad x = 2y$

When $y = 0$, we have

$x = 2 \times 0 = 0$

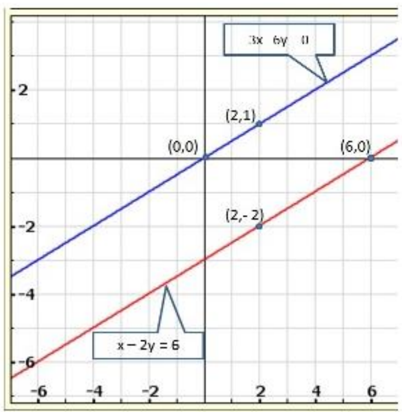
When $y = 1$, we have

$x = 2 \times 1 = 2$

Thus, we have the following table giving points on the line $3x - 6y = 0$

x	0	2
y	0	1

Graph of the given equations:



We find the lines represented by equations $x - 2y = 6$ and $3x - 6y = 0$ are parallel. So, the two lines have no common point.

Hence, the given system of equations is in-consistent.

17. $2y - x = 9$

$6y - 3x = 21$

Sol:

We have

$$2y - x = 9$$

$$6y - 3x = 21$$

Now,

$$2y - x = 9$$

$\Rightarrow \quad 2y - 9 = x$

$\Rightarrow \quad x = 2y - 9$

When $y = 3$, we have

$x = 2\times3-9 = -3$

When $y = 4,$ we have

$x = 2\times4-9 = -1$

Thus, we have the following table giving points on the line $2x - x = 9$

x	-3	-1
y	3	4

Now,

$$6y-3x = 21$$

$$\Rightarrow \quad 6y-21 = 3x$$

$$\Rightarrow \quad 3x = 6y-21$$

$$\Rightarrow \quad x = \frac{3(2y-7)}{3}$$

$$\Rightarrow \quad x = 2y-7$$

When $y = 2,$ we have

$x = 2\times2-7 = -3$

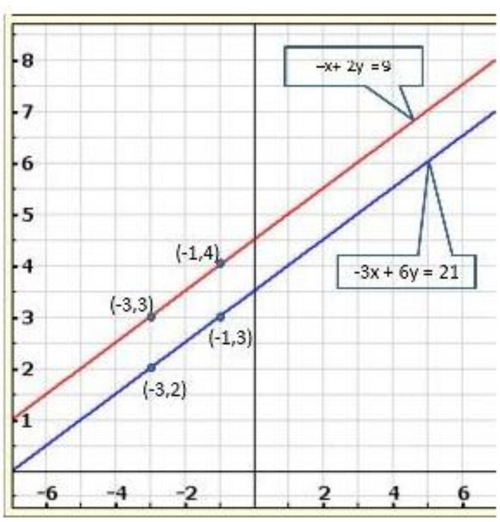
When $y = 3,$ we have

$x = 2\times3-7 = -1$

Thus, we have the following table giving points on the line $6y - 3x = 21$.

x	-3	-1
y	2	3

Graph of the given equations:



We find the lines represented by equations $2y - x = 9$ and $6y - 3x = 21$ are parallel. So, the two lines have no common point.

Hence, the given system of equations is in-consistent.

18. $$3x-4y-1=0$$
$$2x-\frac{8}{3}y+5=0$$

Sol:

We have

$$3x-4y-1=0$$
$$2x-\frac{8}{3}y+5=0$$

Now,

$$3x-4y-1=0$$
$\Rightarrow$ $$3x=1+4y$$
$\Rightarrow$ $$x=\frac{1+4y}{3}$$

When $y=2$, we have

$$x=\frac{1+4\times 2}{3}=3$$

When $y=-1$, we have

$$x=\frac{1+4\times(-1)}{3}=-1$$

Thus, we have the following table giving points on the line $3x-4y-1=0$.

x	-1	3
y	-1	2

Now,

$$2x-\frac{8}{3}y+5=0$$
$\Rightarrow$ $$\frac{6x-8y+15}{3}=0$$
$\Rightarrow$ $$6x-8y+15=0$$
$\Rightarrow$ $$6x=8y-15$$
$\Rightarrow$ $$x=\frac{8y-15}{6}$$

When $y=0$, we have

$$x=\frac{8\times 0-15}{6}=-2.5$$

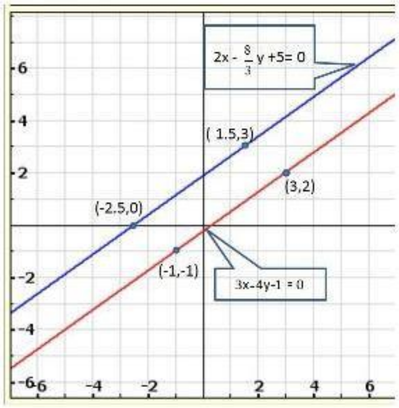
When $y=3$, we have

$$x=\frac{8\times 3-15}{6}=1.5$$

Thus, we have the following table giving points on the line $2x - \frac{8}{3}y + 5 = 0$.

x	-2.5	1.5
y	0	3

Graph of the given equations:



We find the lines represented by equations $3x - 4y - 1 = 0$ *and* $2x - \frac{8}{3}y + 5 = 0$ are parallel. So, the two lines have no common point.
Hence, the given system of equations is in-consistent.

19. Determine graphically the vertices of the triangle, the equations of whose sides are given below:

(i)
$$2y - x = 8$$
$$5y - x = 14$$
$$y - 2x = 1$$

(ii)
$$y = x$$
$$y = 0$$
$$3x + 3y = 10$$

Sol:
We have
$$2y - x = 8$$
$$5y - x = 14$$
$$y - 2x = 1$$
Now,
$$2y - x = 8$$
$$\Rightarrow \quad 2y = 8 = x$$
$$\Rightarrow \quad x = 2y - 8$$

When $y=2,$ we have

$x=2\times2-8=-4$

When $y=4,$ we have

$x=2\times4-8=0$

Thus, we have the following table giving points on the line $2y-x=8.$

x	-4	0
y	2	4

Now,

$$5y-x=14$$

$$\Rightarrow \quad 5y-14=x$$

$$\Rightarrow \quad x=5y-14$$

When $y=2,$ we have

$x=5\times2-14=1$

When $y=3,$ we have

$x=5\times3-14=1$

Thus, we have the following table giving points on the line $5y-x=14.$

x	-4	1
y	2	3

We have

$$y-2x=1$$

$$\Rightarrow \quad y-1=2x$$

$$\Rightarrow \quad x=\frac{y-1}{2}$$

When $y=3,$ we have

$$x=\frac{3-1}{2}=1$$

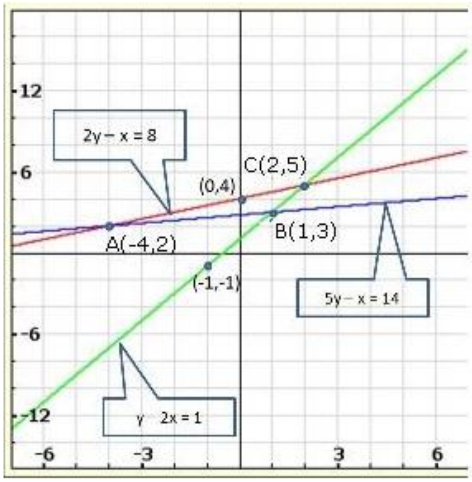
When $y=-1,$ we have

$$x=\frac{-1-1}{2}=1$$

Thus, we have the following table giving points on the line $y-2x=1.$

x	-1	1
y	1	3

Graph of the given equations:



From the graph of the lines represented by the given equations, we observe that the lines taken in pairs intersect each other at points $A(-4,2), B(1,3)$ *and* $C(2,5)$

Hence, the vertices of the triangle are $A(-4,2)$, $B(1,3)$ *and* $C(2,5)$.

The given system of equations is

$y = x$

$y = 0$

$3x + 3y = 10$

We have,

$y = x$

When $x = 1$, we have

$y = 1$

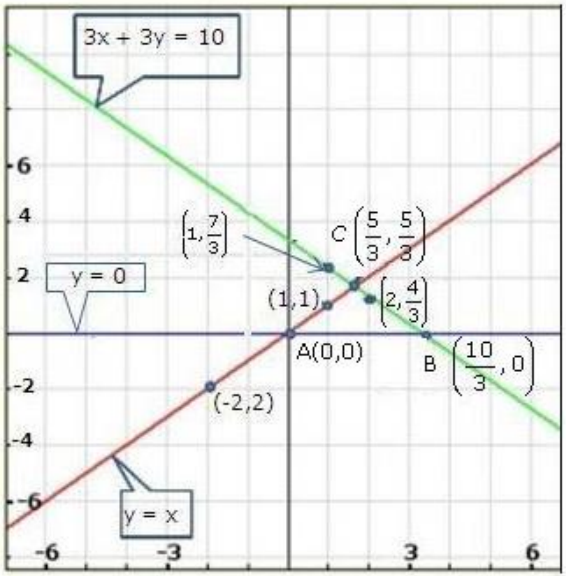
When $x = -2$, we have

$y = -2$

Thus, we have the following table points on the line $y = x$

x	1	-2
y	7/3	4/3

Graph of the given equation:



From the graph of the lines represented by the given equations, we observe that the lines taken in pairs intersect each other at points $A(0,0), B\left(\frac{10}{3},0\right)$ *and* $C\left(\frac{5}{3},\frac{5}{3}\right)$

Hence, the required vertices of the triangle are $A(0,0), B\left(\frac{10}{0},0\right)$ *and* $C\left(\frac{5}{3},\frac{5}{3}\right)$.

20. Determine, graphically whether the system of equations x − 2y = 2, 4x − 2y = 5 is consistent or in-consistent.

Sol:

We have

$$x-2y=2$$
$$4x-2y=5$$

Now

$$x-2y=2$$
$$\Rightarrow \quad x=2+2y$$

When $y=0,$ we have

$x=2+2\times0=2$

When $y=-1,$ we have

$x=2+2\times(-1)=0$

Thus, we have the following table giving points on the line $x-2y=2$

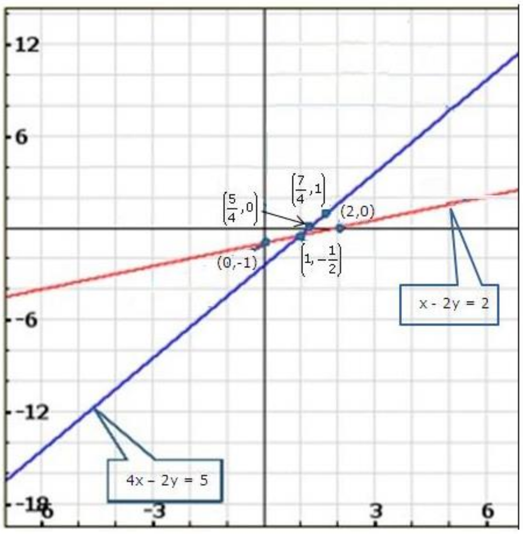
x	2	0
y	0	-1

Now,

$$4x - 2y = 5$$
$$\Rightarrow \quad 4x = 5 + 2y$$
$$\Rightarrow \quad x = \frac{5+2y}{4}$$
When $y = 0$, we have
$$x = \frac{5+2\times 0}{4} = \frac{5}{4}$$
When $y = 1$, we have
$$x = \frac{5+2\times 1}{4} = \frac{7}{4}$$
Thus, we have the following table giving points on the line $4x - 2y = 5$

x	5/4	7/4
y	0	1

Graph of the given equations:



Clearly, the two lines intersect at (i!).
Hence, the system of equations is consistent.

21. Determine, by drawing graphs, whether the following system of linear equations has a unique solution or not:
(i) 2x − 3y = 6, x + y = 1 (ii) 2y = 4x − 6, 2x = y + 3

Sol:
We have
$$2x - 3y = 6$$
$$x + y = 1$$
Now

$$2x - 3y = 6$$

$$\Rightarrow \quad 2x = 6 + 3y$$

When $y = 0$, we have

$$x = \frac{6+3y}{2}$$

When $y = -2$, we have

$$x = \frac{6+3\times(-2)}{2} = 0$$

Thus, we have the following table giving points on the line $2x - 3y = 6$

x	3	0
y	0	-2

Now,

$$x + y = 1$$

$$\Rightarrow \quad x = 1 - y$$

When $y = 1$, we have

$x = 1 - 1 = 0$

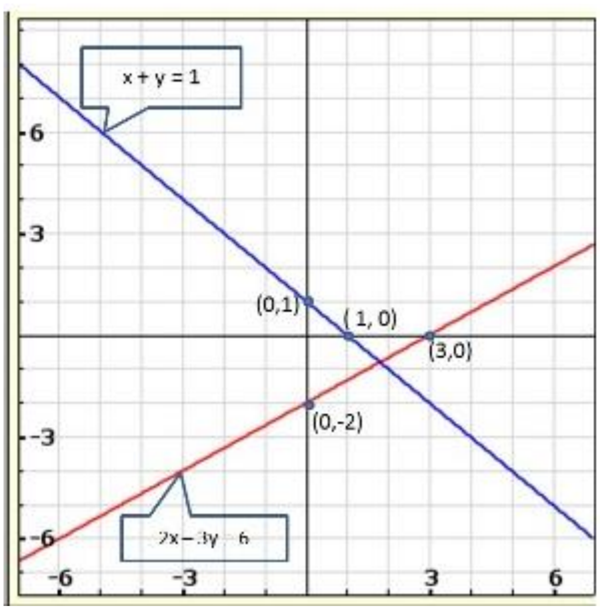
When $y = 0$, we have

$x = 1 - 0 = 1$

Thus, we have the following table giving points on the line $x + y = 1$

x	0	1
y	1	0

Graph of the given equations:



We have,

$2y = 4x - 6$

$2x = y + 3$

Now,

$2y = 4x - 6$

$\Rightarrow \quad 2y + 6 = 4x$

$\Rightarrow \quad 4x = 2y + 6$

$\Rightarrow \quad x = \dfrac{2y+6}{4}$

When $y = -1$, we have

$x = \dfrac{2 \times (-1) + 6}{4} = 1$

When $y = 5$, we have

$x = \dfrac{2 \times 5 + 6}{4} = 4$

Thus, we have the following table giving points on the line $2y = 4x - 6$

x	1	4
y	-1	5

Now,

$2x = y + 3$

$\Rightarrow \quad x = \dfrac{y+3}{2}$

When $y = 1$, we have

$x = \dfrac{1+3}{2} = 2$

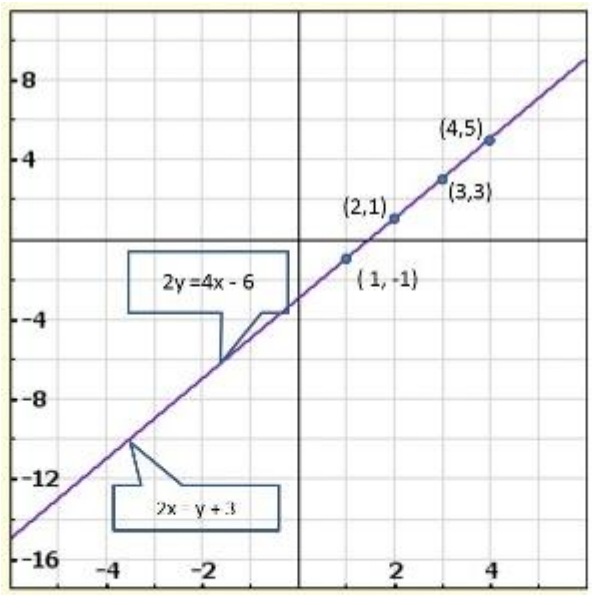
When $y = 3$, we have

$x = \dfrac{3+3}{2} = 3$

Thus, we have the following table giving points on the line $2x = y + 3$

x	2	3
y	1	3

Graph of the given equations:



We find the graphs of the two equations are coincident,

$\therefore$ Hence, the system of equations has infinity many solutions

22. Solve graphically each of the following systems of linear equations. Also find the coordinates of the points where the lines meet axis of y.

(i) $2x-5y+4=0$
$2x+y-8=0$

(ii) $3x+2y=12$
$5x-2y=4$

(iii) $2x+y-11=0$
$x-y-1=0$

(iv) $x+2y-7=0$
$2x-y-4=0$

(v) $3x+y-5=0$
$2x-y-5=0$

(vi) $2x-y-5=0$
$x-y-3=0$

Sol:

We have

$$2x-5y+4=0$$
$$2x+y-8=0$$

Now,

$$2x-5y+4=0$$

$\Rightarrow \quad 2x=5y-4$

$\Rightarrow \quad x=\frac{5y-4}{2}$

When $y=2$, we have

$$x=\frac{5\times2-4}{2}=3$$

When $y=4$, we have

$$x=\frac{5\times4-4}{2}=8$$

Thus, we have the following table giving points on the line $2x-5y+4=0$

x	3	8
y	2	4

Now,

$$2x+y-8=0$$

$\Rightarrow \quad 2x=8-y$

$\Rightarrow \quad x=\frac{8-y}{2}$

When $y=4$, we have

$$x=\frac{8-4}{2}=2$$

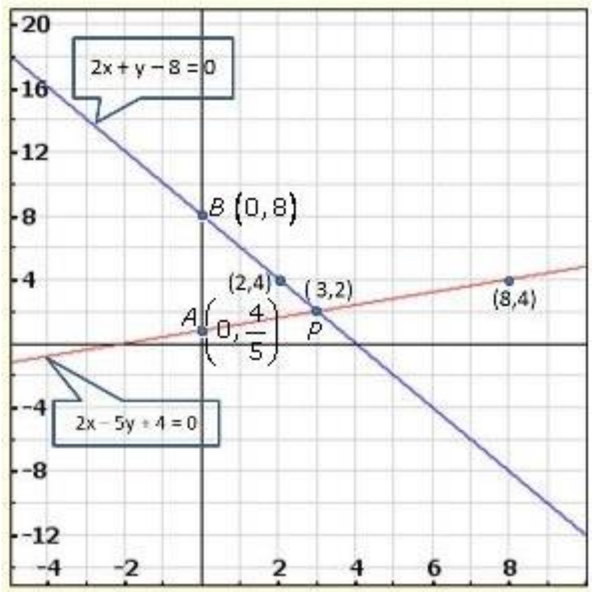
When $y=2$, we have

$$x=\frac{8-2}{2}=3$$

Thus, we have the following table giving points on the line $2x-5y+4=0$

x	3	8
y	2	4

Graph of the given equations:



Clearly, two intersect at $P(3,2)$.

Hence, $x=2,\ y=3$ is the solution of the given system of equations.

We also observe that the lines represented by $2X-5y+4=0$ *and* $2x+y-8=0$ meet y-axis at $A\left(0,\frac{4}{5}\right)$ *and* $B(0,8)$ respectively.

We have,

$$3x+2y=12$$
$$5x-2y=4$$

Now,

$$3x+2y=12$$
$$\Rightarrow \quad 3x=12-2y$$
$$\Rightarrow \quad x=\frac{12-2y}{3}$$

When $y=3$, we have

$$x=\frac{12-2\times 3}{3}=2$$

When $y=-3$, we have

$$x=\frac{12-2\times(-3)}{3}=6$$

Thus, we have the following table giving points on the line $3x+2y=12$

x	2	6
y	3	-3

Now,

$$5x - 2y = 4$$

$$\Rightarrow \quad 5x = 4 + 2y$$

$$\Rightarrow \quad x = \frac{4+2y}{5}$$

When $y = 3$, we have

$$x = \frac{4+2\times3}{5} = 2$$

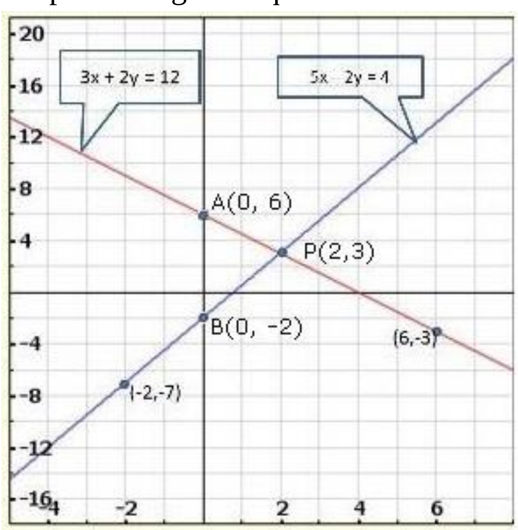
When $y = -7$, we have

$$x = \frac{4+2\times(-7)}{5} = -2$$

Thus, we have the following table giving points on the line $5x - 2y = 4$

x	2	-2
y	3	-7

Graph of the given equation



Clearly, two intersect at $p(2,3)$.

Hence, $x = 2, y = 3$ is the solution of the given system of equations.

We also observe that the lines represented by $3x + 2y = 12$ *and* $5x - 2y = 4$ meet y-axis at $A(0,6)$ *and* $B(0,-2)$ respectively.

We have,

$$2x + y - 11 = 0$$

$$x - y - 1 = 0$$

Now,

$$2x+y-11=0$$
$$\Rightarrow \quad y=11-2x$$
When $x=4$, we have
$y=11-2\times4=3$
When $x=5$, we have
$y=11-2\times5=1$
Thus, we have the following table giving points on the line $2x+y-11=0$

x	4	5
y	3	1

Now,
$$x-y-1=0$$
$$\Rightarrow \quad x-1=y$$
$$\Rightarrow \quad y=x-1$$
When $x=2$, we have
$y=2-1=1$
When $x=3$, we have
$y=3-1=2$
Thus, we have the following table giving points on the line $x-y-1=0$

x	2	3
y	1	2

Graph of the given equation
We have,
$$2x+y-11=0$$
$$x-y-1=0$$
Now,
$$2x+y-11=0$$
$$\Rightarrow \quad y=11-2x$$
When $x=4$, we have
$y=11-2\times4=3$
When $x=5$, we have
$y=11-2\times5=1$
Thus, we have the following table giving points on the line $2x+y-11=0$

x	4	5
y	3	1

Now,

$x - y - 1 = 0$

$\Rightarrow \quad x - 1 = y$

$\Rightarrow \quad y = x - 1$

When $x = 2$, we have

$y = 2 - 1 = 1$

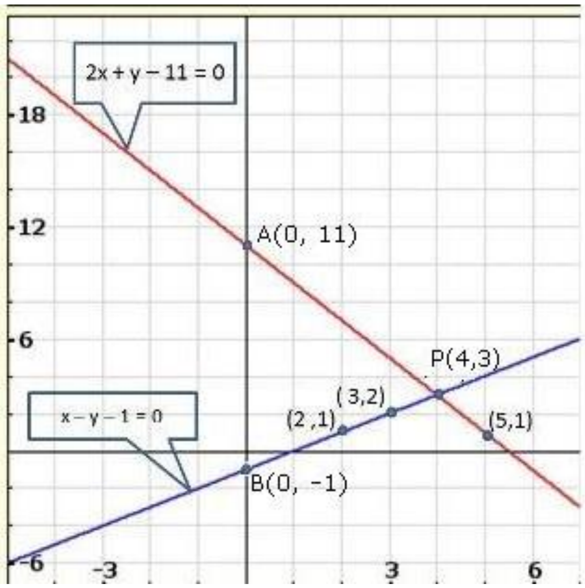
When $x = 3$, we have

$y = 3 - 1 = 2$

Thus, we have the following table giving points on the line $x - y - 1 = 0$

x	2	3
y	1	2

Graph of the given equations:



Clearly, two intersect at $P(4,3)$.

Hence, $x = 4$, $y = 3$ is the solution of the given system of equations.

We also observe that the lines represented by $2x + y - 11 = 0$ *and* $x - y - 1 = 0$ meet y-axis at, $A(0,11)$ *and* $B(0,-1)$ respectively.

We have, $x + 2y - 7 = 0$

Now,

$2x - y - 4 = 0$

$x + 2y - 7 = 0$

$x = 7 - 2y$

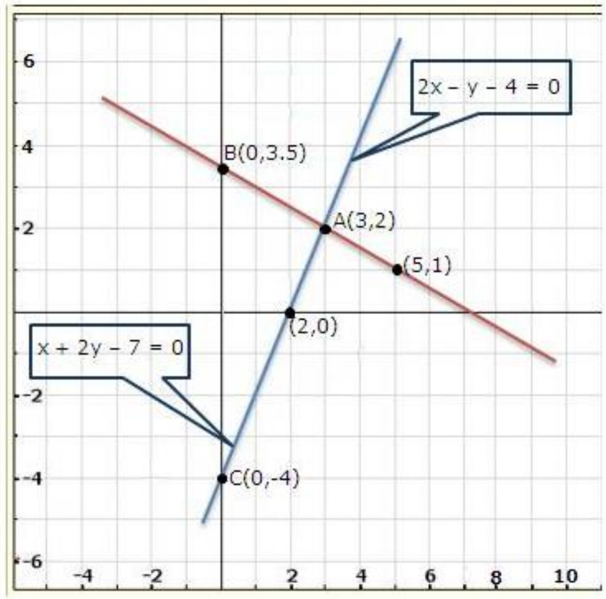
When $\begin{matrix} y = 1, x = 5 \\ y = 2, x = 3 \end{matrix}$

x	5	3
y	1	2

Also, $2x-y-4=0$

$y=2x-4$

x	2	0
y	0	-4



From the graph, the solution is $A(3,2)$.

Also, the coordinates of the points where the lines meet the y-axis are $B(0,3.5)$ *and* $C(0,-4)$.

We have

$3x+y-5=0$

$2x-y-5=0$

Now,

$3x+y-5=0$

$\Rightarrow \quad y=5-3x$

When $x=1$, we have

$y=5,-3\times1=2$

When $x=2$, we have

$y=5,-3\times2=-1$

Thus, we have the following table giving points on the line $3x+y-5=0$

x	1	2

y	2	-1

Now,

$$2x - y - 5 = 0$$

$$\Rightarrow \quad 2x - 5 = y$$

$$\Rightarrow \quad y = 2x - 5$$

When $x = 0$, we have

$y = -5$

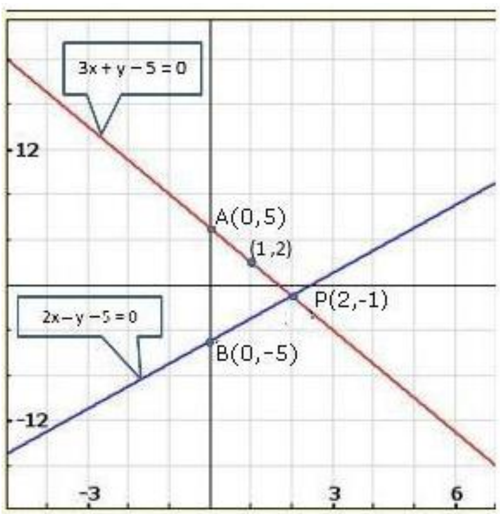
When $x = 2$, we have

$y = 2\times 2 - 5 = -1$

Thus, we have the following table giving points on the line $2x - y - 5 = 0$

x	0	2
y	-5	-1

Graph of the given equations:



Clearly, two intersect at $P(2,-1)$.

Hence, $x = 2,\ y = -1$ is the solution of the given system of equations.

We also observe that the lines represented by $3x + y - 5 = 0$ *and* $2x - y - 5 = 0$ meet y-axis at $A(0,5)$ *and* $8(0,-5)$respectively.

We have,

$$2x - y - 5 = 0$$

$$x - y - 3 = 0$$

Now,

$$2x - y - 5 = 0$$

$$\Rightarrow \quad 2x - 5 = y$$

$$\Rightarrow \quad y = 2x - 5$$

When $x = 1$, we have

$y = 2 \times 1 - 5 = -3$

When $x = 2$, we have

$y = 2 \times 2 - 5 = -1$

Thus, we have the following table giving points on the line $2x - y - 5 = 0$

x	1	2
y	-3	-1

Now,

$$x - y - 3 = 0$$

$$\Rightarrow \quad x - 3 = y$$

$$\Rightarrow \quad y = x - 3$$

When $x = 3$, we have

$y = 3 - 3 = 0$

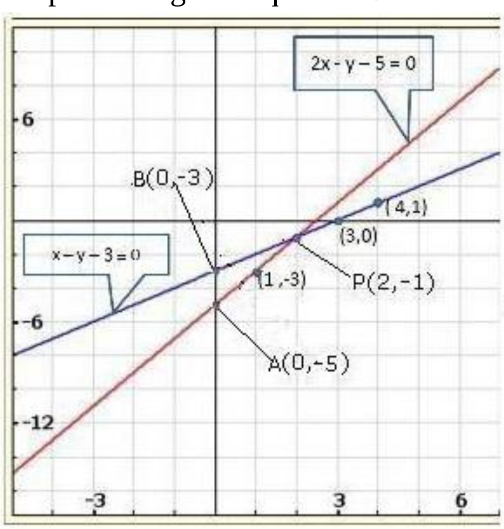
When $x = 4$, we have

$y = 4 - 3 = 1$

Thus, we have the following table giving points on the line $x - y - 3 = 0$

x	3	4
y	0	1

Graph of the given equations:



Clearly, two intersect at $P(2, -1)$.

Hence, $x = 2,\ y = -1$ is the solution of the given system of equations?

We also observe that the lines represented by $2x - y - 5 = 0$ *and* $x - y - 3 = 0$ meet y-axis at $A(0, -5)$ *and* $8(0, -3)$ respectively.

23. Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are:

(i)
$$y = x$$
$$y = 2x$$
$$y + x = 6$$

(ii)
$$y = x$$
$$3y = x$$
$$x + y = 8$$

Sol:

The system of the given equations is,

$y = x$

$y = 2x$

$y + x = 6$

Now,

$y = x$

When $x = 0$, we have

$y = 0$

When $x = -1$, we have

$y = -1$

Thus, we have the following table:

x	0	-1
y	0	-2

We have

$y = 2x$

When $x = 0$, we have

$y = 2 \times 0 = 0$

When $x = -1$, we have

$y = 2(-1) = -2$

Thus, we have the following table:

x	0	-1
y	0	-2

We have

$y + x = 6$

$\Rightarrow \quad y = 6 - x$

When $x = 2$, we have

$y = 6 - 2 = 4$

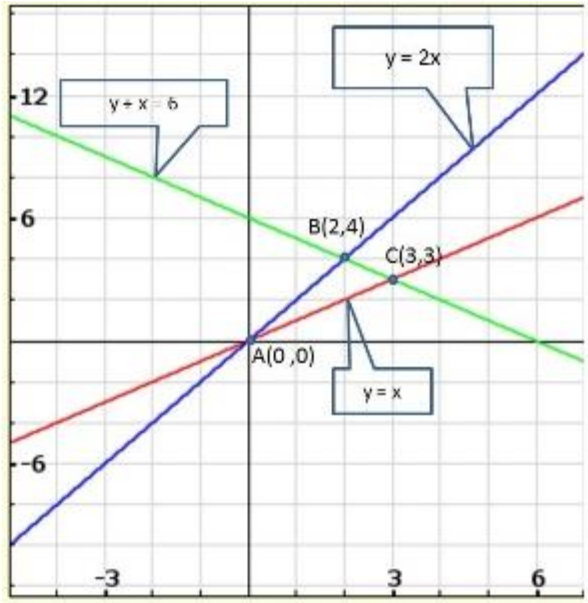
When $x=4$, we have

$y=6-4=2$

Thus, we have the following table:

x	2	4
y	4	2

Graph of the given system of equations:



From the graph of the three equations, we find that the three lines taken in pairs intersect each other at points $A(0,0), B(2,4)$ *and* $C(3,3)$.

Hence, the vertices of the required triangle are $(0,0),(2,4)$ *and* $(3,3)$.

The system of the given equations is,

$y=x$

$3y=x$

$x+y=8$

Now,

$y=x$

$\Rightarrow \quad x=y$

When $y=0$, we have

$x=0$

When $y=-3$, we have

$x=-3$

Thus, we have the following table.

x	0	-3
y	0	-3

We have

$$3y = x$$

$$\Rightarrow \quad x = 3y$$

When $y = 0$, we have

$x = 3\times 0 = 0$

When $y = -1$, we have

$y = 3\times(-1) = -3$

Thus, we have the following table:

x	0	-3
y	0	-1

We have

$$x + y = 8$$

$$\Rightarrow \quad x = 8 - y$$

When $y = 4$, we have

$x = 8 - 4 = 4$

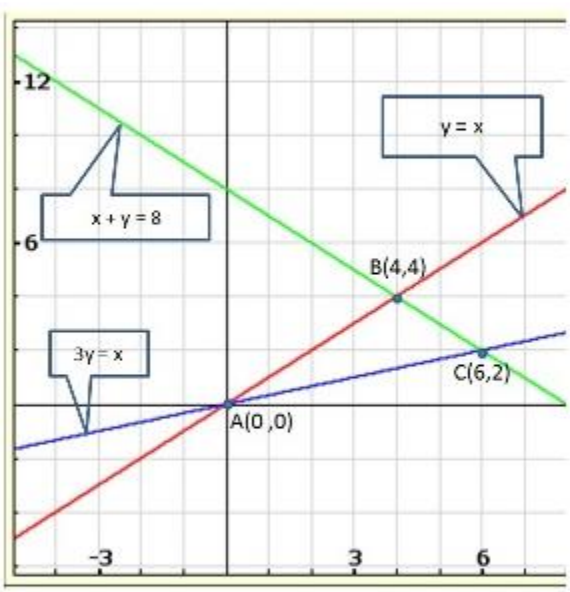
When $y = 5$, we have

$x = 8 - 5 = 3$

Thus, we have the following table:

x	4	5
y	4	3

Graph of the given system of equations:



From the graph of the three equations, we find that the three lines taken in pairs intersect each other at points $A(0,0)$, $B(4,4)$ *and* $C(6,2)$.

Hence, the vertices of the required triangle are $(0,0),(44)$ *and* $(6,2)$.

24. Solve the following system of linear equations graphically and shade the region between the two lines and x-axis:

(i) $2x+3y=12$
$x-y=1$

(ii) $3x+2y-4=0$
$2x-3y-7=0$

(iii) $3x+2y-11=0$
$2x-3y+10=0$

Sol:

The system of given equations is

$$2x+3y=12$$
$$x-y=1$$

Now,

$$2x+3y=12$$
$$\Rightarrow \quad 2x=12-3y$$
$$\Rightarrow \quad x=\frac{12-3\times 2}{2}=3$$

When $y=2,$ we have

$$x=\frac{12-3\times 2}{2}=3$$

When $y=4,$ we have

$$x=\frac{12-3\times 4}{2}=0$$

Thus, we have the following table:

x	0	3
y	4	2

We have,

$$x-y=1$$
$$\Rightarrow \quad x=1+y$$

When $y=0,$ we have

$x=1$

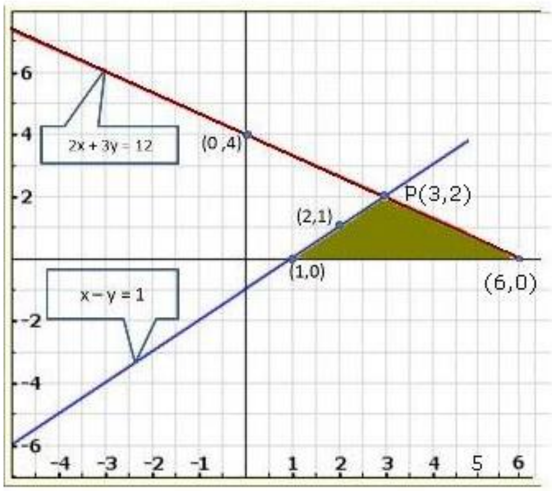
When $y=1,$ we have

$x=1+1=2$

Thus, we have the following table:

x	1	2
y	0	1

Graph of the given system of equations:



Clearly, the two lines intersect at $P(3,2)$.

Hence, $x = 3,\ y = 2$ is the solution of the given system of equations. The system of the given equations is,

$$3x + 2y - 4 = 0$$
$$2x - 3y - 7 = 0$$

Now,

$$3x + 2y - 4 = 0$$
$$\Rightarrow \quad 3x = 4 - 2y$$
$$\Rightarrow \quad x = \frac{4-2y}{3}$$

When $y = 5$, we have

$$x = \frac{4-2\times5}{3} = -2$$

When $y = 8$, we have

$$x = \frac{4-2\times8}{3} = -4$$

Thus, we have the following table:

x	-2	-4
y	5	8

We have,

$$2x-3y-7=0$$

$\Rightarrow \quad 2x=3y+7$

$\Rightarrow \quad x=\frac{3y+7}{2}$

When $y=1$, we have

$x=\frac{3\times 1+7}{2}=5$

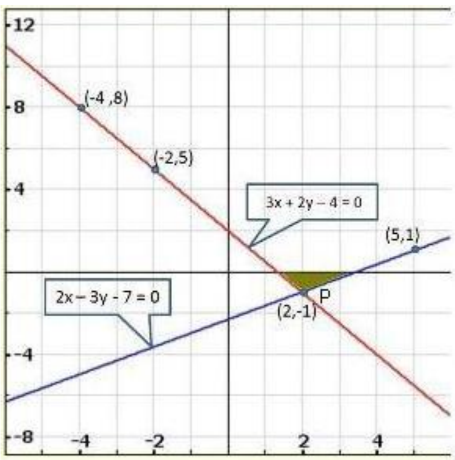
When $y=-1$, we have

$x=\frac{3\times(-1)+7}{2}=2$

Thus, we have the following table:

x	5	2
y	1	-1

Graph of the given system of equations:



Clearly, the two lines intersect at $P(2,-1)$.

Hence, $x=2$, $y=-1$ is the solution of the given system of equations.

The system of the given equations is,

$$3x+2y-11=0$$
$$2x-3y+10=0$$

Now,

$$3x+2y-11=0$$

$\Rightarrow \quad 3x=11-2y$

$\Rightarrow \quad x=\frac{11-2y}{3}$

When $y=1$, we have

$x = \frac{11-2\times1}{3} = 3$

When $y = 4,$ we have

$x = \frac{11-2\times4}{3} = 1$

Thus, we have the following table:

x	3	1
y	1	4

We have,

$$2x-3y+10=0$$

$$\Rightarrow \quad 2x = 3y-10$$

$$\Rightarrow \quad x = \frac{3y-10}{2}$$

When $y = 0,$ we have

$x = \frac{3\times0-10}{2} = -5$

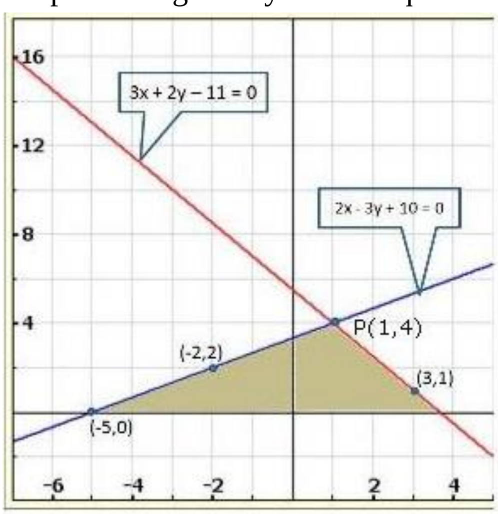
When $y = 2,$ we have

$x = \frac{3\times2-10}{2} = -2$

Thus, we have the following table:

x	-5	-2
y	0	2

Graph of the given system of equations:



Clearly, the two lines intersect at $P(1,4)$.

Hence, $x=1,\ y=4$ is the solution of the given system of equations

25. Draw the graphs of the following equations on the same graph paper:

$$2x+3y=12$$
$$x-y=1$$

Sol:

The system of the given equations is

$$2x+3y=12$$
$$x-y=1$$

Now,

$$2x+3y=12$$
$$\Rightarrow \quad 2x=12-3y$$
$$\Rightarrow \quad x=\frac{12-3y}{2}$$

When $y=0$, we have

$$x=\frac{12-3\times0}{2}=6$$

When $y=2$, we have

$$x=\frac{12-3\times2}{2}=3$$

Thus, we have the following table:

x	6	3
y	0	2

We have

$$x-y=1$$
$$\Rightarrow \quad x=1+y$$

When $y=0$, we have

$x=1$

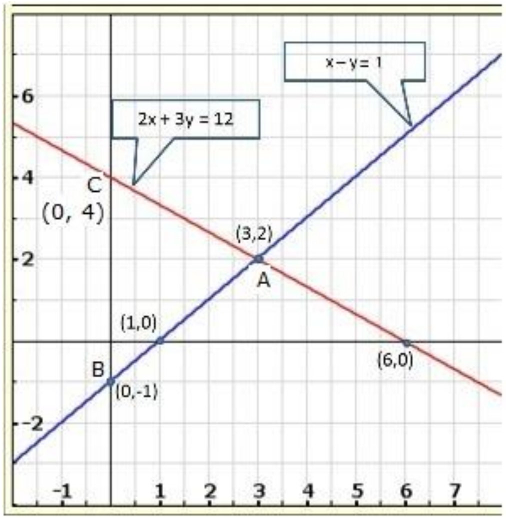
When $y=-1$, we have

$x=1-1=0$

Thus, we have the following table:

x	1	0
y	0	-1

Graph of the given system of equations:



Clearly, the two lines intersect at $A(3,2)$.

We also observe that the lines represented by the equations $2x+3y=12$ *and* $x-y=-1$ meet y-axis at $B(0,-1)$ *and* $C(0,4)$.

Hence, the vertices of the required triangle are $A(3,2)$, $B(0,-1)$ *and* $C(0,4)$.

26. Draw the graphs of x − y + 1 = 0 and 3x + 2y − 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and x- axis and shade the triangular area. Calculate the area bounded by these lines and x-axis.

Sol:

The given system of equations is

$$x-y+1=0$$

$$3x+2y-12=0$$

Now,

$$x-y+1=0$$

$$\Rightarrow \quad x=y-1$$

When $y=3$, we have

$x=3-1=2$

When $y=-1$, we have

$x=-1-1=-2$

Thus, we have the following table:

x	2	-2
y	3	-1

We have

$$3x+2y-12=0$$

$\Rightarrow$ $3x=12-2y$

$\Rightarrow$ $x=\dfrac{12-2y}{3}$

When $y=6$, we have

$$x=\frac{12-2\times 6}{3}=0$$

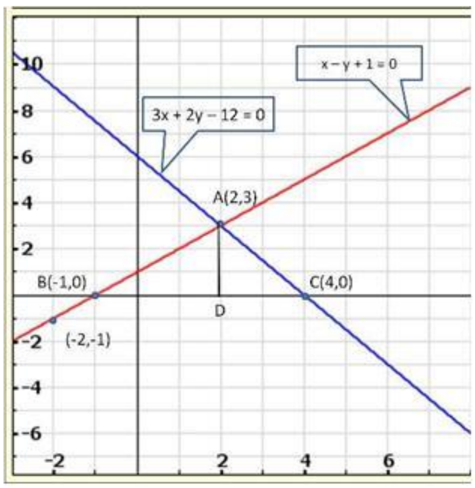
When $y=3$, we have

$$x=\frac{12-2\times 3}{3}=2$$

Thus, we have the following table:

x	0	2
y	6	3

Graph of the given system of equations:



Clearly, the two lines intersect at $A(2,3)$.

We also observe that the lines represented by the equations $x-y+1=0$ *and* $3x+2y-12=0$ meet x-axis at $B(-1,0)$ *and* $C(4,0)$ respectively.

Thus, $x=2,\ y=3$ is the solution of the given system of equations.

Draw AD perpendicular from A on x-axis.

Clearly, we have

$AD = y-$coordinate of point $A(2,3)$

$\Rightarrow$ $AD=3$ and, $BC=4-(-1)=4+1=5$

27. Solve graphically the system of linear equations:

$$4x-3y+4=0$$
$$4x+3y-20=0$$

Find the area bounded by these lines and x-axis.

Sol:

The given system of equation is

$$4x-3y+4=0$$
$$4x+3y-20=0$$

Now,

$$4x-3y+4=0$$
$$\Rightarrow \quad 4x=3y-4$$
$$\Rightarrow \quad x=\frac{3y-4}{4}$$

When $y=0,$ we have

$$x=\frac{3\times 0-4}{4}=-1$$

When $y=4,$ we have

$$x=\frac{3\times 4-4}{4}=2$$

Thus, we have the following table:

x	2	-1
y	4	0

We have

$$4x+3y-20=0$$
$$\Rightarrow \quad 4x=20-3y$$
$$\Rightarrow \quad x=\frac{20-3y}{4}$$

When $y=0,$ we have

$$x=\frac{20-3\times 0}{4}=5$$

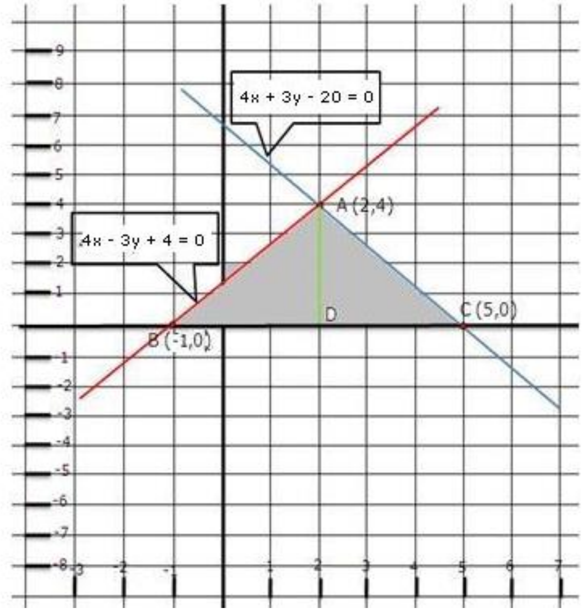
When $y=4,$ we have

$$x=\frac{20-3\times 4}{4}=2$$

Thus, we have the following table:

x	5	2
y	0	4

Graph of the given system of equation:



Clearly, the two lines intersect at $A(2,4)$. Hence $x=2,\ y=4$ is the solution of the given system of equations.

We also observe that the lines represented by the equations

$4x-3y+4=0$ *and* $4x+3y-20=0$ meet x-axis at $B(-1,0)$ *and* $C(5,0)$ respectively.

Thus, $x=2,\ y=4$ is the solution of the given system of equations.

Draw AD perpendicular from A on x-axis.

Clearly, we have

$AD = y -$ coordinate of point $A(2,4)$

$\Rightarrow \quad AD = 4$ and, $BC = 5-(-1) = 5+1 = 6$

$\therefore$ Area of the shaded region = Area of ΔABC

$\Rightarrow$ Area of the shaded region $= \frac{1}{2}(Base \times Height)$

$= \frac{1}{2} \times (BC \times AD)$

$= \frac{1}{2} \times 6 \times 4$

$= 6 \times 2$

$= 12$ sq. units

$\therefore$ Area of shaded region $= 12$ sq. units

28. Solve the following system of linear equations graphically:

$$3x+y-11=0$$
$$x-y-1=0$$

Shade the region bounded by these lines and y -axis. Also, find the area of the region bounded by these lines and y-axis.

Sol:

The given system of equation is

$$3x+y-11=0$$
$$x-y-1=0$$

Now,

$$3x+y-11=0$$
$$\Rightarrow \quad y=11-3x$$

When $x=0$, we have

$y=11-3\times 0=11$

When $x=3$ we have

$y=11-3\times 3=2$

Thus, we have the following table:

x	0	3
y	11	2

We have

$$x-y-1=0$$
$$\Rightarrow \quad x-1=y$$
$$\Rightarrow \quad y=x-1$$

When $x=0$, we have

$y=0-1=-1$

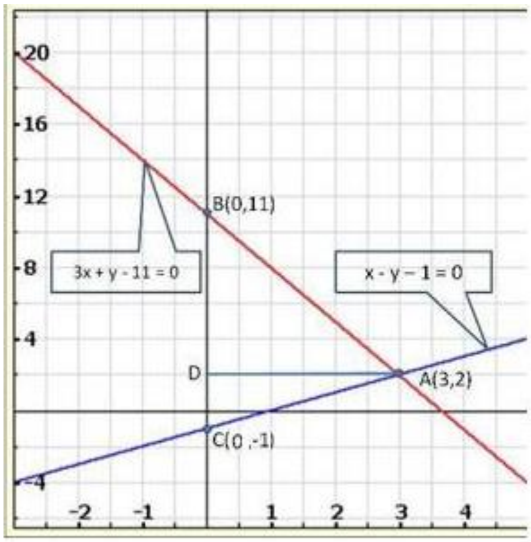
When $x=3$, we have

$y=3-1=2$

Thus, we have the following table:

x	0	3
y	-1	2

Graph of the given system of equations:



Clearly, the two lines intersect at $A(3,2)$. Hence $x=3,\ y=2$ is the solution of the given system of equations.

We so observe that the lines represented by the equations $3x+y-11=0$ *and* $x-y-1=0$ meet y-axis at $B(0,11)$ *and* $C(0,-1)$ respectively.

Thus, $x=3,\ y=2$ is the solution of the given system of equations.

Draw AD perpendicular from A on y-axis.

Clearly, we have

$AD = x-$ coordinate of point $A(3,2)$

$\Rightarrow \quad AD=3$ and, $BC=11-(-1)=11+1=12$

$\therefore$ Area of the shaded region = Area of ΔABC

$\Rightarrow$ Area of the shaded region $=\frac{1}{2}(Base\times Height)$

$=\frac{1}{2}\times(BC\times AD)$

$=\frac{1}{2}\times 12\times 3$

$=6\times 3$

$=18$ sq. units

$\therefore$ Area of the shaded region $=18$ sq. units

29. Solve graphically each of the following systems of linear equations. Also, find the coordinates of the points where the lines meet the axis of x in each system:

(i) $2x-y=2$
$4x-y=8$

(ii) $2x - y = 2$
$4x - y = 8$

(iii) $x + 2y = 5$
$2x - 3y = -4$

(iv) $2x + 3y = 8$
$x - 2y = -3$

Sol:

The given system of equation is

$$2x - y = 2$$
$$4x - y = 8$$

Now,

$$2x + y = 2$$
$$\Rightarrow \quad 2x = y + 2$$
$$\Rightarrow \quad x = \frac{y+2}{2}$$

When $y = 0,$ we have

$$x = \frac{0+2}{2} = 1$$

When $y = 2,$ we have

$$x = \frac{2+2}{2} = 2$$

Thus, we have the following table:

x	1	2
y	0	2

We have,

$$4x - y = 8$$
$$\Rightarrow \quad 4x = y + 8$$
$$\Rightarrow \quad x = \frac{y+8}{4}$$

When $y = 0,$ we have

$$x = \frac{0+8}{4} = 2$$

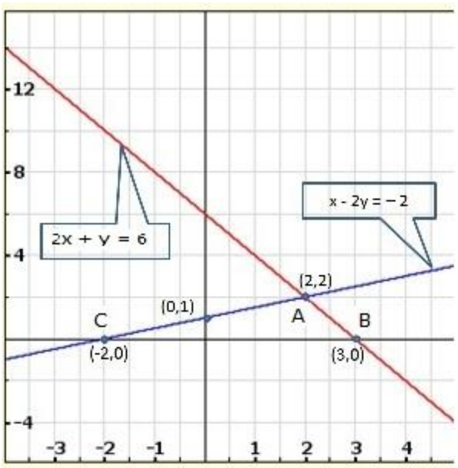
When $y = -4$ we have

$$x = \frac{-4+8}{4} = 1$$

Thus, we have the following table:

x	2	1
y	0	-4

Graph of the given system of equations:



Clearly, the two lines intersect at $A(2,2)$. Hence $x=2,\ y=2$ is the solution of the given system of equations.

We so observe that the lines represented by the equations $2x+y=6$ *and* $x-2y=-2$ meet x-axis at $B(3,0)$ *and* $C(-2,0)$ respectively.

The system of the given equations is

$$2x+y=6$$
$$x-2y=-2$$

Now,

$$2x+y=6$$

$$\Rightarrow \quad x=\frac{6-y}{2}$$

When $y=0,$ we have

$$x=\frac{6-0}{2}=3$$

When $y=2,$ we have

$$x=\frac{6-2}{2}=2$$

Thus, we have the following table:

x	3	2
y	0	2

We have,

$$x - 2y = -2$$

$\Rightarrow \quad y - 2y - 2$

When $y = 0$, we have

$x = 2 \times 0 - 2 = -2$

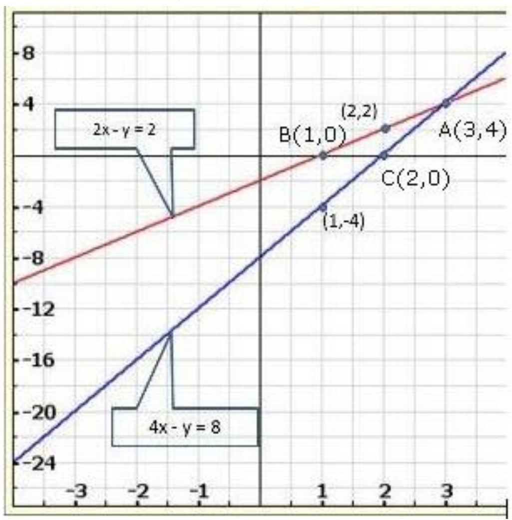
When $y = 1$, we have

$x = 2 \times 1 - 2 = 0$

Thus, we have the following table:

x	-2	0
y	0	1

Graph of the given system of equations:



Clearly the two lines intersect at $A(3,4)$. Hence $x = 3$, $y = 4$ is the solution of the given system of equations.

We so observe that the lines represented by the equations $2x - y = 2$ *and* $4x - y = 8$ meet x-axis at $B(1,0)$ *and* $C(2,0)$ respectively

The system of the given equations is

$$x + 2y = 5$$
$$2x - 3y = -4$$

Now,

$$x + 2y = 5$$

$\Rightarrow \quad x = 5 - 2y$

When $y = 2$, we have

$x = 5 - 2 \times 2 = 1$

When $y = 3$, we have

$x = 5 - 2 \times 3 = -1$

Thus, we have the following table:

x	1	-1
y	2	3

We have,

$$2x - 3y = -4$$

$$\Rightarrow \quad 2x = 3y - 4$$

$$\Rightarrow \quad x = \frac{3y-4}{2}$$

When $y = 0$, we have

$$x = \frac{3\times 0 - 4}{2} = -2$$

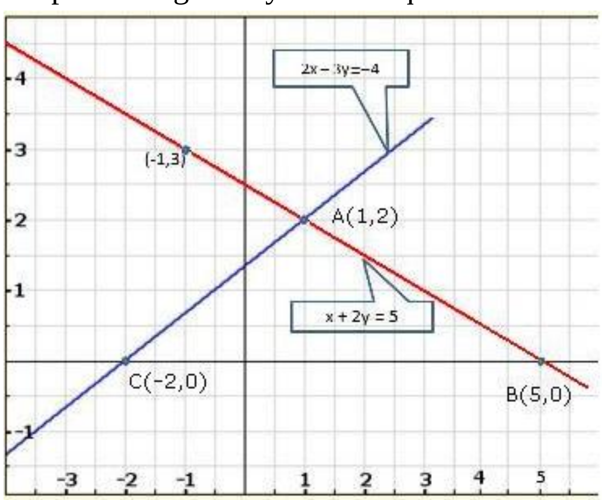
When $y = 2$, we have

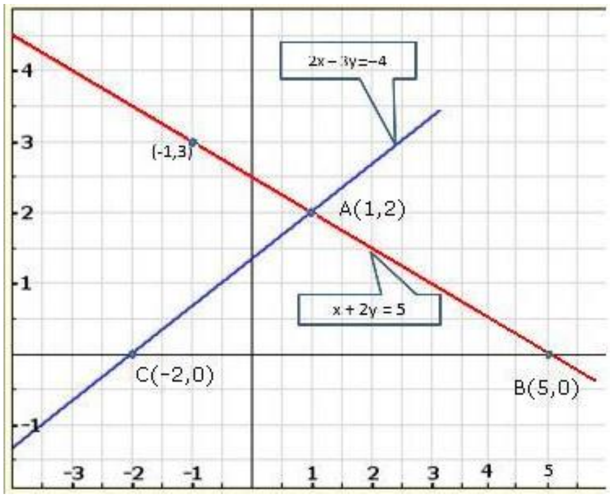
$$x = \frac{3\times 2 - 4}{2} = 1$$

Thus, we have the following table:

x	-2	1
y	0	2

Graph of the given system of equations:





The given system of equation is

$$2x+3y=8$$
$$x-2y=-3$$

Now,

$$2x+3y=8$$
$$\Rightarrow \quad 2x=8-3y$$
$$\Rightarrow \quad x=\frac{8-3y}{2}$$

When $y=2$, we have

$$x=\frac{8-3\times 4}{2}=1$$

When $y=4$, we have

$$x=\frac{8-3\times 4}{2}=-2$$

Thus, we have the following table:

x	1	-2
y	2	4

We have,

$$x-2y=-3$$
$$\Rightarrow \quad x=2y-3$$

When $y=0$, we have

$$x=2\times 0-3=-3$$

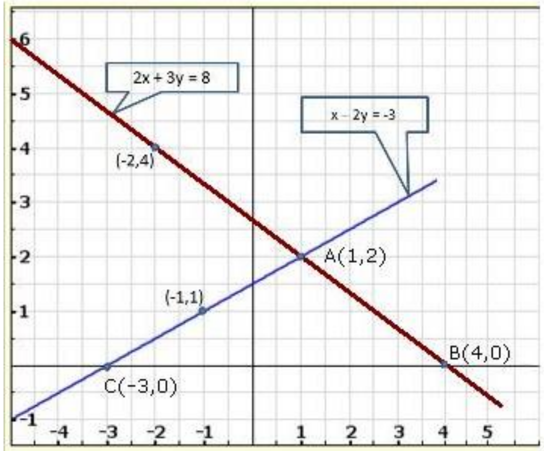
When $y=1$, we have

$$x=2\times 1-3=-1$$

Thus, we have the following table:

x	-3	-1
y	0	1

Graph of the given system of equations:



Clearly, the two lines intersect at $A(1,2)$. Hence $x=1,\ y=2$ is the solution of the given system of equations.

We also observe that the lines represented by the equations $2x+3y=8$ *and* $x-2y=-3$ meet x-axis at $B(4,0)$ *and* $C(-3,0)$ respectively.

30. Draw the graphs of the following equations:

$$2x-3y+6=0$$
$$2x+3y-18=0$$
$$y-2=0$$

Find the vertices of the triangle so obtained. Also, find the area of the triangle.

Sol:

The given system of equation is

$$2x-3y+6=0$$
$$2x+3y-18=0$$
$$y-2=0$$

Now,

$$2x-3y+6=0$$
$$\Rightarrow \quad 2x=3y-6$$
$$\Rightarrow \quad x=\frac{3y-6}{2}$$

When $y=0$, we have

$$x=\frac{3\times 0-6}{2}=-3$$

When $y = 2,$ we have

$$x = \frac{3\times 2-6}{2} = 0$$

Thus, we have the following table:

x	-3	0
y	0	2

We have,

$$2x+3y-18=0$$

$$\Rightarrow \quad 2x = 18-3y$$

$$\Rightarrow \quad x = \frac{18-3y}{2}$$

When $y = 2,$ we have

$$x = \frac{18-3\times 2}{2} = 6$$

When $y = 6,$ we have

$$x = \frac{18-3\times 6}{2} = 0$$

Thus, we have the following table:

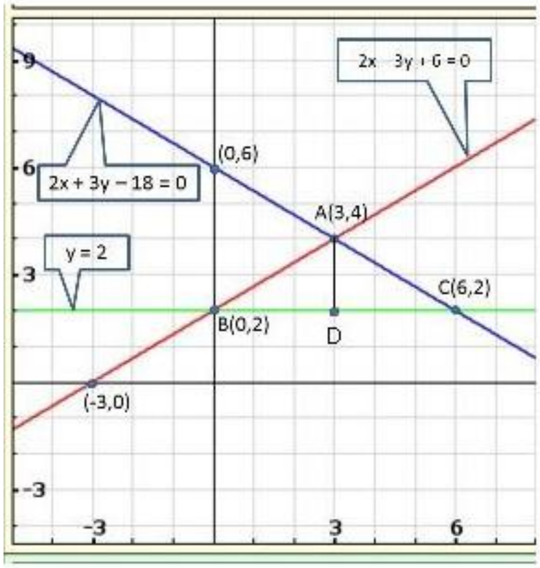
x	6	0
y	2	6

We have

$$y-2=0$$

$$\Rightarrow \quad y=-2$$

Graph of the given system of equations:



From the graph of the three equations, we find that the three lines taken in pairs intersect each other at points $A(3,4),\ B(0,2)\ and\ C(6,2)$.

Hence, the vertices of the required triangle are $(3,4),(0,2)\ and\ (6,2)$.

From graph, we have

$AD = 4-2 = 2$

$BC = 6-0 = 6$

Area of $\Delta ABC = \frac{1}{2}(Base \times Height)$

$= \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 6 \times 2$

$= 6\ sq.\ units$

$\therefore$ Area of $\Delta ABC = 6\ sq.units$

31. Solve the following system of equations graphically:

$$2x-3y+6=0$$
$$2x+3y-18==0$$

Also, find the area of the region bounded by these two lines and y-axis.

Sol:

The given system of equation is

$$2x-3y+6=0$$
$$2x+3y-18==0$$

Now,

$$2x-3y+6=0$$
$$\Rightarrow \quad 2x+6=3y$$
$$\Rightarrow \quad 3y=2x+6$$
$$\Rightarrow \quad y=\frac{2x+6}{3}$$

When $x=0$, we have

$$y=\frac{2\times 0+6}{3}=2$$

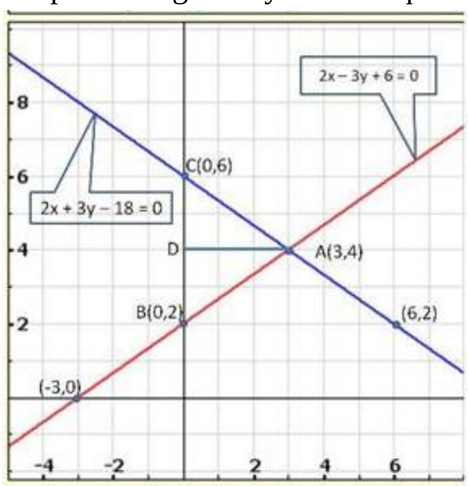
When $x=-3$, we have

$$y=\frac{2\times(-3)+6}{3}=0$$

Thus, we have the following table:

x	0	-3
y	2	6

Graph of the given system of equations:



Clearly, the two lines intersect at $A(3,4)$. Hence, $x=3,\ y=4$ is the solution of the given system of equations.

We also observe that the lines represented by the equations

$2x-3y+6=0$ *and* $2x+3y-18=0$ meet y-axis at $B(0,2)$ *and* $C(0,6)$ respectively.

Thus, $x=3,\ y=4$ is the solution of the given system of equations.

Draw AD perpendicular from A on y-axis.

Clearly, we have,

$$AD = x-\text{coordinate of point } A(3,4)$$
$$\Rightarrow \quad AD=3 \text{ and, } BC=6-2=4$$

Area of the shaded region = Area of ΔABC

Area of the shaded region $= \frac{1}{2}(Base \times Height)$

$= \frac{1}{2}(BC \times AD)$

$= \frac{1}{2} \times 4 \times 3$

$= 2 \times 3$

$= 6\ sq.\ units$

$\therefore$ Area of the region bounded by these two lines and y-axis is 6 sq. units.

32. Solve the following system of linear equations graphically:

$$4x - 5y - 20 = 0$$
$$3x + 5y - 15 = 0$$

Determine the vertices of the triangle formed by the lines representing the above equation and the y-axis.

Sol:

The given system of equation is

$$4x - 5y - 20 = 0$$
$$3x + 5y - 15 = 0$$

Now,

$$4x - 5y - 20 = 0$$

$\Rightarrow \quad 4x = 5y + 20$

$\Rightarrow \quad x = \frac{5y + 20}{4} = 5$

When $y = 0$, we have

$x = \frac{5 \times 0 + 20}{4} = 5$

When $y = -4$, we have

$x = \frac{5 \times (-4) + 20}{4} = 0$

Thus, we have the following table:

x	5	0
y	0	-4

We have,

$$3x + 5y - 15 = 0$$

$\Rightarrow \quad 3x = 15 - 5y$

$\Rightarrow \quad x = \frac{15 - 5y}{3}$

When $y=0$, we have

$$x=\frac{15-5\times3}{3}=0$$

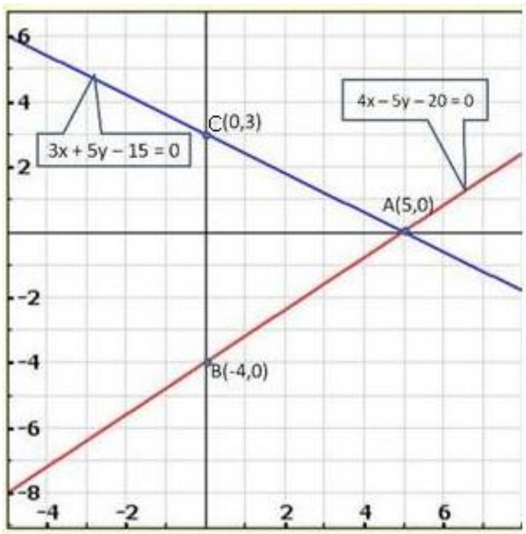
When $y=3$, we have

$$x=\frac{15-5\times3}{3}=0$$

Thus, we have the following table:

x	5	0
y	0	3

Graph of the given system of equations:



Clearly, the two lines intersect at $4(5,0)$. Hence, $x-5$, $y-0$ is the solution of the given system of equations.

We also find that the two lines represented by the equations $4x-5y-20=0$ *and* $3x+5y-15=0$ meet y-axis at $B(0,-4)$ *and* $C(0,3)$ respectively,

$\therefore$ The vertices of the required triangle are $(5,0)$, $(0,-4)$ *and* $(0,3)$.

33. Draw the graphs of the equations 5x − y = 5 and 3x − y = 3. Determine the co-ordinates of the vertices of the triangle formed by these lines and y-axis. Calculate the area of the triangle so formed.

Sol:

$5x-y=5 \Rightarrow y=5x-5$

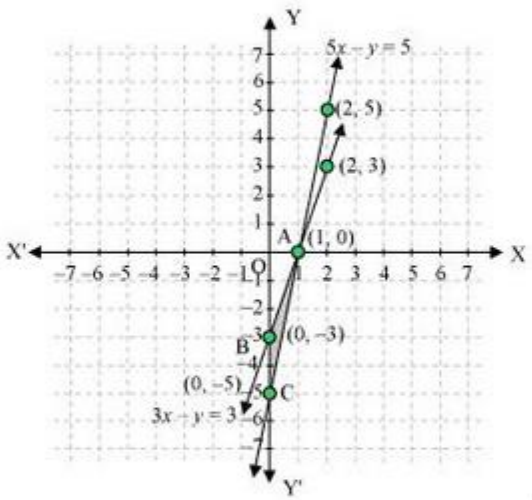
Three solutions of this equation can be written in a table as follows:

x	0	1	2
y	-5	0	5

$3x - y = 3 \Rightarrow y = 3x - 3$

x	0	1	2
y	-3	0	3

The graphical representation of the two lines will be as follows:



It can be observed that the required triangle is ΔABC.

The coordinates of its vertices are $A(1,0)$, $B(0, \ -3)$, $C(0,-5)$.

Concept insight: In order to find the coordinates of the vertices of the triangle so formed. Find the points where the two lines intersects the y-axis and also where the two lines intersect each other. Here, note that the coordinates of the intersection of lines with y-axis is taken and not with x-axis, this is became the question says to find the triangle formed by the two lines and the y-axis.

34. Form the pair of linear equations in the following problems, and find their solution graphically:

(i) 10 students of class X took part in Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs 46. Find the cost of one pencil and a pen.

(iii) Champa went to a ‘sale’ to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, “The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased.” Help her friends to find how many pants and skirts Champa bought.

Sol:

(i) Let the number of girls and boys in the class be *x* and *y* respectively.

According to the given conditions, we have:

$x + y = 10$

$x - y = 4$

$x + y = 10 \Rightarrow x = 10 - y$

Three solutions of this equation can be written in a table as follows:

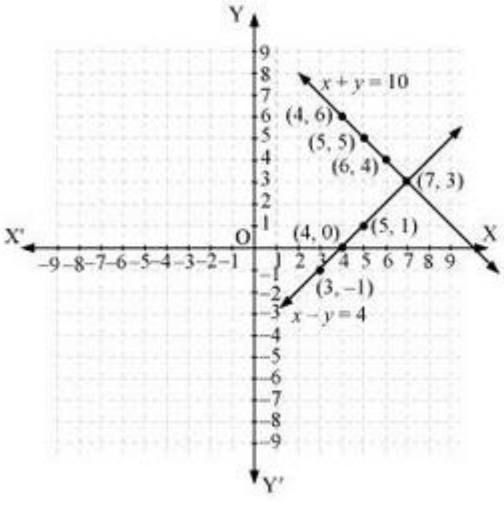
x	4	5	6
y	6	5	4

$x - y = 4 \Rightarrow x = 4 + y$

Three solutions of this equation can be written in a table as follows:

x	5	4	3
y	1	0	-1

The graphical representation is as follows:



From the graph, it can be observed that the two lines intersect each other at the point $(7, 3)$.

So. $x = 7$ *and* $y = 3.$

Thus, the number of girls and boys in the class are 7 and 3 respectively.

(ii) Let the cost of one pencil and one pen be Rs x and Rs y respectively.

According to the given conditions, we have:

$5x + 7y = 50$

$7x + 5y = 46$

$5x + 7y = 50 \Rightarrow x = \frac{50 - 7y}{5}$

Three solutions of this equation can be written in a table as follows:

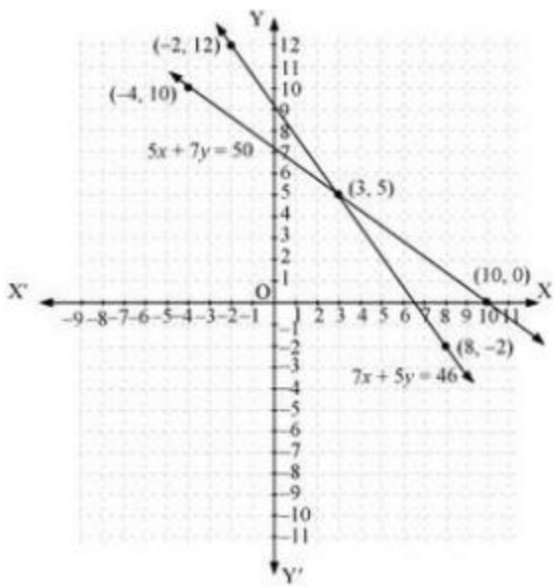
x	3	10	-4
y	5	0	10

$7x+5y=46 \Rightarrow x=\frac{46-5y}{7}$

Three solutions of this equation can be written in a table as follows:

x	8	3	-2
y	-2	5	12

The graphical representation is as follows:



From the graph. It can be observed that the two lines intersect each other at the point $(3,5)$.

So. $x=3$ *and* $y=5$.

Therefore, the cost of one pencil and one pen are Rs 3 and Rs 5 respectively.

(iii) Let us denote the number of pants by x and the number of skirts by y. Then the equations formed are:

$y=2x?2......(1)$ and $y=4x?4......(2)$

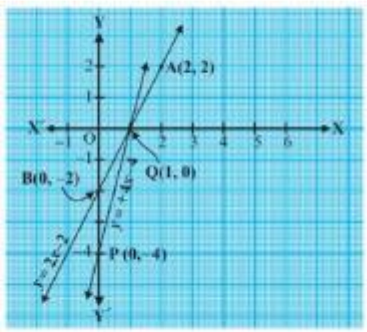
Let us draw the graphs of Equations (1) and (2) by finding two solutions for each of the equations.

They are given in Table

They are giving table

x	2	0
$y-2x?2$	2	-2

x	0	1
$y-2x?2$	-4	0



Plot the point and draw the lines passing through them to represent the equation, as shown in fig.

The t lines intersect at the point $(1,0)$. So. $x-1, y=0$ is the required solution of the pair of linear equations, i.e, the number of pants she purchased island she did not buy any skirt

Concept insight: Read the question carefully and examine what are the unknowns. Represent the given conditions with the help of equations by taking the unknowns quantities as variables. Also carefully state the variables as whole solution is based on it on the graph paper, mark the points accurately and neatly using a sharp pencil. Also take at least three points satisfying the two equations in order to obtain the correct straight line of the equation. Since joining any two points gives a straight line and if one of the points is computed incorrect will give a wrong line and taking third point will give a correct line. The point where the two straight lines will intersect will give the values of the two variables, i.e., the solution of the two linear equations. State the solution point.

35. Solve the following system of equations graphically:
Shade the region between the lines and the y-axis

(i) $3x-4y=7$
$5x+2y=3$

(ii) $4x-y=4$
$3x+2y=14$

Sol:

The given system of equations is

$3x-4y=7$

$5x+2y=3$

Now,

$$3x-4y=7$$

$$\Rightarrow \quad 3x-7=4y$$

$$\Rightarrow \quad 4y=3x-7$$

$$\Rightarrow \quad y=\frac{3x-7}{4}$$

When $x=1,$ we have

$$y=\frac{3\times1-7}{4}=-1$$

When $x=-3,$ we have

$$y=\frac{3\times(-3)-7}{4}=-4$$

Thus, we have the following table:

x	1	-3
y	-1	-4

We have,

$$5x+2y=3$$

$$\Rightarrow \quad 2y=3-5x$$

$$\Rightarrow \quad y=\frac{3-5x}{2}$$

When $x=1,$ we have

$$y=\frac{3-5\times1}{2}=-1$$

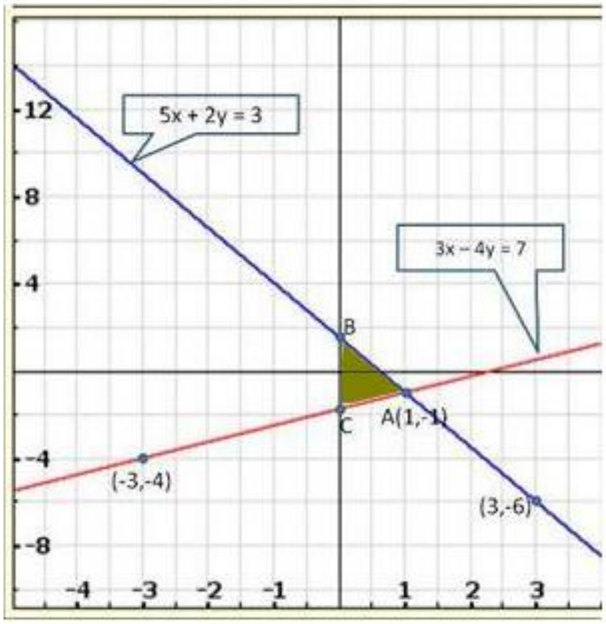
When $x=3,$ we have

$$y=\frac{3-5\times3}{2}=-6$$

Thus, we have the following table:

x	1	3
y	-1	-6

Graph of the given system of equations:



Clearly, the two lines intersect at $A(1,-1)$ Hence, $x=1, y=-1$ is the solution of the given system of equations.
We also observe that the required shaded region is ΔABC

The given system of equations is

$$4x-y=4$$
$$3x+2y=14$$

Now,

$$4x-y=4$$
$$\Rightarrow \quad 4x-4=y$$
$$\Rightarrow \quad y=4x-4$$

When $x=0$, we have

$y=4\times0-4=-4$

When $x=-1$, we have

$y=4\times(-1)-4=-8$

Thus, we have the following table:

x	0	-1
y	-4	-8

We have,

$$3x+2y=14$$
$$\Rightarrow \quad 2y=14-3x$$
$$\Rightarrow \quad y=\frac{14-3x}{2}$$

When $x=0$, we have

$$y=\frac{14-3\times 0}{2}=7$$

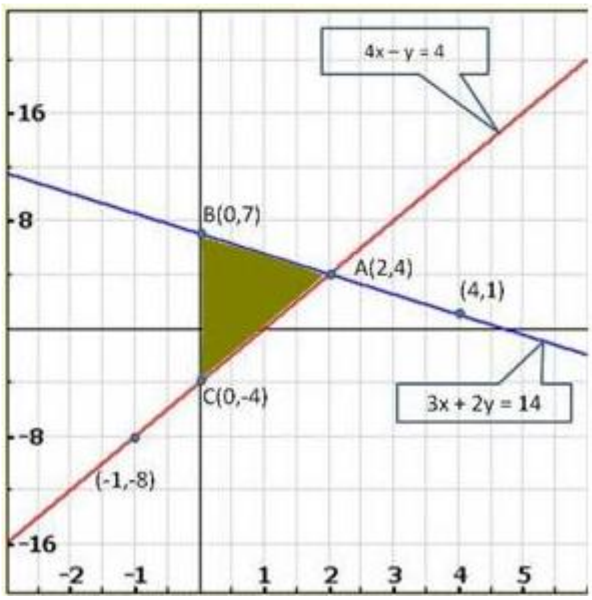
When $x=0$, we have

$$y=\frac{14-3\times 4}{2}=1$$

Thus, we have the following table:

x	0	4
y	7	1

Graph of the given system of equations:



Clearly, the two lines intersect at $A(2,4)$. Hence, $x=2,\ y=4$ is the solution of the given system of equations.

We also observe ΔABC is the required shaded region.

36. Represent the following pair of equations graphically and write the coordinates of points where the lines intersects y-axis

$$x+3y=6$$
$$2x-3y=12$$

Sol:

The given system of equations is

$$x+3y=6$$
$$2x-3y=12$$

Now,

$$x+3y=6$$
$$\Rightarrow \quad 3y=6-x$$
$$\Rightarrow \quad y=\frac{6-x}{3}$$

When $x=0$, we have

$$y=\frac{6-0}{3}=2$$

When $x=3$, we have

$$y=\frac{6-3}{3}=1$$

Thus, we have the following table:

x	0	3
y	2	1

We have,

$$2x+3y=12$$
$$\Rightarrow \quad 2x-12-3x$$
$$\Rightarrow \quad 3y=2x-12$$
$$\Rightarrow \quad y=\frac{2x-12}{3}$$

When $x=0$, we have

$$y=\frac{2\times 0-12}{3}=-4$$

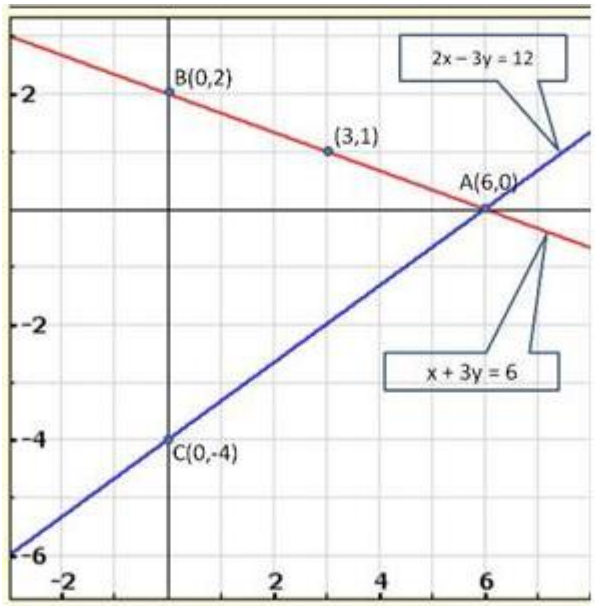
When $x=6$, we have

$$y=\frac{2\times 6-12}{3}=0$$

Thus, we have the following table:

x	0	6
y	-4	0

Graph of the given system of equations:



We observe that the lines represented by the equations $x+3y-6$ *and* $2x-3y-12$ meet y-axis at $B(0,2)$ *and* $C(0,-4)$ respectively.

Hence, the required co-ordinates are $(0,2)$ *and* $(0,-4)$.

37. Given the linear equation 2x + 3y − 8 = 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is (i) intersecting lines (ii) Parallel lines (iii) coincident lines

Sol:

(i) For the two lines $a_1x+b_1x+c_1=0$ and $a_2x+b_2x+c_2=0$, to be intersecting, we must have

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

So, the other linear equation can be $5x+6y-16=0$

As $\frac{a_1}{a_2}=\frac{2}{5},\frac{b_1}{b_2}=\frac{3}{6}=\frac{1}{2},\frac{c_1}{c_2}=\frac{-8}{-16}=\frac{1}{2}$

(ii) For the two lines $a_1x+b_1x+c_1=0$ and $a_2x+b_2x+c_2=0$, to be parallel we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

So, the other linear equation can be $6x+9y+24=0$,

As $\frac{a_1}{a_2}=\frac{2}{6}=\frac{1}{3},\frac{b_1}{b_2}=\frac{3}{9}=\frac{1}{3},\frac{c_1}{c_2}=\frac{-8}{24}=\frac{-1}{3}$

(iii) For the two lines $a_1x+b_1x+c_1=0$ and $a_2x+b_2x+c_2=0,$ to be coincident, we must have $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

So, the other linear equation can be $6x+9y+24=0,$

As $\frac{a_1}{a_2}=\frac{2}{8}=\frac{1}{4},\frac{b_1}{b_2}=\frac{3}{12}=\frac{1}{4},\frac{c_1}{c_2}=\frac{-8}{-32}=\frac{1}{4}$

Concept insight: In orders to answer such type of problems, just remember the conditions for two lines to be intersecting parallel, and coincident

This problem will have multiple answers as their can be marry equations satisfying the required conditions.

Exercise 3.3

Solve the following systems of equations:

1. 11x + 15y + 23 = 0
   7x – 2y – 20 = 0

Sol:

The given system of equation is

$11x+15y+23=0 \qquad ...(i)$

$7x-2y-20=0 \qquad ...(ii)$

From (ii), we get

$2y=7x-20$

$\Rightarrow \quad y=\frac{7x-20}{2}$

Substituting $y=\frac{7x-20}{2}$ in (i) we get

$11x+15\left(\frac{7x-20}{2}\right)+23=0$

$\Rightarrow \quad 11x+\frac{105x-300}{2}+23=0$

$\Rightarrow \quad \frac{22x+105x-300+46}{2}=0$

$\Rightarrow \quad 127x-254=0$

$\Rightarrow \quad 127x=254$

$\Rightarrow \quad x=\frac{254}{127}=2$

Putting $x = 2$ in $y = \frac{7x - 20}{2}$ we get

$\Rightarrow \quad y = \frac{7 \times 2 - 20}{2}$

$= \frac{14 - 20}{2}$

$= \frac{-6}{2}$

$= -3$

Hence, the solution of the given system of equations is $x = 2, y = -3.$

2. 3x – 7y + 10 = 0
y – 2x – 3 = 0
Sol:
The given system of equation is

$3x - 7y + 10 = 0 \quad \ldots(i)$

$y - 2x - 3 = 0 \quad \ldots(ii)$

From (ii), we get

$y = 2x + 3$

Substituting $y = 2x + 3$ in (i) we get

$3x - 7(2x + 3) + 10 = 0$

$\Rightarrow \quad 3x + 14x - 21 + 10 = 0$

$\Rightarrow \quad -11x = 11$

$\Rightarrow \quad x = \frac{11}{-11} = -1$

Putting $x = -1$ in $y = 2x + 3$, we get

$\Rightarrow \quad y = 2 \times (-1) + 3$

$= -2 + 3$

$= 1$

$\Rightarrow \quad y = 1$

Hence, the solution of the given system of equations is $x = -1, y = 1.$

3. 0.4x + 0.3y = 1.7
0.7x + 0.2y = 0.8
Sol:

The given system of equation is

$0.4x+0.3y=1.7 \quad ...(i)$

$0.7x-0.2y=0.8 \quad ...(ii)$

Multiplying both sides of (i) and (ii), by 10, we get

$4x+3y=17 \quad ...(iii)$

$7x-2y=8 \quad ...(iv)$

From (iv), we get

$$7x=8+2y$$

$$\Rightarrow \quad 7x\frac{8+2y}{7}$$

Substituting $x=\frac{8+2y}{7}$ in (iii), we get

$$4\left(\frac{8+2y}{7}\right)+3y=17$$

$$\Rightarrow \quad \frac{32+8y}{7}+3y=17$$

$$\Rightarrow \quad 32+29y=17\times 7$$

$$\Rightarrow \quad 29y=119-32$$

$$\Rightarrow \quad 29y=87$$

$$\Rightarrow \quad y=\frac{87}{29}=3$$

Putting $y=3$ in $x=\frac{8+2y}{7}$, we get

$$x=\frac{8+2\times 3}{7}$$

$$=\frac{8+6}{7}$$

$$=\frac{14}{7}$$

$$=2$$

Hence, the solution of the given system of equation is $x=2, y=3$.

4. $\frac{x}{2}+y=0.8$

Sol:

$$\frac{x}{2}+y=0.8$$

And $\frac{7}{x+\frac{y}{2}}=10$

$\therefore x+2y=1.6 \ \ and \ \ \frac{7\times 2}{2x+y}=10$

$x+2y=1.6 \ and \ 7=10x+5y$

Multiply first equation by 10

$10x+20y=16 \ \ and \ \ 10x+5y=7$

Subtracting the two equations

$15y=9$

$y=\frac{9}{15}=\frac{3}{5}$

$x=1.6-2\left(\frac{3}{5}\right)=1.6-\frac{6}{5}=\frac{2}{5}$

Solution is $\left(\frac{2}{5},\frac{3}{5}\right)$

5. 7(y + 3) – 2 (x + 3) = 14
4(y – 2) + 3 (x – 3) = 2

Sol:

The given system of equations id

$7(y+3)-2(x+3)=14 \qquad ...(i)$

$4(y-2)+3(x-3)=2 \qquad ...(ii)$

From (i), we get

$7x+21-2x-4=14$

$\Rightarrow \quad 7y=14+4-21+2x$

$\Rightarrow \quad y=\frac{2x-3}{7}$

From (ii), we get

$4y-8+3x-9=2$

$\Rightarrow \quad 4y+3x-17-2=0$

$\Rightarrow \quad 4y+3x-19=0 \qquad ...(iii)$

Substituting $y=\frac{2x-3}{7}$ in (iii), we get

$$4\left(\frac{2x-3}{7}\right)+3x-19=0$$

$\Rightarrow \quad \frac{8x-12}{7}+3x-19=0$

$\Rightarrow \quad 8x-12+21x-133=0$

$\Rightarrow \quad 29x-145=0$

$\Rightarrow \quad 29x=145$

$\Rightarrow \quad x=\frac{145}{29}=5$

Putting $x=5$ in $y=\frac{2x-3}{7}$, we get

$y=\frac{2\times 5-7}{7}$

$=\frac{10-3}{7}$

$=\frac{7}{7}$

$=1$

$\Rightarrow y=1$

Hence, the solution of the given system of equations is $x=5, y=1$.

6. $\frac{x}{7}+\frac{y}{3}=5$
$\frac{x}{2}-\frac{y}{9}=6$

Sol:

The given system of equation is

$\frac{x}{7}+\frac{y}{3}=5 \qquad ...(i)$

$\frac{x}{2}-\frac{y}{9}=6 \qquad ...(ii)$

From (i), we get

$\frac{3x+7y}{21}=5$

$\Rightarrow \quad 3x+7y=105$

$\Rightarrow \quad 3x=105-7y$

$\Rightarrow \quad x=\frac{105-7y}{3}$

From (ii), we get

$$\frac{9x-2y}{18}=6$$

$\Rightarrow$ $9x-2y=108$ $\quad ...(iii)$

Substituting $x=\frac{105-7y}{3}$ in (iii), we get

$$9\left(\frac{105-7y}{3}\right)-2y=108$$

$\Rightarrow$ $\frac{948-63y}{3}-2y=108$

$\Rightarrow$ $945-63y-6y=108\times3$

$\Rightarrow$ $945-69y=324$

$\Rightarrow$ $945-324=69y$

$\Rightarrow$ $69y=621$

$\Rightarrow$ $y=\frac{621}{69}=9$

Putting $y=9$ in $x=\frac{1105-7y}{3}$, we get

$$x=\frac{105-7\times9}{3}=\frac{105-63}{3}$$

$\Rightarrow$ $x=\frac{42}{3}=14$

Hence, the solution of thee given system of equations is $x=14, y=9$.

7. $\frac{x}{3}+\frac{y}{4}=11$

$\frac{5x}{6}-\frac{y}{3}=7$

Sol:

The given system of equations is

$$\frac{x}{3}+\frac{y}{4}=11 \quad ...(i)$$

$$\frac{5x}{6}-\frac{y}{3}=7 \quad ...(ii)$$

From (i), we get

$$\frac{4x+3y}{12}=11$$

$\Rightarrow$ $4x+3y=132$ $\quad ...(iii)$

From (ii), we get

$$\frac{5x+2y}{6}=-7$$

$\Rightarrow \quad 5x-2y=-42 \quad ...(iv)$

Let us eliminate y from the given equations. The coefficients of y in the equations(iii) and (iv) are 3 and 2 respectively. The L.C.M of 3 and 2 is 6. So, we make the coefficient of y equal to 6 in the two equations.

Multiplying (iii) by 2 and (iv) by 3, we get

$8x+6y=264 \quad ...(v)$

$15x-6x=-126 \quad ...(vi)$

Adding (v) and (vi), we get

$8x+15x=264-126$

$\Rightarrow \quad 23x=138$

$\Rightarrow \quad x=\frac{138}{23}=6$

Substituting $x=6$ in (iii), we get

$4\times 6+3y=132$

$\Rightarrow \quad 3y=132-24$

$\Rightarrow \quad 3y=108$

$\Rightarrow \quad y=\frac{108}{3}=36$

Hence, the solution of the given system of equations is $x=6, y=36.$

8. $4u+3y=8$

$6u-4y=-5$

Sol:

Taking $\frac{1}{x}=u,$ then given equations become

$4u+3y=8 \quad ...(i)$

$6u-4y=-5 \quad ...(ii)$

From (i), we get

$4u=8-3y$

$\Rightarrow \quad u=\frac{8-3y}{4}$

Substituting $u=\frac{8-3y}{4}$ in (ii), we get

From (ii), we get

$$6\left(\frac{8-3y}{4}\right)-4y=-5$$

$\Rightarrow$ $$\frac{3(8-3y)}{2}-4y=-5$$

$\Rightarrow$ $$\frac{24-9y}{2}-4y=-5$$

$\Rightarrow$ $$\frac{24-9y-8y}{2}=-5$$

$\Rightarrow$ $24-17y=-10$

$\Rightarrow$ $-17y=-10-24$

$\Rightarrow$ $-17y=-34$

$\Rightarrow$ $$y=\frac{-34}{-17}=2$$

Putting $y=2,$ in $u=\frac{8-3y}{4}$, we get

$$u=\frac{8-3\times 2}{4}=\frac{8-6}{4}=\frac{2}{4}=\frac{1}{2}$$

Hence, $x=\frac{1}{u}=2$

So, the solution of the given system of equation is $x=2, y=2.$

9. $$x+\frac{y}{2}=4$$

$$\frac{x}{3}+2y=5$$

Sol:

The given system of equation is

$$x+\frac{y}{2}=4 \qquad ..(i)$$

$$\frac{x}{3}+2y=5 \qquad ..(ii)$$

From (i), we get

$$\frac{2x+y}{2}=4$$

$2x+y=8$

$y=8-2x$

From (ii), we get

$x+6y=15 \qquad ..(iii)$

Substituting $y = 8 - 2x$ in (iii), we get

$$x + 6(8 - 2x) = 15$$

$\Rightarrow \quad x + 48 - 12x = 15$

$\Rightarrow \quad -11x = 15 - 48$

$\Rightarrow \quad -11x = -33$

$\Rightarrow \quad x = \frac{-33}{-11} = 3$

Putting $x = 3,$ in $y = 8 - 2x$, we get

$$y = 8 - 2 \times 3$$

$$= 8 - 6$$

$$= 2$$

$\Rightarrow \quad y = 2$

Hence, solution of the given system of equation is $x = 3, y = 2.$

10. $x + 2y = \frac{3}{2}$

$2x + y = \frac{3}{2}$

Sol:

The given system of equation is

$$x + 2y = \frac{3}{2} \quad ..(i)$$

$$2x + y = \frac{3}{2} \quad ..(ii)$$

Let us eliminate y from the given equations. The Coefficients of y in the given equations are 2 and 1 respectively. The L.C.M of 2 and 1 is 2. So, we make the coefficient of y equal to 2 in the two equations.

Multiplying (i) by 1 and (ii) by 2, we get

$$x + 2y = \frac{3}{2} \quad ..(iii)$$

$$4x + 2y = 3 \quad ..(iv)$$

Subtracting (iii) from (iv), we get

$$4x - x + 2y - 2y = 3 - \frac{3}{2}$$

$\Rightarrow \quad 3x = \frac{6-3}{2}$

$\Rightarrow \quad 3x = \frac{3}{2}$

$\Rightarrow \quad x = \frac{3}{2 \times 3}$

$\Rightarrow \quad x = \frac{1}{2}$

Putting $x = \frac{1}{2}$, in equation (iv), we get

$$4 \times \frac{1}{2} + 2y = 3$$

$\Rightarrow \quad 2 + 2y = 3$

$\Rightarrow \quad 2y = 3 - 2$

$\Rightarrow \quad y = \frac{1}{2}$

Hence, solution of the given system of equation is $x = \frac{1}{2}, y = \frac{1}{2}$.

11. $\sqrt{2}x + \sqrt{3}y = 0$
$\sqrt{3}x - \sqrt{8}y = 0$

Sol:

$\sqrt{2}x + \sqrt{3}y = 0 \qquad ...(i)$

$\sqrt{3}x - \sqrt{8}y = 0 \qquad ...(ii)$

From equation (i), we obtain:

$x = \frac{-\sqrt{3}y}{\sqrt{2}} \qquad ...(iii)$

Substituting this value in equation (ii), we obtain:

$$\sqrt{3}\left(-\frac{\sqrt{3}y}{\sqrt{2}}\right) - \sqrt{8}y = 0$$

$$-\frac{3y}{\sqrt{2}} - 2\sqrt{2}y = 0$$

$$y\left(-\frac{3}{\sqrt{2}} - 2\sqrt{2}\right) = 0$$

$$y = 0$$

Substituting the value of y in equation (iii), we obtain:

$x = 0$

$\therefore x = 0, y = 0$

12. $3x - \frac{y+7}{11} + 2 = 10$

$2y + \frac{x+11}{7} = 10$

Sol:

The given systems of equation is

$3x - \frac{y+7}{11} + 2 = 10 \quad ...(i)$

$2y + \frac{x+11}{7} = 10 \quad ...(ii)$

From (i), we get

$\frac{33x - y - 7 + 22}{11} = 10$

$\Rightarrow \quad 33x - y + 15 = 10 \times 11$

$\Rightarrow \quad 33x + 15 - 110 = y$

$\Rightarrow \quad y = 33x - 95$

From (ii) we get

$\frac{14y + x + 11}{7} = 109$

$\Rightarrow \quad 14y + x + 11 = 10 \times 7$

$\Rightarrow \quad 14y + x + 11 = 70$

$\Rightarrow \quad 14y + x = 70 - 11$

$\Rightarrow \quad 14y + x = 59 \quad ...(iii)$

Substituting $y = 33x - 95$ in (iii), we get

$14(33x - 95) + x = 59$

$\Rightarrow \quad 462x - 1330 + x = 59$

$\Rightarrow \quad 463x = 59 + 1330$

$\Rightarrow \quad 463x = 1389$

$\Rightarrow \quad x = \frac{1389}{463} = 3$

Putting $x = 3$, in $y = 33x - 95$, we get

$y = 33 \times 3 - 95$

$\Rightarrow \quad y = 99 - 95$

$= 4$

$\Rightarrow \quad y = 4$

Hence, solution of the given system of equation is $x = 3, y = 4.$

13. $2x - \frac{3}{y} = 9$

$3x + \frac{7}{y} = 2, y \neq 0$

Sol:

The given systems of equation is

$2x - \frac{3}{y} = 9 \quad ...(i)$

$3x + \frac{7}{y} = 2, y \neq 0 \quad ...(ii)$

Taking $\frac{1}{y} = u,$ the given equations becomes

$2x - 3u = 9 \quad ...(iii)$

$3x + 7u = 2 \quad ...(iv)$

From (iii), we get

$2x = 9 + 3u$

$\Rightarrow \quad x = \frac{9 + 3u}{2}$

Substituting $x = \frac{9 + 3u}{2}$ in (iv), we get

$3\left(\frac{9 + 3u}{2}\right) + 7u = 2$

$\Rightarrow \quad \frac{27 + 9u + 14u}{2} = 2$

$\Rightarrow \quad 27 + 23u = 2 \times 2$

$\Rightarrow \quad 23u = 4 - 27$

$\Rightarrow \quad u = \frac{-23}{23} = -1$

Hence, $y = \frac{1}{u} = \frac{1}{-1} = -1$

Putting $u=-1$ in $x=\frac{9+3u}{2}$, we get

$$x=\frac{9+3(-1)}{2}=\frac{9-3}{2}=\frac{6}{2}=3$$

$\Rightarrow \quad x=3$

Hence, solution of the given system of equation is $x=3, y=-1$.

14. $0.5x+0.7y=0.74$
$0.3x+0.5y=0.5$

Sol:

The given systems of equations is

$0.5x+0.7y=0.74 \quad (i)$

$0.3x+0.5y=0.5 \quad (ii)$

Multiplying (i) and (ii) by 100, we get

$50x+70y=74 \quad ...(iii)$

$30x+50y=50 \quad ...(iv)$

From (iii), we get

$$50x=74-70y$$

$$\Rightarrow \quad x=\frac{74-70y}{50}$$

Substituting $x=\frac{74-70y}{50}$ in equation (iv), we get

$$30\left(\frac{74-70y}{50}\right)+50y=50$$

$$\Rightarrow \quad \frac{3(74-70y)}{5}+50y=50$$

$$\Rightarrow \quad \frac{222-210y}{5}+50y=50$$

$$\Rightarrow \quad 222-210y+250y=250$$

$$\Rightarrow \quad 40y=250-222$$

$$\Rightarrow \quad 40y=28$$

$$\Rightarrow \quad y=\frac{28}{40}=\frac{14}{20}=\frac{7}{10}=0.7$$

Putting $y=0.7$ in $x=\frac{74-70y}{50}$, we get

$$x = \frac{74 - 70 \times 0 \cdot 7}{50}$$
$$= \frac{74 - 49}{50}$$
$$= \frac{25}{50}$$
$$= \frac{1}{2}$$
$$= 0.5$$

Hence, solution of the given system of equation is $x = 0 \cdot 5, y = 0.7$

15. $$\frac{1}{7x} + \frac{1}{6y} = 3$$
$$\frac{1}{2x} - \frac{1}{3y} = 5$$

Sol:

$$\frac{1}{7x} + \frac{1}{6y} = 3 \qquad ....(1)$$
$$\frac{1}{2x} - \frac{1}{3y} = 5 \qquad ....(2)$$

Multiplying (2) by $\frac{1}{2}$, we get

$$\frac{1}{4x} - \frac{1}{6y} = \frac{5}{2} \qquad ....(3)$$

Solving (1) and (3), we get

$$\frac{1}{7x} + \frac{1}{6y} = 3$$
$$\frac{1}{4x} - \frac{1}{6y} = \frac{5}{2}$$
$$\overline{\frac{1}{7x} + \frac{1}{4x} = 3 + \frac{5}{2}} \qquad \text{(Adding the equations)}$$
$$\Rightarrow \frac{4+7}{28x} = \frac{6+5}{2}$$
$$\Rightarrow \frac{11}{28x} = \frac{11}{2}$$
$$\Rightarrow x = \frac{11 \times 2}{28 \times 11} = \frac{1}{14}$$

When $x = \frac{1}{14}$, we get

$$\frac{1}{7\left(\frac{1}{14}\right)} + \frac{1}{6y} = 3 \qquad \text{(Using (1))}$$

$$\Rightarrow 2 + \frac{1}{6y} = 3$$

$$\Rightarrow \frac{1}{6y} = 3 - 2 = 1$$

$$\Rightarrow y = \frac{1}{6}$$

Thus, the solution of given equation is $x = \frac{1}{14}$ *and* $y = \frac{1}{6}$.

16. $\frac{1}{2x} + \frac{1}{3y} = 2$

$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$

Sol:

Let $\frac{1}{x} = u$ *and* $\frac{1}{y} = v$, the given equations become

$$\frac{u}{2} + \frac{v}{3} = 2$$

$$\Rightarrow \quad \frac{3u + 2v}{6} = 2$$

$$\Rightarrow \quad 3u + 2v = 12 \qquad ....(i)$$

And, $\frac{u}{3} + \frac{v}{2} = \frac{13}{6}$

$$\Rightarrow \frac{2u + 3v}{6} = \frac{13}{6}$$

$$\Rightarrow v = \frac{6}{2} = 3$$

Hence, $x = \frac{1}{u} = \frac{1}{2}$ *and* $y = \frac{1}{v} = \frac{1}{3}$

So, the solution of the given system o equation is $x = \frac{1}{2}, y = \frac{1}{3}$.

17. $\frac{x+y}{xy}=2$

$\frac{x-y}{xy}=6$

Sol:

The given system of equation is

$$\frac{x+y}{xy}=2$$

$$\Rightarrow \quad \frac{x}{xy}+\frac{y}{xy}=2$$

$$\Rightarrow \quad \frac{1}{y}+\frac{1}{x}=2 \qquad .....(i)$$

And, $\frac{x-y}{xy}=6$

$$\Rightarrow \frac{x}{xy}-\frac{y}{xy}=6$$

$$\Rightarrow \frac{1}{y}-\frac{y}{x}=6 \qquad ...(ii)$$

Taking $\frac{1}{y}=v$ and $\frac{1}{x}=u$, the above equations become

$v+u=2 \qquad .....(iii)$

$v-u=6 \qquad ......(iv)$

Adding equation (iii) and equation (iv), we get

$$v+u+v-u=2+6$$

$$\Rightarrow \quad 2v=8$$

$$\Rightarrow \quad v=\frac{8}{2}=4$$

Putting $v=4$ in equation (iii), we get

$$4+u=2$$

$$\Rightarrow \quad u=2-4=-2$$

Hence, $x=\frac{1}{u}=\frac{1}{-2}=\frac{-1}{2}$ *and* $y=\frac{1}{v}=\frac{1}{4}$

So, the solution of the given system of equation is $x=\frac{-1}{2}, y=\frac{1}{4}$

18. $\frac{15}{u}+\frac{2}{v}=17$

Sol:

Let $\frac{1}{u}=x\ and\ \frac{1}{v}=y,$ then, the given system of equations become

$15x+2y=17 \qquad ...(i)$

$x+y=\frac{36}{5} \qquad ...(ii)$

From (i), we get

$2y=17-15x$

$\Rightarrow \quad y=\frac{17-15x}{2}$

Substituting $y=\frac{17-15x}{2}$ in equation (ii), we get

$x+\frac{17-15x}{2}=\frac{36}{5}$

$\Rightarrow \quad \frac{2x+17-15x}{2}=\frac{36}{5}$

$\Rightarrow \quad \frac{-13x+17}{2}=\frac{36}{5}$

$\Rightarrow \quad 5(-13x+17)=36\times 2$

$\Rightarrow \quad -65x+85=72$

$\Rightarrow \quad -65x=72-85$

$\Rightarrow \quad -65x=-13$

$\Rightarrow \quad 65x=\frac{-13}{-65}=\frac{1}{5}$

Putting $x=\frac{1}{5}$ in equation (ii), we get

$\frac{1}{5}+y=\frac{36}{5}$

$\Rightarrow \quad y=\frac{36}{5}-\frac{1}{5}$

$=\frac{36-1}{5}=\frac{35}{5}=7$

Hence, $u=\frac{1}{x}=5\,and\ v=\frac{1}{y}=\frac{1}{7}.$

So, the solution off the given system of equation is $u=5,v=\frac{1}{7}.$

19. $\frac{3}{x} - \frac{1}{y} = -9$
$\frac{2}{x} + \frac{3}{y} = 5$
Sol:
Let $\frac{1}{x} = u$ *and* $\frac{1}{y} = v$, Then, the given system of equations becomes
$3u - v = -9$ ......(i)
$2u + 3v = 5$ ......(ii)
Multiplying equation (i) by 3 an equation (ii) by 1, we get
$9u - 3v = -27$ ....(iii)
$2u + 3v = 5$ ....(iv)
Adding equation (i) and equation (ii), we get
$9u + 2u - 3v + 3v = -27 + 5$
$\Rightarrow \quad 11u = -22$
$\Rightarrow \quad u = \frac{-22}{11} = -2$
Putting $u = -2$ in equation (iv), we get
$2 \times (-2) + 3v = 5$
$\Rightarrow \quad -4 + 3v = 5$
$\Rightarrow \quad 3v = 5 + 4$
$\Rightarrow \quad v = \frac{9}{3} = 3$
Hence, $x = \frac{1}{u} = \frac{1}{-2} = \frac{-1}{2}$ and $y = \frac{1}{v} = \frac{1}{3}$.
So, the solution of the given system of equation is $x = \frac{-1}{2}, y = \frac{1}{3}$.

20. $\frac{2}{x} + \frac{5}{y} = 1$
$\frac{60}{x} + \frac{40}{y} = 19, x \neq 0, y \neq 0$
Sol:
Taking $\frac{1}{x} = u$ *and* $\frac{1}{y} = v$, the given becomes
$2u + 5v = 1$ ......(i)
$60u + 40u = 19$ ......(ii)
Let us eliminate 'u' from equation (i) and (ii), multiplying equation (i) by 60 and equation (ii) by 2, we get

$120u+300v=60 \quad ......(iii)$

$120u+80v=38 \quad ......(iv)$

Subtracting (iv) from (iii), we get

$300v-80v=60-38$

$\Rightarrow \quad 220v=22$

$\Rightarrow \quad v=\frac{22}{220}=\frac{1}{10}$

Putting $v=\frac{1}{10}$ in equation (i), we get

$2u+5\times\frac{1}{10}=1$

$\Rightarrow \quad 2u+\frac{1}{2}=1$

$\Rightarrow \quad 2u=1-\frac{1}{2}$

$\Rightarrow \quad 2u=\frac{2-1}{2}=\frac{1}{2}$

$\Rightarrow \quad 2u=\frac{1}{2}$

$\Rightarrow \quad u=\frac{1}{4}$

Hence, $x=\frac{1}{u}=4$ *and* $y=\frac{1}{v}=10$

So, the solution of the given system of equation is $x=4, y=10.$

21. $\frac{1}{5x}+\frac{1}{6y}=12$

$\frac{1}{3x}-\frac{3}{7y}=8, x\neq 0, y\neq 0$

Sol:

Taking $\frac{1}{x}=u$ *and* $\frac{1}{y}=v,$ the given equations become\

$\frac{u}{5}+\frac{v}{6}=12$

$\Rightarrow \quad \frac{6u+5v}{30}=12$

$\Rightarrow \quad 6u+5v=360 \quad .....(i)$

And, $\frac{u}{3}-\frac{3v}{7}=8$

$\Rightarrow \quad \frac{7u+9v}{21}=8$

$\Rightarrow \quad 7u-9v=168 \qquad ......(ii)$

Let us eliminate 'v' from equation (i) and (ii), Multiplying equation (i) by 9 and equation (ii) by 5, we get

$54u+45v=3240 \qquad ......(iii)$

$35u-45v=840 \qquad ......(iv)$

Adding equation (i) adding equation (ii), we get

$54u+35u=3240+840$

$\Rightarrow \quad 89u=4080$

$\Rightarrow \quad u=\frac{4080}{89}$

Putting $u=\frac{4080}{89}$ in equation (i), we get

$6\times\frac{4080}{89}+5v=360$

$\Rightarrow \quad \frac{24480}{89}+5v=360$

$\Rightarrow \quad 5v=360-\frac{24480}{89}$

$\Rightarrow \quad 5v=\frac{32040-24480}{89}$

$\Rightarrow \quad 5v=\frac{7560}{89}$

$\Rightarrow \quad v=\frac{7560}{5\times 89}$

$\Rightarrow \quad v=\frac{1512}{89}$

Hence, $x=\frac{1}{u}=\frac{89}{4080}$ *and* $y=\frac{1}{v}=\frac{89}{1512}$

So, the solution of the given system of equation is $x=\frac{89}{4080}, y=\frac{89}{1512}$.

22. $$\frac{2}{x}+\frac{3}{y}=\frac{9}{xy}$$
$$\frac{4}{x}+\frac{9}{y}=\frac{21}{xy}, where\ x\neq 0, y\neq 0$$

Sol:

The system of given equation is

$$\frac{2}{x}+\frac{3}{y}=\frac{9}{xy} \qquad .....(i)$$

$$\frac{4}{x}+\frac{9}{y}=\frac{21}{xy}, where\ x\neq 0, y\neq 0 \qquad .....(ii)$$

Multiplying equation (i) adding equation (ii) by xy, we get

$$2y+3x=9 \qquad ....(iii)$$

$$4y+9x=21 \qquad ....(iv)$$

From (iii), we get

$$3x=9-2y$$

$$\Rightarrow \quad x=\frac{9-2y}{3}$$

Substituting $x=\frac{9-2y}{3}$ in equation (iv), we get

$$4x+9\left(\frac{9-2y}{3}\right)=21$$

$$\Rightarrow \quad 4y+3(9-2y)=21$$

$$\Rightarrow \quad 4y+27-6y=21$$

$$\Rightarrow \quad -2y=21-27$$

$$\Rightarrow \quad -2y=-6$$

$$\Rightarrow \quad y=\frac{-6}{-2}=3$$

Putting $y=3$ in $x=\frac{9-2y}{3}$, we get

$$x=\frac{9-2\times 3}{3}$$

$$=\frac{9-6}{3}$$

$$=\frac{3}{3}$$

$$=1$$

Hence, solution of the system of equation is $x=1, y=3$

23. $\frac{6}{x+y} = \frac{7}{x-y} + 3$

$\frac{1}{2(x+y)} = \frac{1}{3(x-y)}$, where x + y ≠ 0 and x – y ≠ 0

Sol:

Let $\frac{1}{x+y} = u \; and \; \frac{1}{x-y} = v.$Then, the given system of equation becomes

$$6u = 7v + 3$$

$$\Rightarrow \quad 6u - 7v = 3 \quad ....(i)$$

And, $\frac{u}{2} = \frac{v}{3}$

$$\Rightarrow \quad 3u = 2v$$

$$\Rightarrow \quad 3u - 2v = 0 \quad ......(ii)$$

Multiplying equation (ii) by 2, and equation (i) by 1, we get

$$6u - 7v = 3 \quad ....(iii)$$

$$6u - 4v = 0 \quad ....(iv)$$

Subtracting equation (iv) from equation (iii), we get

$$-7 + 4v = 3$$

$$\Rightarrow \quad -3v = 3$$

$$\Rightarrow \quad v = -1$$

Putting $v = -1$ in equation (ii), we get

$$3u - 2\times(-1) = 0$$

$$\Rightarrow \quad 3u + 2 = 0$$

$$\Rightarrow \quad 3u = -2$$

$$\Rightarrow \quad u = \frac{-2}{3}$$

Now,

$$u = \frac{-2}{3}$$

$$\Rightarrow \quad \frac{1}{x+2} = \frac{-2}{3}$$

$$\Rightarrow \quad x + y = \frac{-3}{2} \quad ....(v)$$

And, $v = -1$

$$\Rightarrow \quad \frac{1}{x-y} = -1$$

$\Rightarrow \quad x-y=-1 \quad ......(vi)$

Adding equation (v) and equation (vi), we get

$$2x=\frac{-3}{2}-1$$

$$\Rightarrow \quad 2x=\frac{-3-2}{2}$$

$$\Rightarrow \quad 2x=\frac{-5}{2}$$

$$\Rightarrow \quad x=\frac{-5}{4}$$

Putting $x=\frac{-5}{4}$ in equation (vi), we get

$$\frac{-5}{4}-y=-1$$

$$\Rightarrow \quad \frac{-5}{4}+1=y$$

$$\Rightarrow \quad \frac{-5+4}{4}=y$$

$$\Rightarrow \quad \frac{-1}{4}=y$$

$$\Rightarrow \quad y=\frac{-1}{4}$$

Hence, solution of the system of equation is $x=\frac{-5}{4}, y=\frac{-1}{4}$.

24. $$\frac{xy}{x+y}=\frac{6}{5}$$

$$\frac{xy}{y-x}=6$$

Sol:

The given system of equation is

$$\frac{xy}{x+y}=\frac{6}{5}$$

$$\Rightarrow \quad 5xy=6(x+y)$$

$$\Rightarrow \quad 5xy=6x+6y \quad ....(i)$$

And, $\frac{xy}{y-x}=6$

$\Rightarrow \quad xy = 6(y - x)$

$\Rightarrow \quad xy = 6y - 6x \qquad .....(ii)$

Adding equation (i) and equation (ii), we get

$6xy = 6y + 6y$

$\Rightarrow \quad 6xy = 12y$

$\Rightarrow \quad x = \frac{12y}{6y} = 2$

Putting $x = 2$ in equation (i), we get

$5 \times 2 \times y = 6 \times 2 + 6y$

$\Rightarrow \quad 10y = 12 + 6y$

$\Rightarrow \quad 10y - 6y = 12$

$\Rightarrow \quad 4y = 12$

$\Rightarrow \quad y = \frac{12}{4} = 3$

Hence, solution of the given system of equation is $x = 2, y = 3.$

25. $\frac{22}{x+y} + \frac{15}{x-y} = 5$

$\frac{55}{x+y} + \frac{45}{x-y} = 14$

Sol:

Let $\frac{1}{x+y} = u$ *and* $\frac{1}{x-y} = v.$ Then, the given system of equation becomes

$22u + 15v = 5 \qquad ...(i)$

$55u + 45v = 14 \qquad ...(ii)$

Multiplying equation (i) by 3, and equation (ii) by 1, we get

$66u + 45v = 15 \qquad .....(iii)$

$55u + 45v = 14 \qquad ......(iv)$

Subtracting equation (iv) from equation (iii), we get

$66u - 55u = 15 - 4$

$\Rightarrow \quad 11u = 1$

$\Rightarrow \quad u = \frac{1}{11}$

Putting $u = \frac{1}{11}$ in equation (i), we get

$22\times\frac{1}{11}+15v=5$

$\Rightarrow \quad 2+15v=5$

$\Rightarrow \quad 15v=5-2$

$\Rightarrow \quad 15v=3$

$\Rightarrow \quad v=\frac{3}{15}=\frac{1}{5}$

Now, $u=\frac{1}{x+y}$

$\Rightarrow \quad \frac{1}{x+y}=\frac{1}{11}$

$\Rightarrow \quad x+y=11 \qquad ....(v)$

And $\quad v=\frac{1}{x-y}$

$\Rightarrow \quad \frac{1}{x-y}=\frac{1}{5}$

$\Rightarrow \quad x-y=5 \qquad .....(vi)$

Adding equation (v) and equation (vi), we get

$2x=11+5$

$\Rightarrow \quad 2x=16$

$\Rightarrow \quad x=\frac{16}{2}=8$

Putting $x=8$ in equation (v), we get

$8+y=11$

$\Rightarrow \quad y=11-8=3$

Hence, solution of the given system of equation is $x=8, y=3$.

26. $\frac{5}{x+y}-\frac{2}{x-y}=-1$

$\frac{15}{x+y}+\frac{7}{x-y}=10$

Sol:

Let $\frac{1}{x+y}=u$ and $\frac{1}{x-y}=v$. Then, the given system off equations becomes

$5u-2v=-1 \qquad ......(i)$

$15u+7v=10 \qquad .....(ii)$

Multiplying equation (i) by 7, and equation (ii) by 2, we get

$35u - 14v = -7 \quad .....(iii)$

$30u + 14v = 20 \quad ......(iv)$

Adding equation (iii) and equation (iv), we get

$\Rightarrow \quad 35u + 30u = -7 + 20$

$\Rightarrow \quad 65u = 13$

$\Rightarrow \quad u = \frac{13}{65} = \frac{1}{5}$

Putting $u = \frac{1}{5}$ in equation (i), we get

$5 \times \frac{1}{5} - 2v = -1$

$\Rightarrow \quad 1 - 2v = -1$

$\Rightarrow \quad -2v = -1 - 1$

$\Rightarrow \quad -2v = -2$

$\Rightarrow \quad v = \frac{-2}{-2} = 1$

Now, $u = \frac{1}{x+y}$

$\Rightarrow \quad \frac{1}{x+y} = \frac{1}{5}$

$\Rightarrow \quad x + y = 5 \quad ......(v)$

and, $v = \frac{1}{x-y} = 1$

$\Rightarrow \quad x - y = 1 \quad ....(vi)$

Adding equation (v) and equation (vi), we get

$2x = 5 + 1$

$\Rightarrow \quad 2x = 6$

$\Rightarrow \quad x = \frac{6}{2} = 3$

Putting $x = 3$ in equation (v), we get

$3 + y = 5$

$\Rightarrow \quad y = 5 - 3 = 2$

Hence, solution of the given system of equation is $x = 3, y = 2$.

27. $\frac{3}{x+y}+\frac{2}{x-y}=2$
$\frac{9}{x+y}-\frac{4}{x-y}=1$

Sol:

Let $\frac{1}{x+y}=u$ *and* $\frac{1}{x-y}=v.$ Then, the given system of equation becomes

$3u+2v=2 \quad ...(i)$

$9u+4v=1 \quad ...(ii)$

Multiplying equation (i) by 3, and equation (ii) by 1, we get

$6u+4v=4 \quad .....(iii)$

$9u-4v=1 \quad ......(iv)$

Adding equation (iii) and equation (iv), we get

$6u+9u=4+1$

$\Rightarrow \quad 15u=5$

$\Rightarrow \quad u=\frac{5}{15}=\frac{1}{3}$

Putting $u=\frac{1}{3}$ in equation (i), we get

$3\times\frac{1}{3}+2v=2$

$\Rightarrow \quad 1+2v=2$

$\Rightarrow \quad 2v=2-1$

$\Rightarrow \quad v=\frac{1}{2}$

Now, $u=\frac{1}{x+y}$

$\Rightarrow \quad \frac{1}{x+y}=\frac{1}{3}$

$\Rightarrow \quad x+y=3 \quad ......(v)$

And, $v=\frac{1}{x-y}$

$\Rightarrow \quad \frac{1}{x-y}=\frac{1}{2}$

$\Rightarrow \quad x-y=2 \quad ......(vi)$

Adding equation (v) and equation (vi), we get

$$2x = 3 + 2$$

$\Rightarrow \quad x = \frac{5}{2}$

Putting $x = \frac{5}{2}$ in equation (v), we get

$$\frac{5}{2} + y = 3$$

$\Rightarrow \quad y = 3 - \frac{5}{2}$

$\Rightarrow \quad y = \frac{6-5}{2} = \frac{1}{2}$

Hence, solution of the given system of equation is $x = \frac{5}{2}, y = \frac{1}{2}$.

28. $\frac{1}{2(x+2y)} + \frac{5}{3(3x-2y)} = \frac{-3}{2}$ $\quad$ $\frac{5}{4(x+2y)} - \frac{3}{5(3x-2y)} = \frac{61}{60}$

Sol:

Let $\frac{1}{x+2y} = u \; and \; \frac{1}{3x-2y} = v.$

Then, the given system of equation becomes

$$\frac{u}{2} + \frac{5v}{3} = \frac{-3}{2}$$

$\Rightarrow \quad \frac{3u+10v}{6} = \frac{-3}{2}$

$\Rightarrow \quad 3u + 10v = \frac{-3\times 6}{2}$

$\Rightarrow \quad 3u + 10v = -9 \qquad .....(i)$

And, $\frac{5u}{4} - \frac{3v}{5} = \frac{61}{60}$

$\Rightarrow \quad \frac{25u-12v}{20} = \frac{61}{60}$

$\Rightarrow \quad 25u - 12v = \frac{61}{3} \qquad ....(ii)$

Multiplying equation (i) by 12, and equation (ii) by 10, we get

$36u + 120v = -108 \qquad ......(iii)$

$250u - 120v = \frac{610}{3} \qquad ......(iv)$

Adding equation (iii) and equation (iv), we get

$$36u+250u=\frac{610}{3}-108$$

$$\Rightarrow \quad 286u=\frac{610-324}{3}$$

$$\Rightarrow \quad 286u=\frac{286}{3}$$

$$\Rightarrow \quad u=\frac{1}{3}$$

Putting $u=\frac{1}{3}$ in equation (i), we get

$$3\times\frac{1}{3}+10v=-9$$

$$\Rightarrow \quad 1+10v=-9$$

$$\Rightarrow \quad 10v=-9-1$$

$$\Rightarrow \quad v=\frac{-10}{10}=-1$$

Now, $u=\frac{1}{x+2y}$

$$\Rightarrow \quad \frac{1}{x+y}=\frac{1}{3}$$

$$\Rightarrow \quad x+2y=3 \qquad ....(v)$$

And, $v=\frac{1}{3x-2y}$

$$\Rightarrow \quad \frac{1}{3x-2y}=-1$$

$$\Rightarrow \quad 3x-2y=-1 \qquad ....(vi)$$

Putting $x=\frac{1}{2}$ in equation (v), we get

$$\frac{1}{2}+2y=3$$

$$\Rightarrow \quad 2y=3-\frac{1}{2}$$

$$\Rightarrow \quad 2y=\frac{6-1}{2}$$

$$\Rightarrow \quad y=\frac{5}{4}$$

Hence, solution of the given system of equations is $x=\frac{1}{2}, y=\frac{5}{4}$.

29. $\frac{5}{x+1}-\frac{2}{y-1}=\frac{1}{2}$

$\frac{10}{x+1}+\frac{2}{y-1}=\frac{5}{2}, where\ x\ \neq -1\ and\ y\ \neq 1$

Sol:

Let $\frac{1}{x+1}=u\ and\ \frac{1}{y-1}=v.$

Then, the given system of equations becomes

$\Rightarrow\quad 5u-2v=\frac{1}{2}\qquad ....(i)$

$\Rightarrow\quad 10u+2y=\frac{5}{2}\qquad ....(ii)$

Adding equation (i) equation (ii), we get

$5u+10u=\frac{1}{2}+\frac{5}{2}$

$\Rightarrow\quad 15u=\frac{1+5}{2}$

$\Rightarrow\quad 15u=\frac{6}{2}=3$

$\Rightarrow\quad u=\frac{3}{15}=\frac{1}{5}$

Putting $u=\frac{1}{5}$ in equation (i), we get

$5\times\frac{1}{5}-2v=\frac{1}{2}$

$\Rightarrow\quad 1-2v=\frac{1}{2}$

$\Rightarrow\quad -2v=\frac{1}{2}-1$

$\Rightarrow\quad -2v=\frac{1-2}{2}$

$\Rightarrow\quad -2v=\frac{-1}{2}$

$\Rightarrow\quad v=\frac{-1}{-4}=\frac{1}{4}$

Now, $u=\frac{1}{x+1}$

$\Rightarrow \quad \frac{1}{x+1} = \frac{1}{5}$

$\Rightarrow \quad x+1 = 5$

$\Rightarrow \quad x = 5-1 = 4$

And, $v = \frac{1}{y-1}$

$\Rightarrow \quad \frac{1}{y-1} = \frac{1}{4}$

$\Rightarrow \quad y-1 = 4$

$\Rightarrow \quad y = 4+1 = 5$

Hence, solution of the give system of equation is $x = 4, y = 5.$

30. $x + y = 5xy$
$3x + 2y = 13xy$

Sol:

The give system of equation is

$x + y = 5xy \qquad ......(i)$

$3x + 2y = 13xy \qquad .....(ii)$

Multiplying equation (i) by 2 and equation (ii) by , we get

$2x + 2y = 10xy \qquad ....(iii)$

$3x + 2y = 13xy \qquad ....(iv)$

Subtracting equation (iii) from equation (iv), we get

$3x - 2x = 13xy - 10xy$

$\Rightarrow \quad x = 3xy$

$\Rightarrow \quad \frac{x}{3x} = y$

$\Rightarrow \quad y = \frac{1}{3}$

Putting $y = \frac{1}{3}$ in equation (i), we get

$$x+y=5\times x\times\frac{1}{3}$$

$$x+\frac{1}{3}=\frac{5x}{3}$$

$$\Rightarrow \quad \frac{1}{3}=\frac{5x}{3}-x$$

$$\Rightarrow \quad \frac{1}{3}=\frac{5x-3x}{3}$$

$$\Rightarrow \quad 1=2x$$

$$\Rightarrow \quad 2x=1$$

$$\Rightarrow \quad x=\frac{1}{2}$$

Hence, solution of the given system of equations is $x=\frac{1}{2}, y=\frac{1}{3}$.

31. $x+y=2xy$

$$\frac{x-y}{xy}=6 \ x\neq 0,\ y\neq 0$$

Sol:

The system of the given equation is

$$x+y=2xy \qquad ....(i)$$

And, $\frac{x-y}{xy}=6$

$$x-y=6xy \qquad ....(ii)$$

Adding equation (i) and equation (ii), we get

$$2x=2xy+6xy$$

$$\Rightarrow \quad 2x=8xy$$

$$\Rightarrow \quad \frac{2x}{8x}=y$$

$$\Rightarrow \quad y=\frac{1}{4}$$

Putting $y=\frac{1}{4}$ in equation (i), we get

$x+\frac{1}{4}=2x\times\frac{1}{4}$

$\Rightarrow \quad x+\frac{1}{4}=\frac{x}{2}$

$\Rightarrow \quad x-\frac{x}{2}=\frac{-1}{4}$

$\Rightarrow \quad \frac{2x-x}{2}=\frac{-1}{4}$

$\Rightarrow \quad x=\frac{-2}{4}=\frac{-1}{2}$

Hence, solution of the given system of equation is $x=\frac{-1}{2}, y=\frac{1}{4}$,

32. $2(3u-v)=5uv$

$2(u+3v)=5uv$

Sol:

The system of the given equation is

$2(3u-v)=5uv$

$\Rightarrow \quad 6u-2v=5uv \quad ....(i)$

And, $2(u+3v)=5uv$

$\Rightarrow \quad 2u+6v=5uv \quad ....(ii)$

Multiplying equation (i) by 3 and equation (ii) by 1, we get

$18u-6v=15uv \quad ....(iii)$

$2u+6v=5uv \quad ....(iv)$

Adding equation (iii) and equation (iv), we get

$18u+2u=15uv+5uv$

$\Rightarrow \quad 20u=20uv$

$\Rightarrow \quad \frac{20u}{20u}=v$

$\Rightarrow \quad v=1$

Putting $v=1$ in equation (i), we get

$6u-2\times1=5u\times1$

$\Rightarrow \quad 6u-2=5u$

$\Rightarrow \quad 6u-5u=2$

$\Rightarrow \quad u=2$

Hence, solution of the given system of equation is $u=2, v=1$.

33. $\frac{2}{3x+2y}+\frac{3}{3x-2y}=\frac{17}{5}$

$\frac{5}{3x+2y}+\frac{1}{3x-2y}=2$

Sol:

Let $\frac{1}{3x+2y}=u$ and $\frac{1}{3x-2y}=v.$ Then, the given system of equation becomes

$$2u+3v=\frac{17}{5} \qquad ....(i)$$

$$5u+v=2 \qquad ....(ii)$$

Multiplying equation (ii) by 3, we get

$$15u-2u=6-\frac{17}{5}$$

$\Rightarrow \quad 13u=\frac{30-17}{5}$

$\Rightarrow \quad 13u=\frac{13}{5}$

$\Rightarrow \quad u=\frac{13}{5\times 13}=\frac{1}{5}$

Putting $u=\frac{1}{5}$ in equation (ii), we get

$$5\times\frac{1}{5}+v=2$$

$\Rightarrow \quad 1+v=2$

$\Rightarrow \quad v=2-1$

$\Rightarrow \quad v=1$

Now, $u=\frac{1}{3x+2y}$

$\Rightarrow \quad \frac{1}{3x+2y}=\frac{1}{5}$

$\Rightarrow \quad 3x+2y=5 \qquad .....(iv)$

And, $v=\frac{1}{3x+2y}$

$\Rightarrow \quad \frac{1}{3x-2y}=1$

$\Rightarrow \quad 3x-2y=1 \qquad ....(v)$

Adding equation (iv) and (v), we get

$6x = 1 + 5$

$\Rightarrow \quad 6x = 6$

$\Rightarrow \quad x = 1$

Putting $x = 1$ in equation (v), we get

$3 \times 1 + 2y = 5$

$\Rightarrow \quad 2y = 5 - 3$

$\Rightarrow \quad 2y = 2$

$\Rightarrow \quad y = \frac{2}{2} = 1$

Hence, solution of the given system of equation is $x = 1, y = 1$.

34. $\frac{4}{x} + 3y = 14$

$\frac{3}{x} - 4y = 23$

Sol:

$\frac{4}{x} + 3y = 14$

$\frac{3}{x} - 4y = 23$

Let $\frac{1}{x} = p$

The given equations reduce to:

$4p + 3y = 14$

$\Rightarrow \quad 4p + 3y - 14 = 0 \quad .....(1)$

$3p - 4y = 23$

$\Rightarrow \quad 3p - 4y - 23 = 0 \quad .....(2)$

Using cross-multiplication method, we obtain

$$\frac{p}{-69 - 56} = \frac{y}{-42 - (-92)} = \frac{1}{-16 - 9}$$

$$\frac{p}{-125} = \frac{y}{50} = \frac{-1}{25}$$

$$\frac{p}{-125} = \frac{-1}{25}, \frac{y}{50} = \frac{-1}{25}$$

$p = 5, y = -2$

$\therefore p = \frac{1}{x} = 5$

$x = \frac{1}{5}$

35. $$99x+101y=499$$
$$101x+99y=501$$

Sol:

The given system of equation is

$$99x+101y=499 \quad .....(i)$$

$$101x+99y=501 \quad ....(ii)$$

Adding equation (i) and equation (ii), we get

$$99x+101x+101y+99y=499+501$$

$\Rightarrow \quad 200x+200y=1000$

$\Rightarrow \quad 200(x+y)=1000$

$\Rightarrow \quad x+y=\frac{1000}{200}=5$

$\Rightarrow \quad x+y=5 \quad .....(iii)$

Subtracting equation (i) by equation (ii), we get

$$101x-99x+99y-101y=501-499$$

$\Rightarrow \quad 2x-2y=2$

$\Rightarrow \quad 2(x-y)=2$

$\Rightarrow \quad x-y=\frac{2}{2}$

$\Rightarrow \quad x-y=1 \quad ....(iv)$

Adding equation (iii) and equation (iv), we get

$$2x=5+1$$

$\Rightarrow \quad x=\frac{6}{2}=3$

Putting $x=3$ in equation (iii), we get

$$3+y=5$$

$\Rightarrow \quad y=5-3=2$

Hence, solution of the given system of equation is $x=3, y=2$.

36. $$23x-29y=98$$
$$29x-23y=110$$

Sol:

The given system of equation is

$$23x-29y=98 \quad .....(i)$$

$$29x-23y=110 \quad .....(ii)$$

Adding equation (i) and equation (ii), we get

$$23x+29x-29y-23y=98+110$$

$\Rightarrow$ $52x-52y=208$

$\Rightarrow$ $52(x-y)=208$

$\Rightarrow$ $x-y=\frac{208}{52}=4$

$\Rightarrow$ $x-y-4$ ....(iii)

Subtracting equation (i) by equation (ii), we get

$$29x-23x-23y+29y=110-98$$

$\Rightarrow$ $6x+6y=12$

$\Rightarrow$ $6(x+y)=12$

$\Rightarrow$ $x+y=\frac{12}{6}=2$

$\Rightarrow$ $x+y=2$ ......(iv)

Adding equation (iii) and equation (iv), we get

$$2x=2+4=6$$

$\Rightarrow$ $x=\frac{6}{2}=3$

Putting $x=3$ in equation (iv), we get

$$3+y=2$$

$\Rightarrow$ $y=2-3=-1$

Hence, solution of the given system of equation is $x=3, y=-1$.

37. $x-y+z=4$
$x-2y-2z=9$
$2x+y+3z=1$

Sol:

We have,

$x-y+z=4$ ......(i)

$x-2y-2z=9$ ......(ii)

$2x+y+3z=1$ .....(iii)

From equation (i), we get

$$z=4-x+y$$

$\Rightarrow$ $z=-x+y+4$

Subtracting the value of z in equation (ii), we get

$x-2y-2(-x+y+4)=9$

$\Rightarrow \quad x-2y+2x-2y-8=8$

$\Rightarrow \quad 3x-4y=9+8$

$\Rightarrow \quad 3x-4y=17 \quad \quad .....(iv)$

Subtracting the value of z in equation (iii), we get

$2x+y+3(-x+y+4)=1$

$\Rightarrow \quad 2x+y+3x+3y+12=1$

$\Rightarrow \quad -x+4y=1-12$

$\Rightarrow \quad -x+4y=-11 \quad \quad ......(v)$

Adding equations (iv) and (v), we get

$3x-x-4y+4y=17-11$

$\Rightarrow \quad 2x=6$

$\Rightarrow \quad x=\frac{6}{2}=3$

Putting $x=3$ in equation (iv), we get

$3\times3-4y=17$

$\Rightarrow \quad 9-4y=17$

$\Rightarrow \quad -4y=17-9$

$\Rightarrow \quad -4y=8$

$\Rightarrow \quad y=\frac{8}{-4}=-2$

Putting $x=3$ and $y=-2$ in $z=-x+y+4$, we get

$z=-3-2+4$

$\Rightarrow \quad z=-5+4$

$\Rightarrow \quad z=-1$

Hence, solution of the giving system of equation is $x=3, y=-2, z=-1.$

38. $x-y+z=4$
$x+y+z=2$
$2x+y-3z=0$

Sol:

We have,

$x-y+z=4 \quad \quad ......(i)$

$x+y+z=2 \quad \quad ......(ii)$

$2x+y-3z=0 \quad \quad ......(iii)$

From equation (i), we get

$z = 4 - x + y$

$\Rightarrow \quad z = -x + y + 4$

Substituting $z = -x + y + 4$ in equation (ii), we get

$x + y + (-x + y + 4) = 2$

$\Rightarrow \quad x + y - x + y + 4 = 2$

$\Rightarrow \quad 2y + 4 = 2$

$\Rightarrow \quad 2y = 2 - 4 = -2$

$\Rightarrow \quad 2y = -2$

$\Rightarrow \quad y = \frac{-2}{2} = -1$

Substituting the value of z in equation (iii), we get

$2x + y - 3(-x + y + 4) = 0$

$\Rightarrow \quad 2x + y + 3x - 3y - 12 = 0$

$\Rightarrow \quad 5x - 2y - 12 = 0$

$\Rightarrow \quad 5x - 2y = 12 \qquad ....(iv)$

Putting $y = -1$ in equation (iv), we get

$5x - 2 \times (-1) = 12$

$\Rightarrow \quad 5x + 2 = 12$

$\Rightarrow \quad 5x = 12 - 2 = 10$

$\Rightarrow \quad x = \frac{10}{5} = 2$

Putting $x = 2$ and $y = -1$ in $z = -x + y + 4$, we get

$z = -2 + (-1) + 4$

$= -2 - 1 + 4$

$= -3 + 4$

$= 1$

Hence, solution of the giving system of equation is $x = 2, y = -1, z = 1$.

39. $\frac{44}{x+y} + \frac{30}{x-y} = 4$

$\frac{55}{x+y} + \frac{40}{x-y} = 13$

Sol:

Let $\frac{1}{x+y} = u$ *and* $\frac{1}{x-y} = v$.

Then, the system of the given equations becomes

$44u+30v=10 \qquad .....(i)$

$55u+40v=13 \qquad .....(ii)$

Multiplying equation (i) by 4 and equation (ii) by 3, we get

$176u+120v=40 \qquad ....(iii)$

$165u+120v=39 \qquad ....(iv)$

Subtracting equation (iv) by equation (iii), we get

$176-165u=40-39$

$\Rightarrow \quad 11u=1$

$\Rightarrow \quad u=\frac{1}{11}$

Putting $u=\frac{1}{11}$ in equation (i), we get

$44\times\frac{1}{11}+30v=10$

$4+30v=10$

$\Rightarrow \quad 30v=10-4$

$\Rightarrow \quad 30v=6$

$\Rightarrow \quad v=\frac{6}{30}=\frac{1}{5}$

Now, $u=\frac{1}{x+y}$

$\Rightarrow \quad \frac{1}{x+y}=\frac{1}{11}$

$\Rightarrow \quad x+y=11 \qquad ....(v)$

Adding equation (v) and (vi), we get

$2x=11+5$

$\Rightarrow \quad 2x=16$

$\Rightarrow \quad x=\frac{16}{2}=8$

Putting $x=8$ in equation (v),, we get

$8+y=11$

$\Rightarrow \quad y=11-8-3$

Hence, solution of the given system of equations is $x=8, y=3.$

40. $\frac{4}{x}+15y=21$

$\frac{3}{x}+4y=5$

Sol:

The given system of equation is

$\frac{4}{x}+15y=21 \quad ......(i)$

$\frac{3}{x}+4y=5 \quad ......(ii)$

Multiplying equation (i) by 3 and equation (ii) by 4, we get

$\frac{12}{x}+15y=21 \quad ......(iii)$

$\frac{12}{x}+16y=20 \quad ......(iv)$

Subtracting equation (iii) from equation (iv), we get

$\frac{12}{x}-\frac{12}{x}+16y-15y=20-21$

$\Rightarrow \quad y=-1$

Putting $y=-1$ in equation (i), we get

$\frac{4}{x}+5\times(-1)=7$

$\Rightarrow \quad \frac{4}{x}-5=7$

$\Rightarrow \quad \frac{4}{x}=7+5$

$\Rightarrow \quad \frac{4}{x}=12$

$\Rightarrow \quad 4=12x$

$\Rightarrow \quad \frac{4}{12}=x$

$\Rightarrow \quad x=\frac{4}{12}$

$\Rightarrow \quad x=\frac{1}{3}$

Hence, solution of the given system of equation is $x=\frac{1}{3}, y=-1.$

41. $2\left(\frac{1}{x}\right)+3\left(\frac{1}{y}\right)=13$

$5\left(\frac{1}{x}\right)-4\left(\frac{1}{y}\right)=-2$

Sol:

Let us write the given pair of equation as

$2\left(\frac{1}{x}\right)+3\left(\frac{1}{y}\right)=13 \qquad (1)$

$5\left(\frac{1}{x}\right)-4\left(\frac{1}{y}\right)=-2 \qquad (2)$

These equation are not in the form $ax+by+c=0.$ However, if we substitute

$\frac{1}{x}=p \text{ and } \frac{1}{y}=q$ in equations (1) and (2), we get

$2p+3q=13$

$5p-4q=-2$

So, we have expressed the equations as a pair of linear equations. Now, you can use any method to solve these equations, and get $p=2, q=3$

You know that $p=\frac{1}{x} \text{ and } q=\frac{1}{y}.$

Substitute the values of p and q to get

$\frac{1}{x}=2, \text{ i.e., } x=\frac{1}{2} \text{ and } \frac{1}{y}=3 \text{ i.e., } y=\frac{1}{3}.$

42. $\frac{5}{x-1}+\frac{1}{y-2}=2$

Sol:

x = 4, y = 5

Detailed answer not given in website

43. $\frac{10}{x+y}+\frac{2}{x-y}=4$

$\frac{15}{x+y}-\frac{5}{x-y}=-2$

Sol:

$\frac{10}{x+y}+\frac{2}{x-y}=4$

$$\frac{15}{x+y}-\frac{5}{x-y}=-2$$

Let $\frac{1}{x+y}=p$ *and* $\frac{1}{x-y}=q$

The given equations reduce to:

$$10p+2q=4$$

$$\Rightarrow \quad 10p+2q-4=0 \quad .....(1)$$

$$15p-5q=-2$$

$$\Rightarrow \quad 15p-5q+2=0 \quad .....(2)$$

Using cross-multiplication method, we obtain:

$$\frac{p}{4-20}=\frac{q}{-60-20}=\frac{1}{-50-30}$$

$$\frac{p}{-16}=\frac{q}{-80}=\frac{1}{-80}$$

$$\frac{p}{-16}=\frac{1}{-80} \text{ and } \frac{q}{-80}=\frac{1}{-80}$$

$$p=\frac{1}{5} \text{ and } q=1$$

$$p=\frac{1}{x+y}=\frac{1}{5} \text{ and } q=\frac{1}{x-y}=1$$

$$x+y=5 \quad ......(3)$$

$$x-y=1 \quad ......(4)$$

Adding equation (3) and (4), we obtain:

$$2x=6$$

$$x=3$$

Substituting the value of x in equation (3), we obtain:

$$y=2$$

$$\therefore x=3, y=2$$

44. $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$

$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8}$

Sol:

Let us put $\frac{1}{x-1}=p$ *and* $\frac{1}{y-2}=q.$ Then the given equations

$5\left(\frac{1}{x-1}\right)+\frac{1}{y-2}=2$ ......(1)

$6\left(\frac{1}{x-1}\right)-3\left(\frac{1}{y-2}\right)=1$ ......(2)

Can be written as: $5p+q=2$ ....(3)

$6p-3q=1$ ......(4)

Equations (3) and (4) from a pair of linear equations in the general form. Now, you can use any method to solve these equations. We get $p=\frac{1}{3}$ *and* $q=\frac{1}{3}$.

Substituting $\frac{1}{x-1}$ for p, we have

$\frac{1}{x-1}=\frac{1}{3},$

i.e., $x-1=3$, *i.e.*, $x=4$.

Similarly, substituting $\frac{1}{y-2}$ for q, we get

$\frac{1}{y-2}=\frac{1}{3}$

i.e., $x-1=3$, *i.e.*, $x=4$

Similarly, substituting $\frac{1}{y-2}$ for q, we get

$\frac{1}{y-2}=\frac{1}{3}$

i.e., $3=y-2$, *i.e.*, $y=5$

Hence, $x=4, y=5$ is the required solution of the given pair of equations.

45. $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

Sol:

$\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

Let $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$

The given equations reduce to:

$2p + 3q = 2 \quad .....(1)$

$4p - 9q = -1 \quad .....(2)$

Multiplying equation (1) by (3), we obtain:

$6p + 9q = 6 \quad .....(3)$

Adding equation (2) and (3), we obtain:

$10p = 5$

$p = \frac{1}{2}$

Putting the value of p in equation (1), we obtain:

$2 \times \frac{1}{2} + 3q = 2$

$3q = 1$

$q = \frac{1}{3}$

$\therefore p = \frac{1}{\sqrt{x}} = \frac{1}{2}$

$\sqrt{x} = 2$

$x = 4$

$q = \frac{1}{\sqrt{y}} = \frac{1}{3}$

$\sqrt{y} = 3$

$y = 9$

$\therefore x = 4, y = 9$

46. $$\frac{7x - 2y}{xy} = 5$$
$$\frac{8x + 7y}{xy} = 15$$

Sol:

$$\frac{7x-2y}{xy}=5$$

$$\Rightarrow \quad \frac{7}{y}-\frac{2}{x}=5 \quad ...(1)$$

$$\frac{8x+7y}{xy}=15$$

$$\Rightarrow \quad \frac{8}{y}+\frac{7}{x}=15 \quad .....(2)$$

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$

The given equations reduce to:

$$-2p+7q=5$$

$$\Rightarrow \quad -2p+7q-5=0 \quad ....(3)$$

$$7p+8q=15$$

$$\Rightarrow \quad 7p+8q-15=0 \quad ....(4)$$

Using cross multiplication method, we obtain:

$$\frac{p}{-105-(-40)}=\frac{q}{-35-30}=\frac{1}{-16-49}$$

$$\frac{p}{-65}=\frac{1}{-65},\frac{q}{-65}=\frac{1}{-65}$$

$$p=1, q=1$$

$$p=\frac{1}{x}=1, q=\frac{1}{y}=1$$

$$x=1, y=1$$

47. $152x-378y=-74$

$-378x+152y=-604$

Sol:

$$152x-378y=-74 \quad ....(1)$$

$$-378x+152y=-604 \quad ....(2)$$

Adding the equations (1) and (2), we obtain:

$$-226x-226y=-678$$

$$\Rightarrow x+y=3 \quad .....(3)$$

Subtracting the equation (2) from equation (1), we obtain

$$530x-530y=530$$

$\Rightarrow x - y = 1 \qquad .....(4)$

Adding equations (3) and (4), we obtain:

$2x = 4$

$x = 2$

Substituting the value of x in equation (3), we obtain:

$y = 1$

Exercise 3.4

Solve each of the following systems of equations by the method of cross-multiplication:

1. $x + 2y + 1 = 0$
   $2x - 3y - 12 = 0$

Sol:

The given system of equation is

$x + 2y + 1 = 0$

$2x - 3y - 12 = 0$

Here,

$a_1 = 1, b_1 = 2, c_1 = 1$

$a_2 = 2, b_2 = -3 \text{ and } c_2 = -12$

By cross-multiplication, we get

$$\Rightarrow \frac{x}{2\times(-12) - 1\times(-3)} = \frac{-y}{1\times(-12) - 1\times 2} = \frac{1}{1\times(-3) - 2\times 2}$$

$$\Rightarrow \frac{x}{-24+3} = \frac{-y}{-12-2} = \frac{1}{-3-4}$$

$$\Rightarrow \frac{x}{-21} = \frac{-y}{-14} = \frac{1}{-7}$$

Now,

$$\frac{x}{-21} = \frac{1}{-7}$$

$$\Rightarrow \quad x = \frac{-21}{-7} = 3$$

And,

$$\frac{-y}{-14}=\frac{1}{-7}$$

$$\Rightarrow \quad \frac{y}{14}=\frac{-1}{7}$$

$$\Rightarrow \quad y=\frac{-14}{7}=-2$$

Hence, the solution of the given system of equations is $x=3, y=-2.$

2. $3x+2y+25=0$
$2x+y+10=0$

Sol:

The given system of equation is

$3x+2y+25=0$

$2x+y+10=0$

Here,

$a_1=3, b_1=2, c_1=25$

$a_2=2, b_2=1$ *and* $c_2=10$

By cross-multiplication, we have

$$\Rightarrow \frac{x}{2\times10-25\times1}=\frac{-y}{3\times10-25\times2}=\frac{1}{3\times1-2\times2}$$

$$\Rightarrow \frac{x}{20-25}=\frac{-y}{30-50}=\frac{1}{3-4}$$

$$\Rightarrow \frac{x}{-5}=\frac{-y}{-20}=\frac{1}{-1}$$

Now, $\frac{x}{-5}=\frac{1}{-1}$

$$\Rightarrow x=\frac{-5}{-1}=5$$

And,

$$\frac{-y}{-20}=\frac{1}{-1}$$

$$\Rightarrow \quad \frac{y}{20}=1$$

$$\Rightarrow \quad y=-20$$

Hence, $x=5, y=-20$ is the solution of the given system of equations.

3. $2x+y-35=0$
$3x+4y-65=0$

Sol:

The given system of equations may be written as

$2x+y-35=0$

$3x+4y-65=0$

Here,

$a_1=2, b_1=1, c_1=-35$

$a_2=3, b_2=4 \text{ and } c_2=-65$

By cross multiplication, we have

$$\Rightarrow \frac{x}{1\times(-65)-(-35)\times 4}=\frac{-y}{2\times(-65)-(-35)\times 3}=\frac{1}{2\times 4-1\times 3}$$

$$\Rightarrow \frac{x}{-65+140}=\frac{-y}{-130+105}=\frac{1}{8-3}$$

$$\Rightarrow \frac{x}{75}=\frac{-y}{-25}=\frac{1}{5}$$

$$\Rightarrow \frac{x}{75}=\frac{y}{25}=\frac{1}{5}$$

Now,

$$\frac{y}{25}=\frac{1}{5}$$

$$\Rightarrow \quad y=\frac{25}{5}=5$$

Hence, $x=15, y=5$ is the solution of the given system of equations.

4. $2x-y-6=0$
$x-y-2=0$

Sol:

The given system of equations may be written as

$2x-y-6=0$

$x-y-2=0$

Here,

$a_1=2, b_1=-1, c_1=-6$

$a_2=1, b_2=-1 \text{ and } c_2=-2$

By cross multiplication, we get

$$\Rightarrow \frac{x}{(-1)\times(-2)-(-6)\times(-1)} = \frac{-y}{2\times(-2)-(-6)\times 1} = \frac{1}{2\times(-1)-(-1)\times 1}$$

$$\Rightarrow \frac{x}{2-6} = \frac{-y}{-4+6} = \frac{1}{-2+1}$$

$$\Rightarrow \frac{x}{-4} = \frac{-y}{2} = \frac{1}{-1}$$

$$\Rightarrow \frac{x}{-4} = \frac{-y}{2} = -1$$

Now,

$$\frac{x}{-4} = -1$$

$$\Rightarrow \quad x = (-4)\times(-1) = 4$$

And,

$$\frac{-y}{2} = -1$$

$$\Rightarrow \quad -y = (-1)\times 2$$

$$\Rightarrow \quad -y = -2$$

$$\Rightarrow \quad y = 2$$

Hence, $x = 4, y = 2$ is the solution of the given system of the equations.

5. $\frac{x+y}{xy} = 2$

$\frac{x-y}{xy} = 6$

Sol:

The given system of equations is

$$\frac{x+y}{xy} = 2$$

$$\Rightarrow \quad \frac{x}{xy} + \frac{y}{xy} = 2$$

$$\Rightarrow \quad \frac{1}{y} + \frac{1}{x} = 2$$

$$\Rightarrow \quad \frac{1}{x} + \frac{1}{y} = 2 \qquad ......(i)$$

And,

$$\frac{x-y}{xy}=6$$

$$\Rightarrow \quad \frac{x}{xy}-\frac{y}{xy}=6$$

$$\Rightarrow \quad \frac{1}{y}-\frac{1}{x}=6$$

$$\Rightarrow \quad \frac{1}{x}-\frac{1}{y}=6 \qquad \text{......}(ii)$$

Taking $u=\frac{1}{x}$ *and* $v=\frac{1}{y}$, we get

$u+v=2\Rightarrow u+v-2=0 \qquad \text{.....}(iii)$

And, $u-v=-6\Rightarrow u-v+6=0 \qquad \text{.....}(iv)$

Here,

$a_1=1, b_1=1, c_1=-2$

$a_2=1, b_2=-1$ *and* $c_2=6$

By cross multiplication

$$\Rightarrow \frac{u}{1\times6-(-2)\times(-1)}=\frac{v}{1\times6-(-2)\times1}=\frac{1}{1\times(-1)-1\times1}$$

$$\Rightarrow \frac{u}{6-2}=\frac{-v}{6+2}=\frac{1}{-1-1}$$

$$\Rightarrow \frac{u}{4}=\frac{-v}{8}=\frac{1}{-2}$$

Now, $\frac{u}{4}=\frac{1}{-2}$

$$\Rightarrow u=\frac{4}{-2}=-2$$

And, $\frac{-v}{8}=\frac{1}{-2}$

$$\Rightarrow -v=\frac{8}{-2}=-4$$

$$\Rightarrow -v=-4$$

$$\Rightarrow v=4$$

Now, $x=\frac{1}{u}=\frac{-1}{2}$ *and* $y=\frac{1}{v}=\frac{1}{4}$

Hence, $x=\frac{-1}{2}, y=\frac{1}{4}$ is the solution of the given system of equations.

6. $ax+by=a-b$
$bx-ay=a+b$

Sol:

The given system of equations is

$ax+by=a-b \quad ....(i)$

$bx-ay=a+b \quad ....(ii)$

Here,

$a_1=a, b_1=b, c_1=b-a$

$a_2=b, b_2=-a \text{ and } c_2=-a-b$

By cross multiplication, we get

$$\Rightarrow \frac{x}{(-a-b)\times(b)-(b-a)\times(-a)}=\frac{-y}{(-a-b)\times(a)-(b-a)\times(-b)}=\frac{1}{-a\times a-b\times b}$$

$$\Rightarrow \frac{x}{-ab-b^2+ab-a^2}=\frac{-y}{-a^2-ab-b^2+ab}=\frac{1}{-a^2-b^2}$$

$$\Rightarrow \frac{x}{-b^2-a^2}=\frac{-y}{-a^2-b^2}=\frac{1}{-a^2-b^2}$$

Now,

$$\frac{x}{-b^2-a^2}=\frac{1}{-a^2-b^2}$$

$$\Rightarrow \quad x=\frac{-b^2-a^2}{-a^2-b^2}$$

$$=\frac{-(b^2+a^2)}{(a^2+b^2)}$$

$$=\frac{(a^2+b^2)}{(a^2+b^2)}$$

$$\Rightarrow \quad x=1$$

And,

$$\frac{-y}{-a^2-b^2}=\frac{1}{-a^2-b^2}$$

$$\Rightarrow \quad -y=\frac{-(a^2+b^2)}{-(a^2+b^2)}$$

$$\Rightarrow \quad -y=1$$

$$\Rightarrow \quad y=-1$$

Hence, $x=1, y=-1$ is the solution of the given system of the equations.

7. $x + ay - b = 0$
$ax - by - c = 0$

Sol:

The given system of equations may be written as

$x + ay - b = 0$

$ax - by - c = 0$

Here,

$a_1 = 1, b_1 = a, c_1 = -b$

$a_2 = a, b_2 = -b \ and \ c_2 = -c$

By cross multiplication, we get

$$\Rightarrow \frac{x}{(a)\times(-c)-(-b)\times(-b)} = \frac{-y}{1\times(-c)-(-b)\times a} = \frac{1}{1\times(-b)-a\times a}$$

$$\Rightarrow \frac{x}{-ac-b^2} = \frac{-y}{-c+ab} = \frac{1}{-b-a^2}$$

Now,

$$\frac{x}{-ac-b^2} = \frac{1}{-b-a^2}$$

$$\Rightarrow \quad x = \frac{-ac-b^2}{-b-a^2}$$

$$\Rightarrow \quad x = \frac{-(b^2+ac)}{-(a^2+b)}$$

$$= \frac{b^2+ac}{a^2+b}$$

And

$$\frac{-y}{-c+ab} = \frac{1}{-b-a^2}$$

$$\Rightarrow \quad -y = \frac{ab-c}{-(a^2+b)}$$

$$\Rightarrow \quad y = \frac{ab-c}{a^2+b}$$

Hence, $x = \dfrac{ac+b^2}{a^2+b}, y = \dfrac{ab-c}{a^2+b}$ is the solution of the given system of the equations.

8. $ax + by = a^2$
$bx + ay = b^2$

Sol:

The system of the given equations may be written as

$ax+by-a^2=0$

$bx+ay-b^2=0$

Here,

$a_1=a, b_1=b, c_1=-a^2$

$a_2=b, b_2=a \text{ and } c_2=-b^2$

By cross multiplication, we get

$$\Rightarrow \frac{x}{b\times\left(-b^2\right)-\left(-a^2\right)\times a}=\frac{-y}{a\times\left(-b^2\right)-\left(-a^2\right)\times b}=\frac{1}{a\times a-b\times b}$$

$$\Rightarrow \frac{x}{-b^3+a^3}=\frac{-y}{-ab^2+a^2b}=\frac{1}{a^2-b^2}$$

Now,

$$\frac{x}{-b^3+a^3}=\frac{1}{a^2-b^2}$$

$$\Rightarrow \quad x=\frac{a^3-b^3}{a^2-b^2}$$

$$=\frac{\left(a-b\right)\left(a^2+ab+b^2\right)}{\left(a-b\right)\left(a+b\right)}$$

$$=\frac{a^2+ab+b^2}{a+b}$$

And,

$$\frac{-y}{-ab^2+a^2b}=\frac{1}{a^2-b^2}$$

$$\Rightarrow \quad -y=\frac{a^2b-ab^2}{a^2-b^2}$$

$$\Rightarrow \quad y=\frac{ab^2-a^2b}{a^2-b^2}$$

$$=\frac{ab\left(b-a\right)}{\left(a-b\right)\left(a+b\right)}$$

$$=\frac{-ab\left(a-b\right)}{\left(a-b\right)\left(a+b\right)}$$

$$=\frac{-ab}{a+b}$$

Hence, $x=\dfrac{a^2+ab+b^2}{a+b}, y=\dfrac{-ab}{a+b}$ is the solution of the given system of the equations.

9. $\frac{x}{a}+\frac{y}{b}=2$

$ax-by=a^2-b^2$

Sol:

The system of the given equations may be written as

$\frac{1}{a}x\times+\frac{1}{b}\times y-2=0$

$ax-by+b^2-a^2=0$

Here,

$a_1=\frac{1}{a},b_1=\frac{1}{b},c_1=-2$

$a_2=a,b_2=-b \text{ and } c_2=b^2-a^2$

By cross multiplication, we get

$$\Rightarrow \frac{x}{\frac{1}{b}\times\left(b^2-a^2\right)-(-2)\times(-b)}=\frac{-y}{\frac{1}{a}\times\left(b^2-a^2\right)-(-2)\times a}=\frac{1}{\frac{-b\times1}{a}-\frac{a\times1}{b}}$$

$$\Rightarrow \frac{x}{\frac{b^2-a^2}{b}-2b}=\frac{-y}{\frac{b^2-a^2}{b}+2b}=\frac{1}{\frac{-b}{a}-\frac{a}{b}}$$

$$\Rightarrow \frac{x}{\frac{b^2-a^2-2b^2}{b}}=\frac{-y}{\frac{b^2-a^2+2b^2}{a}}=\frac{1}{\frac{-b^2-a^2}{ab}}$$

$$\Rightarrow \frac{x}{\frac{-a^2-b^2}{b}}=\frac{-y}{\frac{b^2+a^2}{a}}=\frac{1}{\frac{-b^2-a^2}{ab}}$$

Now,

$$\frac{x}{\frac{-a^2-b^2}{b}}=\frac{1}{\frac{-b^2-a^2}{ab}}$$

$$\Rightarrow \quad x=\frac{-a^2-b^2}{b}\times\frac{ab}{-b^2-a^2}$$

And,

$$\frac{-y}{\frac{b^2+a^2}{a}}=\frac{1}{\frac{-b^2-a^2}{ab}}$$

$$\Rightarrow \quad -y=\frac{b^2+a^2}{a}\times\frac{ab}{-b^2-a^2}$$

$\Rightarrow \quad -y = \dfrac{\left(b^2 + a^2\right) \times b}{-\left(b^2 + a^2\right)}$

$\Rightarrow \quad y = b$

Hence, $x = a, y = b$ is the solution of the given system of the equations.

10. $\frac{x}{a} + \frac{y}{b} = a + b$

Sol:

The given system of equation may be written as

$\frac{1}{a} x \times + \frac{1}{b} \times y - (a + b) = 0$

$\frac{1}{a^2} x \times + \frac{1}{b^2} \times y - 2 = 0$

Here,

$a_1 = \frac{1}{a}, b_2 = \frac{1}{b}, c_1 = -(a + b)$

$a_2 = \frac{1}{a^2}, b_2 = \frac{1}{b^2}, \textit{ and } c_2 = -2$

By cross multiplication, we get

$$\Rightarrow \frac{x}{\frac{1}{b} \times (-2) - \frac{1}{b^2} x - (a + b)} = \frac{-y}{\frac{1}{a} \times -2 - \frac{1}{a^2} x - (a + b)} = \frac{1}{\frac{1}{a} \times \frac{1}{b^2} - \frac{1}{a^2} \times \frac{1}{b}}$$

$$\Rightarrow \frac{x}{-\frac{2}{b} + \frac{a}{b^2} + \frac{1}{b}} = \frac{-y}{-\frac{2}{a} + \frac{1}{a} + \frac{b}{a^2}} = \frac{1}{-\frac{1}{ab^2} - \frac{1}{a^2 b}}$$

$$\Rightarrow \frac{x}{\frac{a}{b^2} - \frac{1}{b}} = \frac{-y}{-\frac{1}{a} + \frac{b}{a^2}} = \frac{1}{\frac{1}{ab^2} - \frac{1}{a^2 b}}$$

$$\Rightarrow \frac{x}{\frac{a-b}{b^2}} = \frac{y}{\frac{a-b}{a^2}} = \frac{1}{\frac{a-b}{a^2 b^2}}$$

$$\Rightarrow x = \frac{a-b}{b^2} \times \frac{1}{\frac{a-b}{a^2 b^2}} = a^2 \textit{ and } y = \frac{a-b}{a^2} \times \frac{1}{\frac{a-b}{a^2 b^2}} = b^2$$

Hence, $x = a^2, y = b^2$ is the solution of the given system of the equtaions.

11. $\frac{x}{a} = \frac{y}{b}$

$ax + by = a^2 + b^2$

Sol:

$$\frac{x}{a}=\frac{y}{b}$$

$$ax+by=a^2+b^2$$

Here $a_1=\frac{1}{a}, b_1=\frac{-1}{b}, c_1=0$

$$a_2=a, b_2=b, c_2=-\left(a^2+b^2\right)$$

By cross multiplication, we get

$$\frac{x}{-\frac{1}{b}\left(-\left(a^2+b^2\right)\right)-b(0)}=\frac{-y}{\frac{1}{a}\left(-\left(a^2+b^2\right)\right)-a(0)}=\frac{1}{\frac{1}{a}(b)-a\times\left(\frac{-1}{b}\right)}$$

$$\frac{x}{\frac{a^2+b^2}{b}}=\frac{y}{\frac{a^2+b^2}{a}}=\frac{1}{\frac{b}{a}+\frac{a}{b}}$$

$$x=\frac{\frac{a^2+b^2}{b}}{\frac{b}{a}+\frac{a}{b}}=\frac{\frac{a^2+b^2}{b}}{\frac{b^2+a^2}{ab}}=a$$

$$y=\frac{\frac{a^2+b^2}{a}}{\frac{b}{a}+\frac{a}{b}}=\frac{\frac{a^2+b^2}{b}}{\frac{b^2+a^2}{ab}}=b$$

Solution is (a, b)

12. $\frac{5}{x+y}-\frac{2}{x-y}=-1$

$\frac{15}{x+y}+\frac{7}{x-y}=10, where\ x\ \neq 0\ and\ y\ \neq 0$

Sol:

Let $\frac{1}{x+y}=u$ and $\frac{1}{x-y}=v$. Then, the given system of equations becomes

$$5u-2v=-1$$

$$15u+7v=10$$

Here

$$a_1=5, b_1=-2, c_1=1$$

$$a_2=15, b_2=7 \text{ and } c_2=-10$$

By cross multiplication, we get

$$\Rightarrow \frac{u}{(-2)\times(-10)-1\times 7}=\frac{u}{5\times(-10)-1\times 15}=\frac{1}{5\times 7-(-2)\times 15}$$

$$\Rightarrow \frac{u}{20-7} = \frac{-v}{-50-15} = \frac{1}{35+30}$$

$$\Rightarrow \frac{u}{13} = \frac{-v}{-65} = \frac{1}{65}$$

$$\Rightarrow \frac{u}{13} = \frac{v}{65} = \frac{1}{65}$$

Now,

$$\frac{u}{13} = \frac{1}{65}$$

$$\Rightarrow \quad u = \frac{13}{65} = \frac{1}{5}$$

And,

$$\frac{v}{65} = \frac{1}{65}$$

$$\Rightarrow \quad v = \frac{65}{65} = 1$$

Now,

$$u = \frac{1}{x+y}$$

$$\Rightarrow \quad \frac{1}{x+y} = \frac{1}{5} \quad ......(i)$$

And,

$$v = \frac{1}{x-y}$$

$$\Rightarrow \quad \frac{1}{x-y} = 1$$

$$\Rightarrow \quad x - y = 1 \quad ......(ii)$$

Adding equation (i) and (ii), we get

$$2x = 5 + 1$$

$$\Rightarrow \quad 2x = 6$$

$$\Rightarrow \quad x = \frac{6}{2} = 3$$

13. $\frac{2}{x} + \frac{3}{y} = 13$

$\frac{5}{x} - \frac{4}{y} = -2$, where $x \neq 0$ *and* $y \neq 0$

Sol:

The given system of equation is

$$\frac{2}{x}+\frac{3}{y}=13$$

$$\frac{5}{x}-\frac{4}{y}=-2, \text{where } x \neq 0 \text{ and } y \neq 0$$

Let $\frac{1}{x}=u$ *and* $\frac{1}{y}=v$, Then, the given system of equations becomes

$$2u+3v=13$$

$$5u-4v=-2$$

Here,

$$a_1=2, b_1=3, c_1=-13$$

$$a_2=5, b_2=-4 \text{ and } c_2=2$$

By cross multiplication, we have

$$\Rightarrow \frac{u}{3\times 2-(-13)\times(-4)}=\frac{-v}{2\times 2-(-13)\times 5}=\frac{1}{2\times(-4)-3\times 5}$$

$$\Rightarrow \frac{u}{6-52}=\frac{-v}{4+65}=\frac{1}{-8-15}$$

$$\Rightarrow \frac{u}{-46}=\frac{-v}{69}=\frac{1}{-23}$$

Now,

$$\frac{u}{-46}=\frac{1}{-23}$$

$$\Rightarrow \quad u=\frac{-46}{-23}=2$$

And

$$\frac{-v}{69}=\frac{1}{-23}$$

$$\Rightarrow \quad v=\frac{-69}{-23}=3$$

Now,

$$x=\frac{1}{u}=\frac{1}{2}$$

And,

$$y=\frac{1}{v}=\frac{1}{3}$$

Hence, $x=\frac{1}{2}, y=\frac{1}{3}$ is the solution of the given system of equations.

14. $ax+by=\frac{a+b}{2}$

$3x+5y=4$

Sol:

The given system of equation is

$ax+by=\frac{a+b}{2}$ ......(i)

$3x+5y=4$ ......(ii)

From (i), we get

$$2(ax+by)=a+b$$

$\Rightarrow 2ax+2by-(a+b)=0$ .....(iii)

From (ii), we get

$3x+5y-4=0$

Here,

$a_1=2a, b_1=2b, c_1=-(a+b)$

$a_2=3, b_2=5, c_2=-4$

By cross multiplication, we have

$$\Rightarrow \frac{x}{2b\times(-4)-[-(a+b)]\times 5}=\frac{-y}{2a\times(-4)-[-(a+b)]\times 3}=\frac{1}{2a\times 5-2b\times 3}$$

$$\Rightarrow \frac{x}{-8b+5(a+b)}=\frac{-y}{-8a+3(a+b)}=\frac{1}{10a-6b}$$

$$\Rightarrow \frac{x}{-8b+5a+5b}=\frac{-y}{-8a+3a+3b}=\frac{1}{10a-6b}$$

$$\Rightarrow \frac{x}{5a-3b}=\frac{-y}{-5a+3b}=\frac{1}{10a-6b}$$

Now,

$$\frac{x}{5a-3b}=\frac{-y}{-5a+3b}=\frac{1}{10a-6b}$$

$$\Rightarrow \quad x=\frac{5a-3b}{10a-6b}=\frac{5a-3b}{2(5a-3b)}=\frac{1}{2}$$

And,

$$\frac{-y}{-5a+3b}=\frac{1}{10a-6b}$$

$$\Rightarrow \quad -y=\frac{-5a+3b}{2(5a-3b)}$$

$$\Rightarrow \quad y=\frac{-(-5a+3b)}{2(5a-3b)}$$

$$=\frac{5a-3b}{2(5a-3b)}$$

$$\Rightarrow \quad y=\frac{1}{2}$$

Hence, $x=\frac{1}{2}, y=\frac{1}{2}$ is the solution of the given system of equations.

15. $2ax+3by=a+2b$
$3ax+2by=2a+b$

Sol:

The given system of equations is

$$2ax+3by=a+2b \qquad ....(i)$$

$$3ax+2by=2a+b \qquad ...(ii)$$

Here,

$a_1=2a, b_1=3b, c_1=-(a+2b)$

$a_2=3a, b_2=2b, c_2=-(2a+b)$

By cross multiplication we have

$$\Rightarrow \frac{x}{-3b\times(2a+b)-[-(a+2b)]\times 2b}=\frac{-y}{-2a\times(2a+b)-[-(a+2b)]\times 3a}=\frac{1}{2a\times 2b-3b\times 3a}$$

$$\Rightarrow \frac{x}{-3b+(2a+b)+2b(a+2b)}=\frac{-y}{-2a(2a+b)+3a(a+2b)}=\frac{1}{4ab-9ab}$$

$$\Rightarrow \frac{x}{-6ab-3b^2+2ab+4b^2}=\frac{-y}{-4a^2-2ab+3a^2+6ab}=\frac{1}{4ab-9ab}$$

$$\Rightarrow \frac{x}{-4ab+b^2}=\frac{-y}{-a^2+4ab}=\frac{1}{-5ab}$$

Now,

$$\frac{x}{-4ab+b^2}=\frac{1}{-5ab}$$

$$\Rightarrow x=\frac{-4ab+b^2}{-5ab}$$

$$=\frac{-b(4a-b)}{-5ab}$$

$$=\frac{4a-b}{5a}$$

And, $\frac{-y}{-a^2+4ab}=\frac{1}{-5ab}$

$\Rightarrow \quad -y=\frac{-a^2+4ab}{-5ab}$

$\Rightarrow \quad -y=\frac{-a(a-4b)}{-5ab}$

$\Rightarrow \quad -y=\frac{a-4b}{5b}$

$\Rightarrow \quad y=\frac{4b-a}{5b}$

Hence, $x=\frac{4a-b}{5a}, y=\frac{4b-a}{5b}$ is the solution of the given system of equation.

16. $5ax+6by=28$
$3ax+4by-18=0$

Sol:

The given system of equation is

$5ax+6by=28$

$\Rightarrow \quad 5ax+6by-28=0 \quad .....(i)$

and, $3ax+4by-18=0$

$\Rightarrow \quad 3ax+4by-18=0 \quad .....(ii)$

Here,

$a_1=5a, b_1=6b, c_1=-28$

$a_2=3a, b_2=4b$ *and* $c_2=-18$

By cross multiplication we have

$\Rightarrow \frac{x}{6b\times(-18)-(-28)\times 4b}=\frac{-y}{5a\times(-18)-(-28)\times 3a}=\frac{1}{5a\times 4b-6b\times 3a}$

$\Rightarrow \frac{x}{-108b+112b}=\frac{-y}{-90a+84a}=\frac{1}{20ab-18ab}$

$\Rightarrow \frac{x}{4b}=\frac{-y}{-6a}=\frac{1}{2ab}$

Now,

$\frac{x}{4b}=\frac{1}{2ab}$

$\Rightarrow \quad x=\frac{5b-2a}{10ab}$

And,

$$\frac{-y}{-6a} = \frac{1}{2ab}$$

$$\Rightarrow \quad y = \frac{6a}{2ab} = \frac{3}{b}$$

Hence, $x = \frac{2}{a}, y = \frac{3}{b}$ is the solution of the given system of equations.

17. $(a+2b)x + (2a-b)y = 2$
$(a-2b)x + (2a+b)y = 3$

Sol:

The given system of equations may be written as

$$(a+2b)x + (2a-b)y - 2 = 0$$

$$(a-2b)x + (2a+b)y - 3 = 0$$

Here,

$a_1 = a+2b, b_1 = 2a-b, c_1 = -2$

$a_2 = a-2b, b_2 = 2a+b$ *and* $c_2 = -3$

By cross multiplication, we have

$$\Rightarrow \frac{x}{-3(2a-b)-(-2)(2a+b)} = \frac{-y}{3(a+2b)-(-2)(a-2b)} = \frac{1}{(a+2b)(2a+b)-(2a-b)(a-2b)}$$

$$\Rightarrow \frac{x}{-6a+3b+4a+2b} = \frac{-y}{-3a-6b+2a-4b} = \frac{1}{2a^2+ab+4ab+2b^2-(2a^2-4ab-ab+2b^2)}$$

$$\Rightarrow \frac{x}{-2a+5b} = \frac{-y}{-a-10b} = \frac{1}{2a^2+ab+4ab+2b^2-(2a^2-4ab-ab+2b^2)}$$

$$\Rightarrow \frac{x}{-2a+5b} = \frac{-y}{-(a+10b)} = \frac{1}{10ab}$$

$$\Rightarrow \frac{x}{-2a+5b} = \frac{y}{a+10b} = \frac{1}{10ab}$$

Now,

$$\frac{x}{-2a+5b} = \frac{1}{10ab}$$

$$\Rightarrow y = \frac{a+10b}{10ab}$$

And,

$$\frac{y}{a+10b} = \frac{1}{10ab}$$

$$\Rightarrow \quad y = \frac{a+10b}{10ab}$$

Hence, $x=\frac{5b-2a}{10ab}, y=\frac{a+10b}{10ab}$ is the solution of the given system of equations.

18. $x\left(a-b+\frac{ab}{a-b}\right)=y\left(a+b-\frac{ab}{a+b}\right)$

$x+y=2a^2$

Sol:

The given system of equation is

$$x\left(a-b+\frac{ab}{a-b}\right)=y\left(a+b-\frac{ab}{a+b}\right) \quad ......(i)$$

$$x+y=2a^2 \quad ......(ii)$$

From equation (i), we get

$$x\left(a-b+\frac{ab}{a-b}\right)-y\left(a+b-\frac{ab}{a+b}\right)=0$$

$$\Rightarrow \quad x\left(\frac{(a-b)^2+ab}{a-b}\right)-y\left(\frac{(a+b)^2-ab}{a+b}\right)=0$$

$$\Rightarrow \quad x\left(\frac{a^2+b^2-2ab+ab}{a-b}\right)-y\left(\frac{a^2+b^2+2ab-ab}{a+b}\right)=0$$

$$\Rightarrow \quad x\left(\frac{a^2+b^2-ab}{a-b}\right)-y\left(\frac{a^2+b^2+ab}{a+b}\right)=0 \quad .....(iii)$$

From equation (ii), we get

$x+y-2a^2=0$

Here,

$a_1=\frac{a^2+b^2-ab}{a-b}, b_1=-\left(\frac{a^2+b^2+ab}{a+b}\right), c_1=0$

$a_2=1, b_2=1$ *and* $c_2=-2a^2$

By cross multiplication, we get

$$\Rightarrow \frac{x}{(-2a^2)\left[-\left(\frac{a^2+b^2+ab}{a+b}\right)\right]-0\times 1}=\frac{-y}{(-2a^2)\left[-\left(\frac{a^2+b^2-ab}{a-b}\right)\right]-0\times 1}=\frac{1}{\frac{a^2+b^2-ab}{a-b}\left[-\frac{(a^2+b^2+ab)}{a+b}\right]}$$

$$\Rightarrow \frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)}=\frac{y}{(2a^2)\left(\frac{a^2+b^2-ab}{a-b}\right)}=\frac{1}{\frac{a^2+b^2-ab}{a-b}+\frac{a^2+b^2-ab}{a+b}}$$

$$\Rightarrow \frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)} = \frac{y}{\left(2a^2\right)\left(\frac{a^2+b^2-ab}{a-b}\right)} = \frac{1}{\frac{(a+b)\left(a^2+b^2-ab\right)+(a-b)\left(a^2+b^2+ab\right)}{(a-b)(a+b)}}$$

$$\Rightarrow \frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)} = \frac{y}{2a^2\left(\frac{a^2+b^2-ab}{a-b}\right)} = \frac{1}{\frac{a^3+b^3+a^3-b^3}{(a-b)(a+b)}}$$

$$\Rightarrow \frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)} = \frac{y}{2a^2\left(\frac{a^2+b^2-ab}{a-b}\right)} = \frac{1}{\frac{2a^3}{(a-b)(a+b)}}$$

Now,

$$\frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)} = \frac{1}{\frac{2a^3}{(a-b)(a+b)}}$$

$$\Rightarrow \quad x = \frac{2a^2\left(a^2+b^2+ab\right)}{a+b} \times \frac{(a-b)(a+b)}{2a^3}$$

$$= \frac{(a-b)\left(a^2+b^2+ab\right)}{a}$$

$$= \frac{a^3-b^3}{a} \qquad \left[\because a^3-b^3 = (a-b)\left(a^2+b^2+ab\right)\right]$$

And,

$$\frac{y}{2a^2\left(\frac{a^2+b^2-ab}{a-b}\right)} = \frac{1}{\frac{2a^3}{(a-b)(a+b)}}$$

$$\Rightarrow \quad y = \frac{2a^2\left(a^2+b^2-ab\right)}{a-b} \times \frac{(a-b)(a+b)}{2a^3}$$

$$= \frac{(a+b)\left(a^2+b^2-ab\right)}{a}$$

$$= \frac{a^3+b^3}{a} \qquad \left[\because a^3+b^3 - (a-b)\left(a^2+b^2-ab\right)\right]$$

Hence, $x = \frac{a^3-b^3}{a}, y = \frac{a^2+b^2}{a}$ is the solution of the given system of equaatiions.

The given system of equation id

$$x\left(a-b+\frac{ab}{a-b}\right) = y\left(a+b-\frac{ab}{a+b}\right) \qquad \ldots\text{(i)}$$

$$x+y = 2a^2 \qquad \ldots\text{(ii)}$$

From equation (i), w get

$$x\left(a-b+\frac{ab}{a-b}\right)-y\left(a+b+\frac{ab}{a+b}\right)=0$$

$$\Rightarrow \quad x\left(\frac{(a-b)^2+ab}{a-b}\right)-y\left(\frac{(a+b)^2-ab}{a+b}\right)=0$$

$$\Rightarrow \quad x\left(\frac{a^2+b^2-2ab+ab}{a-b}\right)-y\left(\frac{a^2+b^2+2ab-ab}{a+b}\right)=0$$

$$\Rightarrow \quad x\left(\frac{a^2+b^2-ab}{a-b}\right)-y\left(\frac{a^2+b^2-ab}{a+b}\right)=0 \qquad \text{.....(iii)}$$

From equation (ii), we get

$$x+y-2a^2=0 \qquad \text{.....(iv)}$$

Here,

$$a_1=\frac{a^2+b^2-ab}{a-b}, b_1=-\left(\frac{a^2+b^2+ab}{a+b}\right), c_1=0$$

$$a_2=1, b_2=1 \text{ and } c_2=-2a^2$$

By cross multiplication we get

$$\Rightarrow \frac{x}{\left(-2a^2\right)\left[-\left(\frac{a^2+b^2+ab}{a+b}\right)\right]-0\times 1}=\frac{-y}{\left(-2a^2\right)\left(\frac{a^2+b^2-ab}{a-b}\right)-0\times 1}=\frac{1}{\frac{a^2+b^2-ab}{a-b}-\left[-\frac{a^2+b^2-ab}{a-b}\right]}$$

$$\Rightarrow \frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)}=\frac{y}{\left(2a^2\right)\left(\frac{a^2+b^2-ab}{a-b}\right)}=\frac{1}{\frac{a^2+b^2-ab}{a-b}+\frac{a^2+b^2+ab}{a+b}}$$

$$\Rightarrow \frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)}=\frac{y}{\left(2a^2\right)\left(\frac{a^2+b^2-ab}{a-b}\right)}=\frac{1}{\frac{a^2+b^2-ab}{a-b}+\frac{a^2+b^2+ab}{a+b}}$$

$$\Rightarrow \frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)}=\frac{y}{2a^2\left(\frac{a^2+b^2-ab}{a-b}\right)}=\frac{1}{\frac{a^3+b^3+a^3-b^3}{(a-b)(a+b)}}$$

$$\Rightarrow \frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)}=\frac{y}{2a^2\left(\frac{a^2+b^2-ab}{a-b}\right)}=\frac{1}{\frac{2a^3}{(a-b)(a+b)}}$$

Now,

$$\frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)} - \frac{1}{\frac{2a^3}{(a-b)(a+b)}}$$

$$\Rightarrow x = \frac{2a^2\left(a^2+b^2+ab\right)}{a+b} \times \frac{(a-b)(a+b)}{2a^3}$$

$$= \frac{(a-b)\left(a^2+b^2+ab\right)}{a}$$

$$= \frac{a^3-b^3}{a} \qquad \left[\because a^2-b^2 = (a-b)\left(a^2+b^2+ab\right)\right]$$

And,

$$\frac{y}{2a^2\left(\frac{a^2+b^2-ab}{a-b}\right)} = \frac{1}{\frac{2a^3}{(a-b)(a+b)}}$$

$$\Rightarrow y = \frac{2a^2\left(a^2+b^2-ab\right)}{a-b} \times \frac{(a-b)(a+b)}{2a^3}$$

$$= \frac{(a+b)\left(a^2+b^2-ab\right)}{a}$$

$$= \frac{a^3+b^3}{a} \qquad \left[\because a^3+b^3 - (a+b)\left(a^2+b^2-ab\right)\right]$$

Hence, $x = \frac{a^2-b^2}{a}, y = \frac{a^3+b^3}{a}$ is the solution of the given system of equation.

19. $bx + cy = a + b$

$$ax\left(\frac{1}{a-b} - \frac{1}{a+b}\right) + cy\left(\frac{1}{b-a} - \frac{1}{b+a}\right) = \frac{2a}{a+b}$$

Sol:

The given system of equation is

$$bx + cy = a + b \qquad ....(i)$$

$$ax\left(\frac{1}{a-b} - \frac{1}{a+b}\right) + cy\left(\frac{1}{b-a} - \frac{1}{b+a}\right) = \frac{2a}{a+b} \qquad .....(ii)$$

From equation (ii), we get

$$bx + cy - (a+b) = 0 \qquad ....(iii)$$

From equation (ii), we get

$$ax\left[\frac{a+b-(a-b)}{(a-b)(a+b)}\right] + cy\left(\frac{b+a-(b-a)}{(b-a)(b+a)}\right) - \frac{2a}{a+b} = 0$$

$$\Rightarrow \quad ax\left[\frac{a+b-a+b}{(a-b)(a+b)}\right]+cy\left(\frac{b+a-b+a}{(b-a)(b+a)}\right)-\frac{2a}{a+b}=0$$

$$\Rightarrow \quad ax\left[\frac{2b}{(a-b)(a+b)}\right]+cy\left(\frac{2a}{(b-a)(b+a)}\right)-\frac{2a}{a+b}=0$$

$$\Rightarrow \quad x\left[\frac{2ab}{(a-b)(a+b)}\right]+y\left(\frac{2ac}{-(a-b)(a+b)}\right)-\frac{2a}{a+b}=0$$

$$\Rightarrow \quad x\left[\frac{2ab}{(a-b)(a+b)}\right]+y\left(\frac{2ac}{(a-b)(a+b)}\right)-\frac{2a}{a+b}=0$$

$$\Rightarrow \quad \frac{1}{a+b}\left[\frac{2abx}{a-b}-\frac{2acy}{a-b}-2a\right]=0$$

$$\Rightarrow \quad \frac{2abx}{a-b}-\frac{2acy}{a-b}-2a=0$$

$$\Rightarrow \quad \frac{2abx-2acy-2a(a-b)}{a-b}=0$$

$$\Rightarrow \quad 2abx-2acy-2a(a-b)=0 \qquad .....(iv)$$

From equation (i) and equation (ii), we get

$a_1=b, b_1=c, c_1=-(a+b)$

$a_2=2ab, b_2=-2ac \text{ and } c_2=-2a(a-b)$

By cross multiplication, we get

$$\Rightarrow \frac{x}{-2ac(a-b)-[-(a+b)][-2ac]}=\frac{-y}{-2ab(a-b)-[-(a+b)][2ab]}=\frac{1}{-2abc-2abc}$$

$$\Rightarrow \frac{x}{-2a^2c+2abc-\left[2a^2c+2abc\right]}=\frac{-y}{-2a^2b+2ab^2+\left[2a^2b+2ab^2\right]}=\frac{1}{-4abc}$$

$$\Rightarrow \frac{x}{-2a^2c+2abc-2a^2c-2abc}=\frac{-y}{-2a^2b+2ab^2+2a^2b-2ab^2}=\frac{-1}{4abc}$$

$$\Rightarrow \frac{x}{-4a^2c}=\frac{-y}{4ab^2}=\frac{-1}{4abc}$$

Now,

$$\frac{x}{-4a^2c}=\frac{-1}{4abc}$$

$$\Rightarrow \quad x=\frac{4a^2c}{4abc}=\frac{a}{b}$$

And,

$$\frac{-y}{4ab^2}=\frac{-1}{4abc}$$

$$\Rightarrow \quad y=\frac{4ab^2}{4abc}=\frac{b}{c}$$

Hence, $x=\frac{a}{b}, y=\frac{b}{c}$ is the solution of the given system of the equations.

20. $(a-b)x+(a+b)y=2a^2-2b^2$

$(a+b)(x+y)=4ab$

Sol:

The given system of equation is

$$(a-b)x+(a+b)y=2a^2-2b^2 \quad \ldots\ldots(i)$$

$$(a+b)(x+y)=4ab \quad \ldots\ldots(ii)$$

From equation (i), we get

$$(a-b)x+(a+b)y-\left(2a^2-2b^2\right)=0$$

$$\Rightarrow \quad (a-b)x+(a-b)y-2\left(a^2-b^2\right)=0 \quad \ldots\ldots(iii)$$

From equation (ii), we get

$$(a+b)x+(a+b)y-4ab=0 \quad \ldots\ldots(iv)$$

Here,

$$a_1=a-b, b_1=a+b, c_1=-2\left(a^2-b^2\right)$$

$$a_2=a+b, b_2=a+b \text{ and } c_2=-4ab$$

By cross multiplication, we get

$$\Rightarrow \frac{x}{-4ab(a+b)+2\left(a^2-b^2\right)(a+b)}=\frac{-y}{-4ab(a-b)+2\left(a^2-b^2\right)(a+b)}=\frac{1}{(a-b)(a+b)-(a+b)(a+b)}$$

$$\Rightarrow \frac{x}{2(a+b)\left[-2ab+a^2-b^2\right]}=\frac{-y}{-4ab(a-b)+2\left[(a-b)(a+b)\right](a+b)}=\frac{1}{(a+b)\left[(a-b)-(a+b)\right]}$$

$$\Rightarrow \frac{x}{2(a+b)\left(a^2-b^2-2ab\right)}=\frac{-y}{2(a-b)\left[-2ab+(a+b)(a+b)\right]}=\frac{1}{(a+b)\left[a-b-a-b\right]}$$

$$\Rightarrow \frac{x}{2(a+b)\left(a^2-b^2-2ab\right)}=\frac{-y}{2(a-b)\left[-2ab+\left(a^2+b^2+2ab\right)\right]}=\frac{1}{(a+b)(-2b)}$$

$$\Rightarrow \frac{x}{2(a+b)\left(a^2-b^2-2ab\right)}=\frac{-y}{2(a-b)\left(a^2+b^2\right)}=\frac{1}{-2b(a+b)}$$

Now,

$$\frac{x}{2(a+b)(a^2-b^2-2ab)}=\frac{1}{-2b(a+b)}$$

$$\Rightarrow x=\frac{2(a+b)(a^2-b^2-2ab)}{-2b(a+b)}$$

$$\Rightarrow x=\frac{a^2-b^2-2ab}{-b}$$

$$\Rightarrow x=\frac{-a^2+b^2+2ab}{b}$$

$$=\frac{2ab-a^2+b^2}{b}$$

Now,

$$\frac{-y}{2(a-b)(a^2+b^2)}=\frac{1}{-2ab(a+b)}$$

$$\Rightarrow \quad -y=\frac{2(a-b)(a^2+b^2)}{-2b(a+b)}$$

$$\Rightarrow \quad y=\frac{(a-b)(a^2+b^2)}{b(a+b)}$$

Hence, $x=\frac{2ab-a^2+b^2}{b}, y=\frac{(a-b)(a^2+b^2)}{b(a+b)}$ is the solution of the given system of equations.

$$\frac{-y}{-a^2d^2+b^2c^2}=\frac{1}{a^4-b^4}$$

$$\Rightarrow \quad -y=\frac{-a^2d^2+b^2c^2}{a^4-b^4}$$

$$\Rightarrow \quad y=\frac{a^2d^2-b^2c^2}{a^4-b^4}$$

21. $a^2x+b^2y=c^2$

$b^2x+a^2y=d^2$

Sol:

The given system of equations may be written as

$a^2x+b^2y-c^2=0$

$b^2x+a^2y-d^2=0$

Here,

$a_1 = a^2, b_1 = b^2, c_1 = -c^2$

$a_2 = b^2, b_2 = a^2 \ and \ c_2 = -d^2$

By cross multiplication, we have

$$\Rightarrow \frac{x}{-b^2d^2 + a^2c^2} = \frac{-y}{-a^2d^2 + b^2c^2} = \frac{1}{a^4 - b^4}$$

Now,

$$\frac{x}{-b^2d^2 + a^2c^2} = \frac{1}{a^4 - b^4}$$

$$\Rightarrow \quad x = \frac{a^2c^2 - b^2d^2}{a^4 - b^4}$$

And,

$$\frac{-y}{-a^2d^2 + b^2c^2} = \frac{1}{a^4 - b^4}$$

$$\Rightarrow \quad -y = \frac{-a^2d^2 + b^2c^2}{a^4 - b^4}$$

$$\Rightarrow \quad y = \frac{a^2d^2 - b^2c^2}{a^4 - b^4}$$

Hence, $x = \frac{a^2c^2 - b^2d^2}{a^4 - b^4}, y \frac{a^2d^2 - b^2c^2}{a^4 - b^4}$ is the solution of the given system of the equations.

22. $\frac{57}{x+y} + \frac{6}{x-y} = 5$

$\frac{38}{x+y} + \frac{21}{x-y} = 9$

Sol:

Let $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v.$ Then, the given system of equations become

$$57u + 6v = 5 \Rightarrow 57u + 6v - 5 = 0$$

$$38u + 21v = 9 \Rightarrow 38u + 21v - 9 = 0$$

Here,

$a_1 = 57, b_1 = 6, c_1 = -5$

$a_2 = 38, b_2 = 21, \ and \ c_2 = -9$

By cross multiplication, we have

$$\Rightarrow \quad \frac{u}{-54 + 105} = \frac{-v}{-513 + 190} = \frac{1}{1193 - 228}$$

$$\Rightarrow \quad \frac{u}{51} = \frac{-v}{-323} = \frac{1}{969}$$

$$\Rightarrow \quad \frac{u}{51} = \frac{v}{323} = \frac{1}{969}$$

Now,

$$\frac{u}{51} = \frac{1}{969}$$

$\Rightarrow \quad u = \frac{51}{969}$

$\Rightarrow \quad u = \frac{1}{19}$

And,

$$\frac{v}{323} = \frac{1}{969}$$

$\Rightarrow \quad v = \frac{323}{969}$

$\Rightarrow \quad v = \frac{1}{3}$

Now,

$$u = \frac{1}{x+y}$$

$\Rightarrow \quad \frac{1}{x+y} = \frac{1}{19}$

$\Rightarrow \quad x + y = 19 \qquad ....(i)$

And,

$$v = \frac{1}{x-y}$$

$\Rightarrow \quad \frac{1}{x-y} = \frac{1}{3}$

$\Rightarrow \quad x - y = 3 \qquad ...(ii)$

23. 2(ax – by) + a + 4b = 0
2(bx + ay) + b – 4a = 0

Sol:

The given system of equation may be written as

$$2ax - 2by + a + 4b = 0$$
$$2bx + 2ay + b - 4a = 0$$

Here,

$$a_1 = 2a, b_1 = -2b, c_1 = a + 4b$$
$$a_2 = 2b, b_2 = 2a, c_2 = b - 4a$$

By cross multiplication, we have

$$\Rightarrow \frac{x}{(-2b)(b-4a)-(2a)(a+4b)} = \frac{-y}{(2b)(b-4a)-(2a)(a+4b)} = \frac{1}{4a^2+4b^2}$$

$$\Rightarrow \frac{x}{-2b^2+8ab-2a^2-8ab}=\frac{-y}{2ab-8a^2-2ab-8b^2}=\frac{1}{4a^2+4b^2}$$

$$\Rightarrow \frac{x}{-2a^2-2b^2}=\frac{-y}{-8a^2-8b^2}=\frac{1}{4a^2+4b^2}$$

Now,

$$\frac{x}{-2a^2-2b^2}=\frac{1}{4a^2+4b^2}$$

$$\Rightarrow \quad x=\frac{-2a^2-2b^2}{4a^2+4b^2}$$

$$=\frac{-2\left(a^2-b^2\right)}{4\left(a^2+b^2\right)}$$

$$=\frac{-1}{2}$$

And,

$$\frac{-y}{-8a^2-8b^2}=\frac{1}{4a^2+4b^2}$$

$$\Rightarrow \quad -y=\frac{-8a^2-8b^2}{4a^2+4b^2}$$

$$\Rightarrow \quad -y=\frac{-8\left(a^2-b^2\right)}{4\left(a^2+b^2\right)}$$

$$\Rightarrow \quad -y=\frac{-8}{4}$$

$$\Rightarrow \quad y=2$$

Hence, $x=\frac{-1}{2}, y=2$ is the solution of the given system of the equations.

The given system of equations may be written as

$$2ax-2by+a+4b=0$$

$$2bx+2ay+b-4a=0$$

Here,

$a_1=2a, b_1=-2b, c_1=a+4b$

$a_2=2b, b_2=2a, c_2=b-4a$

By cross multiplication, we have

$$\Rightarrow \frac{x}{(-2b)(b-4a)-(2a)(a+4b)}=\frac{-y}{(2a)(b-4a)-(2b)(a+4b)}=\frac{1}{4a^2+4b^2}$$

$$\Rightarrow \frac{x}{-2b^2+8ab-2a^2-8ab}=\frac{1}{4a^2+4b^2}$$

$$\Rightarrow \frac{x}{-2a^2-2b^2}=\frac{-y}{-8a^2-8b^2}=\frac{1}{4a^2+4b^2}$$

Now,

$$\frac{x}{-2a^2-2b^2}=\frac{1}{4a^2+4b^2}$$

$$\Rightarrow \quad x=\frac{-2a-2b^2}{4a^2+4b^2}$$

$$=\frac{-2\left(a^2-b^2\right)}{4a^2+4b^2}$$

$$=\frac{-1}{2}$$

And,

$$\frac{-y}{-8a^2-8b^2}=\frac{1}{4a^2+4b^2}$$

$$\Rightarrow \quad -y=\frac{-8a^2-8b^2}{4a^2+4b^2}$$

$$\Rightarrow \quad -y=\frac{-8\left(a^2-b^2\right)}{4\left(a^2+b^2\right)}$$

$$\Rightarrow \quad -y=\frac{-8}{4}$$

$$\Rightarrow \quad y=2$$

Hence, $x=\frac{-1}{2}, y=2$ is the solution of the given system of the equations.

24. $6\left(ax+by\right)=3a+2b$

$6\left(bx-ay\right)=3b-2a$

Sol:

The given system of equation is

$$6\left(ax+by\right)=3a+2b \qquad .....(i)$$

$$6\left(bx-ay\right)=3b-2a \qquad .....(ii)$$

From equation (i), we get

$$6ax+6by-\left(3a+2b\right)=0 \qquad ......(iii)$$

From equation (ii), we get

$$6bx-6ay-\left(3b-2a\right)=0 \qquad ......(iv)$$

Here,

$a_1 = 6a, b_1 = 6b, c_1 = -(3a+2b)$

$a_2 = 6b, b_2 = -6a$ and $c_2 = -(3b-2a)$

By cross multiplication, we have

$$\frac{x}{-6b(3b-2a)-6a(3a+2b)} = \frac{-y}{-6a(3b-2a)+6b(3a+2b)} = \frac{1}{-36a^2-36b^2}$$

$$\Rightarrow \quad \frac{x}{-18b^2+12ab-18a^2-12ab} = \frac{-y}{-18ab+12a^2+18ab+12b^2} = \frac{1}{-36(a^2+b^2)}$$

$$\Rightarrow \quad \frac{x}{-18a^2-18b^2} = \frac{-y}{12a^2+12b^2} = \frac{1}{-36(a^2+b^2)}$$

$$\Rightarrow \quad \frac{x}{-18(a^2+b^2)} = \frac{-y}{12(a^2+b^2)} = \frac{-1}{36(a^2+b^2)}$$

Now,

$$\frac{x}{-18(a^2+b^2)} = \frac{-1}{36(a^2+b^2)}$$

$$\Rightarrow \quad x = \frac{18(a^2+b^2)}{36(a^2+b^2)}$$

$$= \frac{1}{2}$$

And,

$$\frac{-y}{12(a^2+b^2)} = \frac{-1}{36(a^2+b^2)}$$

$$\Rightarrow \quad y = \frac{12(a^2+b^2)}{36(a^2+b^2)}$$

$$\Rightarrow \quad y = \frac{1}{3}$$

Hence, $x = \frac{1}{2}, y = \frac{1}{3}$ is the solution of the given system of equations.

25. $\frac{a^2}{x} - \frac{b^2}{y} = 0$

$\frac{a^2 b}{x} + \frac{b^2 a}{y} = a + b, x, y \neq 0$

Sol:

Taking $\frac{1}{x} = u$ and $\frac{1}{y} = v$. Then, the given system of equations become

$a^2u - b^2v = 0$

$a^2bu + b^2av - (a+b) = 0$

Here,

$a_1 = a^2, b_1 = -b^2, c_1 = 0$

$a_2 = a^2b, b_2 = b^2a, \text{ and } c_2 = -(a+b)$

By cross multiplication, we have

$$\Rightarrow \frac{u}{b^2(a+b) - 0 \times b^2a} = \frac{-v}{-a^2(a+b) - 0 \times a^2b} = \frac{1}{a^3b^2 + a^2b^3}$$

$$\Rightarrow \frac{u}{b^2(a+b)} = \frac{v}{a^2(a+b)} = \frac{1}{a^2b^2(a+b)}$$

Now,

$$\frac{u}{b^2(a+b)} = \frac{1}{a^2b^2(a+b)}$$

$$\Rightarrow \quad u = \frac{b^2(a+b)}{a^2b^2(a+b)}$$

$$\Rightarrow \quad u = \frac{1}{a^2}$$

And,

$$\frac{v}{a^2(a+b)} = \frac{1}{a^2b^2(a+b)}$$

$$\Rightarrow \quad v = \frac{a^2(a+b)}{a^2b^2(a+b)}$$

$$\Rightarrow \quad v = \frac{1}{b^2}$$

Now,

$$x = \frac{1}{u} = a^2$$

And,

$$y = \frac{1}{v} = b^2$$

Hence, $x = a^2, y = b^2$ is the solution of the given system of equations.

26. $mx - my = m^2 + n^2$

$x + y = 2m$

Sol:

The given system of equations may be written as

$mx-ny-\left(m^2+n^2\right)=0$

$x+y-2m=0$

Here,

$a_1=m, b_1=-n, c_1=-\left(m^2+n^2\right)$

$a_2=1, b_2=1, \text{ and } c_2=-2m$

By cross multiplication, we have

$$\frac{x}{2mn+\left(m^2+n^2\right)}=\frac{-y}{-2m^2+\left(m^2+n^2\right)}=\frac{1}{m+n}$$

$$\Rightarrow \quad \frac{x}{2mn+m^2+n^2}=\frac{-y}{-m^2+n^2}=\frac{1}{m+n}$$

$$\Rightarrow \quad \frac{x}{\left(m+n\right)^2}=\frac{-y}{-m^2+n^2}=\frac{1}{m+n}$$

Now,

$$\frac{x}{\left(m+n\right)^2}=\frac{1}{m+n}$$

$$\Rightarrow \quad x=\frac{\left(m+n^2\right)}{m+n}$$

$$\Rightarrow \quad x=m+n$$

And,

$$\frac{-y}{-m^2+n^2}=\frac{1}{m+n}$$

$$\Rightarrow \quad -y=\frac{-m^2+n^2}{m+n}$$

$$\Rightarrow \quad y=\frac{m^2-n^2}{m+n}$$

$$\Rightarrow \quad y=\frac{\left(m-n\right)\left(m+n\right)}{m+n}$$

$$\Rightarrow \quad y=m-n$$

Hence, $x=m+n, y=m-n$ is the solution of the given system of equation.

27. $\frac{ax}{b}-\frac{by}{a}=a+b$

$ax-by=2ab$

Sol:

The given system of equation may be written as

$$\frac{a}{b}x\times-\frac{b}{a}\times y-(a+b)=0$$
$$ax-by-2ab=0$$

Here,

$$a_1=\frac{a}{b}, b_1=-\frac{b}{a}, c_1=-(a+b)$$

$$\Rightarrow \quad \frac{x}{2b^2-ab-b^2}=\frac{-y}{-2a^2+a^2+ab}=\frac{1}{-a+b}$$

$$\Rightarrow \quad \frac{x}{b^2-ab}=\frac{-y}{-a^2+ab}=\frac{1}{-a+b}$$

$$\Rightarrow \quad \frac{x}{b(b-a)}=\frac{-y}{a(-a+b)}=\frac{1}{b-a}$$

Now,

$$\frac{x}{b(b-a)}=\frac{1}{b-a}$$

$$\Rightarrow \quad x=\frac{b(b-a)}{b-a}=b$$

And,

$$\frac{-y}{a(b-a)}=\frac{1}{b-a}$$

$$\Rightarrow \quad -y=\frac{a(b-a)}{b-a}$$

$$\Rightarrow \quad -y=a$$

$$\Rightarrow \quad y=-a$$

Hence, $x=b, y=-a$ is the solution of the given system of equations.

28. $\frac{b}{a}x+\frac{a}{b}y-(a^2+b^2)=0$

$x+y-2ab=0$

Sol:

The given system of equation may be written as

$$\frac{b}{a}x+\frac{a}{b}y-(a^2+b^2)=0$$
$$x+y-2ab=0$$

Here,

$$a_1=\frac{b}{a}, b_1=\frac{a}{b}, c_1=-(a^2+b^2)$$
$$a_2=1, b_2=1, \ and \ c_2=-2ab$$

By cross multiplication, we have

$$\frac{x}{-2ab\times\frac{a}{b}+a^2+b^2}=\frac{-y}{-2ab\times\frac{a}{b}+a^2+b^2}=\frac{1}{\frac{b}{a}-\frac{a}{b}}$$

$$\Rightarrow \quad \frac{x}{-2a^2+a^2+b^2}=\frac{-y}{-2b^2+a^2+b^2}=\frac{1}{\frac{b^2-a^2}{ab}}$$

$$\Rightarrow \quad \frac{x}{b^2-a^2}=\frac{-y}{-b^2+a^2}=\frac{1}{\frac{b^2-a^2}{ab}}$$

Now,

$$\frac{x}{b^2-a^2}=\frac{1}{\frac{b^2-a^2}{ab}}$$

$$\Rightarrow \quad x=b^2-a^2\times\frac{ab}{b^2-a^2}$$

$$\Rightarrow \quad x=ab$$

And,

$$\frac{-y}{-b^2+a^2}=\frac{1}{\frac{b^2-a^2}{ab}}$$

$$\Rightarrow \quad -y=-b^2+a^2\times\frac{ab}{b^2-a^2}$$

$$\Rightarrow \quad -y=-\left(b^2-a^2\right)\times\frac{ab}{b^2-a^2}$$

$$\Rightarrow \quad -y=-ab$$

$$\Rightarrow \quad y=ab$$

Hence, $x=ab, y=ab$ is the solution of the given system of equations.

Exercise 3.5

In each of the following systems of equations determine whether the system has a unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it: (1 −4)

1. $x-3y-3=0$
$3x-9y-2=0$

Sol:

The given system of equations may be written as

$x-3y-3=0$

$3x-9y-2=0$

The given system of equations is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=1, b_1=-3, c_1=-3$

And $a_2=3, b_2=-9, c_2=-2$

We have,

$\frac{a_1}{a_2}=\frac{1}{3}$

$\frac{b_1}{b_2}=\frac{-3}{-9}=\frac{1}{3}$

And, $\frac{c_1}{c_2}=\frac{-3}{-2}=\frac{3}{2}$

Clearly, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

So, the given system of equation has no solutions.

2. $2x+y-5=0$

$4x+2y-10=0$

Sol:

The given system of equation may be written as

$2x+y-5=0$

$4x+2y-10=0$

The given system of equations is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=2, b_1=1, c_1=-5$

And $a_2=4, b_2=2, c_2=-10$

We have,

$\frac{a_1}{a_2}=\frac{2}{4}=\frac{1}{2}$

$\frac{b_1}{b_2}=\frac{1}{2}$

And, $\frac{c_1}{c_2}=\frac{-5}{-10}=\frac{1}{2}$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, the given system of equation has infinity many solutions.

3. $3x - 5y = 20$
$6x - 10y = 40$

Sol:

$3x - 5y = 20$

$6x - 10y = 40$

Compare it with

$a_1 x + by_1 + c_1 = 0$

$a_1 x + by_2 + c_2 = 0$

We get

$a1 = 3, b1 = -5 \ and \ c1 = -20$

$a2 = 6, b2 = -10 \ and \ c2 = -40$

$\frac{a_1}{a_2} = \frac{3}{6}, \frac{b_1}{b_2} = \frac{-5}{-10} \ and \ \frac{c_1}{c_2} = \frac{-20}{-40}$

Simplifying it we get

$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2} \ and \ \frac{c_1}{c_2} = \frac{1}{2}$

Hence

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So both lines are coincident and overlap with each other

So, it will have infinite or many solutions

4. $x - 2y - 8 = 0$
$5x - 10y - 10 = 0$

Sol:

The given system of equation may be written as

$x - 2y - 8 = 0$

$5x - 10y - 10 = 0$

The given system if equation is of the form

$a_1 x + b_1 y + c_1 = 0$

$a_2 x + b_2 y + c_2 = 0$

Where, $a_1 = 1, b_1 = -2, c_1 = -8$

And, $a_2 = 5, b_2 = -10, c_2 = -10$

We have,

$$\frac{a_1}{a_2} = \frac{1}{5}$$

$$\frac{b_1}{b_2} = \frac{-2}{-10} = \frac{1}{5}$$

And, $\frac{c_1}{c_2} = \frac{-8}{-10} = \frac{4}{5}$

Clearly, $\frac{a_1}{a_2} = \frac{b_2}{b_2} \neq \frac{c_1}{c_2}$

So, the given system of equation has no solution.

5. $kx + 2y - 5 = 0$
$3x + y - 1 = 0$

Sol:

The given system of equation is

$$kx + 2y - 5 = 0$$
$$3x + y - 1 = 0$$

The system of equation is of the form

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = k, b_1 = 2, c_1 = -5$

And, $a_2 = 3, b_2 = 1, c_2 = -1$

For a unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\therefore \quad \frac{k}{3} \neq \frac{2}{1}$

$\Rightarrow \quad k \neq 6$

So, the given system of equations will have a unique solution for all real values of k other than 6.

6. 4x + ky + 8 = 0
2x + 2y + 2 = 0

Sol:

Here $a_1 = 4, a_2 = k, b_1 = 2, b_2 = 2$

Now for the given pair to have a unique solution: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

i.e., $\frac{4}{2} \neq \frac{k}{2}$

i.e., $k \neq 4$

Therefore, for all values of k, except 4, the given pair of equations will have a unique solution.

7. $4x - 5y = k$
$2x - 3y = 12$

Sol:

The given system of equation is

$4x - 5y - k = 0$

$2x - 3y - 12 = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 4, b_1 = -5, c_1 = -k$

And, $a_2 = 2, b_2 = -3, c_2 = -12$

For a unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\therefore \quad \frac{4}{2} \neq \frac{-5}{-3}$

$\Rightarrow \quad k$ is any real number.

So, the given system of equations will have a unique solution for all real values of k.

8. $x + 2y = 3$
$5x + ky + 7 = 0$

Sol:

The given system of equation is

$x + 2y - 3 = 0$

$5x + ky + 7 = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 1, b_1 = 2, c_1 = -3$

And, $a_2 = 5, b_2 = k, c_2 = 7$

For a unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\therefore \quad \frac{1}{5} \neq \frac{2}{k}$

$\Rightarrow \quad k \neq 10$

So, the given system of equations will have a unique solution for all real values of k other than 10.

Find the value of k for which each of the following systems of equations have definitely many solution: (9-19)

9. $2x + 3y - 5 = 0$
$6x - ky - 15 = 0$

Sol:

The given system of equation is

$2x + 3y - 5 = 0$

$6x - ky - 15 = 0$

The system of equation is of the form

$a_1 x + b_1 y + c_1 = 0$

$a_2 x + b_2 y + c_2 = 0$

Where, $a_1 = 2, b_1 = 3, c_1 = -5$

And, $a_2 = 6, b_2 = k, c_2 = -15$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{2}{6} = \frac{3}{k}$

$\Rightarrow \quad 2k = 18$

$\Rightarrow \quad k = \frac{18}{2} = 9$

Hence, the given system of equations will have infinitely many solutions, if $k = 9$.

10. $4x + 5y = 3$
$kx + 15y = 9$

Sol:

The given system of equation is

$4x + 5y - 3 = 0$

$kx + 15y - 9 = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 4, b_1 = 5, c_1 = -3$

And, $a_2 = k, b_2 = 15, c_2 = -9$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{4}{k} = \frac{5}{15} = \frac{-3}{-9}$$

Now,

$$\frac{4}{k} = \frac{5}{15}$$

$$\Rightarrow \quad \frac{4}{k} = \frac{1}{3}$$

$$\Rightarrow \quad k = 12$$

Hence, the given system of equations will have infinitely many solutions, if $k = 12$.

11. $kx - 2y + 6 = 0$
$4x + 3y + 9 = 0$

Sol:

The given system of equation is

$kx - 2y + 6 = 0$

$4x + 3y + 9 = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = k, b_1 = -2, c_1 = 6$

And, $a_2 = 4, b_2 = -3, c_2 = 9$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{k}{4} = \frac{-2}{-3} = \frac{6}{9}$$

Now,

$$\frac{k}{4}=\frac{6}{9}$$

$\Rightarrow$ $$\frac{k}{4}=\frac{2}{3}$$

$\Rightarrow$ $$k=\frac{2\times 4}{3}$$

$\Rightarrow$ $$k=\frac{8}{3}$$

Hence, the given system of equations will have infinitely many solutions, if $k=\frac{8}{3}$.

12. $8x+5y=9$
$kx+10y=18$

Sol:

The given system of equation is

$8x+5y-9=0$

$kx+10y-18=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=8, b_1=5, c_1=-9$

And, $a_2=k, b_2=10, c_2=-18$

For a unique solution, we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$\Rightarrow$ $$\frac{8}{k}=\frac{5}{10}=\frac{-9}{-18}$$

Now,

$$\frac{8}{k}=\frac{5}{10}$$

$\Rightarrow$ $8\times 10=5\times k$

$\Rightarrow$ $$\frac{8\times 10}{5}=k$$

$\Rightarrow$ $k=8\times 2=16$

Hence, the given system of equations will have infinitely many solutions, if $k=16$.

13. $2x-3y=7$
$(k+2)x-(2k+1)y-3(2k-1)$

Sol:

The given system of equation may be written as

$2x-3y-7=0$

$(k+2)x-(2k+1)y-3(2k-1)=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=2, b_1=-3, c_1=-7$

And, $a_2=k, b_2=-(2k+1), c_2=-3(2k-1)$

For a unique solution, we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{k+2}=\frac{3}{-(2k+1)}=\frac{-7}{-3(2k-1)}$$

$$\Rightarrow \quad \frac{2}{k+2}=\frac{-3}{-(2k+1)} \text{ and } \frac{-3}{-(2k+1)}=\frac{-7}{-3(2k-1)}$$

$$\Rightarrow \quad 2(2k+1)=3(k+2) \text{ and } 3\times3(2k-1)=7(2k+1)$$

$$\Rightarrow \quad 4k+2=3k+6 \text{ and } 18k-9=14k+7$$

$$\Rightarrow \quad 4k-3k=6-2 \text{ and } 18k-14k=7+9$$

$$\Rightarrow \quad k=4 \text{ and } 4k=16 \Rightarrow k=4$$

$$\Rightarrow \quad k=4 \text{ and } k=4$$

Hence, the given system of equations will have infinitely many solutions, if $k=4$.

14. $2x+3y=2$
$(k+2)x+(2k+1)y-2(k-1)$

Sol:

The given system of equation may be written as

$2x+3y-2=0$

$(k+2)x+(2k+1)y-2(k-1)=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=2, b_1=3, c_1=-2$

And, $a_2 = k+2, b_2 = (2k+1), c_2 = -2(k-1)$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{k+2} = \frac{3}{(2k+1)} = \frac{-2}{-2(k-1)}$$

$$\Rightarrow \quad \frac{2}{k+2} = \frac{3}{(2k+1)} \text{ and } \frac{3}{(2k+1)} = \frac{2}{2(k-1)}$$

$$\Rightarrow \quad 2(2k+1) = 3(k+2) \text{ and } 3(k-1) = (2k+1)$$

$$\Rightarrow \quad 4k+2 = 3k+6 \text{ and } 3k-3 = 2k+1$$

$$\Rightarrow \quad 4k-3k = 6-2 \text{ and } 3k-2k = 1+3$$

$$\Rightarrow \quad k = 4 \text{ and } k = 4$$

Hence, the given system of equations will have infinitely many solutions, if $k = 4$.

15. $x + (k+1)y = 4$
$(k+1)x + 9y - (5k+2)$

Sol:

The given system of equation may be written as

$x + (k+1)y - 4 = 0$

$(k+1)x + 9y - (5k+2) = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 1, b_1 = k+1, c_1 = -4$

And, $a_2 = k+1, b_2 = 9, c_2 = -(5k+2)$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{1}{k+1} = \frac{k+1}{9} = \frac{-4}{-(5k+2)}$$

$$\Rightarrow \quad \frac{1}{k+1} = \frac{k+1}{9} \text{ and } \frac{k+1}{9} = \frac{4}{5k+2}$$

$$\Rightarrow \quad 9 = (k+1)^2 \text{ and } (k+1)(5k+2) = 36$$

$$\Rightarrow \quad 9 = k^2 + 1 + 2k \text{ and } 5k^2 + 2k + 5k + 2 = 36$$

$$\Rightarrow \quad k^2 + 2k + 1 - 9 = 0 \text{ and } 5k^2 + 7k + 2 - 36 = 0$$

$\Rightarrow \quad k^2+2k-8=0 \text{ and } 5k^2+7k-34=0$

$\Rightarrow \quad k^2+4k-2k-8=0 \text{ and } 5k^2+17k-10k-34=0$

$\Rightarrow \quad k(k+4)-2(k+4)=0 \text{ and } (5k+17)-2(5k+17)=0$

$\Rightarrow \quad (k+4)(k-2)=0 \text{ and } (5k+17)(k-2)=0$

$\Rightarrow \quad (k=-4 \text{ or } k=2) \text{ and } \left(k=\frac{-17}{5} \text{ or } k=2\right)$

$\Rightarrow \quad k=2$ satisfies both the conditions

Hence, the given system of equations will have infinitely many solutions, if $k=2$.

16. $kx+3y-2k+1$
$2(k+1)x+9y-(7k+1)$

Sol:

The given system of equation may be written as

$kx+3y-(2k+1)=0$

$2(k+1)x+9y-(7k+1)=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=k, b_1=3, c_1=-(2k+1)$

And, $a_2=2(k+1), b_2=9, c_2=-(7k+1)$

For a unique solution, we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{1}{2(k+1)}=\frac{3}{9}=\frac{-(2k+1)}{-(7k+1)}$

$\Rightarrow \quad \frac{k}{2(k+1)}=\frac{3}{9} \text{ and } \frac{3}{9}=\frac{2k+1}{7k+1}$

$\Rightarrow \quad 9k=3\times 2(k+1) \text{ and } 3(7k+1)=9(2k+1)$

$\Rightarrow \quad 9k=6(k+1) \text{ and } 21k+3=18k+9$

$\Rightarrow \quad 9k-6k=6 \text{ and } 21k-18k=9-3$

$\Rightarrow \quad 3k=6 \text{ and } 3k=6$

$\Rightarrow \quad k=\frac{6}{3} \text{ and } k=\frac{6}{3}$

$\Rightarrow \quad k=2 \text{ and } k=2$

$\Rightarrow \quad k=2$ satisfies both the conditions

Hence, the given system of equations will have infinitely many solutions, if $k=2$.

17. $2x+(k-2)y=k$
$6x+(2k-1)y-(2k+5)$

Sol:

The given system of equation may be written as

$2x+(k-2)y-k=0$

$6x+(2k-1)y-(2k+5)=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=2, b_1=k-2, c_1=-k$

And, $a_2=6, b_2=2k-1, c_2=-(2k+5)$

For a unique solution, we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{6}=\frac{k-2}{2k-1}=\frac{-k}{-2(2k+5)}$$

$$\Rightarrow \quad \frac{2}{6}=\frac{k-2}{2k-1} \text{ and } \frac{k-2}{2k-1}=\frac{k}{2k+5}$$

$$\Rightarrow \quad \frac{1}{3}=\frac{k-2}{2k-1} \text{ and } (k-2)(2k+5)=k(2k-1)$$

$$\Rightarrow \quad 2k-1=3(k-2) \text{ and } 2k^2+5k-4k-10=2k^2-k$$

$$\Rightarrow \quad 2k-3k-6 \text{ and } k-10=-k$$

$$\Rightarrow \quad 2k-3k=-6+1 \text{ and } k+k=10$$

$$\Rightarrow \quad -k=-5 \text{ and } 2k=10$$

$$\Rightarrow \quad k=\frac{-5}{-1} \text{ and } k=\frac{10}{2}$$

$$\Rightarrow \quad k=5 \text{ and } k=5$$

$\Rightarrow$ $k=5$ satisfies both the conditions

Hence, the given system of equations will have infinitely many solutions, if $k=5$.

18. $2x+3y=7$
$(k+1)x+(2k-1)y-(4k+1)$

Sol:

The given system of equation may be written as

$2x+3y-7=0$

$(k+1)x+(2k-1)y-(4k+1)=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=2, b_1=3, c_1=-7$

And, $a_2=k+1, b_2=2k-1, c_2=-(4k+1)$

For a unique solution, we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{k+1}=\frac{3}{2k-1}=\frac{-7}{-(4k+1)}$$

$$\Rightarrow \quad \frac{2}{k+1}=\frac{3}{2k-1} \text{ and } \frac{3}{2k-1}=\frac{7}{4k+1}$$

$$\Rightarrow \quad 2(2k-1)=3(k+1) \text{ and } 3(4k+1)=7(2k-1)$$

$$\Rightarrow \quad 4k-2=3k+3 \text{ and } 12k+3=14k-7$$

$$\Rightarrow \quad 4k-3k=3+2 \text{ and } 12k-14k=-7-3$$

$$\Rightarrow \quad k=5 \text{ and } -2k=-10$$

$$\Rightarrow \quad k=5 \text{ and } k=\frac{10}{2}=5$$

$\Rightarrow \quad k=5$ satisfies both the conditions

Hence, the given system of equations will have infinitely many solutions, if $k=5$.

19. $2x+3y=k$

$(k-1)x+(k+2)y-3k$

Sol:

The given system of equation may be written as

$2x+3y-k=0$

$(k-1)x+(k+2)y-3k=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=2, b_1=3, c_1=-k$

And, $a_2=k-1, b_2=k+2, c_2=3k$

For a unique solution, we must have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{2}{k-1} = \frac{3}{k+1} = \frac{-k}{-3k}$

$\Rightarrow \quad \frac{2}{k-1} = \frac{3}{k+1} \text{ and } \frac{3}{k+1} = \frac{-k}{-3k}$

$\Rightarrow \quad 2(k+2) = 3(k-1) \text{ and } 3\times 3 = k+2$

$\Rightarrow \quad 2k+4 = 3k-3 \text{ and } 9 = k+2$

$\Rightarrow \quad 4+3 = 3k-2k \text{ and } 9-2 = k$

$\Rightarrow \quad 7 = k \text{ and } 7 = k$

$\Rightarrow \quad k = 7 \text{ and } k = 7$

$\Rightarrow \quad k = 7$ satisfies both the conditions

Hence, the given system of equations will have infinitely many solutions, if $k = 7$.

Find the value of k for which the following system of equations has no solution: (20 – 25)

20. $kx - 5y = 2$
$6x + 2y = 7$

Sol:

Given

$kx - 5y = 2$

$6x + 2y = 7$

Condition for system of equations having no solution

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{k}{6} = \frac{-5}{2} \neq \frac{2}{7}$

$\Rightarrow 2k = -30$

$\Rightarrow k = -15$

21. $x + 2y = 0$
$2x + ky - 5 = 0$

Sol:

The given system of equation may be written as

$x + 2y = 0$

$2x + ky - 5 = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 1, b_1 = 2, c_1 = 0$

And, $a_2 = 2, b_2 = k, c_2 = -5$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

We have,

$$\frac{a_1}{a_2} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{2}{k}$$

And, $\frac{c_1}{c_2} = \frac{0}{-5}$

Now, $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\Rightarrow \quad \frac{1}{2} = \frac{2}{k}$

$\Rightarrow \quad k = 4$

Hence, the given system of equations has no solutions, when $k = 4$.

22. $3x - 4y + 7 = 0$
$kx + 3y - 5 = 0$

Sol:

The given system of equation may be written as

$3x - 4y + 7 = 0$

$kx + 3y - 5 = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 3, b_1 = -4, c_1 = 7$

And, $a_2 = k, b_2 = 3, c_2 = -5$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

We have,

$$\frac{b_1}{b_2} = \frac{-4}{3}$$

$$and, \quad \frac{c_1}{c_2} = \frac{-7}{5}$$

Clearly, $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system will have no solution.

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{3}{k} = \frac{-4}{3} \Rightarrow k = \frac{-9}{4}$

Clearly, for this value of k, we have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given system of equations has no solutions, when $k = \frac{-9}{4}$.

23. $2x - ky + 3 = 0$
$3x + 2y - 1 = 0$

Sol:

The given system of equation may be written as

$2x - ky + 3 = 0$

$3x + 2y - 1 = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 2, b_1 = -k, c_1 = 3$

And, $a_2 = 3, b_2 = 2, c_2 = -1$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

We have,

$$\frac{a_1}{a_2} = \frac{2}{3}$$

$$and, \quad \frac{c_1}{c_2} = \frac{3}{-1}$$

Clearly, $\frac{a_1}{a_2} \neq \frac{c_1}{c_2}$

So, the given system will have no solution. If

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$ *i.e.*, $\frac{2}{k} = \frac{-k}{2} \Rightarrow k = \frac{-4}{3}$

Hence, the given system of equations has no solutions, $k = \frac{-4}{3}$.

24. $2x + ky - 11 = 0$
$5x - 7y - 5 = 0$

Sol:

The given system of equation is

$2x + ky - 11 = 0$

$5x - 7y - 5 = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 2, b_1 = k, c_1 = -11$

And, $a_2 = 5, b_2 = -7, c_2 = -5$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{5} = \frac{k}{-7} \neq \frac{-11}{-5}$$

$$\Rightarrow \quad \frac{2}{5} = \frac{k}{-7} \text{ and } \frac{k}{-7} \neq \frac{-11}{-5}$$

Now,

$$\frac{2}{5} = \frac{k}{-7}$$

$$\Rightarrow \quad 2 \times (-7) = 5k$$

$$\Rightarrow \quad 5k = -14$$

$$\Rightarrow \quad k = \frac{-14}{5}$$

Clearly, for $\frac{-14}{5}$ we have $\frac{k}{-7} \neq \frac{-11}{-5}$

Hence, the given system of equation will have no solution, if $k = \frac{-14}{5}$

25. $kx + 3y = 3$
$12x + ky = 6$

Sol:

$kx+3y=3$

$12x+ky=6$

For no solution $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

$\Rightarrow \quad \frac{k}{12}=\frac{2}{k}\neq\frac{3}{6}$

$\frac{k}{12}=\frac{3}{k}$

$k^2=36$

$k=\pm 6\, i.e., \quad k=6,-6$

Also,

$\frac{3}{k}\neq\frac{3}{6}$

$\frac{3\times 6}{3}\neq k$

$k\neq 6$

$k=-6$ satisfies both the condition

Hence, $k=-6$

26. For what value of α, the following system of equations will be inconsistent?

$4x+6y-11=0$

$2x+ky-7=0$

Sol:

The given system of equation may be written as

$4x+6y-11=0$

$2x+ky-7=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=4, b_1=6, c_1=-11$

And, $a_2=2, b_2=k, c_2=-7$

For a unique solution, we must have

$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

Now,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$\Rightarrow \quad \frac{4}{2} = \frac{6}{k}$

$\Rightarrow \quad 4k = 12$

$\Rightarrow \quad k = \frac{12}{4} = 3$

Clearly, for this value of k, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the given system of equation is inconsistent, when $k = 3$.

27. For what value of α, the system of equations

$\alpha x + 3y = \alpha - 3$

$12x + \alpha y = \alpha$

will have no solution?

Sol:

The given system of equation may be written as

$\alpha x + 3y - (\alpha - 3) = 0$

$12x + \alpha y - \alpha = 0$

The system of equation is of the form

$a_1 x + b_1 y + c_1 = 0$

$a_2 x + b_2 y + c_2 = 0$

Where, $a_1 = \alpha, b_1 = 3, c_1 = -(\alpha - 3)$

And, $a_2 = 12, b_2 = \alpha, c_2 = -\alpha$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{\alpha}{12} = \frac{3}{\alpha} \neq \frac{-(\alpha - 3)}{-\alpha}$

Now,

$$\frac{3}{\alpha} \neq \frac{-(\alpha-3)}{-\alpha}$$

$\Rightarrow \quad \frac{3}{\alpha} \neq \frac{\alpha-3}{\alpha}$

$\Rightarrow \quad 3 \neq \alpha - 3$

$\Rightarrow \quad 3+3 \neq \alpha$

$\Rightarrow \quad 6 \neq \alpha$

$\Rightarrow \quad \alpha \neq 6$

And,

$$\frac{\alpha}{12} = \frac{3}{\alpha}$$

$\Rightarrow \quad \alpha^2 = 36$

$\Rightarrow \quad \alpha = \pm 6$

$\Rightarrow \quad \alpha = -6 \qquad [\because \alpha \neq 6]$

Hence, the given system of equation will have no solution, if $\alpha = -6$.

28. Find the value of k for which the system
kx + 2y = 5
3x + y = 1
has (i) a unique solution, and (ii) no solution.

Sol:

The given system of equation may be written as

$kx + 2y - 5 = 0$

$3x + y - 1 = 0$

It is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = k, b_1 = 2, c_1 = -5$

And, $a_2 = 3, b_2 = 1, c_2 = -1$

(i) The given system will have a unique solution, if

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\Rightarrow \quad \frac{k}{3} \neq \frac{2}{1}$

$\Rightarrow \quad k \neq 6$

So, the given system of equations will have a unique solution, if $k \neq 6$

(ii) The given system will have no solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

We have

$$\Rightarrow \quad \frac{b_1}{b_2} = \frac{2}{1} \; and \; \frac{c_1}{c_2} = \frac{-5}{-1} = \frac{5}{1}$$

Clearly, $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system of equations will have no solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$$\Rightarrow \quad \frac{k}{3} = \frac{2}{1}$$

$$\Rightarrow \quad k = 6$$

Hence, the given system of equations will have no solution, if $k = 6$.

29. Prove that there is a value of c ($\neq 0$) for which the system

$6x + 3y = c - 3$

$12x + cy = c$

has infinitely many solutions. Find this value.

Sol:

The given system of equation may be written as

$6x + 3y - (c - 3) = 0$

$12x + cy - c = 0$

This is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 6, b_1 = 3, c_1 = -(c - 3)$

And, $a_2 = 12, b_2 = c, c_2 = -c$

For infinitely many solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{6}{12} = \frac{13}{c} = \frac{-(c-3)}{-c}$

$\Rightarrow \quad \frac{6}{12} = \frac{13}{c} \text{ and } \frac{3}{c} = \frac{c-3}{c}$

$\Rightarrow \quad 6c = 12 \times 3 \text{ and } 3 = (c-3)$

$\Rightarrow \quad c = \frac{36}{6} \text{ and } c - 3 = 3$

$\Rightarrow \quad c = 6 \text{ and } c = 6$

Now,

$$\frac{a_1}{a_2} = \frac{6}{12} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{-(6-3)}{-6} = \frac{1}{2}$$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Clearly, for this value of c, we have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given system of equations has infinitely many solutions, if $c = 6$.

30. Find the values of k for which the system
2x + ky = 1
3x – 5y = 7
will have (i) a unique solution, and (ii) no solution. Is there a value of k for which the system has infinitely many solutions?

Sol:

The given system of equation may be written as

$2x + ky - 1 = 0$

$3x - 5y - 7 = 0$

It is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 2, b_1 = k, c_1 = -1$

And, $a_2 = 3, b_2 = -5, c_2 = -7$

(i) The given system will have a unique solution, if

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\Rightarrow \quad \frac{2}{3} \neq \frac{k}{-5}$

$\Rightarrow \quad -10 \neq 3k$

$\Rightarrow \quad 3k \neq -10$

$\Rightarrow \quad k \neq \frac{-10}{3}$

So, the given system of equations will have a unique solution, if $k = \frac{-10}{3}$.

(ii) The given system will have no solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

We have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$\Rightarrow \quad \frac{2}{3} = \frac{k}{-5}$

$\Rightarrow \quad -10 = 3k$

$\Rightarrow \quad 3k = -10$

$\Rightarrow \quad k = \frac{-10}{3}$

We have

$$\frac{b_1}{b_2} = \frac{k}{-5} = \frac{-10}{3 \times -5} = \frac{2}{3}$$

And, $\frac{c_1}{c_2} = \frac{-1}{-7} = \frac{1}{7}$

Clearly, $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system of equations will have no solution, if $k = \frac{-10}{3}$

For the given system to have infinite number of solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

We have,

$$\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{k}{-5}$$

And, $\frac{c_1}{c_2} = \frac{-1}{-7} = \frac{1}{7}$

Clearly, $\frac{a_1}{a_2} \neq \frac{c_1}{c_2}$

So, whatever be the value of *k*, we cannot have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, there is no value of k, for which the given system of equations has infinitely many solutions.

31. For what value of k, the following system of equations will represent the coincident lines?

$x + 2y + 7 = 0$

$2x + ky + 14 = 0$

Sol:

The given system of equations may be written as

$x + 2y + 7 = 0$

$2x + ky + 14 = 0$

The given system of equations is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 1, b_1 = 2, c_1 = 7$

And $a_2 = 2, b_2 = k, c_2 = 14$

The given equations will represent coincident lines if they have infinitely many solutions, The condition for which is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{1}{2} = \frac{2}{k} = \frac{7}{14} \Rightarrow k = 4$$

Hence, the given system of equations will represent coincident lines, if $k = 4$.

32. Obtain the condition for the following system of linear equations to have a unique solution

$ax + by = c$

$lx + my = n$

Sol:

The given system of equations may be written as

$ax + by - c = 0$

$lx + my - n = 0$

It is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 1, b_1 = 2, c_1 = -c$

And $a_2 = l, b_2 = m, c_2 = -n$

For unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\Rightarrow \quad \frac{a}{l} \neq \frac{b}{m}$

$\Rightarrow \quad am \neq bl$

Hence, $am \neq bl$ is the required condition.

33. Determine the values of a and b so that the following system of linear equations have infinitely many solutions:

$(2a-1)x + 3y - 5 = 0$

$3x + (b-1)y - 2 = 0$

Sol:

The given system of equations may be written as

$(2a-1)x + 3y - 5 = 0$

$3x + (b-1)y - 2 = 0$

It is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 2a, b_1 = 3, c_1 = -5$

And $a_2 = 3, b_2 = b-1, c_2 = -2$

The given system of equations will have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{2a-1}{3} = \frac{3}{b-1} = \frac{-5}{-2}$

$\Rightarrow \quad 2(2a-1) = \frac{-5}{-2} \text{ and } \frac{3}{b-1} = \frac{-5}{-2}$

$\Rightarrow \quad 2(2a-1) = 5 \times 3 \text{ and } 3 \times 2 = 5(b-1)$

$\Rightarrow \quad 4a - 2 = 15 \text{ and } 6 = 5b - 5$

$\Rightarrow \quad 4a = 15 + 2 \text{ and } 6 + 5 = 5b$

$\Rightarrow \quad a = \frac{17}{4} \text{ and } \frac{11}{5} = b$

$\Rightarrow \quad a = \frac{17}{4} \text{ and } b = \frac{11}{5}$

Hence, the given system of equations will have infinitely many solutions,

If $a = \frac{17}{4} \text{ and } b = \frac{11}{5}$.

34. Find the values of a and b for which the following system of linear equations has infinite number of solutions:

$2x - 3y = 7$

$(a+b)x - (a+b-3)y = 4a + b$

Sol:

The given system of equations may be written as

$2x - 3y - 7 = 0$

$(a+b)x - (a+b-3)y - (4a+b) = 0$

It is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 2, b_1 = -3, c_1 = -7$

And $a_2 = a+b, b_2 = -(a+b-3), c_2 = -(4a+b)$

The given system of equations will have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{2}{a+b} = \frac{-3}{-(a+b-3)} = \frac{-7}{-(4a+b)}$

$\Rightarrow \quad \frac{2}{a+b} = \frac{3}{(a+b-3)}$ *and* $\frac{3}{a+b-3} = \frac{7}{4a+b}$

$\Rightarrow \quad 2(a+b-3) = 3(a+b)$ *and* $3(4a+b) = 7(a+b-3)$

$\Rightarrow \quad -6 = 3a-2a+3b-2b$ *and* $12a-7a+3b-7b = 21$

$\Rightarrow \quad -6 = a+b$ *and* $5a-4b = -21$

Now,

$a+b = -6$

$\Rightarrow \quad a = -6-b$

Substituting the value of 'a' in $5a-4b = -2$, we get

$5(-b-6)-4b = -21$

$\Rightarrow \quad -5b-30-4b = -21$

$\Rightarrow \quad -9b = -21+30$

$\Rightarrow \quad -9b = 9$

$\Rightarrow \quad b = \frac{9}{-9} = -1$

Putting $b=-1$ in $a=-b-6$, we get

$a = -(-1)-6 = 1-6 = -5$

Hence, the given system of equations will have infinitely many solutions,

If $a=-5$ *and* $b=-1$.

35. Find the values of p and q for which the following system of linear equations has infinite number of solutions:

$2x-3y=9$

$(p+q)x+(2p-q)y = 3(p+q+1)$

Sol:

The given system of equations may be written as

$2x-3y-9=0$

$(p+q)x+(2p-q)y-3(p+q+1)=0$

It is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1 = 2, b_1 = 3, c_1 = -9$

And $a_2 = p+q, b_2 = 2p-q, c_2 = -3(p+q+1)$

The given system of equations will have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{2}{p+q} = \frac{2}{2p-q} = \frac{-9}{-3(p+q+1)}$

$\Rightarrow \quad \frac{2}{p+q} = \frac{3}{2p-q} = \frac{3}{p+q+1}$

$\Rightarrow \quad \frac{2}{p+q} = \frac{3}{2p-q} \textit{ and } \frac{3}{2p-q} = \frac{3}{p+q+1}$

$\Rightarrow \quad 2(2p-q) = 3(p+q) \textit{ and } p+q+1 = 2p-q$

$\Rightarrow \quad 4p-2q = 3p+3q \textit{ and } -2p+p+q+q = -1$

$\Rightarrow \quad p = 5q = 0 \textit{ and } -p+2q = -1$

$\Rightarrow \quad p-5q-p+2q = -1$ [On adding]

$\Rightarrow \quad -3q = -1$

$\Rightarrow \quad q = \frac{1}{3}$

Putting $q = \frac{1}{3}$ in $p-5q$, we get

$$p - 5\left(\frac{1}{3}\right) = 0$$

$\Rightarrow \quad p = \frac{5}{3}$

Hence, the given system of equations will have infinitely many solutions,

If $p = \frac{5}{3.} \textit{ and } q = \frac{1}{3}$

36. Find the values of a and b for which the following system of equations has infinitely many solutions:

$2x + 3y = 7$

$(a-b)x + (a+b)y = 3a + b - 2$

Sol:

$2x + 3y - 7 = 0$

$(a-b)x + (a+b)y - (3a+b-2) = 0$

Here, $a_1 = 2, b_1 = 3, c_1 = -7$

$a_2 = (a-b), b_2 = (a+b), c_2 = -(3a+b-2)$

$$\frac{a_1}{a_2}=\frac{2}{a-b},\frac{b_1}{b_2}=\frac{3}{a+b},\frac{c_1}{c_2}=\frac{-7}{-(3a+b-2)}=\frac{-7}{(3a+b-2)}$$

For the equation to have infinitely many solutions, we have:

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\frac{2}{a-b}=\frac{7}{3a+b-2}$$

$$6a+2b-4=7a-7b$$

$$a-9b=-4 \qquad ....(1)$$

$$\frac{2}{a-b}=\frac{3}{a+b}$$

$$2a+2b=3a-3b$$

$$a-5b=0 \qquad ....(2)$$

Subtracting (1) from (2), we obtain:

$$4b=4$$

$$b=1$$

Substituting the value of b in equation (2), we obtain

$$a-5\times1=0$$

$$a=5$$

Thus, the values of a and b are 5 and 1 respectively.

(i)

$$(2a-1)x-3y=5$$

$$3x+(b-2)y=3$$

Sol:

The given system of equations is

$$(2a-1)x-3y-5=0$$

$$3x+(b-2)y-3=0$$

It is of the form

$$a_1x+b_1y+c_1=0$$

$$a_2x+b_2y+c_2=0$$

Where, $a_1=2a-1,b_1=-3,c_1=-5$

And, $a_2=3,b_2=b-2,c_2=-3$

The given system of equations will have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{2a-1}{3} = \frac{-3}{b-2} = \frac{-5}{-3}$

$\Rightarrow \quad \frac{2a-1}{3} = \frac{-3}{b-2} = \frac{5}{3}$

$\Rightarrow \quad \frac{2a-1}{3} = \frac{5}{3} \; and \; \frac{-3}{b-2} = \frac{5}{3}$

$\Rightarrow \quad \frac{3(2a-1)}{3} = 5 \; and \; -9 = 5(b-2)$

$\Rightarrow \quad 2a-1 = 5 \; and \; -9 = 5b(b-2)$

$\Rightarrow \quad 2a = 5+1 \; and \; -9+10 = 5b$

$\Rightarrow \quad a = \frac{6}{2} \; and \; 1 = 5b$

$\Rightarrow \quad a = 3 \; and \; \frac{1}{5} = b$

$\Rightarrow \quad a = 3 \; and \; b = \frac{1}{5}$

Hence, the given system of equations will have infinitely many solutions,

If $a = 3$ and $b = \frac{1}{5}$

(ii)

$2x - (2a+5)y = 5$

$(2b+1)x - 9y = 15$

Sol:

The given system of equations is

$2x - (2a+5)y - 5 = 0$

$(2b+1)x - 9y - 15 = 0$

It is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = x = b_1 = -(2a+5), c_1 = -5$

And, $a_2 = (2b+1), b_2 = -9, c_2 = -15$

The given system of equations will be have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{2b+1} = \frac{-(2a+5)}{-9} = \frac{-5}{-15}$$

$$\Rightarrow \quad \frac{2}{2b+1} = \frac{2a+5}{9} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{2}{2b+1} = \frac{1}{3} \; and \; \frac{2a+5}{9} = \frac{1}{3}$$

$$\Rightarrow \quad 6 = 2b+1 \; and \; \frac{3(2a+5)}{9} = 1$$

$$\Rightarrow \quad 6-1 = 2b \; and \; 2a+5 = 3$$

$$\Rightarrow \quad 5 = 2b \; and \; 2a = -2$$

$$\Rightarrow \quad \frac{5}{2} = b \; and \; a = \frac{-2}{2} = -1$$

Hence, the given system of equations will have infinitely many solutions,

If $a = -1 \; and \; b = \frac{5}{2}$.

(iii)

$(a-1)x + 3y = 2$

$6x + (1+2b)y = 6$

Sol:

The given system of equations is

$(a-1)x + 3y - 2 = 0$

$6x + (1+2b)y - 6 = 0$

It is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = a-1, b_1 = 3, c_1 = -2$

And, $a_2 = 6, b_2 = 1-2b, c_2 = -6$

The given system of equations will be have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{a-1}{6} = \frac{3}{1-2b} = \frac{-2}{-6}$

$\Rightarrow \quad \frac{a-1}{6} = \frac{3}{1-2b} = \frac{1}{3}$

$\Rightarrow \quad \frac{a-1}{b} = \frac{1}{3} \ and \ \frac{3}{1-2b} = \frac{1}{3}$

$\Rightarrow \quad 3(a-1) = 6 \ and \ 3 \times 3 = 1-2b$

$\Rightarrow \quad a-1 = 2 \ and \ 9 = 1-2b$

$\Rightarrow \quad a = 2+1 \ and \ 2b = 1-9$

$\Rightarrow \quad a = 3 \ and \ 2b = -8$

$\Rightarrow \quad a = 3 \ and \ b = \frac{-8}{2} = -4$

Hence, the given system of equations will have infinitely many solutions,
If $a = 3 \ and \ b = -4$.

(iv)
$3x + 4y = 12$
$(a+b)x + 2(a-b)y = 5a-1$

Sol:
The given system of equations is
$3x + 4y - 12 = 0$
$(a+b)x + 2(a-b)y - (5a-1) = 0$
It is of the form
$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$
Where, $a_1 = 3, b_1 = 4, c_1 = -12$
And, $a_2 = a+b, b_2 = 2(a-b), c_2 = -(5a-1)$
The given system of equations will be have infinite number of solutions, if

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{3}{a+b} = \frac{4}{2(a-b)} = \frac{12}{5a-1}$

$\Rightarrow \quad \frac{3}{a+b} = \frac{2}{a-b} \text{ and } \frac{2}{a-b} = \frac{12}{5a-1}$

$\Rightarrow \quad 3(a-b) = 2(a+b) \text{ and } 2(5a-1) = 12(a-b)$

$\Rightarrow \quad 3a-3b = 2a+2b \text{ and } 10a-2-12a-12b$

$\Rightarrow \quad 3a-2a = 2b+3b \text{ and } 10a-12a = -12b+2$

$\Rightarrow \quad a = 5b \text{ and } -2a = -12b+2$

Substituting $a = 5b$ in $-2a = -12b+2,$ we get

$-2(5b) = -12b+2$

$\Rightarrow \quad -10b = -12b+2$

$\Rightarrow \quad 12b-10b = 2$

$\Rightarrow \quad 2b = 2$

$\Rightarrow \quad b = 1$

Putting $b = 1$ in $a = 5b,$ we get

$a = 5 \times 1 = 5$

Hence, the given system of equations will have infinitely many solutions,
If $a = 5 \text{ and } b = 1.$

(v)

$2x + 3y = 7$

$(a-1)x + (a+1)y = (3a-1)$

Sol:

The given system of equations is

$2x + 3y - 7 = 0$

$(a-1)x + (a+1)y - (3a-1) = 0$

It is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 2, b_1 = 3, c_1 = -7$

And, $a_2 = a-1, b_2 = a+1, c_2 = -(3a-1)$

The given system of equations will be have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{2}{a-b} = \frac{3}{a+1} = \frac{-7}{-(3a-1)}$

$\Rightarrow \quad \frac{2}{a-1} = \frac{3}{a+1} = \frac{-7}{3a-1}$

$\Rightarrow \quad \frac{3}{a-1} = \frac{3}{a+1} \text{ and } \frac{3}{a+1} = \frac{7}{3a-1}$

$\Rightarrow \quad 2(a+1) = 3(a-1) \text{ and } 3(3a-1) = 7(a+1)$

$\Rightarrow \quad 2a+2 = 3a-3 \text{ and } 9a-3 = 7a+7$

$\Rightarrow \quad 2a-3a = -3 \text{ and } 9a-3 = 7a+7$

$\Rightarrow \quad -a = -5 \text{ and } 2a = 10$

$\Rightarrow \quad a = 5 \text{ and } a = \frac{10}{2} = 5$

$\Rightarrow \quad a = 5$

Hence, the given system of equations will have infinitely many solutions,
If $a = 5$.

(vi)
$2x + 3y = 7$
$(a-1)x + (a+2)y = 3a$

Sol:
The given system of equations is
$2x + 3y - 7 = 0$
$(a-1)x + (a+2)y - 3a = 0$
It is of the form
$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$
Where, $a_1 = 2, b_1 = 3, c_1 = -7$
And, $a_2 = a-1, b_2 = a+1, c_2 = -3a$
The given system of equations will be have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{2}{a-b} = \frac{3}{a+1} = \frac{-7}{-3a}$

$\Rightarrow \quad \frac{2}{a-1} = \frac{3}{a+2} = \frac{7}{3a}$

$\Rightarrow \quad \frac{2}{a-1}=\frac{3}{a+2}\ and\ \frac{3}{a+2}=\frac{7}{3a}$

$\Rightarrow \quad 2(a+2)=3(a-1)\ and\ 3\times 3a=7(a+2)$

$\Rightarrow \quad 2a-4a=-3\ and\ 9a=7a+14$

$\Rightarrow \quad 2a-3a=-3\ and\ 9a-7a=14$

$\Rightarrow \quad -a=-7\ and\ 2a=14$

$\Rightarrow \quad a=7\ and\ a=\frac{14}{2}=7$

$\Rightarrow \quad a=7$

Hence, the given system of equations will have infinitely many solutions,
If $a=7$.

Exercise 3.6

1. 5 pens and 6 pencils together cost Rs 9 and 3 pens and 2 pencils cost Rs 5. Find the cost of 1 pen and 1 pencil.

Sol:

Let the cost of a pen be Rs x and that of a pencil be Rs y. Then,

$5x+6y=9 \quad ....(i)$

$and \quad 3x+2y=5 \quad ......(ii)$

Multiplying equation (i) by 2 and equation (ii) by 6, we get

$10x+12y=18 \quad ......(iii)$

$18x+12y=30 \quad ......(iv)$

Subtracting equation (iii) by equation (iv), we get

$18x-10x+12y-12y=30-18$

$\Rightarrow \quad 8x=12$

$\Rightarrow \quad x=\frac{12}{8}=\frac{3}{2}=1.5$

Substituting $x=1.5$ in equation (i), we get

$5\times 1.5+6y=9$

$\Rightarrow \quad 7.5+6y=9$

$\Rightarrow \quad 6y=9-7.5$

$\Rightarrow \quad 6y=1.5$

$\Rightarrow \quad y=\frac{1.5}{6}=\frac{1}{4}=0.25$

Hence, cost of one pen $=Rs\ 1.50$ and cost of one pencil $=Rs\ 0.25$

2. 7 audio cassettes and 3 video cassettes cost Rs 1110, while 5 audio cassettes and 4 video cassettes cost Rs 1350. Find the cost of an audio cassette and a video cassette.

Sol:

Let the cost of a audio cassette be $Rs\ x$ and that of a video cassette be $Rs\ y$. Then,

$$7x+3y=1110 \qquad ....(i)$$

and $$5x+4y=1350 \qquad ....(ii)$$

Multiplying equation (i) by 4 and equation (ii) by 3, we get

$$28x+12y=4440 \qquad ......(iii)$$

$$15x+12y=4050 \qquad ......(iv)$$

Subtracting equation (iv) from equation (iii), we get

$$28x-15x+12y-12y=4440-4050$$

$$\Rightarrow \quad 13x=390$$

$$\Rightarrow \quad x=\frac{390}{13}=30$$

Substituting equation (iv) from equation (iii), we get

$$28x-15x+12y-12y=4440-4050$$

$$\Rightarrow \quad 13x=390$$

$$\Rightarrow \quad x=\frac{390}{13}=30$$

Substituting $x=30$ in equation (i), w get

$$7\times30+3y=1110$$

$$\Rightarrow \quad 210+3y=1110$$

$$\Rightarrow \quad 3y=1110-210$$

$$\Rightarrow \quad 3y=900$$

$$\Rightarrow \quad y=\frac{900}{3}=300$$

Hence, cost of one audio cassette $=Rs\,30$ and cost of one video cassette $=Rs\,300$

3. Reena has pens and pencils which together are 40 in number. If she has 5 more pencils and 5 less pens, then nuimber of pencils would become 4 times the number of pens. Find the original number of pens and pencils.

Sol:

Let the number of pens be x and that of pencil be y. then,

$$x+y=40 \qquad ....(i)$$

and $$(y+5)=4(x-5)$$

$\Rightarrow \quad y+5=4x-20$

$\Rightarrow \quad 5+20=4x-y$

$\Rightarrow \quad 4x-y=25 \quad ......(ii)$

Adding equation (i) and equation (ii), we get

$x+4x=40+25$

$\Rightarrow \quad 5x=65$

$\Rightarrow \quad x=\frac{65}{5}=13$

Putting $x=13$ in equation (i), we get

$13+y=40$

$\Rightarrow \quad y=40-13-27$

Hence, Reena has 13 pens 27 pencils.

4. 4 tables and 3 chairs, together, cost Rs 2,250 and 3 tables and 4 chairs cost Rs 1950. Find the cost of 2 chairs and 1 table.

Sol:

Let the cost of a table be $Rs\,x$ and that of a chairs be $Rs\ y.$ Then,

$4x+3y=2,250 \quad .....(i)$

$and, \quad 3x+4y=1950 \quad .....(ii)$

Multiplying equation (i) by 4 and equation (ii) by 3, we get

$16x+12y=9000 \quad ......(iii)$

$9x+12y=5850 \quad ....(iv)$

Subtracting equation (iv) by equation (iii), we get

$16x-9x=9000-5850$

$\Rightarrow \quad 7x=3150$

$\Rightarrow \quad x=\frac{3150}{7}=450$

Putting $x=450$ in equation (i), we get

$4\times 450+3y=2,250$

$\Rightarrow \quad 1800+3y=2250$

$\Rightarrow \quad 3y=2250-1800$

$\Rightarrow \quad 3y=450$

$\Rightarrow \quad y=\frac{450}{3}=150$

$\Rightarrow \quad 2y=2\times 150=300$

Cost of 2 chairs $=$ $Rs\ 300$ and cost of 1 table $=$ $Rs\ 450$

$\therefore$ The cost of 2 chairs and 1 table $=300+450=Rs\ 750$

5. 3 bags and 4 pens together cost Rs 257 whereas 4 bags and 3 pens together cost R 324. Find the total cost of 1 bag and 10 pens.

Sol:

Let the cost of a bag be $Rs\ x$ and that of a pen be $Rs\ y$. Then,

$$3x+4y=257 \quad .....(i)$$

$$and, \quad 4x+3y=324 \quad .....(ii)$$

Multiplying equation (i) by 3 and equation (ii) by 4, we get

$$9x+12y=770 \quad ....(iii)$$

$$16x+12y=1296 \quad ......(iv)$$

Subtracting equation (iii) by equation (iv), we get

$$16x-9x=1296-771$$

$$\Rightarrow \quad 7x=525$$

$$\Rightarrow \quad x=\frac{525}{7}=75$$

Cost of a bag $=Rs\,75$

Putting $x=75$ in equation (i), we get

$$3\times 75+4y=257$$

$$\Rightarrow \quad 225+4y=257$$

$$\Rightarrow \quad 4y=257-225$$

$$\Rightarrow \quad 4y=32$$

$$\Rightarrow \quad y=\frac{32}{4}=8$$

$\therefore$ Cost of a pen $=Rs\ 8$

$\therefore$ Cost of 10 pens $=8\times 10=Rs\ 80$

Hence, the total cost of 1 bag and 10 pens $=75+80=Rs$ 155.

6. 5 books and 7 pens together cost Rs 79 whereas 7 books and 5 pens together cost Rs 77. Find the total cost of 1 book and 2 pens.

Sol:

Let the cost of a book be Rs x and that of a pen be Rs y. Then,

$$5x+7y=79 \quad .....(i)$$

$$and, \quad 7x+5y=77 \quad ....(ii)$$

Multiplying equation (i) by 5 and equation (ii) by 7, we get

$$25+35y=395 \quad ......(iii)$$

$$49x+35y=539 \quad ....(iv)$$

Subtracting equation (iii) by equation (iv), we get

$$49x - 25x = 539 - 395$$

$\Rightarrow \quad 24x = 144$

$\Rightarrow \quad x = \frac{144}{24} = 6$

$\therefore$ Cost of a book $= Rs\ 6$

Putting $x = 6$ in equation (i), we get

$$5 \times 6 + 7y = 79$$

$\Rightarrow \quad 30 + 7y = 79$

$\Rightarrow \quad 7y = 79 - 30$

$\Rightarrow \quad 7y = 49$

$\Rightarrow \quad y = \frac{79}{7} = 7$

$\therefore$ Cost of a pen $= Rs\ 7$

$\therefore$ Cost of 2 pens $= 2 \times 7 = Rs\ 14$

Hence, the total cost of 1 book and 2 pens $= 6 + 14 = Rs\ 20$

7. A and B each have a certain number of mangoes. A says to B, "if you give 30 of your mangoes, I will have twice as many as left with you." B replies, "if you give me 10, I will have thrice as many as left with you." How many mangoes does each have?

Sol:

Suppose A has x mangoes and B has y mangoes

According to the given conditions, we have

$$x + 30 = 2(y - 30)$$

$\Rightarrow \quad x + 30 = 2y - 60$

$\Rightarrow \quad x - 2y = -60 - 30$

$\Rightarrow \quad x - 2y = -90 \qquad ....(i)$

And, $y + 10 = 3(x - 10)$

$\Rightarrow \quad y + 10 = 3x - 30$

$\Rightarrow \quad 10 + 30 = 3x - y$

$\Rightarrow \quad 3x - y = -40 \qquad ....(ii)$

Multiplying equation (i) by 3 and equation (ii) by 1, we get

$$3x - 6y = -270 \qquad .....(iii)$$

$$3x - y = 40 \qquad ......(iv)$$

Subtracting equation (iv) by equation (iii), we get

$$-6y-(-y)=-270-40$$
$$\Rightarrow \quad -6y+y=-310$$
$$\Rightarrow \quad -5y=-310$$
$$\Rightarrow \quad y=\frac{310}{5}=62$$

Putting $x=62$ in equation (i), we get

$$x-2\times 62=-90$$
$$\Rightarrow \quad x-124=-90$$
$$\Rightarrow \quad x=-90+124$$
$$\Rightarrow \quad x=34$$

Hence, *A* has 34 mangoes and *B* has 62 mangoes

8. On selling a T.V. at 5%gain and a fridge at 10% gain, a shopkeeper gains Rs 2000. But if he sells the T.V. at 10% gain and the fridge at 5% loss. He gains Rs 1500 on the transaction. Find the actual prices of T.V. and fridge.

Sol:

Let the price of a T.V. be Rs x and that of a fridge be Rs y. Then, we have

$$\frac{5x}{100}+\frac{10y}{100}=2000$$
$$\Rightarrow \quad 5x+10y=200000$$
$$\Rightarrow \quad 5(x+2y)=200000$$
$$\Rightarrow \quad x+2y=400000 \qquad ...(i)$$

And, $\frac{10x}{100}-\frac{5y}{100}=1500$

$$\Rightarrow 10x-5y=150000$$
$$\Rightarrow 5(2x-y)=150000$$
$$\Rightarrow 2x-y=30000$$

Multiplying equation (ii) by 2, we get

$$4x-2y=6000 \qquad .....(iii)$$

Adding equation (i) and equation (iii), we get

$$x+4x=40000+60000$$
$$\Rightarrow \quad 5x=100000$$
$$\Rightarrow \quad x=20000$$

Putting $x=20000$ in equation (i), we get

$20000+2y=40000$

$\Rightarrow \quad 2y=40000-20000$

$\Rightarrow \quad y=\dfrac{20000}{2}=10000$

Hence, the actual price of T.V $= Rs\ 20,000$ and, the actual price of fridge $= Rs\ 10,000$

9. The coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, he buys 3 bats and 5 balls for Rs 1750. Find the cost of each bat and each ball.

Sol:

Let the cost of bat and a ball be x and y respectively

According to the given information

$7x+6y=3800$ …….(1)

$3x+5y=1750$ ……..(2)

From (1), we obtain

$y=\dfrac{3800-7x}{6}$ ……..(3)

Substituting this value in equation (2), we obtain

$3x+5\left(\dfrac{3800-7x}{6}\right)=1750$

$3x+\dfrac{9500}{3}-\dfrac{35x}{6}=1750$

$3x-\dfrac{35x}{6}=1750=\dfrac{9500}{3}$

$3x-\dfrac{35x}{6}=\dfrac{5250-9500}{3}$

$-\dfrac{17x}{6}=\dfrac{-4250}{3}$

$-17x=-8500$

$x=500 \qquad (4)$

Substituting this in equation (3), we obtain

$y=\dfrac{3800-7\times 500}{6}$

$=\dfrac{300}{6}=50$

Hence, the cost of a bat is Rs 500 and that of a ball is Rs 50.

Concept Insight: Cost of bats and balls needs to be found so the cost of a ball and bat will be taken as the variables. Applying the conditions of total cost of bats and balls algebraic equations will be obtained. The pair of equations can then be solved by suitable substitution.

10. One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their respective capital?

Sol:

Let the money with the first person and second person be Rs x and Rs y respectively.

According to the question

$x+100=2(y-100)$

$x+100=2y-200$

$x-2y=-300$ ……(1)

$6(x-10)=(y+10)$

$6x-30=y+10$

$6x-y=70$ …….(2)

Multiplying equation (2) by 2, we obtain

$12x-2y=140$ …….(3)

Subtracting equation (1) from equation (3), we obtain:

$11x=140+300$

$11x=440$

$x=40$

Putting the value of x in equation (1), we obtain

$40-2y=-300$

$40+300=2y$

$2y=340$

$y=170$

Thus, the two friends had $Rs\,40$ and $Rs\,170$ with them.

11. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Sol:

Let the fixed charge for first three days and each day charge thereafter be Rs x and Rs y respectively.

According to the question,

$x+4y=27 \quad ...(1)$

$x+2y=21 \quad ...(2)$

Subtracting equation (2) from equation (1), we obtain:

$2y=6$

$y=3$

Subtracting the value of y in equation (1), we obtain

$x+12=27$

$x=15$

Hence, the fixed charge is Rs 15 and the charge per day is Rs 3.

Exercise 4.1

1. (i) All circles are .... (congruent, similar).
   (ii) All squares are .... (similar, congruent).
   (iii) All .... triangles are similar (isosceles, equilaterals):
   (iv) Two triangles are similar, if their corresponding angles are .... (proportional, equal)
   (v) Two triangles are similar, if their corresponding sides are .... (proportional, equal)
   (vi) Two polygons of the same number of sides are similar, if (a) their corresponding angles are and (b) their corresponding sides are .... (equal, proportional).

   Sol:
   (i) All circles are similar
   (ii) All squares are similar
   (iii) All equilateral triangles are similar
   (iv) Two triangles are similar, if their corresponding angles are equal
   (v) Two triangles are similar, if their corresponding sides are proportional
   (vi) Two polygons of the same number of sides are similar, if (a) their corresponding angles are equal and (b) their corresponding sides are proportional.

2. Write the truth value (T/F) of each of the following statements:
   (i) Any two similar figures are congruent.
   (ii) Any two congruent figures are similar.
   (iii) Two polygons are similar, if their corresponding sides are proportional.
   (iv) Two polygons are similar if their corresponding angles are proportional.
   (v) Two triangles are similar if their corresponding sides are proportional.
   (vi) Two triangles are similar if their corresponding angles are proportional.

   Sol:
   (i) False
   (ii) True
   (iii) False
   (iv) False
   (v) True
   (vi) True

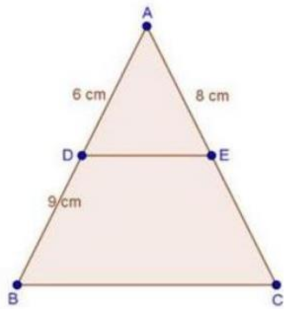
Exercise 4.2

1. In ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC
   (i) If AD = 6 cm, DB = 9 cm and AE = 8 cm, find AC.
   (ii) If $\frac{AD}{DB} = \frac{3}{4}$ and AC = 15 cm, find AE
   (iii) If $\frac{AD}{DB} = \frac{2}{3}$ and AC = 18 cm, find AE

(iv) If AD = 4, AE = 8, DB = x – 4, and EC = 3x – 19, find x.
(v) If AD = 8cm, AB = 12 cm and AE = 12 cm, find CE.
(vi) If AD = 4 cm, DB = 4.5 cm and AE = 8 cm, find AC.
(vii) If AD = 2 cm, AB = 6 cm and AC = 9 cm, find AE.
(viii) If $\frac{AD}{BD} = \frac{4}{5}$ $and\ EC = 2.5\ cm, find\ AE$
(ix) If AD = x, DB = x − 2, AE = x + 2 and EC = x − 1, find the value of x.
(x) If AD = 8x − 7, DB = 5x − 3, AE = 4x − 3 and EC = (3x − 1), find the value of x.
(xi) If AD = 4x − 3, AE = 8x – 7, BD = 3x – 1 and CE = 5x − 3, find the volume of x.
(xii) If AD = 2.5 cm, BD = 3.0 cm and AE = 3.75 cm, find the length of AC.

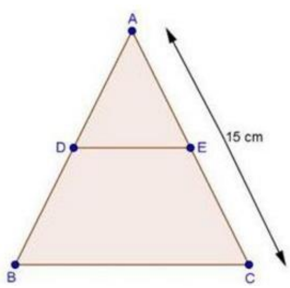
Sol:

(i)



We have,
DE || BC
Therefore, by basic proportionally theorem,
We have $\frac{AD}{DB} = \frac{AE}{EC}$
⇒ $\frac{6}{9} = \frac{8}{EC}$
⇒ $\frac{2}{3} = \frac{8}{EC}$
⇒ EC = $\frac{8\times3}{2}$
⇒ EC = 12 cm
⇒ Now, AC = AE + EC = 8 + 12 = 20 cm
∴ AC = 20 cm

(ii)



We have,

$\frac{AD}{DB} = \frac{3}{4}$ $and\ DE\ ||BC$

$Therefore, by\ basic\ proportionality\ theorem, we\ have$

$\frac{AD}{DB} = \frac{AE}{EC}$

Adding 1 on both sides, we get

$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$

$\frac{3}{4} + 1 = \frac{AE+EC}{EC}$

$\Rightarrow \frac{3+4}{4} = \frac{AC}{EC}$ [∵ AE + EC = AC]

$\Rightarrow \frac{7}{4} = \frac{15}{EC}$

$\Rightarrow \text{EC} = \frac{15\times4}{7}$

$\Rightarrow \text{EC} = \frac{60}{7}$

Now, $\text{AE} + \text{EC} = \text{AC}$

$\Rightarrow \text{AE} + \frac{60}{7} = 15$

$\Rightarrow \text{AE} = 15 - \frac{60}{7}$

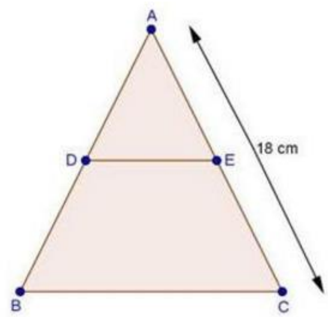
$= \frac{105-60}{7}$

$= \frac{45}{7}$

$= 6.43$ cm

∴ AE = 6.43 cm

(iii)



We have,

$\frac{AD}{DB} = \frac{2}{3}$ $and\ DE\ ||\ BC$

$Therefore, by\ basic\ proportionality\ theorem, we\ have,$

$\frac{AD}{DB} = \frac{EC}{AE}$

$\Rightarrow \frac{3}{2} = \frac{EC}{AE}$

Adding 1 on both sides, we get

$\frac{3}{2}+1=\frac{EC}{AE}+1$

$\Rightarrow \frac{3+2}{2}=\frac{EC+AE}{AE}$

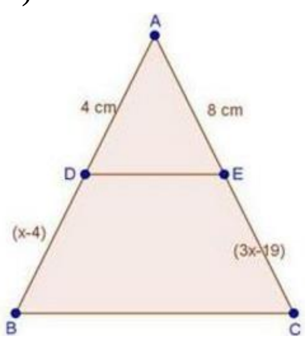
$\Rightarrow \frac{5}{2}=\frac{AC}{AE}$ [∵ AE + EC = AC]

$\Rightarrow \frac{5}{2}=\frac{18}{AE}$ [∵ AC = 18]

$\Rightarrow AE=\frac{18\times 2}{5}$

$\Rightarrow AE=\frac{36}{5}=7.2\ cm$

(iv)



We have,

DE || BC

Therefore, by basic proportionality theorem, we have,

$\frac{AD}{DB}=\frac{AE}{EC}$

$\frac{4}{x-4}=\frac{8}{3x-19}$

⇒ 4(3x – 19) = 8(x – 4)

⇒ 12x – 76 = 8x – 32

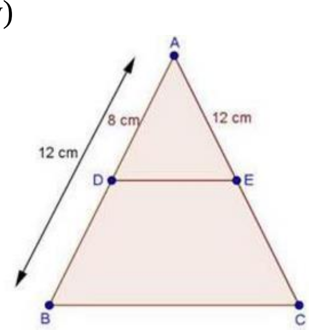
⇒ 12x – 8x = −32 + 76

⇒ 4x = 44

$\Rightarrow x=\frac{44}{4}=11cm$

∴ x = 11 cm

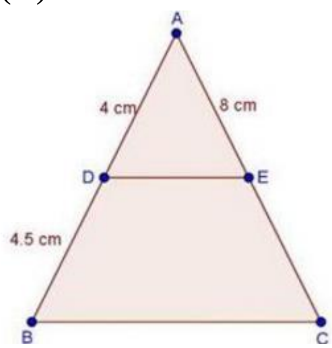
(v)



We have,

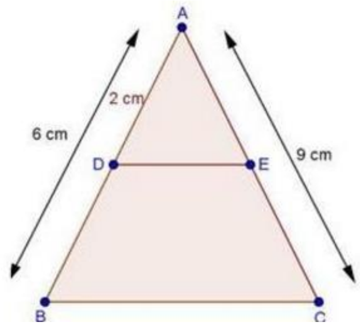
AD = 8cm, AB = 12 cm
∴ BD = AB – AD
= 12 – 8
⇒ BD = 4 cm
And, DE || BC
Therefore, by basic proportionality theorem, we have,
$\frac{AD}{BD} = \frac{AE}{CE}$
⇒ $\frac{8}{4} = \frac{12}{CE}$
⇒ CE = $\frac{12 \times 4}{8} = \frac{12}{2}$
⇒ CE = 6cm
∴ CE = 6cm

(vi)



We have,
DE || BC
Therefore, by basic proportionality theorem, we have,
$\frac{AD}{DB} = \frac{AE}{EC}$
⇒ $\frac{4}{4.5} = \frac{8}{EC}$
⇒ $EC = \frac{8 \times 4.5}{4}$
⇒ EC = 9cm
Now, AC = AE + EC
= 8 + 9
= 17 cm
∴ AC = 17 cm

(vii)



We have,

AD = 2 cm, AB = 6 cm

∴ DB = AB – AD

= 6 – 2

⇒ DB = 4 cm

And, DE || BC

Therefore, by basic proportionality theorem, we have,

$\frac{AD}{DB} = \frac{AE}{EC}$

Taking reciprocal on both sides, we get,

$\frac{DB}{AD} = \frac{EC}{AE}$

$\frac{4}{2} = \frac{EC}{AE}$

Adding 1 on both sides, we get

$\frac{4}{2} + 1 = \frac{EC}{AE} + 1$

$\Rightarrow \frac{4+2}{2} = \frac{EC+AE}{AE}$

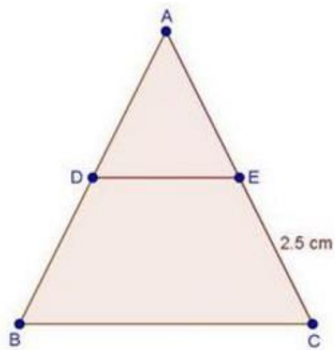
$\Rightarrow \frac{6}{2} = \frac{AC}{AE}$ [∵ EC + AE = AC]

$\Rightarrow \frac{6}{2} = \frac{9}{AE}$ [∵ AC = 9cm]

$AE = \frac{9\times 2}{6}$

$\Rightarrow AE = 3cm$

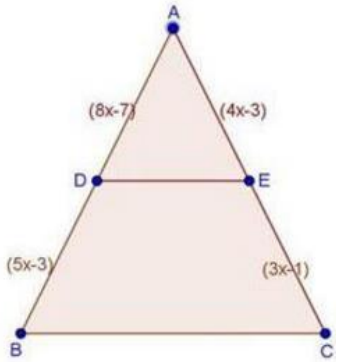
(viii)



We have, DE || BC

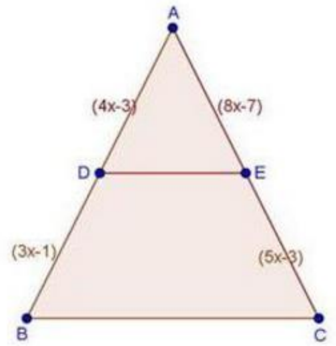
Therefore, by basic proportionality theorem,
We have,
$\frac{AD}{BD} = \frac{AE}{EC}$
$\Rightarrow \frac{4}{5} = \frac{AE}{2.5}$
$\Rightarrow$ AE = $\frac{4 \times 2.5}{5}$
$\Rightarrow$ AE = 2cm

(ix)



We have,
DE || BC
Therefore, by basic proportionality theorem,
We have,
$\frac{AD}{DB} = \frac{AE}{EC}$
$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$
$\Rightarrow x(x-1) = (x+2)(x-2)$
$\Rightarrow x^2 - x = x^2 - (2)^2$ [∵ (a – b) (a + b) = $a^2 - b^2$]
$\Rightarrow -x = -4$
$\Rightarrow$ x = 4 cm
∴ x = 4 cm

(x)



We have,

DE || BC

Therefore, by basic proportionality theorem, we have,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$

$\Rightarrow (8x-7)(3x-1) = (4x-3)(5x-3)$

$\Rightarrow 24x^2 - 8x - 21x + 7 = 20x^2 - 12x - 15x + 9$

$\Rightarrow 24x^2 - 20x^2 - 29x + 27x + 7 - 9 = 0$

$\Rightarrow 4x^2 - 2x - 2 = 0$

$\Rightarrow 2[2x^2 - x - 1] = 0$

$\Rightarrow 2x^2 - x - 1 = 0$

$\Rightarrow 2x^2 - 2x + 1x - 1 = 0$

$\Rightarrow 2x(x-1) + 1(x-1) = 0$

$\Rightarrow (2x + 1)(x - 1) = 0$

$\Rightarrow 2x + 1 = 0$ or $x - 1 = 0$

$\Rightarrow x = -\frac{1}{2}$ *or* $x = 1$

$x = -\frac{1}{2}$ *is not possible*

$\therefore x = 1$

(xi)

We have, DE || BC

Therefore, by basic proportionality theorem,

We have,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$

$\Rightarrow (4x-3)(5x-3) = (8x-7)(3x-1)$

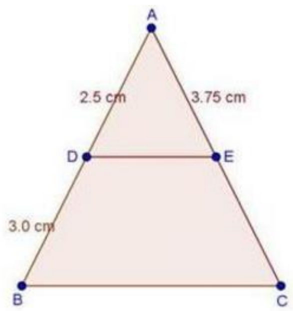
$\Rightarrow 4x(5x-3) - 3(5x-3) = 8x(3x-1) - 7(3x-1)$

$\Rightarrow 20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$

$\Rightarrow 4x^2 - 2x - 2 = 0$

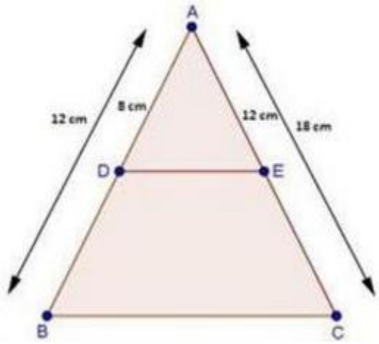
$\Rightarrow 2(2x^2 - x - 1) = 0$
$\Rightarrow 2x^2 - x - 1 = 0$
$\Rightarrow 2x^2 - 2x + 1x - 1 = 0$
$\Rightarrow 2x(x - 1) + 1(x - 1) = 0$
$\Rightarrow (2x + 1)(x - 1) = 0$
$\Rightarrow 2x + 1 = 0$ or $x - 1 = 0$
$\Rightarrow x = -\frac{1}{2}$ or $x = 1$
$x = -\frac{1}{2}$ is not possible
$\therefore x = 1$

(xii)

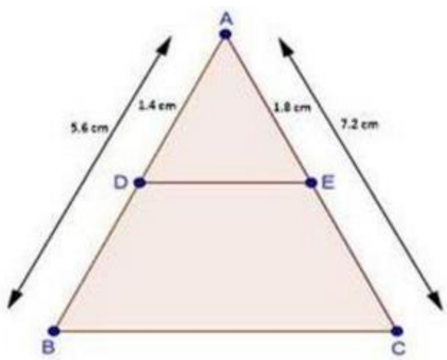


We have, DE || BC
Therefore, by basic proportionality theorem, we have,
$\frac{AD}{DB} = \frac{AE}{EC}$
$\Rightarrow \frac{2.5}{3.0} = \frac{3.75}{EC}$
$\Rightarrow EC = \frac{3.75 \times 3}{2.5} = \frac{375 \times 3}{250}$
$\Rightarrow EC = \frac{15 \times 3}{10}$
$= \frac{45}{10} = 4.5\ cm$
Now, $AC = AE + EC = 3.75 + 4.5 = 8.25$
$\therefore AC = 8.25$ cm

2. In a ΔABC, D and E are points on the sides AB and AC respectively. For each of the following cases show that DE || BC:
(i) AB = 2cm, AD = 8cm, AE = 12 cm and AC = l8cm.
(ii) AB = 5.6cm, AD = 1.4cm, AC= 7.2 cm and AE = 1.8 cm.
(iii) AB = 10.8 cm, BD = 4.5 cm, AC = 4.8 cm and AE = 2.8 cm.
(iv) AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm and EC = 5.5 cm.
Sol:



AB = 12 cm, AD = 12 cm and AC = 18 cm.
∴ DB = AB – AD
= 12 – 8
⇒ DB = 4 cm
And, EC = AC – AE
= 18 – 12
⇒ EC = 6 cm
Now, $\frac{AD}{DB} = \frac{8}{4} = \frac{2}{1}$ [∵ DB = 4 cm]
And, $\frac{AE}{EC} = \frac{12}{6} = \frac{2}{1}$ [∵ EC = 6 cm]
⇒ $\frac{AD}{DB} = \frac{AE}{EC}$
Thus, DE divides sides AB and AC of ΔABC in the same ratio.
Therefore, by the converse of basic proportionality theorem,
(ii)
We have, DE || BC



We have,
AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm
∴ DB = AB – AD
= 5.6 – 1.4
⇒ DB = 4.2 cm
And, EC = AC – AE
= 7.2 – 1.8
⇒ EC = 5.4 cm

Now, $\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3}$ [∵ DB = 4.2 cm]

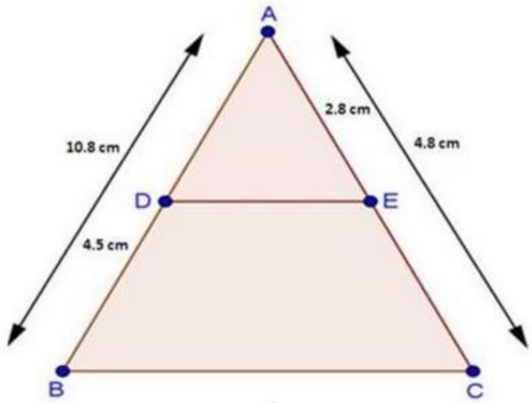
And, $\frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$ [∵ EC = 5.4 cm]

Thus, DE divides sides AB and AC of ΔABC in the same ratio.

Therefore, by the converse of basic proportionality theorem,

(iii)

We have,



We have,

AB = 10.8cm, BD = 4.5cm, AC = 4.8 cm and AE = 2.8cm

∴ AD = AB – DB = 10.8 – 4.5

⇒ AD = 6.3 cm

And, EC = AC – AE

= 4.8 – 2.8

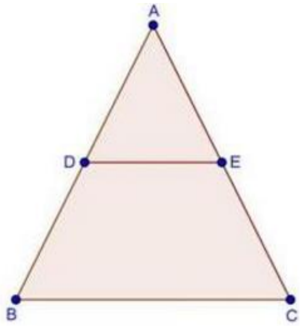
⇒ EC = 2 cm

Now, $\frac{AD}{DB} = \frac{6.3}{4.5} = \frac{7}{5}$ [∵ AD = 6.3 cm]

And, $\frac{AE}{EC} = \frac{2.8}{2} = \frac{28}{20} = \frac{75}{5}$ [∵ EC = 2 cm]

Thus, DE divides sides AB and AC of ΔABC in the same ratio. Therefore, by the converse of basic proportionality theorem.

(iv)



We have,

DE || BC

We have, AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm and EC = 5.5 cm

Now $\frac{AD}{BD} = \frac{5.7}{9.5} = \frac{57}{95}$

$\Rightarrow \frac{AD}{BD} = \frac{3}{5}$

And, $\frac{AE}{EC} = \frac{3.3}{5.5} = \frac{33}{55}$

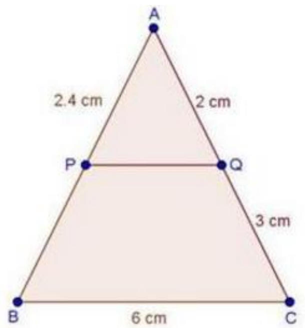
$\Rightarrow \frac{AE}{EC} = \frac{3}{5}$

Thus DE divides sides AB and AC of ΔABC in the same ratio.

Therefore, by the converse of basic proportionality theorem. We have DE || BC

3. In a ΔABC, P and Q are points on sides AB and AC respectively, such that PQ || BC. If AP = 2.4 cm, AQ = 2 cm, QC = 3 cm and BC = 6 cm, find AB and PQ.

Sol:



We have || BC

Therefore, by BPT

We have,

$\frac{AP}{PB} = \frac{AQ}{QC}$

$\frac{2.4}{PB} = \frac{2}{3}$

$\Rightarrow$ PB = $\frac{3\times2.4}{2} = \frac{3\times24}{2} = \frac{3\times6}{5} = \frac{18}{5}$

$\Rightarrow$ PB = 3.6 cm

Now, AB = AP + PB

= 2.4 + 3.6 = 6cm

Now, In ΔAPQ and ΔABC

∠A = ∠A [common]

∠APQ = ∠ABC [∵ PQ || BC ⇒ Corresponding angles are equal]

⇒ ΔAPQ ~ ΔABC [By AA criteria]

$\Rightarrow \frac{AB}{AP} = \frac{BC}{PQ}$ [corresponding sides of similar triangles are proportional]

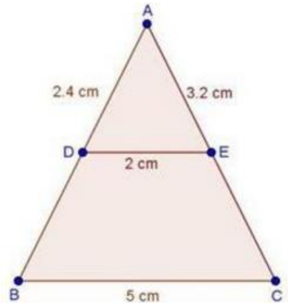
$\Rightarrow$ PQ = $\frac{6\times2.4}{6}$

$\Rightarrow$ PQ = 2.4 cm

Hence, AB = 6 cm and PO = 2.4 cm

4. In a ΔABC, D and E are points on AB and AC respectively such that DE || BC. If AD = 2.4cm, AE = 3.2 cm, DE = 2cm and BC = 5 cm, find BD and CE.

Sol:



We have,

DE || BC

Now, In ΔADE and ΔABC

∠A = ∠A [common]

∠ADE = ∠ABC [∵ DE || BC ⇒ Corresponding angles are equal]

⇒ ΔADE ~ ΔABC [By AA criteria]

$\Rightarrow \frac{AB}{BC} = \frac{AD}{DE}$ [corresponding sides of similar triangles are proportional]

$\Rightarrow AB = \frac{2.4 \times 5}{2}$

⇒ AB = 1.2 × 5 = 6.0 cm

⇒ AB = 6 cm

∴ BD = 6 cm

BD = AB – AD

= 6 – 2.4 = 3.6 cm

⇒ DB = 3.6 cm

Now,

$\frac{AC}{BC} = \frac{AE}{DE}$ [∵ Corresponding sides of similar triangles are equal]

$\Rightarrow \frac{AC}{5} = \frac{3.2}{2}$

$\Rightarrow AC = \frac{3.2 \times 5}{2} = 1.6 \times 5 = 8.0\ cm$

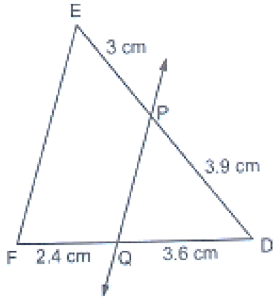
⇒ AC = 8 cm

∴ CE = AC – AE

= 8 – 3.2 = 4.8 cm

Hence, BD = 3.6 cm and CE = 4.8 cm

5. In below Fig., state if PQ || EF.



Sol:

We have,

DP = 3.9 cm, PE = 3cm, DQ = 3.6 cm and QF = 2.4 cm

Now, $\frac{DP}{PE} = \frac{3.9}{3} = \frac{1.3}{1} = \frac{13}{10}$

And, $\frac{DQ}{QF} = \frac{3.6}{2.4} = \frac{36}{24} = \frac{3}{2}$

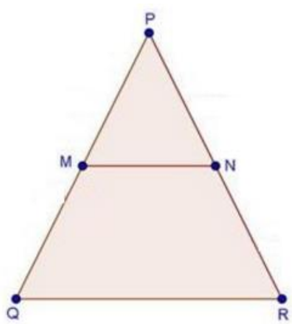
$\Rightarrow \frac{DP}{PE} \neq \frac{DQ}{QF}$

So, PQ is not parallel to EF

6. M and N are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether MN || QR

(i) PM = 4cm, QM = 4.5 cm, PN = 4 cm and NR = 4.5 cm

Sol:



(i) We have, PM = 4cm, QM = 4.5 cm, PN = 4 cm and NR = 4.5 cm

Hence, $\frac{PM}{QM} = \frac{4}{4.5} = \frac{8}{9}$

Also, $\frac{PN}{NR} = \frac{4}{4.5} = \frac{8}{9}$

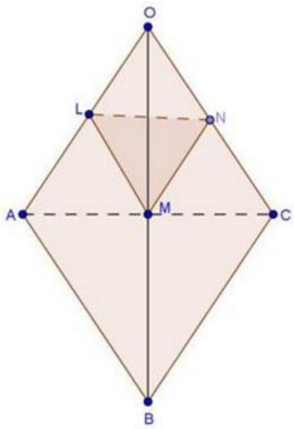
Hence, $\frac{PM}{QM} = \frac{PN}{NR}$

By converse of proportionality theorem

MN || QR

7. In three line segments OA, OB, and OC, points L, M, N respectively are so chosen that LM || AB and MN || BC but neither of L, M, N nor of A, B, C are collinear. Show that LN ||AC.

Sol:



We have,
LM || AB and MN || BC
Therefore, by basic proportionality theorem,
We have,

$\frac{QL}{AL} = \frac{OM}{MB}$ ...(i)

$and, \frac{ON}{NC} = \frac{OM}{MB}$ ...(ii)

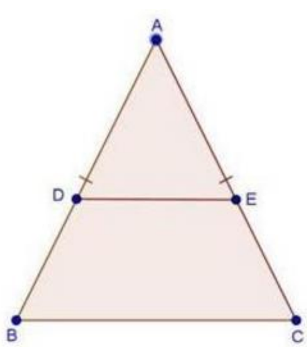
$Comparing\ equation\ (i) and\ equation\ (ii), we\ get,$

$\frac{ON}{AL} = \frac{ON}{NC}$

Thus, LN divides sides OA and OC of ΔOAC in the same ratio. Therefore, by the converse of basic proportionality theorem,
we have, LN || AC

8. If D and E are points on sides AB and AC respectively of a ΔABC such that DE || BC and BD = CE. Prove that ΔABC is isosceles.

Sol:

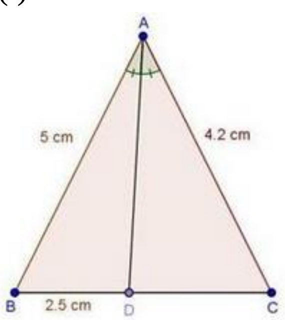


We have, DE || BC
Therefore, by BPT, we have,
$\frac{AD}{DB} = \frac{AE}{EC}$
$\Rightarrow \frac{AD}{DB} = \frac{AE}{DB}$ [∵BD = CE]
⇒ AD = AE
Adding DB on both sides
⇒ AD + DB = AE + DB
⇒ AD + DB = AE + EC [∴ BD = CE]
⇒ AB = AC
⇒ Δ ABC is isosceles

Exercise 4.3

1. In a ΔABC, AD is the bisector of ∠A, meeting side BC at D.
(i) If BD = 2.5cm, AB = 5cm and AC = 4.2cm, find DC.
(ii) If BD = 2cm, AB = 5cm and DC = 3cm, find AC.
(iii) If AB = 3.5 cm, AC = 4.2 cm and DC = 2.8 cm, find BD.
(iv) If AB = lo cm, AC =14 cm and BC =6 cm, find BD and DC.
(v) If AC = 4.2 cm, DC = 6 cm and 10 cm, find AB
(vi) If AB = 5.6 cm, AC = 6cm and DC = 3cm, find BC.
(vii) If AD = 5.6 cm, BC = 6cm and BD = 3.2 cm, find AC.
(viii) If AB = 10cm, AC = 6 cm and BC = 12 cm, find BD and DC.

Sol:
(i)



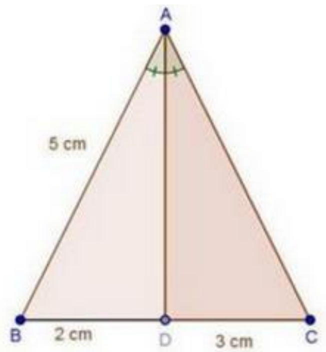
We have,
∠BAD = ∠CAD
We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.
$\therefore \frac{BD}{DC} = \frac{AB}{AC}$
$\Rightarrow \frac{2.5}{DC} = \frac{5}{4.2}$

$\Rightarrow DC = \frac{2.5 \times 4.2}{5}$

$= \frac{25 \times 42}{5 \times 100} = \frac{5 \times 42}{100} = \frac{210}{100} = 2.1\ cm$

∴ DC = 2.1 cm

(ii)



We have,

AD is the bisector of ∠A

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

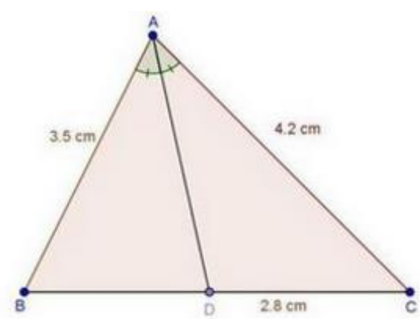
$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{2}{3} = \frac{5}{AC}$

$\Rightarrow$ AC = $\frac{5 \times 3}{2} = \frac{15}{2}$

$\Rightarrow$ AC = 7.5 cm

(iii)



In ΔABC, AD is the bisector of ∠A.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

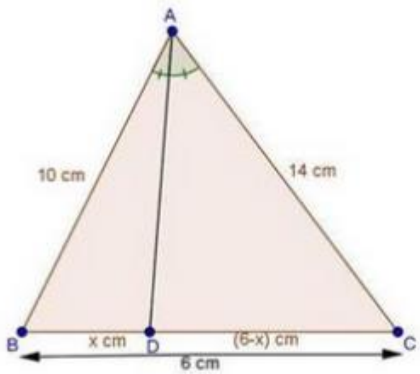
$\Rightarrow \frac{BD}{2.8} = \frac{3.5}{4.2}$

$= \frac{3.5 \times 2}{3}$

$= \frac{7}{3} = 2.33\ cm$

$\therefore BD = 2.3\ cm$

(iv)



In ΔABC, AD is the bisector of ∠A

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{x}{6-x} = \frac{10}{14}$

⇒ 14x = 10(6 – x)

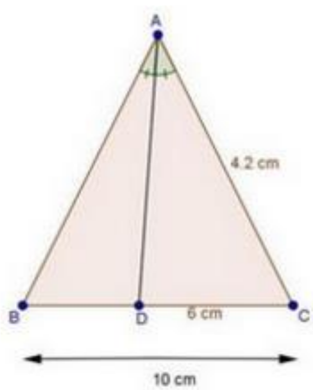
⇒ 24x = 60

$\Rightarrow x = \frac{60}{24} = \frac{5}{2} = 2.5cm$

Since, DC = 6 – x = 6 – 2.5 = 3.5 cm

Hence, BD = 2.5cm, and DC = 3.5 cm

(v)



We have,

BC = 10 cm, DC = 6 cm and AC = 4.2 cm

∴ BD = BC – DC = 10 – 6 = 4 cm

⇒ BD = 4 cm

In ΔABC, AD is the bisector of ∠A.

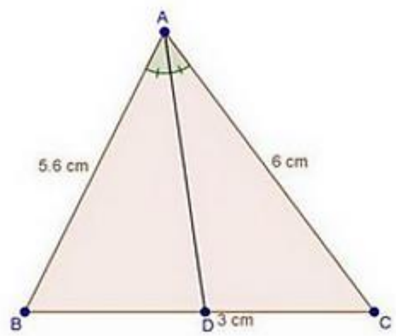
We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{4}{6} = \frac{AB}{4.2}$ [∵ BD = 4 cm]

⇒ AB = 2.8 cm

(vi)



We have, In ΔABC, AD is the bisector of ∠A.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{BD}{3} = \frac{5.6}{6}$

⇒ BD = $\frac{5.6\times3}{6} = \frac{5.6}{2} = 2.8cm$

⇒ BD = 2.8 cm

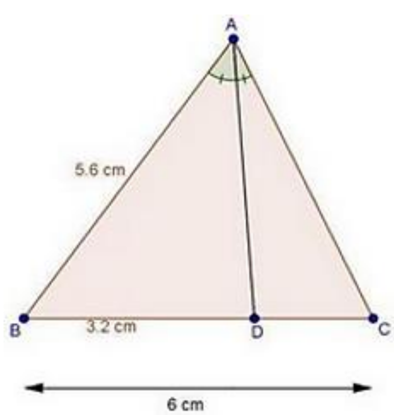
Since, BC = BD + DC

= 2.8 + 3

= 5.8 cm

∴ BC = 5.8 cm

(vii)



We have,

In ΔABC, AD is the bisector of ∠A.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the containing the angle.

$\therefore \frac{AB}{AC} = \frac{BD}{DC}$

$\frac{5.6}{AC} = \frac{3.2}{6-3.2}$ [∵ DC = BC – BD]

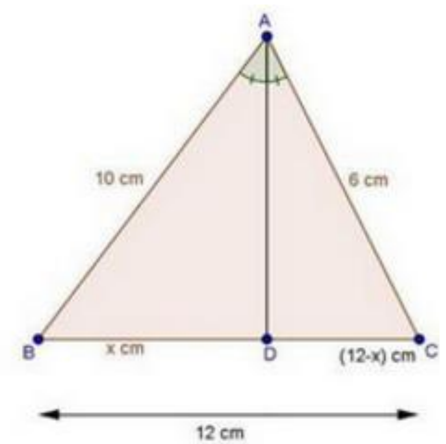
$\Rightarrow \frac{5.6}{AC} = \frac{3.2}{2.8}$

$\Rightarrow AC = \frac{5.6 \times 2.8}{3.2}$

$= \frac{5.6\times7}{8} = 0.7 \times 7$

$= 4.9\ cm$

(viii)



In ΔABC, AD is the bisector of ∠A.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{x}{12-z} = \frac{10}{6}$

⇒ 6x = 10(12 – x)

⇒ 6x = 120

$\Rightarrow x = \frac{120}{16} = 7.5\ cm$

∴ BD = 7.5 cm and DC = 12 – x = 12 – 7.5 = 4.5 cm

Hence, BD = 7.5 cm and DC = 4.5 cm

2. In Fig. 4.57, AE is the bisector of the exterior ∠CAD meeting BC produced in E. If AB = 10cm, AC = 6cm and BC = 12 cm, find CE.

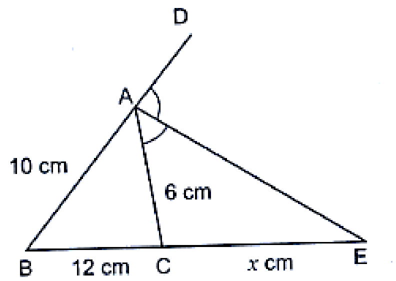


Fig. 4.57

Sol:

In ΔABC, AD is the bisector of ∠A.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC} \Rightarrow \frac{x}{12-x} = \frac{10}{6}$

$\Rightarrow 6(12 + x) = 10x$

$\Rightarrow 72 + 6x = 10x$

$\Rightarrow 4x - 72$

$\Rightarrow x = \frac{72}{4} = 18\ cm$

∴ CE = 18 cm

3. In Fig. 4.58, ΔABC is a triangle such that $\frac{AB}{AC} = \frac{BD}{DC}, \angle B = 70°, \angle C = 50°. Find\ \angle BAD.$

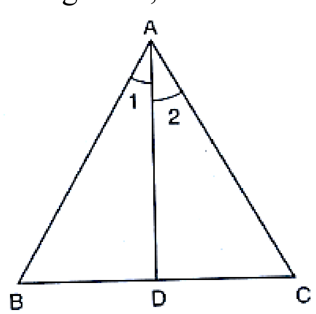


Fig. 4.58

Sol:

We have, if a line through one vertex of a triangle divides the opposite side in the ratio of the other two sides, then the line bisects the angle at the vertex.

$\therefore \angle 1 = \angle 2$

In ΔABC

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + 70° + 50° = 180°$ [$\because \angle B = 70°$ and $\angle C = 50°$]

$\Rightarrow \angle A = 180° - 120° = 60°$

$\Rightarrow \angle 1 + \angle 2 = 60°$

$\Rightarrow \angle 1 + \angle 1 = 60°$ [$\because \angle 1 = \angle 2$]

$\Rightarrow 2\angle 1 = 60°$
$\Rightarrow \angle 1 = 30°$
$\therefore \angle BAD = 30°$

4. In ΔABC (Fig., 4.59), if ∠1 = ∠2, prove that $\frac{AB}{AC} = \frac{BD}{DC}$.

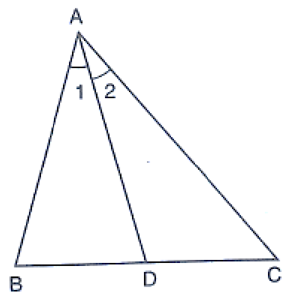
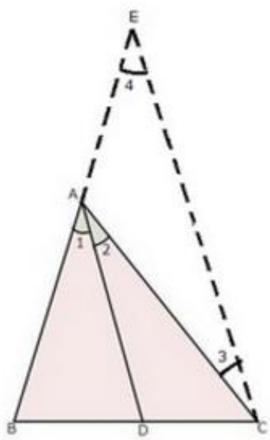


Fig. 4.59

Sol:



Given: A ΔABC in which ∠1 = ∠2
To prove: $\frac{AB}{AC} = \frac{BD}{DC}$
Construction: Draw CE || DA to meet BA produced in E.
Proof: since, CE || DA and AC cuts them.
$\therefore \angle 2 - \angle 3$ .... (i) [Alternate angles]
And, $\angle 1 - \angle 4$ ....(ii) [Corresponding angles]
But, $\angle 1 - \angle 2$ [Given]
From (i) and (ii), we get
$\angle 3 - \angle 4$
Thus, in ΔACE, we have
$\angle 3 = \angle 4$
$\Rightarrow AE = AC$ ... (iii) [Sides opposite to equal angles are equal]
Now, In ΔBCE, we have
DA || CE

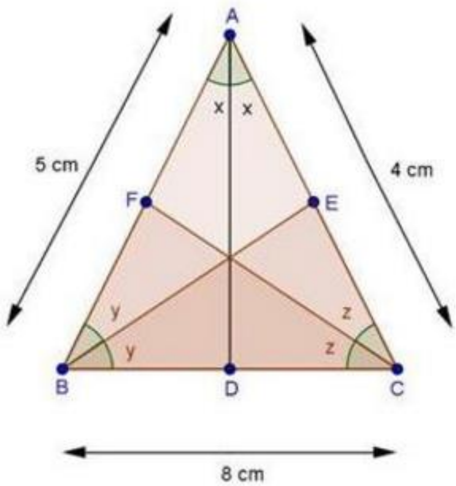
$\Rightarrow \frac{BD}{\boldsymbol{DC}} = \frac{BA}{\boldsymbol{AE}}$ [Using basic proportionality theorem]

$\Rightarrow \frac{BD}{DC} = \frac{AB}{AC}$ [∵ BA – AB and AE – AC from (iii)]

Hence, $\frac{AB}{AC} = \frac{BD}{DC}$

5. D, E and F are the points on sides BC, CA and AB respectively of ΔABC such that AD bisects ∠A, BE bisects ∠B and CF bisects ∠C. If AB = 5 cm, BC = 8 cm and CA = 4 cm, determine AP, CE and BD.

Sol:



In ΔABC, CF bisects ∠C.

We know that, the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{AF}{FB} = \frac{AC}{BC}$

$\Rightarrow \frac{AF}{5-AF} = \frac{4}{8}$ [∵ FB = AB – AF = 5 – AF]

$\Rightarrow \frac{AF}{5-AF} = \frac{1}{2}$

⇒ 2AF = 5 – AF

⇒ 2AF + AF = 5

⇒ 3AF = 5

⇒ AF = $\frac{5}{3}$ cm

Again, In ΔABC, BE bisects ∠B.

$\therefore \frac{AE}{EC} = \frac{AB}{BC}$

$\Rightarrow \frac{4-CE}{CE} = \frac{5}{8}$ [∵ AE = AC – CE = 4 – CE]

$\Rightarrow 8(4 - CE) = 5 \times CE$

⇒ 32 – 8CE = 5CE

⇒ 32 = 13CE

⇒ CE = $\frac{32}{13}$ cm

Similarly,

$\frac{BD}{DC} = \frac{AD}{AC}$

⇒ $\frac{BD}{8-BD} = \frac{5}{4}$ [∵ DC = BC – BD = 8 – BD]

⇒ 4BD = 40 – 5BD

⇒ 9BD = 40

⇒ BD = $\frac{40}{9}$ cm

Hence, AF = $\frac{5}{3}$ cm, CE = $\frac{32}{13}$ cm and BD = $\frac{40}{9}$ cm.

6. In fig., 4.60, check whether AD is the bisector of ∠A of ∆ABC in each of the following:

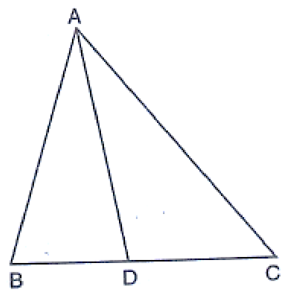


Fig. 4.60

(i) AB = 5cm, AC = 10cm, BD = 1.5 cm and CD = 3.5 cm

(ii) AB = 4 cm, AC = 6 cm, BD = 1.6 cm and CD = 2.4 cm

(iii) AB = 8 cm, AC = 24 cm, BD = 6 cm and BC = 24cm

(iv) AB = 6 cm, AC = 8cm, BD = l.5 cm and CD= 2 cm.

(v) AB = 5 cm, AC = l2 cm, BD = 2.5 cm and BC = 9cm

Sol:

Now,

$\frac{BD}{CD} = \frac{1.5}{3.5} = \frac{3}{7}$

And, $\frac{AB}{AC} = \frac{5}{10} = \frac{1}{2}$

⇒ $\frac{BD}{CD} \neq \frac{AB}{AC}$

⇒ AD is not the bisector of ∠A.

Now,

$\frac{AB}{AC} = \frac{4}{6} = \frac{2}{3}$

And, $\frac{BD}{CD} = \frac{1.6}{2.4} = \frac{2}{3}$

⇒ $\frac{AB}{AC} = \frac{BD}{CD}$

⇒ AD is the bisector of ∠A.

Now, $\frac{AB}{AC} = \frac{8}{24} = \frac{1}{3}$

And, $\frac{BD}{CD} = \frac{BD}{BC-BD}$ [∵ CD = BC – BD]

$= \frac{BD}{24-6}$

$= \frac{6}{18}$

$= \frac{1}{3}$

$\therefore \frac{AB}{AC} = \frac{BD}{CD}$

$\therefore AD\ is\ the\ bisector\ of\ \angle A\ of\ \Delta ABC.$

$\frac{AB}{AC} = \frac{6}{8} = \frac{3}{4}$

And, $\frac{BD}{CD} = \frac{2.5}{BC-BD}$ [∵ CD = BC – BD]

$= \frac{2.5}{9-2.5}$

$= \frac{2.5}{6.5}$

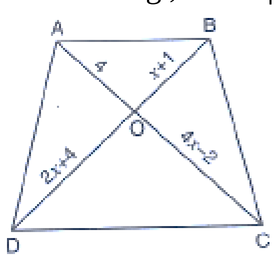
$= \frac{1}{3}$

$\therefore \frac{AB}{AC} \neq \frac{BD}{CD}$

∴ AD is not the bisector of ∠A of ΔABC.

Exercise 4.4

1. (i) In below fig., If AB || CD, find the value of x.



Sol:

Since diagonals of a trapezium divide each other proportionally.

$\therefore \frac{AO}{OC} = \frac{BO}{OD}$

$\Rightarrow \frac{4}{4x-2} = \frac{x+1}{2x+4}$

$\Rightarrow 4(2x+4) = (x+1)(4x-2)$

⇒ 8x + 16 = x(4x – 2) +1(4x – 2)

⇒ $8x + 16 = 4x^2 + 2x – 2$

$\Rightarrow 4x^2 + 2x - 8x - 2 - 16 = 0$

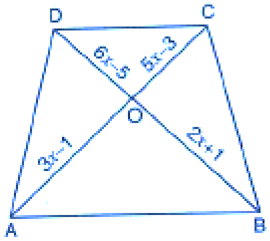
$\Rightarrow 4x^2 - 6x - 18 = 0$

$\Rightarrow 2[2x^2 - 3x - 9] = 0$

$\Rightarrow 2x^2 - 3x - 9 = 0$

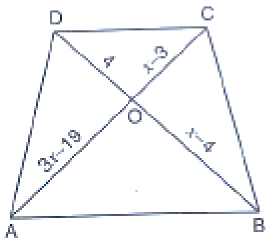
$\Rightarrow 2x(x-3)+3(x-3)=0$
$\Rightarrow (x-3)(2x+3)=0$
$\Rightarrow x-3=0$ or $2x+3=0$
$\Rightarrow x=3$ or $x=-\frac{3}{2}$
$\Rightarrow$ x = 3 or $x\ =-\frac{3}{2}$
$x\ =-\frac{3}{2}$ is not possible, because OB = x + 1 = $-\frac{3}{2}+1=-\frac{1}{2}$
Length cannot be negative
$\therefore \frac{AO}{OC}=\frac{BO}{OD}$

(ii) In the below fig., If AB || CD, find the value of x.



$\Rightarrow \frac{3x-1}{5x-3}=\frac{2x+1}{6x-5}$
$\Rightarrow$ (3x – 1) (6x – 5) = (2x + 1) (5x – 3)
$\Rightarrow$ 3x (6x – 5) – 1(6x – 5) = 2x (5x – 3) + 1 (5x – 3)
$\Rightarrow 18x^2-15x-6x+5=10x^2-6x+5x-3$
$\Rightarrow 8x^2-20x+8=0$
$\Rightarrow 4(2x^2-5x+2)=0$
$\Rightarrow 2x^2-4x-1x+2=0$
$\Rightarrow 2x(x-2)-1(x-2)=0$
$\Rightarrow (2x-1)(x-2)=0$
$\Rightarrow$ 2x – 1 = 0 or x – 2 = 0
$\Rightarrow x=\frac{1}{2}\ or\ x=2$
$x=\frac{1}{2}$ is not possible, because, OC = 5x – 3
$=5\left(\frac{1}{2}\right)-3$
$=\frac{5-6}{2}=-\frac{1}{2}$

(iii) In below fig., AB || CD. If OA = 3x – 19, OB = x – 4, OC = x – 3 and OD = 4, find x.



Since diagonals of a trapezium divide each other proportionally.

$\therefore \frac{AO}{OC} = \frac{BO}{OD}$

$\Rightarrow \frac{3x-19}{x-3} = \frac{x-4}{4}$

$\Rightarrow 4(3x-19) = (x-4)(x-3)$

$\Rightarrow$ 12x – 76 = x (x – 3) −4(x – 3)

$\Rightarrow 12x - 76 = x^2 - 3x - 4x + 12$

$\Rightarrow x^2 - 7x - 12x + 12 + 76 = 0$

$\Rightarrow x^2 - 19x + 88 = 0$

$\Rightarrow x^2 - 11z - 8z + 88 = 0$

$\Rightarrow x(x-11) - 8(x-11) = 0$

$\Rightarrow (x-11)(x-8) = 0$

$\Rightarrow x - 11 = 0$ or x – 8 = 0

$\Rightarrow$ x = 11 or x = 8

Exercise 4.5

1. In fig. 4.136, ΔACB ~ ΔAPQ. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ.

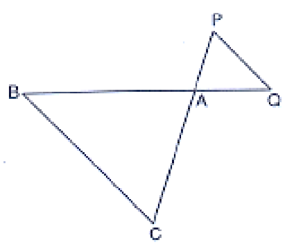


Fig. 4.136

Sol:

Given ΔACB ~ ΔAPQ

Then, $\frac{AC}{AP} = \frac{BC}{PQ} = \frac{AB}{AQ}$ [corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{AC}{2.8} = \frac{8}{4} = \frac{6.5}{AQ}$

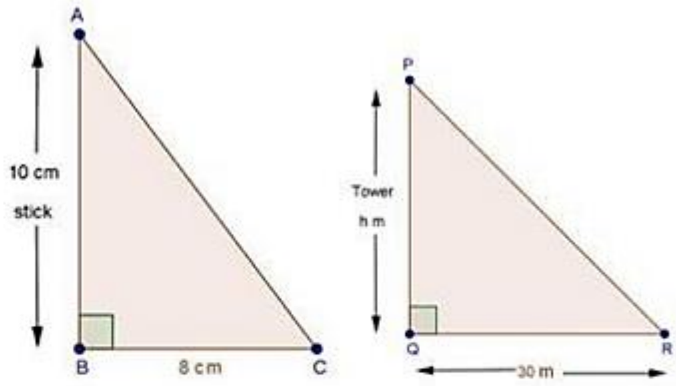
$\Rightarrow \frac{AC}{2.8} = \frac{8}{4} and \frac{8}{4} - \frac{6.5}{AQ}$

$\Rightarrow$ AC = $\frac{8}{4} \times 2.8\ and\ AQ = 6.5 \times \frac{4}{8}$

⇒ AC = 5.6 cm and AQ = 3.25 cm

2. A vertical stick 10 cm long casts a shadow 8 cm long. At the same time a shadow 30 m long. Determine the height of the tower.

Sol:



Length of stick = 10 cm
Length of shadow of stick = 8 cm
Length of shadow of tower = h cm
In ΔABC and ΔPQR
∠B = ∠Q = 90°
And, ∠C = ∠R [Angular elevation of sun]
Then, ΔABC ~ ΔPQR [By AA similarity]

$\therefore \frac{AB}{PQ} = \frac{BC}{QR}$

$\Rightarrow \frac{10cm}{8\ cm} = \frac{h\ cm}{3000}$

$\Rightarrow h = \frac{10}{8} \times 3000 = 3750\ cm = 37.5\ m$

3. In Fig. 4.137, AB || QR. Find the length of PB.

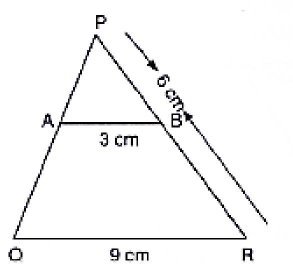


Fig. 4.137

Sol:

We have, ΔPAB and ΔPQR
∠P = ∠P [common]
∠PAB = ∠PQR [corresponding angles]

Then, ΔPAB ~ ΔPQR [By AA similarity]

$\therefore \frac{PB}{PR} = \frac{AB}{QR}$ [Corresponding parts of similar Δare proportional]

$\Rightarrow \frac{PB}{6} = \frac{3}{9}$

$\Rightarrow$ PB $= \frac{3}{9} \times 6 = 2\ cm$

4. In fig. 4.138, XY || BC. Find the length of XY

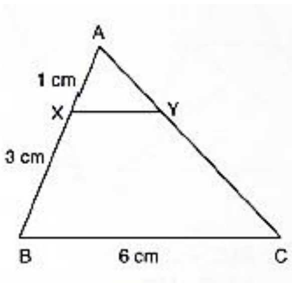


Fig. 4.138

Sol:

We have, XY || BC

In ΔAXY and ΔABC

∠A = ∠A [common]

∠AXY = ∠ABC [corresponding angles]

Then, ΔAXY ~ ΔABC [By AA similarity]

$\therefore \frac{AX}{AB} = \frac{XY}{BC}$ [Corresponding parts of similar Δ are proportional]

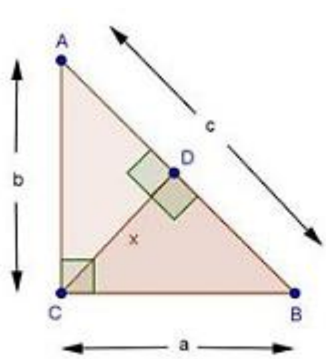
$\Rightarrow \frac{1}{4} = \frac{XY}{6}$

$\Rightarrow$ XY $= \frac{6}{4} = 1.5cm$

5. In a right angled triangle with sides a and b and hypotenuse c, the altitude drawn on the hypotenuse is x. Prove that ab = cx.

Sol:

We have: ∠C = 90° and CD ⊥ AB



In ΔACB and ΔCDB

∠B = ∠B [common]
∠ACB = ∠CDB [Each 90°]
Then, ΔACB ~ ΔCDB [By AA similarity]
$\therefore \frac{AC}{CD} = \frac{AB}{CB}$ [Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{b}{x} = \frac{c}{a}$
⇒ ab = cx

6. In Fig. 4.139, ∠ABC = 90° and BD ⊥ AC. If BD = 8 cm and AD = 4 cm, find CD.

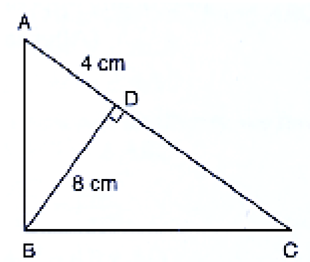


Fig. 4.139

Sol:
We have, ∠ABC = 90° and BD ⊥ AC
Now, ∠ABD + ∠DBC − 90° ...(i) [∵ ∠ABC − 90°]
And, ∠C + ∠DBC − 90° ...(ii) [By angle sum prop. in ΔBCD]
Compare equations (i) & (ii)
∠ABD = ∠C ...(iii)
In ΔABD and ΔBCD
∠ABD = ∠C [From (iii)]
∠ADB = ∠BDC [Each 90°]
Then, ΔABD ~ ΔBCD [By AA similarity]
$\therefore \frac{BD}{CD} = \frac{AD}{BD}$ [Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{8}{CD} = \frac{4}{8}$
$\Rightarrow \text{CD} = \frac{8\times 8}{4} = 16\ cm$

7. In Fig. 4.14, ∠ABC = 90° and BD ⊥ AC. If AB = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm, find BC.

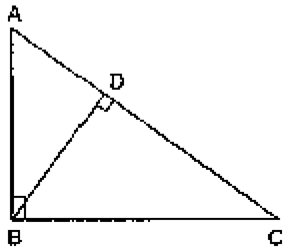


Fig. 4.140

Sol:

We have, ∠ABC = 90° and BD ⊥ AC

In ΔABC and ΔBDC

∠ABC = ∠BDC [Each 90°]

∠C = ∠C [Common]

Then, ΔABC ~ ΔBDC [By AA similarity]

$\therefore \frac{AB}{BD} = \frac{BC}{DC}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{5.7}{3.8} = \frac{BC}{5.4}$

⇒ BC = $\frac{5.7}{3.8} \times 8.1\ cm$

8. In Fig. 4.141, DE || BC such that AE = (1/4) AC. If AB = 6 cm, find AD.

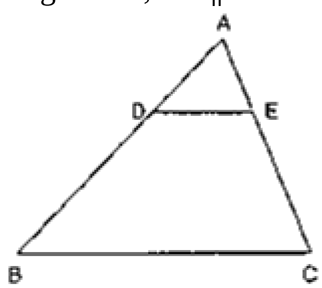


Fig. 4.141

Sol:

We have, DE || BC, AB = 6 cm and AE = $\frac{1}{4}\ AC$

In ΔADE and ΔABC

∠A = ∠A [Common]

∠ADE = ∠ABC [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{AD}{6} = \frac{\frac{1}{4}AC}{AC}$ [∵ AE = $\frac{1}{4}\ AC\ given$]

$\Rightarrow \frac{AD}{6} = \frac{1}{4}$

⇒ AD = $\frac{6}{4}$ = 1.5 cm

9. In fig., 4.142, PA, QB and RC are each perpendicular to AC. Prove that $\frac{1}{x}+\frac{1}{z}+\frac{1}{y}$

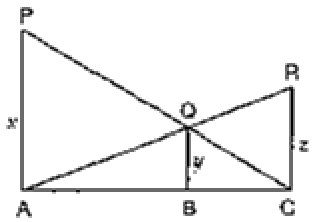


Fig. 4.142

Sol:

We have, PA ⊥ AC, QB ⊥ AC and RC ⊥ AC

Let, AB = a and BC = b

In ΔCQB and ΔCPA

∠QCB = ∠PCA [Common]

∠QBC = ∠PAC [Each 90°]

Then, ΔCQB ~ ΔCPA [By AA similarity]

$\therefore \frac{QB}{PA}=\frac{CB}{CA}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{y}{x}=\frac{b}{a+b}$ ….(i)

In Δ AQB and ΔARC

∠QAB = ∠RAC [common]

∠ABQ = ∠ ACR [Each 90°]

Then, ΔAQB ~ ΔARC [By AA similarity]

$\therefore \frac{QB}{RC}=\frac{AB}{AC}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{y}{z}=\frac{a}{a+b}$ ….(ii)

Adding equations (i) & (ii)

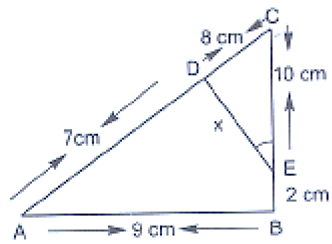
$\frac{y}{x}+\frac{y}{z}=\frac{b}{a+b}+\frac{a}{a+b}$

$\Rightarrow y\left(\frac{1}{x}+\frac{1}{z}\right)=\frac{b+a}{a+b}$

$\Rightarrow y\left(\frac{1}{x}+\frac{1}{z}\right)=1$

$\Rightarrow \frac{1}{x}+\frac{1}{z}=\frac{1}{y}$

10. In below fig., ∠A = ∠CED, Prove that ΔCAB ~ ΔCED. Also, find the value of x.



Sol:

We have, ∠A = ∠CED

In ΔCAB and ΔCED

∠C = ∠C [Common]

∠A = ∠CED [Given]

Then, ΔCAB ~ ΔCED [By AA similarity]

$\therefore \frac{CA}{CE} = \frac{AB}{ED}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{15}{10} = \frac{9}{x}$

$\Rightarrow x = \frac{10\times9}{15} = 6\ cm$

11. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of first triangle is 9 cm, what is the corresponding side of the other triangle?

Sol:

Assume ABC and PQR to be 2 triangles

We have,

ΔABC ~ ΔPQR

Perimeter of Δ ABC = 25 cm

Perimeter of Δ PQR = 15 cm

AB = 9 cm

PQ = ?

Since, ΔABC ~ ΔPQR

Then, ratio of perimeter of triangles = ratio of corresponding sides

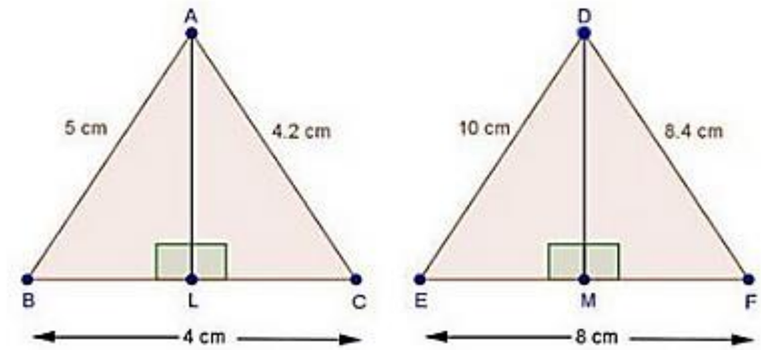
$\Rightarrow \frac{25}{12} = \frac{AB}{PQ}$

$\Rightarrow \frac{25}{15} = \frac{9}{PQ}$

$\Rightarrow$ PQ = $\frac{15\times9}{25} = 5.4\ cm$

12. In ΔABC and ΔDEF, it is being given that: AB = 5 cm, BC = 4 cm and CA = 4.2 cm; DE=10cm, EF = 8 cm and FD = 8.4 cm. If AL ⊥ BC and DM ⊥ EF, find AL: DM.

Sol:



Since, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DE} = \frac{1}{2}$

Then, ΔABC ~ ΔDEF [By SSS similarity]

Now, In ΔABL ~ ΔDEM

∠B = ∠E [Δ ABC ~ ΔDEF]

∠ALB = ∠DME [Each 90°]

Then, ΔABL ~ ΔDEM [By AA similarity]

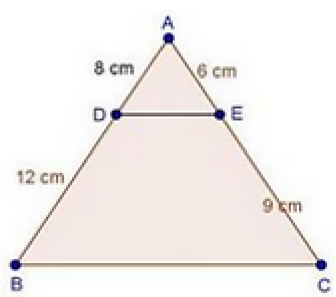
$\therefore \frac{AB}{DE} = \frac{AL}{DM}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{5}{10} = \frac{AL}{DM}$

$\Rightarrow \frac{1}{2} = \frac{AL}{DM}$

13. D and E are the points on the sides AB and AC respectively of a ΔABC such that: AD = 8 cm, DB = 12 cm, AE = 6 cm and CE = 9 cm. Prove that BC = 5/2 DE.

Sol:



We have,

$\frac{AD}{DB} = \frac{8}{12} = \frac{2}{3}$

And, $\frac{AE}{EC} = \frac{6}{9} = \frac{2}{3}$

Since, $\frac{AD}{DB} = \frac{AE}{EC}$

Then, by converse of basic proportionality theorem

DE || BC

In ΔADE and ΔABC

$\angle A = \angle A$ [Common]

$\angle ADE = \angle B$ [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

$\therefore \frac{AD}{AB} = \frac{DE}{BC}$ [Corresponding parts of similar Δ are proportional]

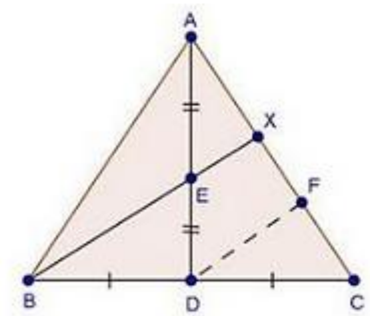
$\Rightarrow \frac{8}{20} = \frac{DE}{BC}$

$\Rightarrow \frac{2}{5} = \frac{DE}{BC}$

$\Rightarrow BC = \frac{5}{2}$ DE

14. D is the mid-point of side BC of a ΔABC. AD is bisected at the point E and BE produced cuts AC at the point X. Prove that BE : EX = 3 : 1

Sol:



Given: In ΔABC, D is the mid-point of BC and E is the mid-point of AD.

To prove: BE : EX = 3 : 1

Const: Through D, draw DF || BX

Proof: In ΔEAX and ΔADF

$\angle EAX = \angle ADF$ [Common]

$\angle AXE = \angle DAF$ [Corresponding angles]

Then, ΔAEX ~ ΔADF [By AA similarity]

$\therefore \frac{EX}{DF} = \frac{AE}{AD}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{EX}{DF} = \frac{AE}{2AE}$ [AE = ED given]

$\Rightarrow DF = 2EX$ …. (i)

In ΔCDF and ΔCBX [By AA similarity]

$\therefore \frac{CD}{CB} = \frac{DF}{BX}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{1}{2} = \frac{DF}{BE+EX}$ [BD = DC given]

$\Rightarrow BE + EX = 2DF$

$\Rightarrow BE + EX = 4EX$

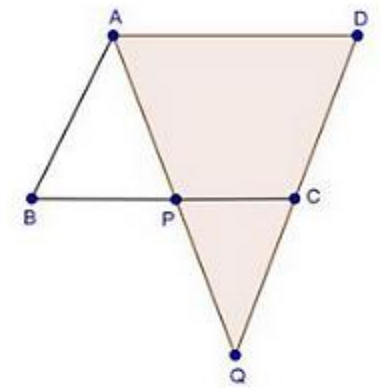
$\Rightarrow BE = 4EX – EX$ [By using (i)]

$\Rightarrow BE = 4EX – EX$

$\Rightarrow \frac{BE}{EX} = \frac{3}{1}$

15. ABCD is a parallelogram and APQ is a straight line meeting BC at P and DC produced at Q. Prove that the rectangle obtained by BP and DQ is equal to the AB and BC.

Sol:



Given: ABCD is a parallelogram

To prove: BP × DQ = AB × BC

Proof: In ΔABP and ΔQDA

$\angle B = \angle D$ [Opposite angles of parallelogram]

$\angle BAP = \angle AQD$ [Alternate interior angles]

Then, ΔABP ~ ΔQDA [By AA similarity]

$\therefore \frac{AB}{QD} = \frac{BP}{DA}$ [Corresponding parts of similar Δ are proportional]

But, DA = BC [Opposite sides of parallelogram]

Then, $\frac{AB}{QD} = \frac{BP}{BC}$

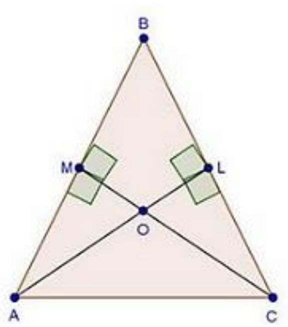
⇒ AB × BC = QD × BP

16. In ΔABC, AL and CM are the perpendiculars from the vertices A and C to BC and AB respectively. If AL and CM intersect at O, prove that:

(i) Δ OMA and ΔOLC

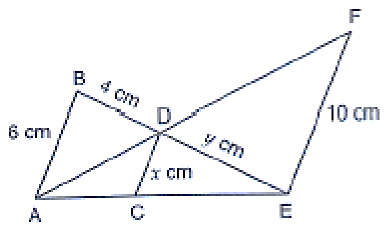
(ii) $\frac{OA}{OC} = \frac{OM}{OL}$

Sol:



We have,
AL ⊥ BC and CM ⊥ AB
In Δ OMA and ΔOLC
∠MOA = ∠LOC [Vertically opposite angles]
∠AMO = ∠CLO [Each 90°]
Then, ΔOMA ~ ΔOLC [By AA similarity]
$\therefore \frac{OA}{OC} = \frac{OM}{OL}$ [Corresponding parts of similar Δ are proportional]

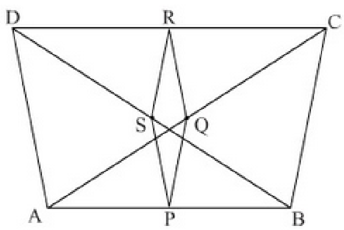
17. In Fig below we have AB || CD || EF. If AB = 6 cm, CD = x cm, EF = 10 cm, BD = 4 cm and DE = y cm, calculate the values of x and y.



Sol:
We have AB || CD || EF. If AB = 6 cm, CD = x cm, EF = 10 cm, BD = 4 cm and DE = y cm
In ΔECD and ΔEAB
∠CED = ∠AEB [common]
∠ECD = ∠EAB [corresponding angles]
Then, ΔECD ~ ΔEAB ….(i) [By AA similarity]
$\therefore \frac{EC}{EA} = \frac{CD}{AB}$ [Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{EC}{EA} = \frac{x}{6}$ ….(ii)
In ΔACD and ΔAEF
∠CAD = ∠EAF [common]
∠ACD = ∠AEF [corresponding angles]
Then, ΔACD ~ ΔAEF [By AA similarity]
$\therefore \frac{AC}{AE} = \frac{CD}{EF}$
$\Rightarrow \frac{AC}{AE} = \frac{x}{10}$ …(iii)
Add equations (iii) & (ii)
$\therefore \frac{EC}{EA} + \frac{AC}{AE} = \frac{x}{6} + \frac{x}{10}$
$\Rightarrow \frac{AE}{AE} = \frac{5x+3x}{30}$
$\Rightarrow 1 = \frac{8x}{30}$
$\Rightarrow x = \frac{30}{8} = 3.75$ cm
From (i) $\frac{DC}{AB} = \frac{ED}{BE}$

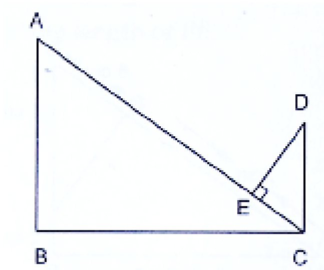
$\Rightarrow \frac{3.75}{6} = \frac{y}{y+4}$
⇒ 6y = 3.75y + 15
⇒ 2.25y = 15
$\Rightarrow y = \frac{15}{2.25} = 6.67\ cm$

18. ABCD is a quadrilateral in which AD = BC. If P, Q, R, S be the mid-points of AB, AC, CD and BD respectively, show that PQRS is a rhombus.
Sol:



AD = BC and P, Q, R and S are the mid-points of sides AB, AC, CD and BD respectively, show that PQRS is a rhombus.
In ΔBAD, by mid-point theorem
PS ∥ AD and PS = $\frac{1}{2}$ AD …(i)
In ΔCAD, by mid-point theorem
QR ∥ AD and QR = $\frac{1}{2}AD$ …(ii)
Compare (i) and (ii)
PS ∥ QR and PS = QR
Since one pair of opposite sides is equal as well as parallel then
PQRS is a parallelogram ...(iii)
Now, In ΔABC, by mid-point theorem
PQ ∥ BC and PQ = $\frac{1}{2}$ BC …(iv)
And, AD = BC …(v) [given]
Compare equations (i) (iv) and (v)
PS = PQ …(vi)
From (iii) and (vi)
Since, PQRS is a parallelogram with PS = PQ then PQRS is a rhombus

19. In Fig. below, if AB ⊥ BC, DC ⊥ BC and DE ⊥ AC, Prove that Δ CED ~ ABC.



Sol:

Given: AB ⊥ BC, DC ⊥ BC and DE ⊥ AC

To prove: ΔCED ~ ΔABC

Proof:

$\angle BAC + \angle BCA = 90°$ ...(i) [By angle sum property]

And, $\angle BCA + \angle ECD = 90°$ ...(ii) [DC ⊥ BC given]

Compare equation (i) and (ii)

$\angle BAC = \angle ECD$ ...(iii)

In ΔCED and ΔABC

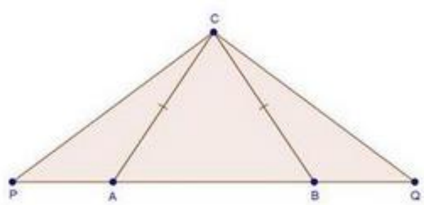
$\angle CED = \angle ABC$ [Each 90°]

$\angle ECD = \angle BAC$ [From (iii)]

Then, ΔCED ~ ΔABC [By AA similarity]

20. In an isosceles ΔABC, the base AB is produced both the ways to P and Q such that AP × BQ = AC^2. Prove that ΔAPC ~ ΔBCQ.

Sol:



Given: In ΔABC, CA = CB and AP × BQ = AC2

To prove: ΔAPC ~ ΔBCQ

Proof:

AP × BQ = AC^2 [Given]

⇒ AP × BQ = AC × AC

⇒ AP × BQ = AC × BC [AC = BC given]

$\Rightarrow \frac{AP}{BC} = \frac{AC}{BQ}$ …(i)

Since, CA = CB [Given]

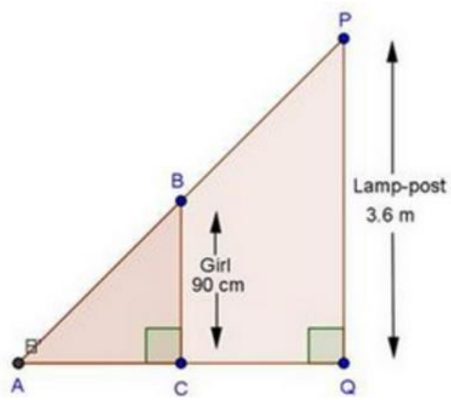
Then, ∠CAB = ∠CBA …(ii) [Opposite angles to equal sides]

Now, ∠CAB + ∠CAP = 180° …(iii) [Linear pair of angles]

And, ∠CBA + ∠CBQ = 180° …(iv) [Linear pair of angles]

Compare equation (ii) (iii) & (iv)
∠CAP = ∠CBQ …(v)
In ΔAPC and ΔBCQ
∠CAP = ∠CBQ [From (v)]
$\frac{AP}{BC} = \frac{AC}{BQ}$ [From (i)]
$Then, \Delta APC \sim \Delta BCQ$ [By SAS similarity]

21. A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2m/sec. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.
Sol:

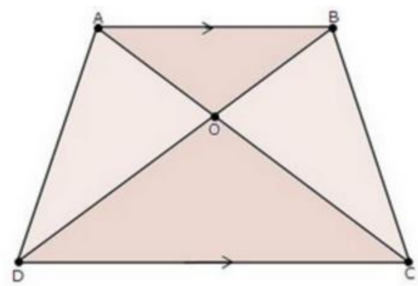


We have,
Height of girl = 90 cm = 0.9 m
Height of lamp-post = 3.6 m
Speed of girl = 1.2 m/sec
∴ Distance moved by girl (CQ) = Speed × Time
= 1.2 × 4 = 4.8m
Let length of shadow (AC) = x cm
In ΔABC and ΔAPQ
∠ACB = ∠AQP [Each 90°]
∠BAC = ∠PAQ [Common]
Then, ΔAB ~ ΔAPQ [By AA similarity]
$\therefore \frac{AC}{AQ} = \frac{BC}{PQ}$ [Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{x}{x+4.8} = \frac{0.9}{3.6}$
$\Rightarrow \frac{x}{x+4.8} = \frac{1}{4}$
⇒ 4x = x + 4.8
⇒ 4x – x = 4.8
⇒ 3x = 4.8
$\Rightarrow x = \frac{4.8}{3} = 1.6\ m$

∴ Length of shadow = 1.6m

22. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Sol:



We have,

ABCD is a trapezium with AB || DC

In ΔAOB and ΔCOD

∠AOB = ∠COD [Vertically opposite angles]

∠OAB = ∠OCD [Alternate interior angles]

Then, ΔAOB ~ ΔCOD [By AA similarity]

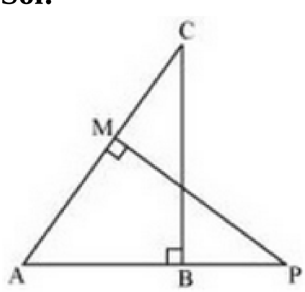
∴ $\frac{OA}{OC} = \frac{OB}{OD}$ [Corresponding parts of similar Δ are proportional]

23. If ΔABC and ΔAMP are two right triangles, right angled at B and M respectively such that ∠MAP = ∠BAC. Prove that

(i) ΔABC ~ ΔAMP

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Sol:



We have,

∠B = ∠M = 90°

And, ∠BAC = ∠MAP

In ΔABC and ΔAMP

∠B = ∠M [Each 90°]

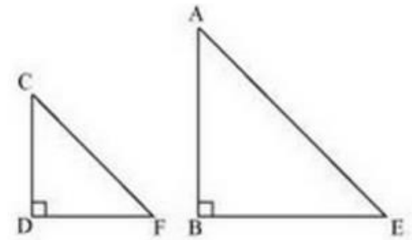
∠BAC = ∠MAP [Given]

Then, ΔABC ~ ΔAMP [By AA similarity]

$\therefore \frac{CA}{PA} = \frac{BC}{MP}$ [Corresponding parts of similar Δ are proportional]

24. A vertical stick of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Sol:



Let AB be a tower
CD be a stick, CD = 6m
Shadow of AB is BE = 28m
Shadow of CD is DF = 4m
At same time light rays from sun will fall on tower and stick at same angle.
So, ∠DCF = ∠BAE
And ∠DFC = ∠BEA
∠CDF = ∠ABE (tower and stick are vertical to ground)
Therefore Δ ABE ~ ΔCDF (By AA similarity)
So,

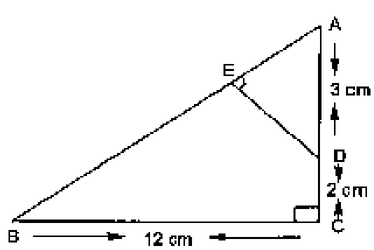
$\frac{AB}{CD} = \frac{BE}{DF}$

$\frac{AB}{6} = \frac{28}{4}$

AB = $28 \times \frac{6}{4} = 42m$

So, height of tower will be 42 metres.

25. In below Fig., ΔABC is right angled at C and DE ⊥ AB. Prove that ΔABC ~ ΔADE and Hence find the lengths of AE and DE.



Sol:

In ΔACB, by Pythagoras theorem

$AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = (5)^2 + (12)^2$

$\Rightarrow AB^2 = 25 + 144 = 169$

$\Rightarrow$ AB $= \sqrt{169} = 13\ cm$

In ΔAED and ΔACB

∠A = ∠A [Common]

∠AED = ∠ACB [Each 90°]

Then, ΔAED ~ ΔACB [By AA similarity]

$\therefore \frac{AE}{AC} = \frac{DE}{CB} = \frac{AD}{AB}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{AE}{5} = \frac{DE}{12} = \frac{3}{13}$

$\Rightarrow \frac{AE}{5} = \frac{3}{13}$ and $\frac{DE}{12} = \frac{3}{13}$

$\Rightarrow$ AE $= \frac{15}{13}$ cm and DE $= \frac{36}{13}$ cm

Exercise 4.6

1. Triangles ABC and DEF are similar

(i) If area $(\Delta ABC) = 16 cm^2$, area $(\Delta DEF) = 25\ cm^2$ and BC = 2.3 cm, find EF.

(ii) If area $(\Delta ABC) = 9cm^2$, area $(\Delta DEF) = 64\ cm^2$ and DE = 5.1 cm, find AB.

(iii)If AC = 19cm and DF = 8 cm, find the ratio of the area of two triangles.

(iv)If area $(\Delta ABC) = 36cm^2$, area $(\Delta DEF) = 64\ cm^2$ and DE = 6.2 cm, find AB.

(v) If AB = 1.2 cm and DE = 1.4 cm, find the ratio of the areas of ΔABC and ΔDEF.

Sol:

(i)

We have,

ΔABC ~ΔDEF

Area $(\Delta ABC) = 16\ cm^2$,

Area $(\Delta DEF) = 25\ cm^2$

And BC = 2.3 cm

Since, ΔABC ~ ΔDEF

Then, $\frac{Area\ (\Delta ABC)}{Area\ (\Delta\ DEF)} = \frac{BC^2}{EF^2}$ [By area of similar triangle theorem]

$\Rightarrow \frac{16}{25} = \frac{(2.3)^2}{EF^2}$

$\Rightarrow \frac{4}{5} = \frac{2.3}{EF}$ [By taking square root]

$\Rightarrow$ EF $= \frac{11.5}{4} = 2.875\ cm$

(ii)

We have,

ΔABC ~ ΔDEF

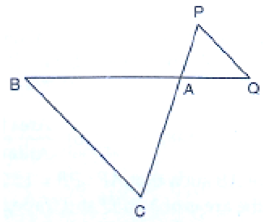
Area(ΔABC) = 9 cm^2

Area (ΔDEF) = 64 cm^2

And DE = 5.1 cm
Since, ΔABC ~ ΔDEF
Then, $\frac{Area\ (\Delta ABC)}{Area\ (\Delta DEF)} = \frac{AB^2}{DE^2}$ [By area of similar triangle theorem]
$\Rightarrow \frac{9}{64} = \frac{AB^2}{(5.1)^2}$
$\Rightarrow \frac{3}{8} = \frac{AB}{5.1}$ [By taking square root]
$\Rightarrow$ AB $= \frac{3\times 5.1}{8} = 1.9125\ cm$
(iii)
We have,
ΔABC ~ ΔDEF
AC = 19 cm and DF = 8 cm
By area of similar triangle theorem
$\frac{Area\ (\Delta\ ABC)}{Area\ (\Delta DEF)} = \frac{AC^2}{DF^2} = \frac{(19)^2}{8^2} = \frac{361}{64}$
We have,
ΔABC ~ ΔDEF
AC = 19 cm and DF = 8 cm
By area of similar triangle theorem
$\frac{Area\ (\Delta ABC)}{Area\ (\Delta DEF)} = \frac{AC^2}{DF^2} = \frac{(19)^2}{8^2} = \frac{361}{64}$
(iv)
We have, Area (ΔABC) = 36 cm^2
Area $(\Delta DEF) = 64\ cm^2$
DE = 6.2 cm
And, ΔABC ~ ΔDEF
By area of similar triangle theorem
$\frac{Area(\ \Delta ABC)}{Area\ (\Delta DEF)} = \frac{AB^2}{DE^2}$
$\Rightarrow \frac{36}{64} = \frac{AB^2}{(6.2)^2}$ [By taking square root]
$\Rightarrow$ AB = $\frac{6\times 6.2}{8} = 4.65\ cm$
(v)
We have,
ΔABC ~ ΔDEF
AB = 1.2 cm and DF = 1.4 cm
By area of similar triangle theorem
$\frac{Area\ (\Delta\ ABC)}{Area\ (\Delta\ DEF)} = \frac{AB^2}{DE^2}$
$= \frac{(1.2)^2}{(1.4)^2}$

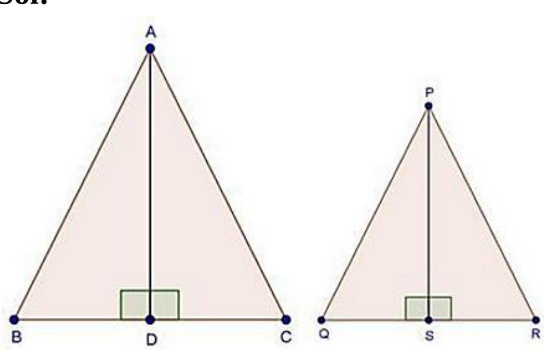
$= \frac{1.44}{1.96}$
$= \frac{36}{49}$

2. In fig. below ΔACB ~ ΔAPQ. If BC = 10 cm, PQ = 5 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ. Also, find the area (ΔACB): $area\ (\Delta APQ)$



Sol:
We have,
ΔACB ~ ΔAPQ
Then, $\frac{AC}{AP} = \frac{CB}{PQ} = \frac{AB}{AQ}$ [Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{AC}{2.8} = \frac{10}{5} = \frac{6.5}{AQ}$
$\Rightarrow \frac{AC}{2.8} = \frac{10}{5}\ and\ \frac{10}{5} = \frac{6.5}{AQ}$
$\Rightarrow$ AC = $\frac{10}{5} \times 2.8\ and\ AQ = 6.5 \times \frac{5}{10}$
$\Rightarrow$ AC = 5.6 cm and AQ = 3.25 cm
By area of similar triangle theorem
$\frac{Area\ (\Delta ACB)}{Area\ (\Delta APQ)} = \frac{BC^2}{PQ^2}$
$= \frac{(10)^2}{(5)^2}$
$= \frac{100}{25}$
$= \frac{4}{1}$

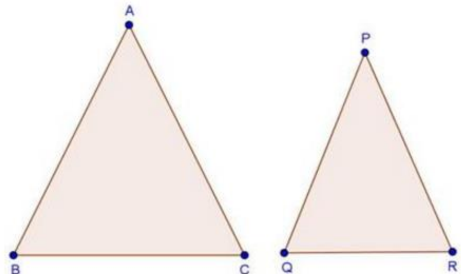
3. The areas of two similar triangles are 81 cm^2 and 49 cm^2 respectively. Find the ratio of their corresponding heights. What is the ratio of their corresponding medians?
Sol:



We have,
ΔABC ~ ΔPQR
Area (ΔABC) = 81 cm^2,
Area (ΔPQR) = 49 cm^2
And AD and PS are the altitudes
By area of similar triangle theorem
$\frac{Area\ (\Delta ABC)}{Area\ (\Delta PQR)} = \frac{AB^2}{PQ^2}$
$\Rightarrow \frac{81}{49} = \frac{AB^2}{PQ^2}$
$\Rightarrow \frac{9}{7} = \frac{AB}{PQ}$ ….(i) [Taking square root]
$In\ \Delta ABD\ and\ \Delta PQS$
∠B = ∠Q [ΔABC ~ ΔPQR]
∠ ADB = ∠PSQ [Each 90°]
Then, ΔABD ~ ΔPQS [By AA similarity]
$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$ …(ii) [Corresponding parts of similar Δ are proportional]
Compare (1) and (2)
$\frac{AD}{PS} = \frac{9}{7}$
∴ Ratio of altitudes = $\frac{9}{7}$
Since, the ratio of the area of two similar triangles is equal to the ratio of the squares of the squares of their corresponding altitudes and is also equal to the squares of their corresponding medians.
Hence, ratio of altitudes = Ratio of medians = 9 : 7

4. The areas of two similar triangles are 169 cm^2 and 121 cm^2 respectively. If the longest side of the larger triangle is 26 cm, find the longest side of the smaller triangle.

Sol:



We have,
ΔABC ~ ΔPQR
Area(ΔABC) = 169 cm^2
Area(ΔPQR) = 121 cm2
And AB = 26 cm

By area of similar triangle theorem

$\frac{Area\ (\Delta ABC)}{Area\ (\Delta\ PQR)} = \frac{AB^2}{PQ^2}$

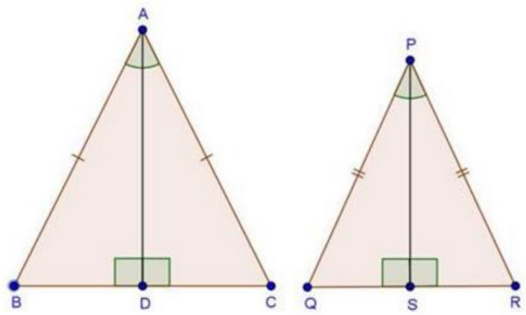
$\Rightarrow \frac{169}{121} = \frac{(26)^2}{PQ^2}$

$\Rightarrow \frac{13}{11} = \frac{26}{PQ}$ [Taking square root]

⇒ PQ = $\frac{11}{13} \times 26 = 22\ cm$

5. Two isosceles triangles have equal vertical angles and their areas are in the ratio 36 : 25. Find the ratio of their corresponding heights.

Sol:



Given: AB = AC, PQ = PQ and ∠A = ∠P

And, AD and PS are altitudes

And, $\frac{Area\ (\Delta ABC)}{Area\ (\Delta\ PQR)} = \frac{36}{25}$ …(i)

To find: $\frac{AD}{PS}$

Proof: Since, AB = AC and PQ = PR

Then, $\frac{AB}{AC} = 1$ and $\frac{PQ}{PR} = 1$

$\therefore \frac{AB}{AC} = \frac{PQ}{PR}$

$\Rightarrow \frac{AB}{PQ} = \frac{AC}{PR}$ …(ii)

In ΔABC and ΔPQR

∠A = ∠P [Given]

$\frac{AB}{PQ} = \frac{AC}{PR}$ [From (2)]

$Then, \Delta ABC \sim \Delta PQR$ [By SAS similarity]

$\therefore \frac{Area\ (\Delta ABC)}{Area\ (\Delta\ PQR)} = \frac{AB^2}{PQ^2}$ ….(iii) [By area of similar triangle theorem]

$Compare\ equation\ (i) and\ (iii)$

$\frac{AB^2}{PQ^2} = \frac{36}{25}$

$\Rightarrow \frac{AB}{PQ} = \frac{6}{5}$ ….(iv)

In ΔABD and ΔPQS

∠B = ∠Q [ΔABC ~ ΔPQR]

∠ADB = ∠PSQ [Each 90°]

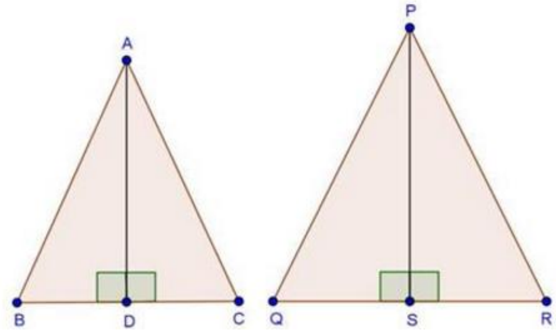
Then, ΔABD ~ ΔPQS [By AA similarity]

$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$

$\Rightarrow \frac{6}{5} = \frac{AD}{PS}$ [From (iv)]

6. The areas of two similar triangles are 25 cm^2 and 36 cm^2 respectively. If the altitude of the first triangle is 2.4 cm, find the corresponding altitude of the other.

Sol:



We have,

ΔABC ~ ΔPQR

Area (ΔABC) = 25 cm^2

Area (ΔPQR) = 36 cm^2

AD = 2.4 cm

And AD and PS are the altitudes

To find: PS

Proof: Since, ΔABC ~ ΔPQR

Then, by area of similar triangle theorem

$\frac{Area\ (\Delta ABC)}{Area\ (\Delta PQR)} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{25}{36} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{5}{6} = \frac{AB}{PQ}$ ….(i)

$In\ \Delta ABD\ and\ \Delta PQS$

∠B = ∠Q [Δ ABC ~ ΔPQR]

∠ADB ~ ∠PSQ [Each 90°]

Then, ΔABD ~ ΔPQS [By AA similarity]

$\therefore \frac{AB}{PS} = \frac{AD}{PS}$ ….(ii) [Corresponding parts of similar Δ are proportional]

Compare (i) and (ii)

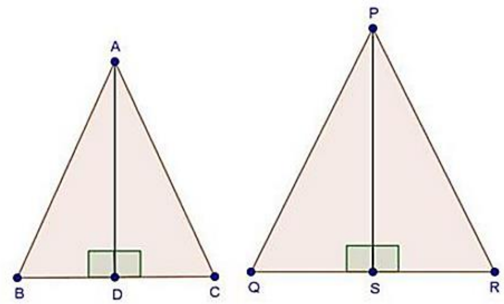
$\frac{AD}{PS} = \frac{5}{6}$

$\Rightarrow \frac{2.4}{PS} = \frac{5}{6}$

$\Rightarrow PS = \frac{2.4\times6}{5} = 2.88\ cm$

7. The corresponding altitudes of two similar triangles are 6 cm and 9 cm respectively. Find the ratio of their areas.

Sol:



We have,

ΔABC ~ ΔPQR

AD = 6 cm

And, PS = 9 cm

By area of similar triangle theorem

$\frac{Area\ (\Delta ABC)}{Area\ (\Delta PQR)} = \frac{AB^2}{PQ^2}$ …(i)

$In\ \Delta ABD\ and\ \Delta PQS$

∠B = ∠Q [ΔABC ~ ΔPQR]

∠ADB = ∠PSQ [Each 90°]

Then, ΔABD ~ ΔPQS [By AA similarity]

$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{AB}{PQ} = \frac{6}{9}$

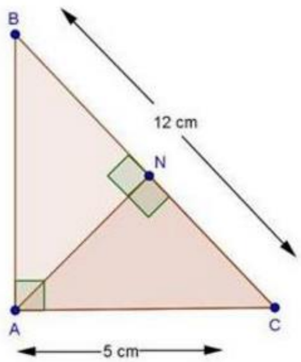
$\Rightarrow \frac{AB}{PQ} = \frac{2}{3}$ …(ii)

Compare equations (i) and (ii)

$\frac{Area(\Delta ABC)}{Area\ (\Delta PQR)} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

8. ABC is a triangle in which ∠A =90°, AN⊥ BC, BC = 12 cm and AC = 5cm. Find the ratio of the areas of ΔANC and ΔABC.

Sol:



In ΔANC and ΔABC

∠C = ∠C [Common]

∠ANC = ∠BAC [Each 90°]

Then, ΔANC ~ ΔBAC [By AA similarity]

By area of similarity triangle theorem

$\frac{Area\ (\Delta\ ANC)}{Area\ (\Delta BAC)} = \frac{AC^2}{BC^2}$

$= \frac{5^2}{12^2}$

$= \frac{25}{144}$

9. In Fig. 4.178, DE || BC

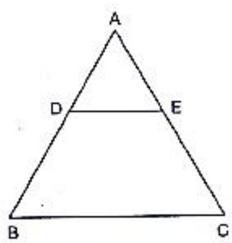


Fig. 4.178

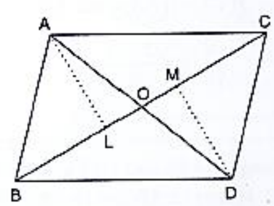


Fig. 4.179

(i) If DE = 4 cm, BC = 6 cm and Area (ΔADE) = 16 cm^2, find the area of ΔABC.

(ii) If DE = 4cm, BC = 8 cm and Area (ΔADE) = 25 cm^2, find the area of ΔABC.

(iii)If DE : BC = 3 : 5. Calculate the ratio of the areas of ΔADE and the trapezium BCED.

Sol:

We have, DE || BC, DE = 4 cm, BC = 6 cm and area $(\Delta ADE) = 16cm^2$

In ΔADE and ΔABC

∠A = ∠A [Common]

∠ADE = ∠ABC [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

∴ By area of similar triangle theorem

$\frac{Area\ (\Delta ADE)}{Area\ (\Delta ABC)} = \frac{DE^2}{BC^2}$

$\Rightarrow \frac{16}{Area\ (\Delta ABC)} = \frac{4^2}{6^2}$

$\Rightarrow Area\ (\Delta ABC) = \frac{16\times36}{16} = 36cm^2$

$we\ have, DE\ ||BC, DE = 4\ cm, BC = 8\ cm\ and\ area\ (\Delta ADE) = 25\ cm^2$

$In\ \Delta ADE\ and\ \Delta ABC$

∠A = ∠A [Common]

∠ADE = ∠ABC [Corresponding angles]

Then, $\Delta ADE \sim \Delta ABC$ [By AA similarity]

By area of similar triangle theorem

$\frac{Area\ (\Delta ADE)}{Area\ (\Delta\ ABC)} = \frac{DE^2}{BC^2}$

$\Rightarrow \frac{16}{Area\ (\Delta ABC)} = \frac{4^2}{6^2}$

⇒ Area (ΔABC) = $\frac{16\times36}{16} = 36\ cm^2$

We have, DE || BC, DE = 4 cm, BC = 8 cm and area (ΔADE) = $25cm^2$

In ΔADE and ΔABC

∠A = ∠A [Common]

∠ADE = ∠ABC [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

By area of similar triangle theorem

$\Rightarrow \frac{Area\ (\Delta ADE)}{Area\ (\Delta ABC)} = \frac{DE^2}{BC^2}$

$\frac{25}{Area\ (\Delta ABC)} = \frac{4^2}{8^2}$

$\Rightarrow Area\ (\Delta ABC) = \frac{25\times64}{16} = 100\ \ cm^2$

We have, DE || BC, and $\frac{DE}{BC} = \frac{3}{5}$ ….(i)

In ΔADE and ΔABC

∠A = ∠A [Common]

∠ADE = ∠B [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

By area of similar triangle theorem

$\Rightarrow \frac{Area\ (\Delta ADE)}{Area\ (\Delta ABC)} = \frac{DE^2}{BC^2}$

$\Rightarrow \frac{ar\ (\Delta ADE)}{ar(\Delta ADE)+ar(trap.DECB)} = \frac{3^2}{5^2}$ [From (i)]

⇒ 25ar (ΔADE) = $9ar\ (\Delta ADE) + 9ar\ (trap.DECB)$

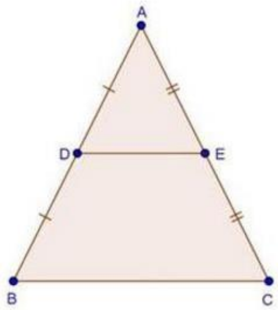
⇒ 25 ar (ΔADE – 9ar) (ΔADE) = 9ar (trap.DECB)

⇒ 16 ar(ΔADE) = 9 ar (trap. DECB)

$\Rightarrow \frac{ar\ (\Delta ADE)}{ar\ (trap.DECB)} = \frac{9}{16}$

10. In ΔABC, D and E are the mid-points of AB and AC respectively. Find the ratio of the areas of ΔADE and ΔABC

Sol:



We have, D and E as the mid-points of AB and AC

So, according to the mid-point theorem

DE || BC and DE = $\frac{1}{2}\ BC$ …(i)

In ΔADE and ΔABC

∠A = ∠A [Common]

∠ADE = ∠B [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

By area of similar triangle theorem

$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{DE^2}{BC^2}$

$= \frac{\left(\frac{1}{2}BC\right)^2}{BC^2}$ [From (i)]

$= \frac{\frac{1}{4}BC^2}{BC^2}$

$= \frac{1}{4}$

11. In Fig., 4.179, ΔABC and ΔDBC are on the same base BC. If AD and BC intersect at O, prove that $\frac{area\ (\Delta ABC)}{area\ (\Delta DBC)} = \frac{AO}{DO}$

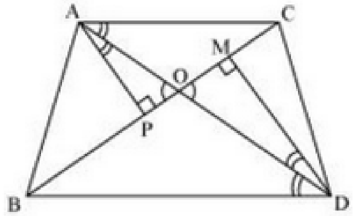
Sol:

We know that area of a triangle = $\frac{1}{2} \times Base \times height$

Since ΔABC and ΔDBC are one same base,

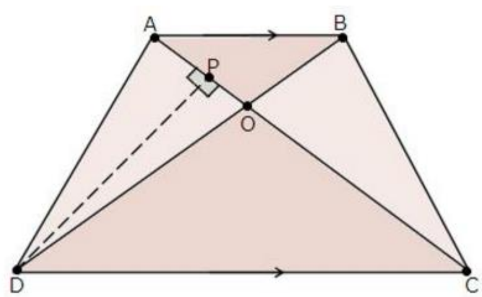
Therefore ratio between their areas will be as ratio of their heights.

Let us draw two perpendiculars AP and DM on line BC.



In ΔAPO and ΔDMO,
∠APO = ∠DMO (Each is 90°)
∠AOP = ∠DOM (vertically opposite angles)
∠OAP = ∠ODM (remaining angle)
Therefore ΔAPO ~ ΔDMO (By AAA rule)
Therefore $\frac{AP}{DM} = \frac{AO}{DO}$
Therefore $\frac{area\ (\Delta ABC)}{area\ (\Delta DBC)} = \frac{AO}{DO}$

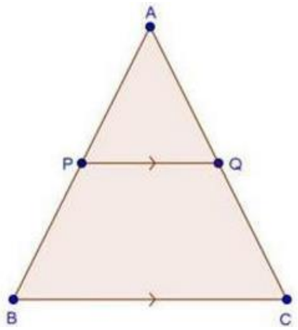
12. ABCD is a trapezium in which AB || CD. The diagonals AC and BD intersect at O. Prove that: (i) ΔAOB and ΔCOD (ii) If OA = 6 cm, OC = 8 cm,
Find:
(a) $\frac{area\ (\Delta AOB)}{area\ (\Delta COD)}$
(b) $\frac{area\ (\Delta AOD)}{area\ (\Delta COD)}$
Sol:



We have,
AB || DC
In ΔAOB and ΔCOD
∠AOB = ∠COD [Vertically opposite angles]
∠OAB = ∠OCD [Alternate interior angles]
Then, ΔAOB ~ ΔCOD [By AA similarity]
(a) By area of similar triangle theorem
$\frac{ar\ (\Delta AOB)}{ar\ (\Delta COD)} = \frac{OA^2}{OC^2} = \frac{6^2}{8^2} = \frac{36}{64} = \frac{9}{16}$
(b) Draw DP ⊥ AC
$\therefore \frac{area\ (\Delta AOD)}{area\ (\Delta COD)} = \frac{\frac{1}{2} \times AO \times DP}{\frac{1}{2} \times CO \times DP}$
$= \frac{AO}{CO}$
$= \frac{6}{8}$
$= \frac{3}{4}$

13. In ABC, P divides the side AB such that AP : PB = 1 : 2. Q is a point in AC such that PQ || BC. Find the ratio of the areas of ΔAPQ and trapezium BPQC.

Sol:



We have,

PQ || BC

And $\frac{AP}{PB} = \frac{1}{2}$

In ΔAPQ and ΔABC

∠A = ∠A [Common]

∠APQ = ∠B [Corresponding angles]

Then, ΔAPQ ~ΔABC [By AA similarity]

By area of similar triangle theorem

$\frac{ar\ (\Delta APQ)}{ar\ (\Delta ABC)} = \frac{AP^2}{AB^2}$

$\Rightarrow \frac{ar\ (\Delta APQ)}{ar(\Delta APQ)+ar(trap.BPQC)} = \frac{1^2}{3^2}\ \left[\frac{AP}{PB} = \frac{1}{2}\right]$

$\Rightarrow$ 9ar $(APQ) = ar(\Delta APQ) + ar(trap.BPQC)$

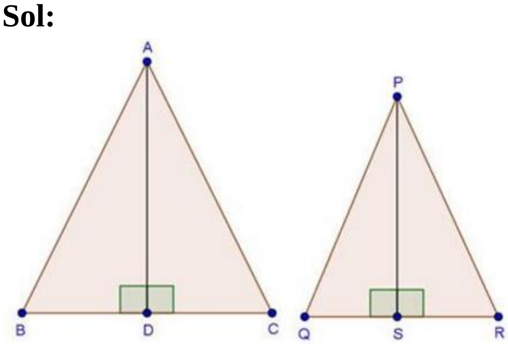
$\Rightarrow$ 9ar $(APQ) - ar(\Delta APQ) + ar(trap.BPQC)$

⇒ 8ar(APQ) = ar(trap. BPQC)

$\Rightarrow \frac{ar\ (\Delta APQ)}{ar(trap.BPQC)} = \frac{1}{8}$

14. The areas of two similar triangles are 100 cm^2 and 49 cm^2 respectively. If the altitude the bigger triangle is 5 cm, find the corresponding altitude of the other.

Sol:



We have, ΔABC ~ ΔPQR

Area(ΔABC) = 100 cm²,
Area (ΔPQR) = 49 cm²
AD = 5 cm
And AD and PS are the altitudes
By area of similar triangle theorem

$\frac{Area\ (\Delta ABC)}{Area\ (\Delta PQR)} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{100}{49} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{10}{7} = \frac{AB}{PQ}$ ...(i)

In ΔABD and ΔPQS

∠B = ∠Q [ΔABC ~ ΔPQR]
∠ADB = ∠PSQ [Each 90°]
Then, ΔABD ~ ΔPQS [By AA similarity]

$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$ ...(ii) [Corresponding parts of similar Δ are proportional]

Compare (i) and (ii)

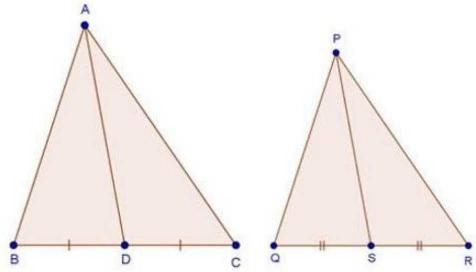
$\frac{AD}{PS} = \frac{10}{7}$

$\Rightarrow \frac{5}{PS} = \frac{10}{7}$

$\Rightarrow PS = \frac{5\times 7}{10} = 3.5\ cm$

15. The areas of two similar triangles are 121 cm² and 64 cm² respectively. If the median of the first triangle is 12.1 cm, find the corresponding median of the other.

Sol:



We have,
ΔABC ~ ΔPQR
Area (ΔABC) = 121 cm²,
Area (ΔPQR) = 64 cm²
AD = 12.1 cm
And AD and PS are the medians
By area of similar triangle theorem

$\frac{Area(\Delta ABC)}{Area(\Delta PQR)} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{121}{64} = \frac{AB^2}{PQ^2}$

$\Rightarrow \frac{11}{8} = \frac{AB}{PQ}$ …(i)

Since, ΔABC ~ ΔPQR

Then, $\frac{AB}{PQ} = \frac{BC}{QR}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QS}$ [AD and PS are medians]

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QS}$ …(ii)

In ΔABD and ΔPQS

∠B = ∠Q [ΔABC ~ ΔPQS]

$\frac{AB}{PQ} = \frac{BD}{QS}$ [From (ii)]

$Then, \Delta ABD \sim \Delta PQS$ [By SAS similarity]

$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$ …(iii) [Corresponding parts of similar Δ are proportional]

$Compare\ (i)\ and\ (iii)$

$\frac{11}{8} = \frac{AD}{PS}$

$\Rightarrow \frac{11}{8} = \frac{12.1}{PS}$

$\Rightarrow$ PS = $\frac{8 \times 12.1}{PS}$

$\Rightarrow$ PS = $\frac{8 \times 12.1}{PS} = 8.8\ cm$

16. If ΔABC ~ ΔDEF such that AB = 5 cm, area (ΔABC) = 20 cm^2 and area (ΔDEF) = 45 cm^2, determine DE.

Sol:

We have,

ΔABC ~ ΔDEF such that AB = 5 cm,

Area (ΔABC) = 20 cm^2 and area(ΔDEF) = 45 cm^2

By area of similar triangle theorem

$\frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{AB^2}{DE^2}$

$\Rightarrow \frac{20}{45} = \frac{5^2}{DE^2}$

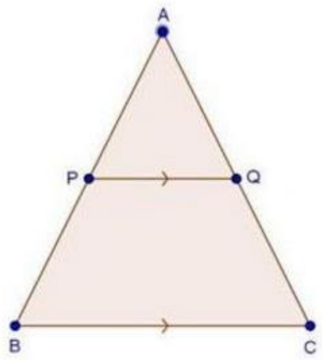
$\Rightarrow \frac{4}{9} = \frac{5^2}{DE^2}$

$\Rightarrow \frac{2}{3} = \frac{5}{DE}$ [Taking square root]

$\Rightarrow DE = \frac{3 \times 5}{2} = 7.5\ cm$

17. In ΔABC, PQ is a line segment intersecting AB at P and AC at Q such that PQ || BC and PQ divides ΔABC into two parts equal in area. Find $\frac{BP}{AB}$

Sol:



We have,

PQ || BC

And ar(ΔAPQ) = ar(trap. PQCB)

⇒ ar(ΔAPQ) = ar(ΔABC) – ar(ΔAPQ)

⇒ 2ar(ΔAPQ) = ar(ΔABC) …(i)

In ΔAPQ and ΔABC

∠A = ∠A [common]

∠APQ = ∠B [corresponding angles]

Then, ΔAPQ ~ ΔABC [By AA similarity]

∴ By area of similar triangle theorem

$\frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2}$

$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta APQ)} = \frac{AP^2}{AB^2}$ [By using (i)]

$\Rightarrow \frac{1}{2} = \frac{AP^2}{AB^2}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AP}{AB^2}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AP}{AB}$ [Taking square root]

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AB-BP}{AB}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AB}{AB} - \frac{BP}{AB}$

$\Rightarrow \frac{1}{\sqrt{2}} = 1 - \frac{BP}{AB}$

$= \frac{BP}{AB} = 1 - \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{BP}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$

18. The areas of two similar triangles ABC and PQR are in the ratio 9:16. If BC = 4.5 cm, find the length of QR.

Sol:

We have,

ΔABC ~ ΔPQR

$\frac{area\ (\Delta ABC)}{area\ (\Delta PQR)} = \frac{BC^2}{QR^2}$

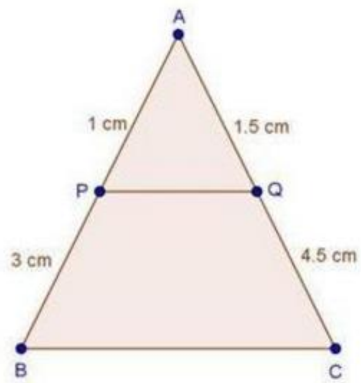
$\Rightarrow \frac{9}{16} = \frac{(4.5)^2}{QR^2}$

$\Rightarrow \frac{3}{4} = \frac{4.5}{QR}$ [Taking square root]

$\Rightarrow$ QR = $\frac{4 \times 4.5}{3} = 6cm$

19. ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If AP = 1 cm, PB = 3 cm, AQ = 1.5 cm, QC = 4.5 m, prove that area of ΔAPQ is one- sixteenth of the area of ABC.

Sol:



We have,

AP = 1 cm, PB = 3 cm, AQ = 1.5 cm and QC = 4.5 m

In ΔAPQ and ΔABC

∠A = ∠A [Common]

$\frac{AP}{AB} = \frac{AQ}{AC}$ [Each equal to $\frac{1}{4}$]

$Then, \Delta APQ \sim \Delta ABC$ [By SAS similarity]

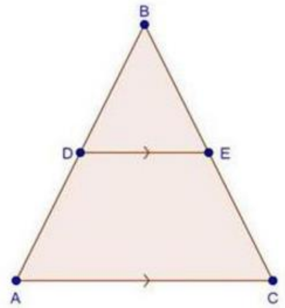
$By\ area\ of\ similar\ triangle\ theorem$

$\frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{1^2}{4^2}$

$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{1}{16} \times ar(\Delta ABC)$

20. If D is a point on the side AB of ΔABC such that AD : DB = 3.2 and E is a Point on BC such that DE || AC. Find the ratio of areas of ΔABC and ΔBDE.

Sol:



We have,

$\frac{AD}{DB} = \frac{3}{2}$

$\Rightarrow \frac{DB}{AD} = \frac{2}{3}$

In ΔBDE and ΔBAC

∠B = ∠B [common]

∠ BDE = ∠A [corresponding angles]

Then, ΔBDE ~ ΔBAC [By AA similarity]

By area of similar triangle theorem

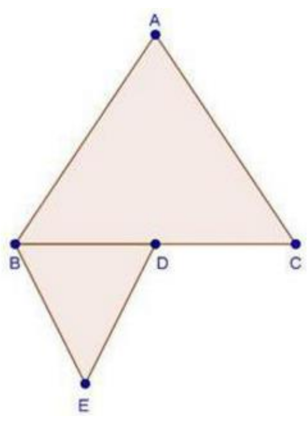
$\frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \frac{AB^2}{BD^2}$

$= \frac{5^2}{2^2}$ $\left[\frac{AD}{DB} = \frac{3}{2}\right]$

$= \frac{25}{4}$

21. If ΔABC and ΔBDE are equilateral triangles, where D is the mid-point of BC, find the ratio of areas of ΔABC and ΔBDE.

Sol:



We have,

ΔABC and ΔBDE are equilateral triangles then both triangles are equiangular

∴ ΔABC ~ ΔBDE [By AAA similarity]

By area of similar triangle theorem

$\frac{ar(\Delta ABC)}{ar\ (\Delta BDE)} = \frac{BC^2}{BD^2}$

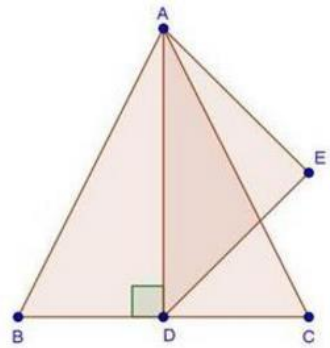
$= \frac{2(BD)^2}{BD^2}$ [D is the mid-point of BC]

$= \frac{4BD^2}{BD^2}$

$= \frac{4}{1}$

22. AD is an altitude of an equilateral triangle ABC. On AD as base, another equilateral triangle ADE is constructed. Prove that Area (ΔADE): Area (ΔABC) = 3: 4

Sol:



We have,

ΔABC is an equilateral triangle

Then, AB = BC = AC

Let, AB = BC = AC = 2x

Since, AD ⊥ BC then BD = DC = x

In ΔADB, by Pythagoras theorem

$AB^2 = (2x)^2 - (x)^2$

$\Rightarrow AD^2 = 4x^2 - x^2 = 3x^2$

$\Rightarrow AD = \sqrt{3}x\ cm$

$Since, \Delta ABC\ and\ \Delta ADE\ both\ are\ equilateral\ triangles\ then\ they\ are\ equiangular$

∴ ΔABC ~ ΔADE [By AA similarity]

By area of similar triangle theorem

$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{AD^2}{AB^2}$

$= \frac{(\sqrt{3}x)^2}{(2x)^2}$

$= \frac{3x^2}{4x^2}$

$= \frac{3}{4}$

Exercise 4.7

1. If the sides of a triangle are 3 cm, 4 cm, and 6 cm long, determine whether the triangle is a right-angled triangle.

Sol:

We have,

Sides of triangle

AB = 3 cm

BC = 4 cm

AC = 6 cm

$\therefore AB^2 = 3^2 = 9$

$BC^2 = 4^2 = 16$

$AC^2 = 6^2 = 36$

Since, $AB^2 + BC^2 \neq AC^2$

Then, by converse of Pythagoras theorem, triangle is not a right triangle.

2. The sides of certain triangles are given below. Determine which of them right triangles are.

(i) a = 7 cm, b = 24 cm and c = 25 cm

(ii) a = 9 cm, b = l6 cm and c = 18 cm

(iii) a = 1.6 cm, b = 3.8 cm and c = 4 cm

(iv) a = 8 cm, b = 10 cm and c = 6 cm

Sol:

We have,

a = 7 cm, b = 24 cm and c = 25 cm

$\therefore a^2 = 49, b^2 = 576\ and\ c^2 = 625$

Since, $a^2 + b^2 = 49 + 576$

= 625

= c^2

Then, by converse of Pythagoras theorem, given triangle is a right triangle.

We have,

a = 9 cm, b = 16 cm and c = 18 cm

$\therefore a^2 = 81, b^2 = 256\ and\ c^2 = 324$

Since, $a^2 + b^2 = 81 + 256 = 337$

$\neq c^2$

Then, by converse of Pythagoras theorem, given triangle is not a right triangle.

We have,

a = 1.6 cm, b = 3.8 cm and C = 4 cm

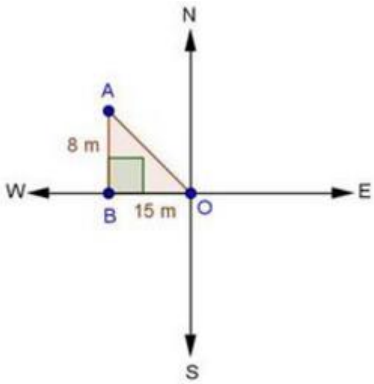
$\therefore a^2 = 64, b^2 = 100\ and\ c^2 = 36$

Since, $a^2 + c^2 = 64 + 36 = 100 = b^2$

Then, by converse of Pythagoras theorem, given triangle is a right triangle.

3. A man goes 15 metres due west and then 8 metres due north. How far is he from the starting point?

Sol:



Let the starting point of the man be O and final point be A.

∴ In ΔABO, by Pythagoras theorem $AO^2 = AB^2 + BO^2$

⇒ $AO^2 = 8^2 + 15^2$

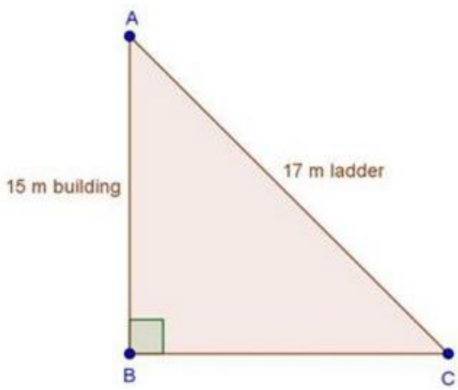
⇒ $AO^2 = 64 + 225 = 289$

⇒ AO = $\sqrt{289} = 17m$

∴ He is 17m far from the starting point.

4. A ladder 17 m long reaches a window of a building 15 m above the ground. Find the distance of the foot of the ladder from the building.

Sol:



In ΔABC, by Pythagoras theorem

$AB^2 + BC^2 = AC^2$

⇒ $15^2 + BC^2 = 17^2$

⇒ $225 + BC^2 = 17^2$

⇒ $BC^3 = 289 - 225$

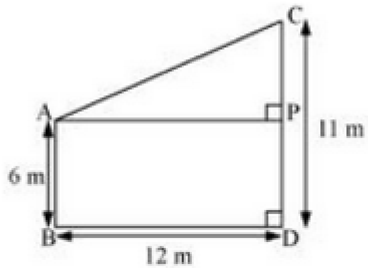
⇒ $BC^2 = 64$

$\Rightarrow BC = 8\,m$

$\therefore Distance\ of\ the\ foot\ of\ the\ ladder\ from\ building = 8\,m$

5. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

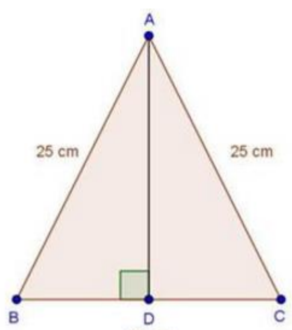
Sol:



Let CD and AB be the poles of height 11 and 6 m.
Therefore CP = 11 – 6 = 5 m
From the figure we may observe that AP = 12m
In triangle APC, by applying Pythagoras theorem
$AP^2 + PC^2 = AC^2$
$12^2 + 5^2 = AC^2$
$AC^2 = 144 + 25 = 169$
AC = 13
Therefore distance between their tops = 13m.

6. In an isosceles triangle ABC, AB = AC = 25 cm, BC = 14 cm. Calculate the altitude from A on BC.

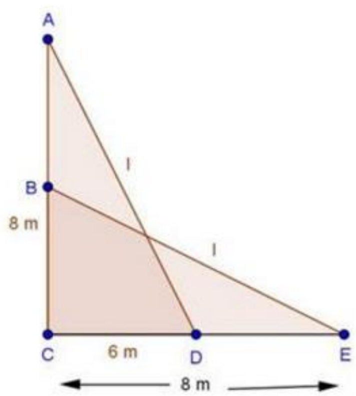
Sol:



We have
AB = AC = 25 cm and BC = 14 cm
In ΔABD and ΔACD
∠ADB = ∠ADC [Each 90°]
AB = AC [Each 25 cm]

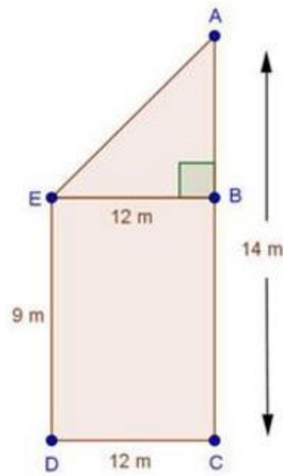
AD = AD [Common]
Then, ΔABD ≅ ΔACD [By RHS condition]
∴ BD = CD = 7 cm [By c.p.c.t]
In ΔADB, by Pythagoras theorem
$AD^2 + BD^2 = AB^2$
$\Rightarrow AD^2 + 7^2 = 25^2$
$\Rightarrow AD^2 = 625 - 49 = 576$
$\Rightarrow AD = \sqrt{576} = 24\ cm$

7. The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its tip reach?
Sol:



Let, length of ladder be AD = BE = l m
In ΔACD, by Pythagoras theorem
$AD^2 = AC^2 + CD^2$
$\Rightarrow\ l^2 = 8^2 + 6^2$ ….(i)
$In\ \Delta BCE, by\ pythagoras\ theorem$
$BE^2 = BC^2 + CE^2$
$\Rightarrow\ l^2 = BC^2 + 8^2$ ….(ii)
$Compare\ (i) and\ (ii)$
$BC^2 + 8^2 = 8^2 + 6^2$
$\Rightarrow BC^2 = 6^2$
$\Rightarrow BC = 6m$

8. Two poles of height 9 m and 14 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.
Sol:



We have,

AC = 14 m, DC = 12m and ED = BC = 9m

Construction: Draw EB ⊥ AC

∴ AB = AC – BC = 14 – 9 = 5m

And, EB = DC = 12 m

In ΔABE, by Pythagoras theorem,

$AE^2 = AB^2 + BE^2$

$\Rightarrow AE^2 = 5^2 + 12^2$

$\Rightarrow AE^2 = 25 + 144 = 169$

$\Rightarrow AE = \sqrt{169} = 13\ m$

$\therefore Distance\ between\ their\ tops = 13\ m$

9. Using Pythagoras theorem determine the length of AD in terms of b and c shown in Fig. 4.219

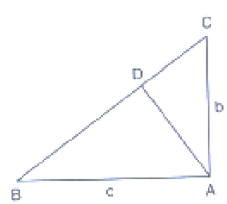


Fig. 4.219

Sol:

We have,

In ΔBAC, by Pythagoras theorem

$BC^2 = AB^2 + AC^2$

$\Rightarrow BC^2 = c^2 + b^2$

$\Rightarrow BC = \sqrt{c^2 + b^2}$ …(i)

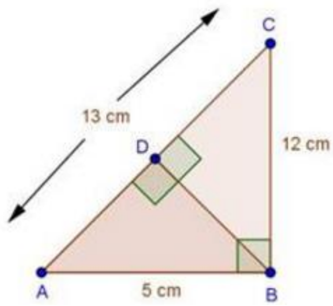
$In\ \Delta ABD\ and\ \Delta CBA$

∠B = ∠B [Common]

$\angle ADB = \angle BAC$ [Each 90°]
Then, $\Delta ABD \sim \Delta CBA$ [By AA similarity]
$\therefore \frac{AB}{CB} = \frac{AD}{CA}$ [Corresponding parts of similar Δ are proportional]
$\Rightarrow \frac{c}{\sqrt{c^2+b^2}} = \frac{AD}{b}$
$\Rightarrow AD = \frac{bc}{\sqrt{c^2+b^2}}$

10. A triangle has sides 5 cm, 12 cm and 13 cm. Find the length to one decimal place, of the perpendicular from the opposite vertex to the side whose length is 13 cm.

Sol:

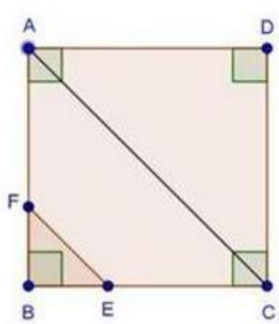


Let, AB = 5cm, BC = 12 cm and AC = 13 cm. Then, $AC^2 = AB^2 + BC^2$. This proves that ΔABC is a right triangle, right angles at B. Let BD be the length of perpendicular from B on AC.

Now, Area ΔABC = $\frac{1}{2}(BC \times BA)$
= $\frac{1}{2}(12 \times 5)$
= 30 cm^2
Also, Area of ΔABC = $\frac{1}{2}AC \times BD = \frac{1}{2}(13 \times BD)$
$\Rightarrow (13 \times BD) = 30 \times 2$
$\Rightarrow$ BD = $\frac{60}{13}$ cm

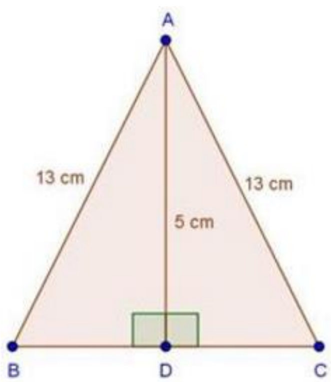
11. ABCD is a square. F is the mid-point of AB. BE is one third of BC. If the area of ΔFBE = 108 cm^2, find the length of AC.

Sol:



Since, ABCD is a square
Then, AB = BC = CD = DA = x cm
Since, F is the mid-point of AB
Then, AF = $FB = \frac{x}{2}\ cm$
Since, BE is one third of BC
Then, BE = $\frac{x}{3}$ cm
We have, area of ΔFBE = 108 cm^2
$\Rightarrow \frac{1}{2} \times BE \times FB = 108$
$\Rightarrow \frac{1}{2} \times \frac{x}{3} \times \frac{x}{2} = 108$
$\Rightarrow x^2 = 108 \times 2 \times 3 \times 2$
$\Rightarrow x^2 = 1296$
$\Rightarrow x = \sqrt{1296} = 36cm$
$In\ \Delta ABC, by\ pythagoras\ theorem\ AC^2 = AB^2 + BC^2$
$\Rightarrow AC^2 = x^2 + x^2$
$\Rightarrow AC^2 = 2x^2$
$\Rightarrow AC^2 = 2 \times (36)^2$
$\Rightarrow AC = 36\sqrt{2} = 36 \times 1.414 = 50.904\ cm$

12. In an isosceles triangle ABC, if AB = AC = 13 cm and the altitude from A on BC is 5 cm, find BC.
Sol:

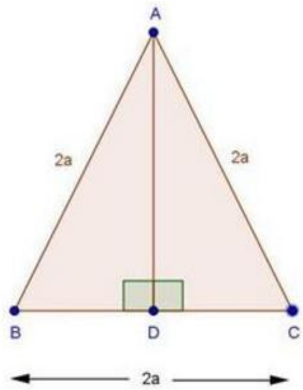


In ΔADB, by Pythagoras theorem
$AD^2 + BD^2 = 13^2$
$\Rightarrow 25 + BD^2 = 169$
$\Rightarrow BD^2 = 169 - 2 = 144$
$\Rightarrow BD = \sqrt{144} = 12\ cm$
$In\ \Delta ADB\ and\ \Delta ADC$
∠ADB = ∠ADC [Each 90°]
AB = AC [Each 13 cm]

AD = AD [Common]
Then, ΔADB ≅ ΔADC [By RHS condition]
∴ BD = CD = 12 cm [By c.p.c.t]
Hence, BC = 12 + 12 = 24 cm

13. In a ΔABC, AB = BC = CA = 2a and AD ⊥ BC. Prove that
(i) AD = $a\sqrt{3}$ (ii) Area (ΔABC) = $\sqrt{3}$ a^2

Sol:

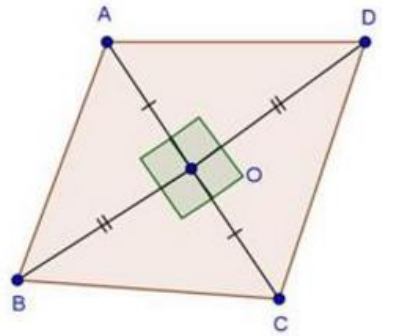


(i) In ΔABD and ΔACD
∠ADB = ∠ADC [Each 90°]
AB = AC [Given]
AD = AD [Common]
Then, ΔABD ≅ ΔACD [By RHS condition]
∴ BD = CD = a [By c.p.c.t]
In ΔADB, by Pythagoras theorem
$AD^2 + BD^2 = AB^2$
$\Rightarrow AD^2 + (a)^2 = (2a)^2$
$\Rightarrow AD^2 + a^2 = 4a^2$
$\Rightarrow AD^2 = 4a^2 - a^2 = 3a^2$
$\Rightarrow AD = a\sqrt{3}$

(ii) Area of ΔABC = $\frac{1}{2} \times BC \times AD$
$= \frac{1}{2} \times 2a \times a\sqrt{3}$
$= \sqrt{3}a^2$

14. The lengths of the diagonals of a rhombus are 24 cm and 10 cm. Find each side of the rhombus.

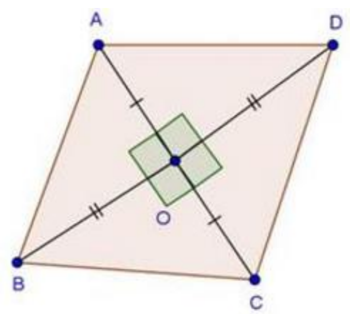
Sol:



We have,
ABCD is a rhombus with diagonals AC = 10 cm and BD = 24 cm
We know that diagonal of a rhombus bisect each other at 90°
∴ AO = OC = 5 cm and BO = OD = 12 cm
In ΔAOB, by Pythagoras theorem
$AB^2 = AO^2 + BO^2$
$\Rightarrow AB^2 = 5^2 + 12^2$
$\Rightarrow AB^2 = 25 + 144 = 169$
$\Rightarrow AB = \sqrt{169} = 13\ cm$

15. Each side of a rhombus is 10 cm. If one of its diagonals is 16 cm find the length of the other diagonal.

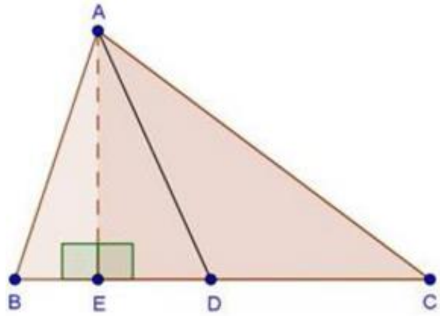
Sol:



We have,
ABCD is a rhombus with side 10 cm and diagonal BD = 16 cm
We know that diagonals of a rhombus bisect each other at 90°
∴ BO = OD = 8 cm
In ΔAOB, by pythagoras theorem
$AO^2 + BO^2 = AB^2$
$\Rightarrow AO^2 + 8^2 = 10^2$
$\Rightarrow AO^2 = 100 - 64 = 36$
$\Rightarrow AO = \sqrt{36} = 6\ cm$ [By above property]
$hence, AC = 6 + 6 = 12\ cm$

16. In an acute-angled triangle, express a median in terms of its sides.

Sol:



We have,

In ΔABC, AD is a median.

Draw AE ⊥ BC

In ΔAEB, by pythagoras theorem

$AB^2 = AE^2 + BE^2$

$\Rightarrow AB^2 = AD^2 - DE^2 + (BD - DE)^2$ [By Pythagoras theorem]

$\Rightarrow AB^2 = AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE$

$\Rightarrow AB^2 = AD^2 + BD^2 - 2BD \times DE$

$\Rightarrow AB^2 = AD^2 + \frac{BC^2}{4} - BC \times DE$ ….(i) [BC = 2BD given]

Again, In ΔAEC, by pythagoras theorem

$AC^2 = AE^2 + EC^2$

$\Rightarrow AC^2 = AD^2 - DE^2 + (DE + CD)^2$ [By Pythagoras theorem]

$\Rightarrow AC^2 = AD^2 + CD^2 + 2CD \times DE$

$\Rightarrow AC^2 = AD^2 + \frac{BC^2}{4} + BC \times DE$ ….(ii) [BC = 2CD given]

$Add\ equations\ (i)\ and\ (ii)$

$AB^2 + AC^2 = 2AD^2 + \frac{BC^2}{2}$

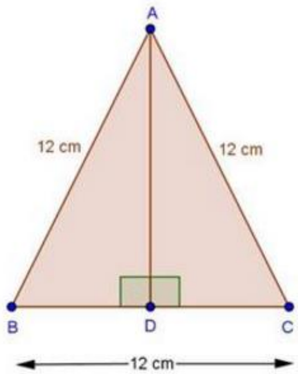
$\Rightarrow 2AB^2 + 2AC^2 = 4AD^2 + BC^2$ [Multiply by 2]

$\Rightarrow 4AD^2 = 2AB^2 + 2AC^2 - BC^2$

$\Rightarrow AD^2 = \frac{2AB^2 + 2AC^2 - BC^2}{4}$

17. Calculate the height of an equilateral triangle each of whose sides measures 12 cm.

Sol:



We have,

ΔABC is an equilateral Δ with side 12 cm.

Draw AE ⊥ BC

In ΔABD and ΔACD

∠ADB = ∠ADC [Each 90°]

AB = AC [Each 12 cm]

AD = AD [Common]

Then, ΔABD ≅ ΔACD [By RHS condition]

$\therefore AD^2 + BD^2 = AB^2$

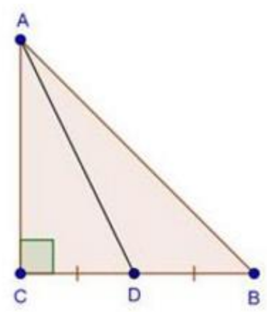
$\Rightarrow AD^2 + 6^2 = 12^2$

$\Rightarrow AD^2 = 144 - 36 = 108$

⇒ AD = $\sqrt{108} = 10.39$ cm

18. In right-angled triangle ABC in which ∠C = 90°, if D is the mid-point of BC, prove that $AB^2 = 4\ AD^2 - 3\ AC^2$.

Sol:



We have,

∠C = 90° and D is the mid-point of BC

In ΔACB, by Pythagoras theorem

$AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = AC^2 + (2CD)^2$ [D is the mid-point of BC]

$AB^2 = AC^2 + 4CD^2$
$\Rightarrow AB^2 = AC^2 + 4(AD^2 - AC^2)$ [In ΔACD, by Pythagoras theorem]
$\Rightarrow AB^2 = AC^2 + 4AD^2 - 4AC^2$
$\Rightarrow AB^2 = 4AD^2 - 3AC^2$

19. In Fig. 4.220, D is the mid-point of side BC and AE ⊥ BC. If BC = a, AC = b, AB = c, ED = x, AD = p and AE = h, prove that:

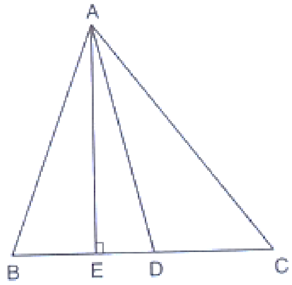


Fig. 4.220

(i) $b^2 = p^2 + ax + \frac{a^2}{4}$
(ii) $c^2 = p^2 - ax + \frac{a^2}{4}$
(iii) $b^2 + c^2 = 2p^2 + \frac{a^2}{2}$

Sol:
We have, D as the mid-point of BC
(i) $AC^2 = AE^2 + EC^2$
$b^2 = AE^2 + (ED + DC)^2$ [By pythagoras theorem]
$b^2 = AD^2 + DC^2 + 2DC \times ED$
$b^2 = p^2 + \left(\frac{a}{2}\right)^2 + 2\left(\frac{a}{2}\right) \times x$ [BC = 2CD given]
$\Rightarrow b^2 = p^2 + \frac{a^2}{4} + ax$ ...(i)
(ii) In ΔAEB, by pythagoras theorem
$AB^2 = AE^2 + BE^2$
$\Rightarrow c^2 = AD^2 - ED^2 + (BD - ED)^2$ [By pythagoras theorem]
$\Rightarrow c^2 = p^2 - ED^2 + BD^2 + ED^2 - 2BD \times ED$
$\Rightarrow c^2 = p^2 + \left(\frac{a}{2}\right)^2 - 2\left(\frac{a}{2}\right) \times x$ ...(ii)
(iii) Add equations (i) and (ii)
$b^2 + c^2 = 2p^2 + \frac{a^2}{2}$

20. In Fig., 4.221, ∠B < 90° and segment AD ⊥ BC, show that

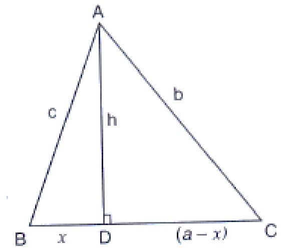


Fig. 4.221

(i) $b^2 = h^2 + a^2 + x^2 - 2ax$

(ii) $b^2 = a^2 + c^2 - 2ax$

Sol:

In ΔADC, by pythagoras theorem

$AC^2 = AD^2 + DC^2$

$\Rightarrow b^2 = h^2 + (a - x)^2$

$\Rightarrow b^2 = h^2 + a^2 + x^2 - 2ax$

$\Rightarrow b^2 = a^2 + (h^2 + x^2) - 2ax$

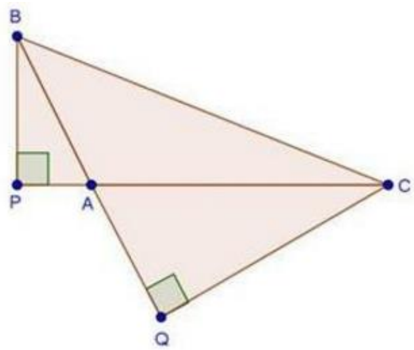
$\Rightarrow b^2 = a^2 + c^2 - 2ax$ by Pythagoras theorem

21. In ΔABC, ∠A is obtuse, PB ⊥AC and QC ⊥ AB. Prove that:

(i) AB × AQ = AC × AP

(ii) $BC^2 = (AC \times CP + AB \times BQ)$

Sol:



Then, ΔAPB ~ ΔAQC [By AA similarity]

$\therefore \frac{AP}{AQ} = \frac{AB}{AC}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow AP \times AC = AQ \times AB$ ...(i)

(ii) In ΔBPC, by pythagoras theorem

$BC^2 = BP^2 + PC^2$

$\Rightarrow BC^2 = AB^2 - AP^2 + (AP + AC)^2$ [By pythagoras theorem]

$\Rightarrow BC^2 = AB^2 + AC^2 + 2AP \times AC$ …(ii)

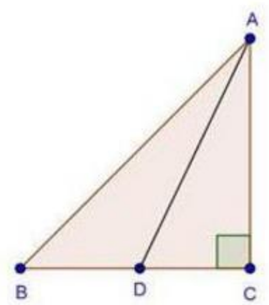
$In\ \Delta BQC, by\ pythagoras\ theorem,$

$BC^2 = CQ^2 + BQ^2$

$\Rightarrow BC^2 = AC^2 - AQ^2 + (AB + AQ)^2$ [By pythagoras theorem]

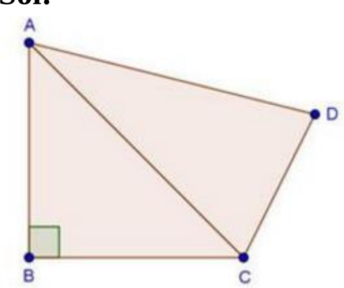
$\Rightarrow BC^2 = AC^2 - AQ^2 + AB^2 + AQ^2 + 2AB \times AQ$
$\Rightarrow BC^2 = AC^2 + AB^2 + 2AB \times AQ$ ...(iii)
$Add\ equations\ (ii) \& (iii)$
$2BC^2 = 2AC^2 + 2AB^2 + 2AP \times AC + 2AB \times AQ$
$\Rightarrow 2BC^2 = 2AC^2 + 2AB^2 + 2AP \times AC + 2AB \times AQ$
$\Rightarrow 2BC^2 = 2AC[AC + AP] + AB[AB + AQ]$
$\Rightarrow 2BC^2 = 2AC \times PC + 2AB \times BQ$
$\Rightarrow BC^2 = AC \times PC + AB \times BQ$ [Divide by 2]

22. In a right ΔABC right-angled at C, if D is the mid-point of BC, prove that $BC^2 = 4(AD^2 - AC^2)$
Sol:



To prove: $BC^2 = 4[AD^2 - AC^2]$
We have, ∠C = 90° and D is the mid-point of BC.
LHS = BC^2
= $(2CD)^2$ [D is the mid-point of BC]
= $4CD^2$
= $4[AD^2 - AC^2]$ [In ΔACD, by pythagoras theorem]
= RHS

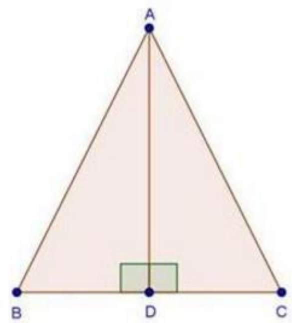
23. In a quadrilateral ABCD, ∠B = 90°, $AD^2 = AB^2 + BC^2 + CD^2$, prove that ∠ACD = 90°.
Sol:



We have, ∠B = 90° and $AD^2 = AB^2 + BC^2 + CD^2$
$\therefore AD^2 = AB^2 + BC^2 + CD^2$ [Given]
But $AB^2 + BC^2 = AC^2$ [By pythagoras theorem]

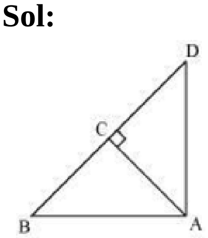
Then, $AD^2 = AC^2 + CD^2$
By converse of by pythagoras theorem
∠ACD = 90°

24. In an equilateral ΔABC, AD ⊥ BC, prove that $AD^2 = 3BD^2$.
Sol:



We have, ΔABC is an equilateral Δ and AD ⊥ BC
In ΔADB and ΔADC
∠ADB = ∠ADC [Each 90°]
AB = AC [Given]
AD = AD [Common]
Then, ΔADB ≅ ΔADC [By RHS condition]
∴ BD = CD = $\frac{BC}{2}$ …(i) [corresponding parts of similar Δ are proportional]
In, ΔABD, by Pythagoras theorem
$AB^2 = AD^2 + BD^2$
⇒ $BC^2 = AD^2 + BD^2$ [AB = BC given]
⇒ $[2BD]^2 = AD^2 + BD^2$ [From (i)]
⇒ $4BD^2 - BD^2 = AD^2$
⇒ $3BD^2 = AD^2$

25. ΔABD is a right triangle right angled at A and AC ⊥ BD. Show that:
(i) $AB^2 = CB \times BD$
(ii) $AC^2 = DC \times BC$
(iii) $AD^2 = BD \times CD$
(iv) $\frac{AB^2}{AC^2} = \frac{BD}{DC}$
Sol:



(i) In ΔADB and ΔCAB

∠DAB = ∠ACB = 90°

∠ABD = ∠CBA (common angle)

∠ADB = ∠CAB (remaining angle)

So, ΔADB ~ ΔCAB (by AAA similarity)

Therefore $\frac{AB}{CB} = \frac{BD}{AB}$

$\Rightarrow AB^2 = CB \times BD$

(ii) Let ∠CAB = x

In ΔCBA

∠CBA = $180° - 90° - x$

∠CBA = $90° - x$

Similarly in ΔCAD

∠CAD = $90° - \angle CAD = 90° - x$

∠CDA = $90° - \angle CAB$

$= 90° - x$

∠CDA = $180° - 90° - (90° - x)$

∠CDA = x

Now in ΔCBA and ΔCAD we may observe that

∠CBA = ∠CAD

∠CAB = ∠CDA

∠ACB = ∠DCA = 90°

Therefore ΔCBA ~ ΔCAD (by AAA rule)

Therefore $\frac{AC}{DC} = \frac{BC}{AC}$

$\Rightarrow AC^2 = DC \times BC$

(iii) In ΔDCA & ΔDAB

∠DCA = ∠DAB (both are equal to 90°)

∠CDA = ∠ADB (common angle)

∠DAC = ∠DBA (remaining angle)

ΔDCA ~ ΔDAB (AAA property)

Therefore $\frac{DC}{DA} = \frac{DA}{DB}$

$\Rightarrow AD^2 = BD \times CD$

(iv) From part (i) $AB^2 = CB \times BD$

From part (ii) $AC^2 = DC \times BC$

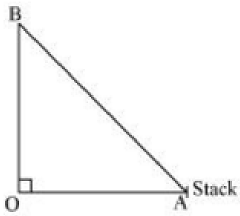
Hence $\frac{AB^2}{AC^2} = \frac{CB \times BD}{DC \times BC}$

$\frac{AB^2}{AC^2} = \frac{BD}{DC}$

Hence proved

26. A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Sol:



Let OB be the pole and AB be the wire. Therefore by pythagoras theorem,

$AB^2 = OB^2 + OA^2$

$24^2 = 18^2 + OA^2$

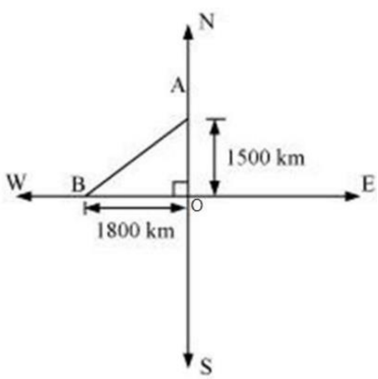
$OA^2 = 576 - 324$

OA = $\sqrt{252} = \sqrt{6 \times 6 \times 7} = 6\sqrt{7}$

Therefore distance from base = $6\sqrt{7}\ m$

27. An aeroplane leaves an airport and flies due north at a speed of 1000km/hr. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km/hr. How far apart will be the two planes after 1 hours?

Sol:



Distance traveled by the plane flying towards north in $1\frac{1}{2}$ hrs

= $1000 \times 1\frac{1}{2} = 1500\ km$

Similarly, distance travelled by the plane flying towards west in $1\frac{1}{2}\ hrs$

= $1200 \times 1\frac{1}{2} = 1800\ km$

Let these distances are represented by OA and OB respectively.

Now applying Pythagoras theorem

Distance between these planes after $1\frac{1}{2}\ hrs$ AB = $\sqrt{OA^2 + OB^2}$

= $\sqrt{(1500)^2 + (1800)^2} = \sqrt{2250000 + 3240000}$

$= \sqrt{5490000} = \sqrt{9 \times 610000} = 300\sqrt{61}$

So, distance between these planes will be $300\sqrt{61}$ km, after $1\frac{1}{2}\ hrs$

28. Determine whether the triangle having sides (a − 1) cm, $2\sqrt{a}$ cm and (a + 1) cm is a right-angled triangle.

Sol:

Let ABC be the Δ with

AB = (a – 1) cm BC = $2\sqrt{a}$ cm, CA = (a + 1) cm

Hence, $AB^2 = (a-1)^2 = a^2 + 1 - 2a$

$BC^2 = \left(2\sqrt{a}\right)^2 = 4a$

$CA^2 = (a+1)^2 = a^2 + 1 + 2a$

Hence $AB^2 + BC^2 = AC^2$

$So\ \Delta\ ABC\ is\ right\ angled\ \Delta\ at\ B.$

Exercise 5.1

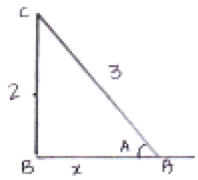
1. In each of the following one of the six trigonometric ratios is given. Find the values of the other trigonometric ratios.

Sol:

(i) Sin A = $\frac{2}{3}$

We know that Sin θ = $\frac{opposite\ side}{hypotenuse}$

Let us Consider a right angled Δ^{le} ABC.



By applying Pythagorean theorem we get

$AC^2 = AB^2 + BC^2$

9 = x^2 + 4

x^2 = 9 - 4

x = $\sqrt{5}$

We know that cos = $\frac{adjacent\ side}{hypotenuse}$ and

tanθ = $\frac{opposite\ side}{adjacent\ side}$

So, cosθ = $\frac{\sqrt{5}}{3}$;

sec = $\frac{1}{cos\theta} = \frac{3}{\sqrt{5}}$

tanθ = $\frac{2}{\sqrt{5}}$;

cot = $\frac{1}{tan\theta}$ = $\frac{\sqrt{5}}{2}$

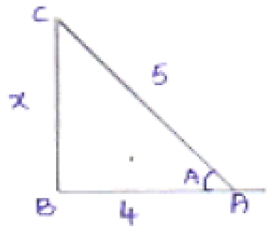
cosecθ = $\frac{1}{sin\theta} = \frac{3}{2}$

(ii)

Cos A = $\frac{4}{5}$

We know that cosθ = $\frac{adjacent\ side}{hypotenuse}$

Let us consider a right angled Δ^{le} ABC.



Let opposite side BC = x.

By applying pythagorn's theorem, we get

$AC^2 = AB^2 + BC^2$

25 = x + 16

x = 25 - 16 = 9

x = $\sqrt{9}$ = 3

We know that cosA = $\frac{4}{5}$

sinA = $\frac{opposite\ side}{hypotenuse} = \frac{3}{5}$

tanA = $\frac{opposite\ side}{adjacent\ side} = \frac{3}{4}$

cosecA = $\frac{1}{sinA} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$

secA = $\frac{1}{cosA} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$

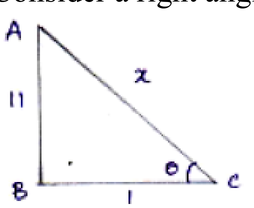
cotA = $\frac{1}{tanA} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$

(iii)

tanθ = 11.

We know that tanθ = $\frac{opposite\ side}{adjacent\ side} = \frac{11}{1}$

Consider a right angled Δ^{le} ABC.



Let hypotenuse AC = x, by applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$x^2 = 11^2 + 1^2$

$x^2 = 121 + 1$

$x = \sqrt{122}$

We know that sinθ = $\frac{opposite\ side}{hypotenuse} = \frac{11}{\sqrt{122}}$

cosθ = $\frac{adjacent\ side}{hypotenuse} = \frac{1}{\sqrt{122}}$

cosecθ = $\frac{1}{sin\theta}$ = $\frac{\frac{1}{11}}{\sqrt{122}}$ = $\frac{\sqrt{122}}{11}$

secθ = $\frac{1}{cos\theta}$ = $\frac{1/1}{\sqrt{122}}$ = $\sqrt{122}$

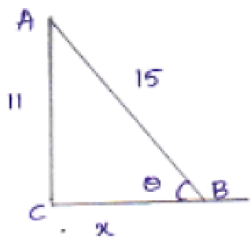
cotθ = $\frac{1}{tan\theta}$ = $\frac{1}{11}$ = $\frac{1}{11}$

(iv)

Sin θ = $\frac{11}{5}$

We know Sin θ = $\frac{opposite\ side}{hypotenuse}$ = $\frac{11}{15}$

Consider right angled Δ^{le} ACB.



Let x = $adjacent\ side$

By applying Pythagoras

$AB^2 = AC^2 + BC^2$

225 = 121+x^2

x^2 = 225 -121

x^2 = 104

x = $\sqrt{104}$

cos = $\frac{adjacent\ side}{hypotenuse}$ = $\sqrt{\frac{104}{15}}$

tan = $\frac{opposite\ side}{adjacent\ side}$ = $\frac{11}{\sqrt{104}}$

cosecθ = $\frac{1}{sin\theta}$ = $\frac{15}{11}$

sec = $\frac{1}{cos\theta}$ = $\frac{15}{\sqrt{104}}$

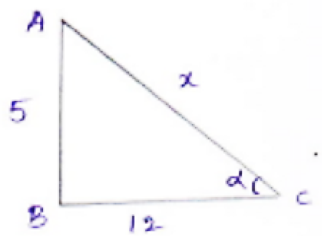
cot = $\frac{1}{tan\theta}$ = $\frac{\sqrt{104}}{11}$

(v)

tanα = $\frac{5}{12}$

We know that tanα= $\frac{opposite\ side}{adjacent\ side}$ = $\frac{5}{12}$

Now consider a right angled Δ^{le} ABC.



Let x = hypotenuse .By applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$x^2 = 5^2 + 12^2$

$x^2 = 25 + 144 = 169$

$x = 13$

sinα $= \frac{opposite\ side}{hypotenuse} = \frac{5}{13}$

cosα $= \frac{adjacent\ side}{hypotenuse} = \frac{12}{13}$

cotα $= \frac{1}{tan\alpha} = \frac{12}{15}$

cosecα $= \frac{1}{sin\alpha} = \frac{1/5}{13} = \frac{13}{5}$

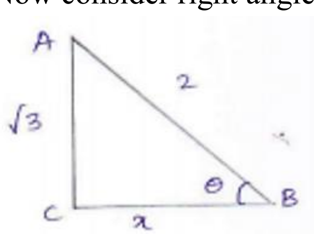
secα $= \frac{1}{cos\alpha} = \frac{\frac{1}{12}}{13} = \frac{13}{12}$.

(vi)

Sin $\theta = \frac{\sqrt{3}}{2}$

We know Sin $\theta = \frac{opposite\ side}{hypotenuse} = \frac{\sqrt{3}}{2}$

Now consider right angled Δ^{le} ABC.



Let x = $adjacent\ side$

By applying Pythagoras

$AB^2 = AC^2 + BC^2$

$4 = 3 + x^2$

$x^2 = 4 - 3$

$x^2 = 1$

$x = 1$

cos $= \frac{adjacent\ side}{hypotenuse} = \frac{1}{2}$

tan $= \frac{opposite\ side}{adjacent\ side} = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\text{cosec}\theta = \frac{1}{sin\theta} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$

sec $= \frac{1}{cos\theta} = \frac{1}{\frac{1}{2}} = 2$

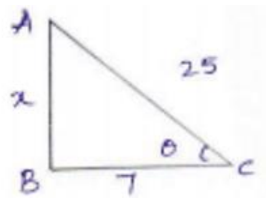
cot $= \frac{1}{tan\theta} = \frac{1}{\sqrt{3}}$

(vii)

Cosθ = $\frac{7}{25}$.

We know that cosθ = $\frac{adjacent\ side}{hypotenuse}$

Now consider a right angled Δ[le] ABC,



Let x be the opposite side.

By applying pythagorn's theorem

$AC^2 = AB^2 + BC^2$

$(25)^2 = x^2 + 7^2$

625 - 49 = x^2

$576 = \sqrt{576} = 24$

sinθ = $\frac{opposite\ side}{hypotenuse} = \frac{24}{25}$

tanθ = $\frac{opposite\ side}{adjacent\ side} = \frac{24}{7}$

cosecθ = $\frac{1}{sin\theta} = \frac{1}{\frac{3}{5}} = \frac{25}{24}$

secθ = $\frac{1}{cos\theta} = \frac{1}{\frac{4}{5}} = \frac{25}{7}$

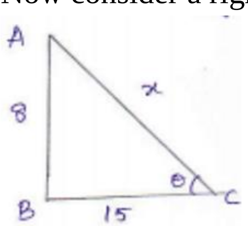
cotθ = $\frac{1}{tan\theta} = \frac{1}{\frac{3}{4}} = \frac{7}{24}$

(viii)

tanθ = $\frac{8}{15}$

We know that tanθ= $\frac{opposite\ side}{adjacent\ side} = \frac{8}{15}$

Now consider a right angled Δ[le] ABC.



By applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$x^2 = 8^2 + 15^2$

$x^2 = 225 + 64 = 289$

$x = \sqrt{289} = 17$

$\sin\theta = \frac{opposite\ side}{hypotenuse} = \frac{8}{17}$

$\cos\theta = \frac{adjacent\ side}{hypotenuse} = \frac{15}{17}$

$\tan\theta = \frac{opposite\ side}{adjacent\ side} = \frac{8}{15}$

$\cot\theta = \frac{1}{tan\theta} = \frac{1}{\frac{8}{15}} = \frac{15}{8}$

$\text{cosec}\theta = \frac{1}{sin\theta} = \frac{\frac{1}{8}}{17} = \frac{17}{8}$

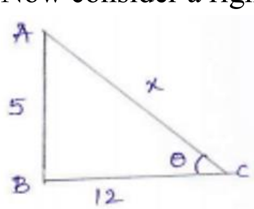
$\sec\theta = \frac{1}{cos\theta} = \frac{\frac{1}{15}}{17} = \frac{17}{15}$

(ix)

$\cot\theta = \frac{12}{5}$

$\cot\alpha = \frac{adjacent\ side}{opposite\ side} = \frac{12}{5}$

Now consider a right angled Δ^{le} ABC,



By applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$x^2 = 25 + 144$

$x^2 = 169 = \sqrt{169}$

$x = 13$

$\tan\theta = \frac{1}{cot\theta} = \frac{\frac{1}{12}}{5} = \frac{5}{12}$

$\sin\theta = \frac{opposite\ side}{hypotenuse} = \frac{5}{13}$

$\cos\theta = \frac{adjacent\ side}{hypotenuse} = \frac{12}{13}$

$\text{cosec}\theta = \frac{1}{sin\theta} = \frac{1}{5/13} = \frac{13}{5}$

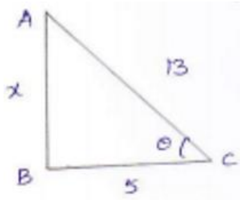
$\sec\theta = \frac{1}{cos\theta} = \frac{1}{12/13} = \frac{13}{12}$

(x)

$\sec\theta = \frac{13}{5}$

secθ = $\frac{hypotenuse}{adjacent\ side}$ = $\frac{13}{5}$

Now consider a right angled Δ^{le} ABC,



By applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$169 = x^2 + 25$

$x^2 = 169 - 25 = 144$

$x = 12$

cosθ $= \frac{1}{sec\theta} = \frac{1}{\frac{13}{5}} = \frac{5}{13}$

tanθ $= \frac{opposite\ side}{adjacent\ side} = \frac{12}{5}$

sinθ $= \frac{opposite\ side}{hypotenuse} = \frac{12}{13}$

cosecθ $= \frac{1}{sin\theta} = \frac{1}{12/13} = \frac{13}{12}$

secθ $= \frac{1}{cos\theta} = \frac{1}{5/13} = \frac{13}{5}$

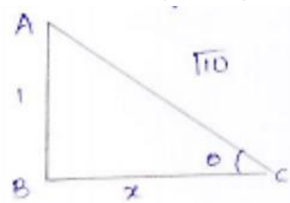
cotθ $= \frac{1}{tan\theta} = \frac{1}{12/5} = \frac{5}{12}$

(xi)

cosecθ = $\sqrt{10}$

cosecθ = $\frac{hypotenuse}{opposite\ side}$ = $\sqrt{10}$

consider a right angled Δ^{le} ABC, we get



Let x be the adjacent side.

By applying pythagora's theorem

$AC^2 = AB^2 + BC^2$

$\left(\sqrt{10}\right)^2 = 1^2 + x^2$

$x^2 = 10 - 1 = 9$

$x = 3$

sinθ $= \frac{1}{cosec\theta} = \frac{1}{\sqrt{10}}$

cosθ $= \frac{adjacent\ side}{hypotenuse} = \frac{3}{\sqrt{10}}$

tanθ $= \frac{opposite\ side}{adjacent\ side} = \frac{1}{3}$

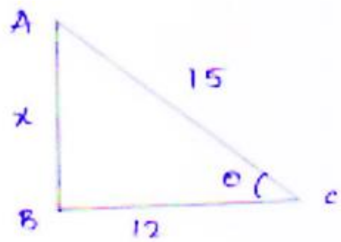
secθ $= \frac{1}{cos\theta} = \frac{\sqrt{10}}{3}$

cotθ $= \frac{1}{tan\theta} = \frac{1}{\frac{1}{3}} = 3.$

(xii)

Cosθ = $\frac{12}{5}$

cosθ = $\frac{adjacent\ side}{hypotenuse} = \frac{12}{15}$.



Let x be the opposite side.

By applying pythagorn's theorem

$AC^2 = AB^2 + BC^2$

$225 = x^2 + 144$

$225 - 144 = x^2$

$x^2 = 81$

$x = 9$

sinθ $= \frac{opposite\ side}{hypotenuse} = \frac{9}{15}$

tanθ $= \frac{opposite\ side}{adjacent\ side} = \frac{9}{12}$

cosecθ = $\frac{1}{sin\theta} = \frac{1}{\frac{9}{15}} = \frac{15}{9}$

secθ $= \frac{1}{cos\theta} = \frac{1}{\frac{12}{15}} = \frac{15}{12}$

cotθ $= \frac{1}{tan\theta} = \frac{1}{\frac{9}{12}} = \frac{12}{9}$

2. In a ΔABC, right angled at B, AB = 24 cm, BC = 7 cm. Determine

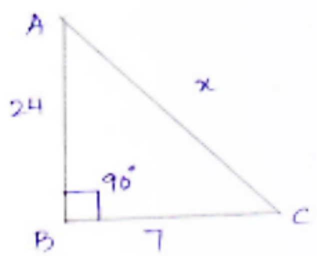
(i) Sin A, Cos A

(ii) Sin C, cos C

Sol:

ΔABC is right angled at B

AB = 24cm, BC = 7cm.

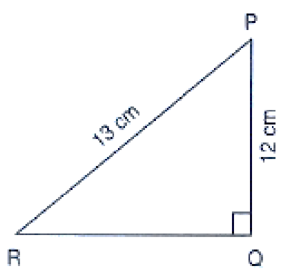


Let 'x' be the hypotenuse,
By applying Pythagoras
$AC^2 = AB^2 + BC^2$
$x^2 = 24^2 + 7^2$
$x^2 = 576 + 49$
$x^2 = 625$
$x = 25$

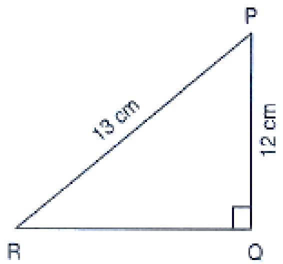
a. Sin A, Cos A
 At ∠A, opposite side = 7
 adjacent side = 24
 hypotenuse = 25
 sin A = $\frac{opposite\ side}{hypotenuse} = \frac{7}{25}$
 cos A = $\frac{adjacent\ side}{hypotenuse} = \frac{24}{25}$

b. Sin C, Cos C
 At ∠C, opposite side = 24
 adjacent side = 7
 hypotenuse = 25
 sin C = $\frac{24}{25}$
 cos C = $\frac{7}{25}$

3. In Fig below, Find tan P and cot R. Is tan P = cot R?



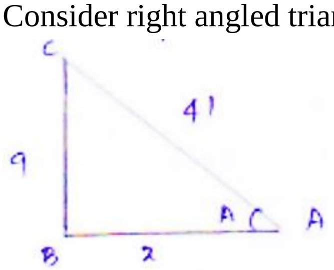
Sol:



Let x be the adjacent side.
By Pythagoras theorem
$PR^2 = PQ^2 + RQ^2$
$169 = x^2 + 144$
$x^2 = 25$
$x = 5$
At LP, opposite side = 5
Adjacent side = 12
Hypotenuse = 13
tan P = $\frac{1}{\frac{12}{5}} \Rightarrow \frac{5}{12}$
At LR, opposite side = 12
Adjacent side = 5
Hypotenuse = 13
cot R = $\frac{1}{\tan R} = \frac{1}{\frac{12}{5}} = \frac{5}{12}$
[∵ Tan R = $\frac{opposite\ side}{adjacent\ side}$]
∵ tan P = cot R

4. If sin A = $\frac{9}{41}$, *compute* cos A *and* tan A

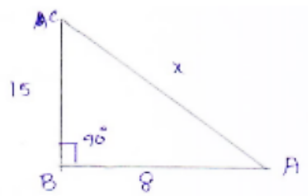
Sol:
$\sin A = \frac{9}{41}$
Sin A = $\frac{opposite\ side}{adjacent\ side} = \frac{9}{41}$
Consider right angled triangle ABC,



Let x be the adjacent side
By applying Pythagorean

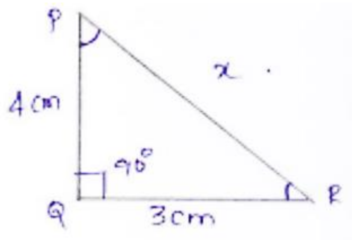
$AC^2 = AB^2 + BC^2$
$41^2 = 12^2 + 9^2$
$x^2 = 41^2 - 9^2$
$x = 40$
$\cos A = \frac{adjacent\ side}{hypotenuse} = \frac{40}{41}$
$\tan A = \frac{opposite\ side}{Hypotenuse\ side} = \frac{9}{40}$

5. Given 15 cot A = 8, find Sin A and sec A.
Sol:
15 cot A = 8, find Sin A and sec A
Cot A = $\frac{8}{15}$



Consider right angled triangle ABC,
Let x be the hypotenuse,
$AC^2 = AB^2 + BC^2$
$x^2 = (8)^2 + (15)^2$
$x^2 = 64 + 225$
$x^2 = 289$
$x = 17$
Sin A = $\frac{opposite\ side}{hypotenuse} = \frac{15}{17}$
Sec A = $\frac{1}{\cos A}$
$\cos A = \frac{adjacent\ side}{Hypotenuse} = \frac{8}{17}$
Sec A = $\frac{1}{\cos A} = \frac{1}{8/17} = \frac{17}{8}$

6. In ΔPQR, right angled at Q, PQ = 4 cm and RQ = 3 cm. Find the values of sin P, sin R, sec P and sec R.
Sol:
ΔPQR, right angled at Q.



Let x be the hypotenuse
By applying Pythagoras
$PR^2 = PQ^2 + QR^2$
$x^2 = 4^2 + 3^2$
$x^2 = 16 + 9$
$\therefore x = \sqrt{25} = 5$
Find $\sin P$, $\sin R, \sec P, \sec R$
At LP, opposite side = 3 cm
Adjacent side = 4 cm
Hypotenuse = 5
$\sin P = \frac{opposite\ side}{Hypotenuse} = \frac{3}{5}$
$\sec P = \frac{Hypotenuse}{adjacent\ side} = \frac{5}{4}$
At LK, opposite side = 4 cm
Adjacent side = 3 cm
Hypotenuse = 5 cm
Sin R = $\frac{4}{5}$
Sec R = $\frac{5}{3}$

7. If cot $\theta = \frac{7}{8}, evaluate$:

(i) $\frac{(1+\sin\theta)\,(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) Cot$^2\theta$

Sol:

Cot $\theta = \frac{7}{8}$

(i) $\frac{(1+\sin\theta)\,(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{1-\sin^2\theta}{1-\cos^2\theta}$ [∵ (a + b) (a – b) = $a^2 - b^2$] a = 1, b = sin θ

We know that $Sin^2\theta + \cos^2\theta = 1$

$1 - \sin^2\theta = \cos^2\theta = \cos^2\theta$

$1 - \cos^2\theta = \sin^2\theta$

$= \frac{\cos^2\theta}{\sin^2\theta}$

$= \cot^2\theta$

$= (\cot\ \theta)^2 = \left[\frac{7}{8}\right]^2$

$= \frac{49}{64}$

(ii) $\cot^2\theta$

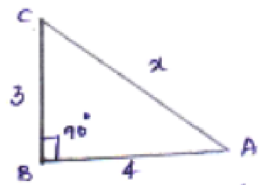
$\Rightarrow (\cot\theta)^2 = \left[\frac{7}{8}\right]^2$

$= \frac{49}{64}$

8. If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Sol:

3 cot A = 4, check $= \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$



Cot A = $\frac{adjacent\ side}{opposite\ side} = \frac{4}{3}$

Let x be the hypotenuse

By Applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$x^2 = 4^2 + 3^2$

$x^2 = 25$

$x = 5$

Tan A = $\frac{1}{\cos^2 A} = \frac{3}{4}$

Cos A = $\frac{adjacent\ side}{hypotenuse} = \frac{4}{5}$

Sin A = $\frac{3}{5}$

LHS = $\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{7}{25}$

RHS $\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16-9}{25}$

$= \frac{7}{251}$

9. If $\tan\theta = \frac{a}{b}$, find the value of $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

Sol:

Tan $\theta = \frac{a}{b}$ find $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$ ….(i)

Divide equation (i) with cos θ, we get

$\Rightarrow \frac{\frac{\cos\theta+\sin\theta}{\cos\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$

$\Rightarrow \frac{1+\frac{\sin\theta}{\cos\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$\Rightarrow \frac{1+\tan\theta}{1-\tan\theta}$

$= \frac{1+\frac{a}{b}}{1-\frac{a}{b}}$

$= \frac{b+a}{b-a}$

10. If 3 tan θ = 4, $find\ the\ value\ of\ \frac{4\cos\theta-\sin\theta}{2\cos\theta+\sin\theta}$

Sol:

$3\tan\theta = 4$ find $\frac{4\cos\theta\ -\sin\theta}{2\cos\theta+\sin\theta}$ ...(i)

Tan $\theta = \frac{4}{3}$

Dividing equation (i) with cos θ we get

$= \frac{\frac{4\cos\theta-\sin\theta}{\cos\theta}}{\frac{2\cos\theta+\sin\theta}{\cos\theta}} = \frac{4-\tan\theta}{2+\tan\theta}\left[\because \frac{\sin\theta}{\cos\theta} = \tan\theta\right]$

$= \frac{4-\tan\theta}{2+\tan\theta}$ $[\because \frac{\sin\theta}{\cos\theta} = \tan\theta]$

$= \frac{4-\frac{4}{1}}{2+\frac{4}{5}}$

$= \frac{12-4}{6+4}$

$= \frac{8}{10}$

$= \frac{4}{5}$

11. If 3 cot θ = 2, find the value of $= \frac{4\sin\theta-3\cos\theta}{2\sin\theta+6\sin\theta}$

Sol:

3 cot θ = 2 find $\frac{4\sin\theta-3\cos\theta}{2\sin\theta+6\cos\theta}$ … (i)

Cot θ = $\frac{2}{3}$

$= \frac{\frac{4\sin\theta-3\cos\theta}{\sin\theta}}{\frac{2\sin\theta+6\cos\theta}{\sin\theta}}$

$= \frac{4-3\cot\theta}{2+6\cot\theta}$

$= \frac{4-3\times\frac{2}{3}}{2+6\times\frac{2}{3}}$

$= \frac{4+2}{2+4} = \frac{2}{6}$
$= \frac{1}{3}$

12. If $\tan\theta = \frac{a}{b}$, prove that $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2-b^2}{a^2+b^2}$

Sol:

$\text{Tan}\theta = \frac{a}{b}$. PT $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2-b^2}{a^2+b^2}$

Let $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$ …(i)

Divide both Nr and Dr with $\cos\theta$ of (a)

$= \frac{\frac{a\sin\theta - b\cos\theta}{\cos\theta}}{\frac{a\sin\theta + b\cos\theta}{\cos\theta}}$

$= \frac{a\tan\theta - b}{a\tan\theta + b}$

$= \frac{a\times\left(\frac{a}{b}\right) - b}{a\times\left(\frac{a}{b}\right) + b}$

$= \frac{a^2-b^2}{a^2+b^2}$

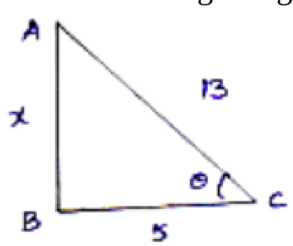
13. If $\sec\theta = \frac{13}{5}$, show that $\frac{2\cos\theta - 3\cos\theta}{4\sin\theta - 9cos\theta} = 3$

Sol:

$Sec\ \theta = \frac{13}{5}$

$\text{Sec}\ \theta = \frac{Hypotenuse}{adjacent\ side} = \frac{13}{5}$

Now consider right angled triangle ABC



By applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$169 = x^2 + 25$

$x^2 = 169 - 25 = 144$

$x = 12$

$cos\theta = \frac{1}{\sec\theta} = \frac{1}{13} = \frac{5}{3}$

$\tan\theta = \frac{opposite\ side}{adjacent\ side} = \frac{12}{13}$

$\sin\theta = \frac{opposite\ side}{hypotenuse} = \frac{12}{13}$

Cosec $\theta = \frac{1}{\sin\theta} = \frac{1}{12/13} = \frac{13}{12}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{5/13} = \frac{13}{5}$

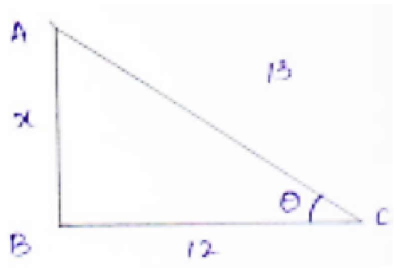
Cot $\theta = \frac{1}{\tan\theta} = \frac{1}{12/5} = \frac{5}{12}$

14. If cos $\theta = \frac{12}{13}$, show that sin θ (1 – tan θ) = $\frac{35}{156}$

Sol:

Cos $\theta = \frac{12}{3}$ S.T Sin $\theta\ (1 - \tan\theta) = \frac{35}{156}$

Cos $\theta = \frac{adjacent\ side}{hypotenuse} = \frac{12}{13}$



Let x be the opposite side

By applying Pythagoras

$AC^2 = AB^2 + BC^2$

169 = $x^2 + 144$

x = 25

x = 5

$\sin\theta = \frac{AB}{AC} = \frac{5}{3}$

Tan $\theta = \frac{AB}{BC} = \frac{5}{12}$

$\sin\theta\,(1 - \tan\theta) = \frac{5}{13}\left(1 - \frac{5}{12}\right)$

$= \frac{5}{13}\left[\frac{7}{12}\right] = \frac{35}{156}$

15. If cot $\theta = \frac{1}{\sqrt{3}}$, show that $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$

Sol:

$\cot\theta = \frac{1}{\sqrt{3}} \frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$

Cot $\theta = \frac{adjacent\ side}{opposite\ side} = \frac{1}{\sqrt{3}}$

Let x be the hypotenuse

By applying Pythagoras

$AC^2 = AB^2 + BC^2$

$x^2 = (\sqrt{3})^2 + 1$

$x^2 = 3 + 1$

$x^2 = 3 + 1 \Rightarrow x = 2$

Cos $\theta = \frac{BC}{AC} = -\frac{1}{2}$

Sin $\theta = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$

$\frac{1-\cos^2\theta}{2-\sin^2\theta} \Rightarrow \frac{1-\left(\frac{1}{2}\right)^2}{2-\left(\frac{\sqrt{3}}{2}\right)^2}$

$\Rightarrow \frac{1-\frac{1}{4}}{2-\frac{3}{4}} \Rightarrow \frac{\frac{3}{4}}{\frac{5}{4}}$

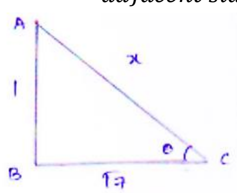
$= \frac{3}{5}$

16. If tan $\theta = \frac{1}{\sqrt{7}}$ $\quad \frac{cosec^2\theta-\sec^2\theta}{cosec^2\theta+\sec^2\theta} = \frac{3}{4}$

Sol:

Tan$\theta = \frac{1}{\sqrt{7}}$ $\quad \frac{cosec^2\theta-\sec^2\theta}{cosec^2\theta+\sec^2\theta} = \frac{3}{4}$

Tan $\theta = \frac{opposite\ side}{adjacent\ side}$



Let 'x' be the hypotenuse

By applying Pythagoras

$AC^2 = AB^2 + BC^2$

$x^2 = 1^2 + (\sqrt{7})^2$

$x^2 = 1 + 7 = 8$

$x = 2\sqrt{2}$

Cosec $\theta = \frac{AC}{AB} = 2\sqrt{2}$

Sec $\theta = \frac{AC}{BC} = \frac{2\sqrt{2}}{\sqrt{7}}$

Substitute, cosec θ, sec θ in equation

$\Rightarrow \frac{(2\sqrt{2})^2-\left(2\sqrt{\frac{2}{7}}\right)^2}{(2\sqrt{2})^2+\left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$

$\frac{8-4\times\frac{2}{7}}{8+4\times\frac{2}{7}}$

$\Rightarrow \frac{8-\frac{8}{7}}{8+\frac{8}{7}}$

$= \frac{\frac{56-8}{7}}{\frac{56+8}{7}}$

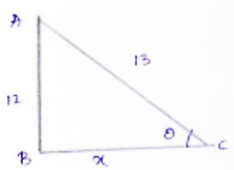
$= \frac{48}{64}$

$= \frac{3}{4}$

$L.H.S = R.H.S$

17. If Sin $\theta = \frac{12}{13}$ find $\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} \times \frac{1}{tan^2\theta}$

Sol:



Let x be the adjacent side

By applying Pythagoras

$AC^2 = AB^2 + BC^2$

$169 = 144 + x$

$x^2 = 25$

$x = 5$

Cos θ $= \frac{BC}{AC} = \frac{5}{13}$

Tan θ = $\frac{AB}{BC} = \frac{12}{5}$

$\Rightarrow \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)}{\alpha \times \frac{2}{13} \times \frac{5}{13}} \times \frac{1}{\left[\frac{12}{5}\right]^2}$

$\Rightarrow \frac{\frac{144-25}{169}}{\frac{24\times 5}{169}} \times \frac{25}{144}$

$\Rightarrow \frac{\frac{119}{169}}{\frac{120}{169}} \times \frac{25}{144} = \frac{129}{120} \times \frac{25}{144} = \frac{595}{3456}$

18. If sec $\theta = \frac{5}{4}$, find the value of $\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$

Sol:

Not given

19. If cos $\theta = \frac{5}{13}$, find the value of $\frac{\sin^2\theta - \cos^2\theta}{2sin\,\theta\cos\theta} = \frac{3}{5}$

Sol:

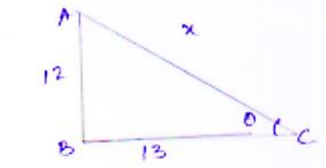
Not given

20. Tan $\theta = \frac{12}{13}$ Find $\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$

Sol:

Tan $\theta = \frac{opposite\ side}{adjacent\ side}$

Let x be, the hypotenuse



By Pythagoras we get

$AC^2 = AB^2 + BC^2$

$x^2 = 144 + 169$

$x = \sqrt{313}$

Sin $\theta = \frac{AB}{AC} = \frac{12}{\sqrt{313}}$

Cos $\theta = \frac{BC}{AC} = \frac{13}{\sqrt{313}}$

Substitute, Sin θ, cos θ in equation we get

$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} \Rightarrow \frac{2\times\frac{12}{\sqrt{313}}\times\frac{13}{\sqrt{313}}}{\frac{169}{313} - \frac{144}{313}}$

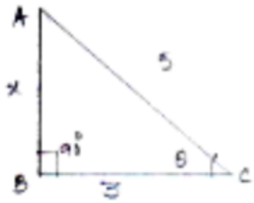
$= \frac{\frac{312}{313}}{\frac{25}{313}} = \frac{312}{25}$

21. If cos $\theta = \frac{3}{5}$, find the value of $\frac{\sin\theta - \frac{1}{\tan\theta}}{2\tan\theta}$

Sol:

Cos $\theta = \frac{3}{5}$ find value of $\frac{\sin\theta - \frac{1}{\tan\theta}}{2\tan\theta}$

We know that cos $\theta = \frac{adjacent\ side}{hypotenuse}$



Let us consider right angled Δle ABC

Let x be the opposite side, By applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$25 = x^2 + 9$

$x^2 = 16 \Rightarrow x = 4$

Sin $\theta = \frac{AB}{AC} = \frac{4}{5}$

Tan $\theta = \frac{AB}{BC} = \frac{4}{3}$

Substitute sin θ, tan θ in equation we get

$\frac{\sin\theta - \frac{1}{\tan\theta}}{2\tan\theta} = \frac{\frac{4}{5}-\frac{3}{4}}{2\times\frac{4}{3}}$

$= \frac{\frac{16-15}{20}}{\frac{8}{3}} = \frac{\frac{1}{20}}{\frac{8}{3}}$

$= \frac{1}{20} \times \frac{3}{8} = \frac{3}{160}$

22. If sin $\theta = \frac{3}{5}, evaluate\ \frac{\cos\theta - \frac{1}{\tan\theta}}{2\cot\theta}$

Sol:

Not given

23. If sec A = $\frac{5}{4}$, verify that $\frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A} = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

Sol:

Not given

24. If sin $\theta = \frac{3}{4}$, prove that $\sqrt{\frac{cosec^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \frac{\sqrt{7}}{3}$

Sol:

Not given

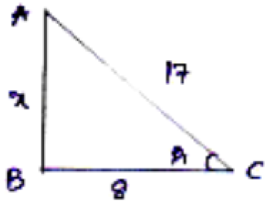
25. If sec A = $\frac{17}{8}$, verify that $\frac{3-4\sin^2 A}{4\cos^2 A - 3} = \frac{3-\tan^2 A}{1-3\tan^2 A}$

Sol:

Sec A = $\frac{17}{8}$ verify that $\frac{3-4\sin^2 A}{4\cos^2 A - 3} = \frac{3-\tan^2 A}{1-3\tan^2 A}$

We know sec A = $\frac{hypotenuse}{adjacent\ side}$

Consider right angled triangle ABC



Let x be the adjacent side

By applying Pythagoras we get

$AC^2 = AB^2 + BC^2$

$(17)^2 = x^2 + 64$

$x^2 = 289 - 64$

$x^2 = 225 \Rightarrow x = 15$

Sin A = $\frac{AB}{BC} = \frac{15}{17}$

Cos A = $\frac{BC}{AC} = \frac{8}{17}$

Tan A = $\frac{AB}{BC} = \frac{15}{8}$

L.H.S = $\frac{3-4\sin^2 A}{4\cos^2 A-3} = \frac{3-4\times\left(\frac{15}{17}\right)^2}{4\times\left(\frac{8}{17}\right)^2-3} = \frac{3-4\times\frac{225}{289}}{4\times\frac{64}{289}-3} = \frac{867-900}{256-867} = \frac{-33}{-611} = \frac{33}{611}$

R.H.S = $\frac{3-\tan^2 A}{1-3\tan^2 A} = \frac{3-\left(\frac{15}{8}\right)^2}{1-3\times\left(\frac{15}{8}\right)^2} = \frac{3-\frac{225}{64}}{1-3\times\frac{225}{64}} = \frac{\frac{-33}{64}}{\frac{-611}{64}} = \frac{-33}{-611} = \frac{33}{611}$

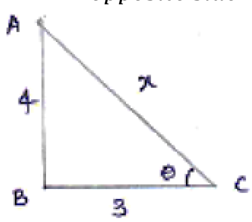
∴ LHS = RHS

26. If cot $\theta = \frac{3}{4}$, prove that $\sqrt{\frac{\sec\theta - cosec\,\theta}{\sec\theta + cosec\,\theta}} = \frac{1}{\sqrt{7}}$

Sol:

Cot $\theta = \frac{3}{4}$ P.T $\sqrt{\frac{\sec\theta - cosec\,\theta}{\sec\theta + cosec\,\theta}} = \frac{1}{\sqrt{7}}$

Cot $\theta = \frac{adjacent\ side}{opposite\ side}$



Let x be the hypotenuse by applying Pythagoras theorem.

$AC^2 = AB^2 + BC^2$

$x^2 = 16 + 9$

$x^2 = 25 \Rightarrow x = 5$

$\sec\theta = \frac{AC}{BC} = \frac{5}{3}$

Cosec $\theta = \frac{AC}{AB} = \frac{5}{4}$

On substituting in equation we get

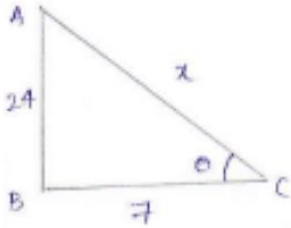
$\sqrt{\frac{\sec\theta - cosec\,\theta}{\sec\theta + cosec\,\theta}} = \sqrt{\frac{\frac{5}{3}-\frac{5}{4}}{\frac{5}{3}+\frac{5}{4}}}$

$= \sqrt{\frac{\frac{20-15}{12}}{\frac{20+15}{12}}} = \sqrt{\frac{5}{35}} = \frac{1}{\sqrt{7}}$

27. If tan $\theta = \frac{24}{7}$, find that sin θ + cos θ

Sol:

Tan $\theta = \frac{24}{7} find \sin\theta + \cos\theta$



Let x − 1 be the hypotenuse By applying Pythagoras theorem we get

$AC^2 = AB^2 + BC^2$

$x^2 = (24)^2 + (7)^2$

$x^2 = 576 + 49 = 62.5$

x = 25

sin $\theta = \frac{AB}{AC} = \frac{24}{25}$

cos $\theta = \frac{BC}{AC} = \frac{7}{25}$

sin θ + cos $\theta = \frac{24}{25} + \frac{7}{25}$

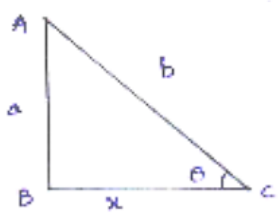
$= \frac{31}{25}$

28. If sin $\theta = \frac{a}{b}$, find sec θ + tan θ in terms of a and b.

Sol:

Sin $\theta = \frac{a}{b}$ find sec θ + tan θ

We know sin $\theta = \frac{opposite\ side}{hypotenuse}$



Let x be the adjacent side

By applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$b^2 = a^2 + x^2$

$x^2 = b^2 - a^2$

$x = \sqrt{b^2 - a^2}$

sec $\theta = \frac{AC}{BC} = \frac{b}{\sqrt{b^2-a^2}}$

Tan $\theta = \frac{AB}{BC} = \frac{a}{\sqrt{b^2-a^2}}$

Sec θ + tan θ = $\frac{b}{\sqrt{b^2-a^2}} + \frac{a}{\sqrt{b^2-a^2}}$

$= \frac{b+a}{\sqrt{b^2-a^2}} = \frac{b+a}{\sqrt{(b+a)(b-a)}} = \frac{b+a}{\sqrt{b+a}} - \frac{1}{\sqrt{b-a}} = \sqrt{\frac{b+a}{b-a}}$

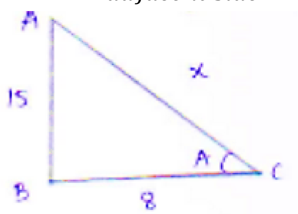
29. If 8 tan A = 15, find sin A – cos A.

Sol:

8 tan A = 15 find. Sin A – cos A

Tan A = $\frac{15}{8}$

Tan A = $\frac{opposite\ side}{adjacent\ side}$



Let x be the hypotenuse By applying theorem.

$AC^2 = AB^2 + BC^2$

$x^2 = 15^2 + 8^2$

$x^2 = 225 + 64$

$x^2 = 289 \Rightarrow$ x = 17

$Sin\ A = \frac{AB}{AC} = \frac{15}{17}$

Sin A – cos A = $\frac{15}{17} - \frac{8}{17}$

$= \frac{7}{17}$

30. If 3cos θ – 4sin θ = 2cos θ + sinθ Find tan θ

Sol:

3 cos θ – 2 cos θ = 4 sin θ + sin θ find tan θ

3 cos θ – 2 cos θ = sin θ + 4 sin θ

Cos θ = 5 sin θ

Dividing both side by use we get

$\frac{\cos\theta}{\cos\theta} = \frac{5\sin\theta}{\cos\theta}$

1 = 5 tan θ

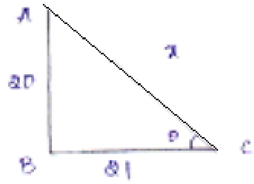
$\Rightarrow \tan\theta = 1$

31. If tan θ = $\frac{20}{21}$, show that $\frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta} = \frac{3}{7}$

Sol:

Tan θ = $\frac{20}{21}$ S.T $\frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta} = \frac{3}{7}$

Tan θ = $\frac{opposite\ side}{efficient\ side} = \frac{20}{21}$



Let x be the hypotenuse By applying Pythagoras we get

$AC^2 + AB^2 + BC^2$

$x^2 = (20)^2 + (21)^2$

$x^2 = 400 + 441$

$x^2 = 841$ ⇒ x = 29

$Sin\ \theta = \frac{AB}{AC} = \frac{20}{29}$

Cos θ = $\frac{BC}{AC} = \frac{21}{29}$

Substitute sin θ, cos θ in equation we get

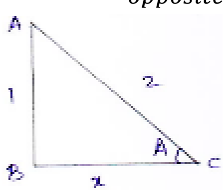
$\Rightarrow \frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta}$

$\Rightarrow \frac{1-\frac{20}{29}+\frac{21}{29}}{1+\frac{20}{29}+\frac{21}{29}} = \frac{\frac{29-20+21}{29}}{\frac{29+20+21}{29}} = \frac{30}{70} = \frac{3}{7}$

32. If Cosec A = 2 find $\frac{1}{Tan\ A} + \frac{\sin A}{1+\cos A}$

Sol:

Cosec A = $\frac{hypotenuse}{opposite\ side} = \frac{2}{1}$



Let x be the adjacent side

By applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$4 = 1 + x^2$

$x^2 = 3 \Rightarrow x = \sqrt{3}$

Sin A = $\frac{1}{cosec\ A} = \frac{1}{2}$

Tan A = $\frac{AB}{BC} = \frac{1}{\sqrt{3}}$

Cos A = $\frac{BC}{AC} = \frac{\sqrt{3}}{2}$

Substitute in equation we get

$$\frac{1}{TanA}+\frac{\sin A}{1+\cos A}=\frac{1}{\frac{1}{\sqrt{3}}}+\frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}$$

$$=\sqrt{3}+\frac{\frac{1}{2}}{\frac{2+\sqrt{3}}{2}}=\sqrt{3}+\frac{1}{2+\sqrt{3}}=\frac{2\sqrt{3}+3+1}{2+\sqrt{3}}=\frac{2\sqrt{3}+4}{2+\sqrt{3}}=\frac{2(2+\sqrt{3})}{2+\sqrt{3}}=2$$

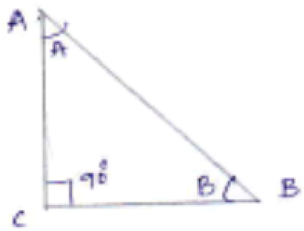
33. If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

Sol:

∠A and ∠B are acute angles.

Cos A = cos B S.T ∠A = ∠B

Let us consider right angled triangle ACB.



We have cos A = $\frac{adjacent\ side}{Hypotenuse}$

= $\frac{AC}{AB}$

Cos B = $\frac{BC}{AB}$

$Cos\ A = \cos B$

$\frac{AC}{AB}=\frac{BC}{AB}$

AC = BC

∠A = ∠B

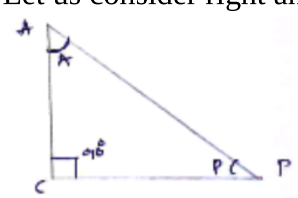
34. If ∠A and ∠P are acute angles such that tan A = tan P, then show that ∠A = ∠P.

Sol:

A and P are acute angle tan A = tan P

S. T. ∠A = ∠P

Let us consider right angled triangle ACP,



We know tan θ = $\frac{opposite\ side}{adjacent\ side}$

Tan A = $\frac{PC}{AC}$

Tan A = $\frac{AC}{PC}$

Tan A = $\frac{AC}{PC}$

Tan = tan P

$\frac{DC}{AC} = \frac{AC}{PC}$

$(PC)^2 = (AC)^2$

$PC = AC$ [∵ Angle opposite to equal sides are equal]

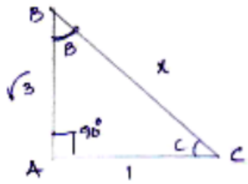
$\angle P = \angle A$

35. In a ΔABC, right angled at A, if tan C = $\sqrt{3}$, find the value of sin B cos C + cos B sin C.

Sol:

In a Δle ABC right angled at A tan C = $\sqrt{3}$

Find sin B cos C + cos B sin C



Tan c = $\sqrt{3}$

Tan C = $\frac{opposite\ side}{adjacent\ side}$

Let x be the hypotenuse By applying Pythagoras we get

$BC^2 = BA^2 + AC^2$

$x^2 = \left(\sqrt{3}\right)^2 + 1^2$

$x^2 = \Delta \Rightarrow x = 2$

At ∠B, sin B $= \frac{AC}{BC} = \frac{1}{2}$

Cos B $= \frac{\sqrt{3}}{2}$

At ∠C, sin $= \frac{\sqrt{3}}{2}$

Cos c $= \frac{1}{2}$

On substitution we get

$\Rightarrow \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{1}{4} + \frac{(\sqrt{3})}{4} \times \left(\sqrt{3}\right) = \frac{\sqrt{3}\times\sqrt{3}+1}{4} = \frac{3+1}{4} = \frac{4}{4} = 1$

36. State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) Sec A = $\frac{12}{5}$ for some value of angle A.

(iii) Cos A is the abbreviation used for the cosecant of angle A.

(iv) Sin θ = $\frac{4}{3}$ for some angle θ.

Sol:

(a) Tan A ∠1

Value of tan A at 45° i.e., tan 45 = 1

As value of A increases to 90°

Tan A becomes infinite

So given statement is false.

(b) Sec A = $\frac{12}{5}$ for some value of angle of

M-I

Sec A = 2.4

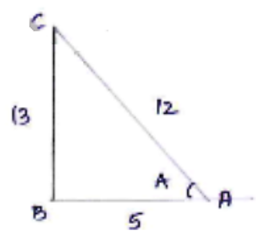
Sec A > 1

So given statement is True

M-II

For sec A = $\frac{12}{5}$

For sec A = $\frac{12}{5}$ we get adjacent side = 13



We get a right angle Δle

Subtending 9i at B.

So, given statement is true

(c) Cos A is the abbreviation used for cosecant of angle A.

The given statement is false. ∴ Cos A is abbreviation used for cos of angle A but not for cosecant of angle A.

(d) Cot A is the product of cot A and A

Given statement is false

∵ cot A is co-tangent of angle A and co-tangent of angle A = $\frac{adjacent\ side}{opposite\ side}$

(e) Sin θ = $\frac{4}{3}$ for some angle θ

Given statement is false

Since value of sin θ is less than (or) equal to one. Here value of sin θ exceeds one, so given statement is false.

Exercise 5.2

Evaluate each of the following (1 – 19):

1. $\sin 45° \sin 30° + \cos 45° \cos 30°$

Sol:

$\sin 45° \sin 30° + \cos 45° \cos 30° \,.... (i)$

We know that by trigonometric ratios we have,

Sin 45° $= \frac{1}{\sqrt{2}}$ $\quad \sin 30° = \frac{1}{2}$

Cos 45° $= \frac{1}{\sqrt{2}}$ $\quad \cos 30° = \frac{\sqrt{3}}{2}$

Substituting the values in (i) we get

$\frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

2. Sin 60° cos 30° + cos 60° sin 30°

Sol:

Sin 60° cos 30° + cos 60° sin 30° …(i)

By trigonometric ratios we have,

Sin 60° = $\frac{\sqrt{3}}{2}$ sin 30° = $\frac{1}{2}$

Cos 30° = $\frac{\sqrt{3}}{2}$ cos 60° = $\frac{1}{2}$

Substituting above values in (i), we get

$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

3. Cos 60° cos 45° - sin 60° · sin 45°

Sol:

Cos 60° cos 45° - sin 60° · sin 45° …(i)

By trigonometric ratios we know that,

Cos 60° = $\frac{1}{2}$ cos 45° = $\frac{1}{\sqrt{2}}$

Sin 60° = $\frac{\sqrt{3}}{2}$ sin 45° = $\frac{1}{\sqrt{2}}$

By substituting above value in (i), we get

$\frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} \Rightarrow \frac{1-\sqrt{3}}{2\sqrt{2}}$

4. $Sin^2 30° + \sin^2 45° + \sin^2 60° + \sin^2 90°$

Sol:

$Sin^2 30° + \sin^2 45° + \sin^2 60° + \sin^2 90°$ …(i)

$By\ trigonometric\ ratios\ we\ have$

Sin 30° = $\frac{1}{2}$ sin 45° = $\frac{1}{\sqrt{2}}$

Sin 60° = $\frac{\sqrt{3}}{2}$ sin 90° = 1

By substituting above values in (i), we get

$= \left[\frac{1}{2}\right]^2 + \left[\frac{1}{\sqrt{2}}\right]^2 + \left[\frac{\sqrt{3}}{2}\right]^2 + [1]^2$

$= \frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1 \Rightarrow \frac{1+3}{4} + \frac{1+2}{2}$

$\Rightarrow 1 + \frac{3}{2} = \frac{2+3}{2} = \frac{5}{2}$

5. $\cos^2 30° + \cos^2 45° + \cos^2 60° + \cos^2 90°$

Sol:

$\cos^2 30° + \cos^2 45° + \cos^2 60° + \cos^2 90°$ …(i)

By trigonometric ratios we have

Cos 30° = $\frac{\sqrt{3}}{2}$ cos 45° = $\frac{1}{\sqrt{2}}$

Cos 60° = $\frac{1}{2}$ cos 90° = 0

By substituting above values in (i), we get

$\left[\frac{\sqrt{3}}{2}\right]^2 + \left[\frac{1}{\sqrt{2}}\right]^2 + \left[\frac{1}{2}\right]^2 + [1]^2$

$\frac{3}{4} + \frac{1}{2} + \frac{1}{4} = 0 \Rightarrow 1 + \frac{1}{2} = \frac{3}{2}$

6. $\tan^2 30° + \tan^2 60° + \tan^2 45°$

Sol:

$\tan^2 30° + \tan^2 60° + \tan^2 45°$ …(i)

By trigonometric ratios we have

Tan 30° = $\frac{1}{\sqrt{3}}$ tan 60° = $\sqrt{3}$ tan 45° = 1

By substituting above values in (i), we get

$\left[\frac{1}{\sqrt{3}}\right]^2 + \left[\sqrt{3}\right]^2 + [1]^2$

$\Rightarrow \frac{1}{3} + 3 + 1 \Rightarrow \frac{1}{3} + 4$

$\Rightarrow \frac{1+12}{3} = \frac{13}{3}$

7. $2\sin^2 30° - 3\cos^2 45° + \tan^2 60°$

Sol:

$2\sin^2 30° - 3\cos^2 45° + \tan^2 60°$ …(i)

By trigonometric ratios we have

Sin 30° = $\frac{1}{2}$ cos 45° $\frac{1}{\sqrt{2}}$ tan 60° = $\sqrt{3}$

By substituting above values in (i), we get

$2\cdot\left[\frac{1}{2}\right]^2 - 3\left[\frac{1}{\sqrt{2}}\right]^2 + \left[\sqrt{3}\right]^2$

$2.\frac{1}{4} - 3.\frac{1}{2} + 3$

$\frac{1}{2} - \frac{3}{2} + 3 \Rightarrow \frac{3}{2} + 2 = 2$

8. $\sin^2 30° \cos^2 45° + 4\tan^2 30° + \frac{1}{2}\sin^2 90° - 2\,cos^2\,90° + \frac{1}{24}\cos^2 0°$

Sol:

$\sin^2 30° \cos^2 45° + 4\tan^2 30° + \frac{1}{2}\sin^2 90° - 2\,cos^2\,90° + \frac{1}{24}\cos^2 0°$ …(i)

By trigonometric ratios we have

Sin 30° = $\frac{1}{2}$ cos 45° = $\frac{1}{\sqrt{2}}$ tan 30° = $\frac{1}{\sqrt{3}}$ sin 90° = 1 cos 90° = 0 cos 0° = 1

By substituting above values in (i), we get

$\left[\frac{1}{2}\right]^2 \cdot \left[\frac{1}{\sqrt{2}}\right]^2 + 4\left[\frac{1}{\sqrt{3}}\right]^2 + \frac{1}{2}[1]^2 - 2[0]^2 + \frac{1}{24}[1]^2$

$\frac{1}{4}\cdot\frac{1}{2} + 4\cdot\frac{1}{3} + \frac{1}{2} - 0 + \frac{1}{24}$

$\frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24} = \frac{48}{24} = 2$

9. $4(\sin^4 60° + \cos^4 30°) - 3(\tan^2 60° - \tan^2 45°) + 5\cos^2 45°$

Sol:

$4(\sin^4 60° + \cos^4 30°) - 3(\tan^2 60° - \tan^2 45°) + 5\cos^2 45°$ …(i)

By trigonometric ratios we have

Sin 60° = $\frac{\sqrt{3}}{2}$ cos 30° = $\frac{\sqrt{3}}{2}$ tan 60° = $\sqrt{3}$ tan 45° = 1 cos 45 ° = $\frac{1}{\sqrt{2}}$

By substituting above values in (i), we get

$4\left(\left[\frac{\sqrt{3}}{2}\right]^4 + \left[\frac{\sqrt{3}}{2}\right]^4\right) - 3([3]^2 - [1]^2) + 5\left[\frac{1}{\sqrt{2}}\right]^2$

$\Rightarrow 4\left[\frac{9}{16} + \frac{9}{16}\right] - 3[3-1] + 5\left[\frac{1}{2}\right]$

$\Rightarrow 4\cdot\frac{18}{16} - 6 + \frac{5}{2}$

$\Rightarrow \frac{1}{4} - 6 + \frac{5}{2}$

$= \frac{9}{2} + \frac{5}{2} - 6$

$= \frac{14}{2} - 6 = 7 - 6 = 1$

10. $(cosec^2 45° \sec^2 30°)(\sin^2 30° + 4\cot^2 45° - \sec^2 60°)$

Sol:

$(cosec^2 45° \sec^2 30°)(\sin^2 30° + 4\cot^2 45° - \sec^2 60°)$ ...(i)

By trigonometric ratios we have

Cosec 45° = $\sqrt{2}$ sec 30° = $\frac{2}{\sqrt{3}}$ sin 30° = $\frac{1}{2}$ cot 45° = 1 sec 60° = 2

By substituting above values in (i), we get

$$\left([\sqrt{2}]^2 \cdot \left[\frac{2}{\sqrt{3}}\right]^2\right)\left(\left[\frac{1}{2}\right]^2 + 4[1]^2 \cdot [2]^2\right)$$

$$\Rightarrow \left[2 \cdot \frac{4}{3}\right]\left[\frac{1}{4} + 4 - 4\right] \Rightarrow 3 \cdot \frac{4}{3} \cdot \frac{1}{4} = \frac{2}{3}$$

11. $cosec^3 30° \cos 60° \tan^3 45° \sin^2 90° \sec^2 45° \cot 30°$

Sol:

$cosec^3 30° \cos 60° \tan^3 45° \sin^2 90° \sec^2 45° \cot 30°$ ...(i)

By trigonometric ratios we have

Cosec 30° = 2, cos 60° = $\frac{1}{2}$, tan 45° = 1 sin 90° = 1 sec 45° = $\sqrt{2}$ cot 30° = $\sqrt{3}$

By substituting above values in (i), we get

$$[2]^3 \cdot \frac{1}{2} \cdot (1)^3 \cdot (1)^2 (\sqrt{2})^2 \cdot \sqrt{3}$$

$$\Rightarrow 8 \cdot \frac{1}{2} \cdot 1 \cdot 2 \cdot \sqrt{3} \Rightarrow 8\sqrt{3}$$

12. $\cot^2 30° - 2\cos^2 60° - \frac{3}{4}\sec^2 45° - 4\sec^2 30°$

Sol:

$\cot^2 30° - 2\cos^2 60° - \frac{3}{4}\sec^2 45° - 4\sec^2 30°$ ...(i)

By trigonometric ratios we have

cot 30° = $\sqrt{3}$ cos 60° = $\frac{1}{2}$ sec 45° = $\sqrt{2}$ sec 30° = $\frac{2}{\sqrt{3}}$

By substituting above values in (i), we get

$$(\sqrt{3})^2 - 2\left[\frac{1}{2}\right]^2 - \frac{3}{4}(\sqrt{2})^2 - 4\left[\frac{2}{\sqrt{3}}\right]^2$$

$$3 - 2 \cdot \frac{1}{4} - \frac{3}{4} \cdot 2 - 4 \cdot \frac{4}{3}$$

$$3 - \frac{1}{2} - \frac{3}{2} - \frac{8}{3} \Rightarrow -\frac{5}{3}$$

13. $(\cos 0^{\circ} + \sin 45° + \sin 30°)(\sin 90° - \cos 45° + \cos 60°)$

Sol:

$(\cos 0^{\circ} + \sin 45° + \sin 30°)(\sin 90° - \cos 45° + \cos 60°)$ ...(i)

By trigonometric ratios we have

Cos 0° = 1, sin 45° = $\frac{1}{\sqrt{2}}$, sin 30° = $\frac{1}{2}$, sin 90° = 1, cos 45° = $\frac{1}{\sqrt{2}}$ cos 60° = $\frac{1}{2}$

By substituting above values in (i), we get

$\left(1+\frac{1}{\sqrt{2}}+\frac{1}{2}\right)\left(1-\frac{1}{\sqrt{2}}+\frac{1}{2}\right)$

$\left[\frac{3}{2}+\frac{1}{\sqrt{2}}\right]\left[\frac{3}{2}-\frac{1}{\sqrt{2}}\right] \Rightarrow \left[\frac{3}{2}\right]^2-\left[\frac{1}{\sqrt{2}}\right]=\frac{9}{4}-\frac{1}{2}=\frac{7}{4}$

14. $\frac{\sin 30°-\sin 90°+2\cos 0°}{\tan 30° \tan 60°}$

Sol:

$\frac{\sin 30°-\sin 90°+2\cos 0°}{\tan 30° \tan 60°}$ …(i)

By trigonometric ratios we have

Sin 30° = $\frac{1}{2}$ sin 90° = 1 cos 0° = 1 tan 30° = $\frac{1}{\sqrt{3}}$ tan 60° = $\sqrt{3}$

By substituting above values in (i), we get

$\frac{\frac{1}{2}-1+2}{\sqrt{3}\cdot\frac{1}{\sqrt{3}}}=\frac{\frac{3}{2}+1}{1}=\frac{3}{2}$

15. $\frac{4}{\cot^2 30°}+\frac{1}{\sin^2 60°}-\cos^2 45°$

Sol:

$\frac{4}{\cot^2 30°}+\frac{1}{\sin^2 60°}-\cos^2 45°$ …(i)

By trigonometric ratios we have

Cot 30° = $\sqrt{3}$ sin 60° = $\frac{\sqrt{3}}{2}$ cos 45° = $\frac{1}{\sqrt{2}}$

By substituting above values in (i), we get

$\frac{4}{(\sqrt{3})^2}+\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}-\left(\frac{1}{\sqrt{2}}\right)^2$

$\frac{4}{3}+\frac{4}{3}-\frac{1}{2}=\frac{13}{6}$

16. $4(\sin^4 30°+\cos^2 60°)-3(\cos^2 45°-\sin^2 90°)-\sin^2 60°$

Sol:

$4(\sin^4 30°+\cos^2 60°)-3(\cos^2 45°-\sin^2 90°)-\sin^2 60°$ …(i)

By trigonometric ratios we have

Sin 30° = $\frac{1}{2}$ cos 60° = $\frac{1}{2}$ cos 45° = $\frac{1}{\sqrt{2}}$ Sin 90° = 1 sin 60° = $\frac{\sqrt{3}}{2}$

By substituting above values in (i), we get

$4\left[\left(\frac{1}{2}\right)^4+\left(\frac{1}{2}\right)^2\right]-3\left[\left[\frac{1}{\sqrt{2}}\right]^2-1\right]-\left[\frac{\sqrt{3}}{2}\right]^2$

$4\left[\frac{1}{16}+\frac{1}{4}\right]-3\left[\frac{1-[\sqrt{2}]}{(\sqrt{2})^2}\right]-\frac{3}{4}$

$\frac{1}{4}+1-3\left[\frac{1-[\sqrt{2}]}{[\sqrt{2}]}\right]^2-\frac{3}{4}$

$=\frac{1}{4}+1-\frac{3}{4}+\frac{3}{2}=2$

17. $\frac{\tan^2 60°+4\cos^2 45°+3\sec^2 30°+5\cos^2 90°}{cosec\ 30°+\sec 60°-\cot^2 30°}$

Sol:

$\frac{\tan^2 60°+4\cos^2 45°+3\sec^2 30°+5\cos^2 90°}{cosec\ 30°+\sec 60°-\cot^2 30°}$ …(i)

By trigonometric ratios we have

Tan 60° = $\sqrt{3}$ cos 45° = $\frac{1}{\sqrt{2}}$ sec 30° = $\frac{2}{\sqrt{3}}$

cos 90° = 0 cosec 30° = 2 sec 60° = 2 cot 30° = $\sqrt{3}$

By substituting above values in (i), we get

$\frac{(\sqrt{3})^2+4\cdot\left(\frac{1}{\sqrt{3}}\right)^2+2+\left[\frac{2}{\sqrt{3}}\right]^2+5(0)^2}{2+2\sqrt{2}(+\sqrt{3})^2}$

$=\frac{3+4\cdot\frac{1}{2}+3\cdot\frac{4}{3}}{4-3}=\frac{3+2+4}{1}=9$

18. $\frac{\sin 30°}{\sin 45°}+\frac{\tan 45°}{\sec 60°}-\frac{\sin 60°}{\cot 45°}-\frac{\cos 30°}{\sin 90°}$

Sol:

$\frac{\sin 30°}{\sin 45°}+\frac{\tan 45°}{\sec 60°}-\frac{\sin 60°}{\cot 45°}-\frac{\cos 30°}{\sin 90°}$ …(i)

By trigonometric ratios we have

Sin 30° = $\frac{1}{2}$ sin 45° = $\frac{1}{\sqrt{2}}$ tan 45° = 1 sec 60° = 2 sin 60° = $\frac{\sqrt{3}}{2}$

cot 45° = 1 cos 30° = $\frac{\sqrt{3}}{2}$ sin 90° = 1

By substituting above values in (i), we get

$\frac{1}{2}\cdot\sqrt{2}+\frac{1}{2}-\frac{\sqrt{3}}{2}\cdot 1-\frac{\sqrt{3}}{2}.1$

$=\frac{2+1-\frac{2}{3}}{2}$

19. $\frac{Tan\ 45°}{cosec\ 30°}+\frac{\sec 60°}{\cot 45°}-\frac{5\sin 90°}{2\cos 0°}$

Sol:

$\frac{Tan\ 45°}{cosec\ 30°}+\frac{\sec 60°}{\cot 45°}-\frac{5\sin 90°}{2\cos 0°}$ …(i)

By trigonometric ratios we have

Tan 45° = 1 cosec 30° = 2 sec 60° = 2 cot 45° = 1 sin 90° = 1 cos 0° = 1

By substituting above values in (i), we get

$\frac{1}{2}+\frac{2}{1}-5\cdot\frac{1}{2}$

$-\frac{4}{2}+2=-2+2=0$

20. $2\sin 3x = \sqrt{3}$ s = ?

Sol:

Sin 3x = $\frac{\sqrt{3}}{2}$

Sin 3x = sin 60°

Equating angles we get,

3x = 60°

x = 20°

21. $2\sin\frac{x}{2} = 1$ x = ?

Sol:

Sin $\frac{x}{2} = \frac{1}{2}$

Sin $\frac{x}{2} = \sin 30°$

$\frac{x}{2} = 30°$

$x = 60°$

22. $\sqrt{3}\sin x = \cos x$

Sol:

$\sqrt{3}$ tan x = 1

$\tan x = \frac{1}{\sqrt{3}}$

∴ Tan x = Tan 30°

x = 30°

23. Tan x = sin 45° cos 45 ° + sin 30°

Sol:

Tan x = $\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}+\frac{1}{2}$ [∵ sin 45° = $\frac{1}{\sqrt{2}}$ cos 45° = $\frac{1}{\sqrt{2}}$ $\sin 30° = \frac{1}{2}$]

Tan x = $\frac{1}{2}+\frac{1}{2}$

Tan x = 1

Tan x = tan 45°

x = 45°

24. $\sqrt{3}\tan 2x = \cos 60° + \sin 45°\cos 45°$

Sol:

$\sqrt{3}\tan 2x = \frac{1}{2}+\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}$ $\left[\because \cos 60° = \frac{1}{2}\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}\right]$

$\sqrt{3}\tan 2x = \frac{1}{\sqrt{3}} \Rightarrow \tan 2x = \tan 30°$

2x = 30°

x = 15°

25. Cos 2x = cos 60° cos 30° + sin 60° sin 30°

Sol:

Cos 2x = $\frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{2}$ $\left[\because \cos 60° = \sin 30° = \frac{1}{2} \sin 60° = \cos 30° = \frac{\sqrt{3}}{2}\right]$

Cos 2x = $2 \cdot \frac{\sqrt{3}}{4}$

⇒ cos 2x = $\frac{\sqrt{3}}{2}$

Cos 2x = cos 30°

$2x = 30°$

$x = 15°$

26. If θ = 30° verify

(i) Tan 2θ = $\frac{2\tan\theta}{1-\tan^2\theta}$

Sol:

Tan 2θ = $\frac{2\tan\theta}{1-\tan^2\theta}$ …(i)

Substitute θ = 30° in (i)

LHS = Tan 60° = $\sqrt{3}$

RHS = $\frac{2\tan 30°}{1-\tan^2 30°} = \frac{2\cdot\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$

= $\frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \sqrt{3}$

∴ LHS = RHS

(ii) Sin θ = $\frac{2\tan\theta}{1-\tan^2\theta}$

Substitute θ = 30°

Sin 60° = $\frac{2\tan 30°}{1+(\tan 30°)^2}$

= $\frac{\sqrt{3}}{2} = \frac{2\cdot\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$

= $\frac{\sqrt{3}}{2} = \frac{2}{\sqrt{3}} \cdot \frac{3}{4} \Rightarrow \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

∴ LHS = RHS

(iii) Cos 2θ = $\frac{1-\tan^2\theta}{1+\tan^2\theta}$

Substitute θ = 30°

LHS = cosec θ RHS = $\frac{1-\tan^2\theta}{1+\tan^2\theta}$

= cos 2(30°) = $\frac{1-\tan^2 30°}{1+\tan^2 30°}$

Cos 60° = $\frac{1}{2}$ = $\frac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{1-\frac{1}{3}}{1+\frac{1}{3}} = \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{1}{2}$

∴ LHS = RHS

(iv) Cos 30θ = 4 $\cos^3\theta - 3\cos\theta$

LHS = Cos 30° RHS 4 $\cos^3\theta - 3\cos\theta$

Substitute $\theta = 30°$ $4\cos^3 30°$ - 3 cos 30°

Cos 3 (30°) = cos 90° $4\cdot\left[\frac{\sqrt{3}}{2}\right]^3 - 3\cdot\frac{\sqrt{3}}{2}$

$= 0$ $\Rightarrow \frac{3\sqrt{3}}{2} - \frac{3\sqrt{2}}{2} = 0$

27. If A = B = 60°. Verify

(i) Cos (A – B) = Cos A cos B + sin A sin B

Sol:

Cos (A – B) = Cos A cos B + sin A sin B …(i)

Substitute A & B in (i)

⇒ cos (60 - 60°)= cos 60° cos 60° + sin 60° sin 60°

Cos 0° = $\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$

1 = $\frac{1}{4} + \frac{3}{4} = 1 = 1$ LHS = RHS

(ii) Substitute A & B in (i)

Sin (60° - 60°) = Sin 60° Cos 60° − cos 60° sin 60°

= sin 0° = 0 = 0

LHS = RHS

(iii) Tan (A – B) = $\frac{Tan\,A - \tan B}{1 + \tan A \tan B}$

A = 60° B = 60° we get

Tan (60° - 60°) = $\frac{\tan 60° - \tan 60°}{1 - \tan 60 \tan 60°}$

Tan 0° = 0

0 = 0

LHS = RHS

28. If A = 30° B = 60° verify

(i) Sin (A + B) = Sin A Cos B + cos A sin B

Sol:

A = 30°, B = 60° we get

Sin (30° + 60°) = Sin 30° cos 60° + cos 30° sin 60°

Sin 90° = $\frac{1}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}$

Sin 90° = 1 ⇒ 1 = 1

LHS = RHS

(ii) Cos (A + B) = cos A cos B – Sin A Sin B

A = 30° B = 60°

Cos (90°) = Cos 30° cos 60° − sin 30° sin 60°

= cos 90° = $\frac{1}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{1}{2}$

0 = 0

LHS = RHS

29. Sin (A – B) = Sin A Cos B – cos A sin B

Cos (A – B) = cos A Cos B – sin A sin B

Find sin 15° cos 15°

Sol:

Sin (A – B) = Sin A Cos B – cos A sin B …(i)

Cos (A – B) = cos A Cos B – sin A sin B …(ii)

Let A = 45° B = 30° we get on substituting in (i)

⇒ Sin(45° − 30°) = Sin 45° cos 30°

Sin 15° = $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

∴ Sin 15° = $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(ii) A = 45° B = 30° in equation (ii) we get

Cos (45° − 30°) cos 45° cos 30° + sin 45° sin 30°

Cos 15° - $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

Cos 15° ⇒ $\frac{\sqrt{3}+1}{2\sqrt{2}}$

30. In right angled triangle ABC. ∠C = 90°, ∠B = 60°. AB = 15units. Find remaining angles and sides.

Sol:

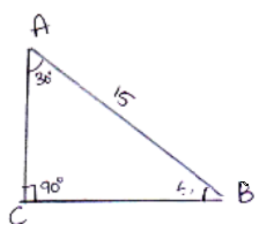
In a Δle sum of all angles = 180°

∠A + ∠B + ∠C = 180°

⇒ 90° + 60° + ∠A = 180°

∠A = 180° − 150°

∴ ∠A = 30°



From above figure

Cos B = $\frac{BC}{AB}$

Cos 60° = $\frac{BC}{15}$

$\frac{1}{2} = \frac{BC}{15}$

BC = $\frac{15}{2}$

Sin B = $\frac{AC}{15}$

Sin 60° = $\frac{AC}{15}$

$\frac{\sqrt{3}}{2} = \frac{AV}{15} = AC = \frac{15\sqrt{3}}{2}$

31. In ΔABC is a right triangle such that $\angle C = 90° \angle A = 45°$, $BC = 7$ units find $\angle B$, AB and AC

Sol:

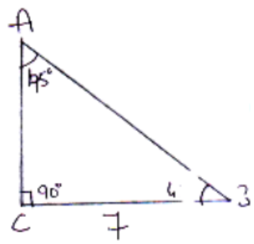
Sum of angles in Δle = 180°

$\angle A + \angle B + \angle C = 180°$

$45° + \angle B + 90° = 180°$

$\angle B = 180° - 135°$

$\angle B = 45°$



From figure cos B $= \frac{BC}{AB}$

Cos 45° = $\frac{7}{AB}$

$\frac{1}{\sqrt{2}} \cdot \frac{7}{AB}$

AB = $7\sqrt{2}$ units

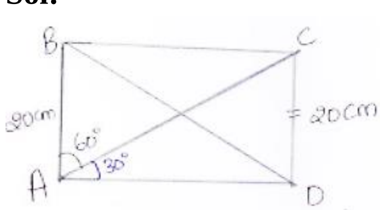
From figure sin B = $\frac{AC}{AB}$

Sin 45° = $\frac{AC}{7\sqrt{2}}$

$\frac{1}{\sqrt{2}} = \frac{AC}{7\sqrt{2}} \therefore AC = 7\ units$

32. In rectangle ABCD AB = 20cm ∠BAC = 60° BC, calculate side BC and diagonals AC and BD.

Sol:



Consider Δle ABC we get

Cos A = $\frac{AB}{AC}$ Sin A = $\frac{BC}{AC}$

∴ cos 60° = $\frac{20}{AC}$ Sin 60° = $\frac{BC}{AC}$

$\frac{1}{a} = \frac{20}{AC}$ ∴ AC = 40 cm $\frac{\sqrt{3}}{2} = \frac{BC}{40}$

∴ AC = 40 cm ∴ BC = $20\sqrt{3}$ cm

Consider Δle ACD we know ∠CAD = 30°

∴ Tan 30° = $\frac{CD}{AD} = \frac{1}{\sqrt{3}} = \frac{20}{AC} = AD = 20\sqrt{3}$

In rectangle diagonals are equal in magnitude

∴ BD = AC = 40 cm

33. If Sin (A + B) = 1 and cos (A – B) = 1, 0° < A + B ≤ 90° A ≥ B. Fin A & B

Sol:

Sin(A + B) = 1

∴ Sin (A + B) = Sin 90°

A + B = 90° …(i)

Cos (A – B) = 1

Cos (A – B) = cos 0°

A – B = 0° …(ii)

Adding (i) & (ii) we get

A + B = 90°

<u>A – B = 0°</u>

A = 90° A = 45°

A – B = 0

A = B ⇒ B = 45°

34. If Tan (A – B) = $\frac{1}{\sqrt{3}}$ and Tan (A + B) = $\sqrt{3}$, $0° < A + B \leq 90°$, A ≥ B, Find A & B

Sol:

Tan (A – B) = Tan 30° Tan (A + B) = Tan 60°

∴ A – B = 30° …(i) A + B = 60° …(ii)

Add (i) & (ii)

A – B = 30°

<u>A + B = 60°</u>

2A = 90° A = 40°

A – B = 30° 45° - B = 30°

B = 45° − 30° = 15°

35. If Sin (A – B) = $\frac{1}{2}$ and Cos (A + B) = $\frac{1}{2}$, 0° < A + B $\leq$ 90°, A $>$ B, Find A & B

Sol:

Sin (A – B) = sin 30° Cos (A + B) = cos 60°

A – B = 30° …(i)

A + B = 60° …(ii)

Add (i) & (ii) we get

2A = 90°, A = 45°.

A – B = 30°

45 – B = 30° B = 45 – 30°

B = 15°

36. In right angled triangle ΔABC at B, ∠A = ∠C. Find the values of

(i) Sin A cos C + Cos A Sin C

Sol:

In Δle ABC ∠A + ∠B + ∠C = 180°

$\angle A + 90° + \angle A = 180°$

$2\angle A = 90°$

$\angle A = 45°$

$\therefore \angle A = 45°$

(ii) $\text{Sin } 45° \cos 45° + \cos 45° \sin 45°$

$\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2} \cdot \frac{1}{2} = 1$

$(ii) Sin\ A\ Sin\ B + \cos A \cos B$

∠A = 45° sin 90° + cos 45° cos 90°

= $\frac{1}{\sqrt{2}} \cdot 1 + 0$

= $\frac{1}{\sqrt{2}}$

37. Find acute angles A & B, if sin (A + 2B) = $\frac{\sqrt{3}}{2}$ Cos (A + 4B) = 0, A > B.

Sol:

Sin (A + 2B) = Sin 60°

Cos (A + 4B) = cos 90°

A + 2B = 60° …(i)

A + 4B = 90° …(ii)

Subtracting (ii) from (i)

A + 4B = 90°

$\underline{-A - 2B = -60}$

2B = 30° ∴ B = 15°

A + 4B = 90°

4B = 4(15°) = 4B = 60°

∴ A + 60° = 90° ∴ A = 30°

38. If A and B are acute angles such that Tan A = $\frac{1}{2}$ Tan B = $\frac{1}{3}$ *and* Tan (A + B) = $\frac{\tan A+Tan\,B}{1-\tan A\,Tan\,B}$ A + B = ?

Sol:

Tan A = $\frac{1}{2}$ Tan B = $\frac{1}{3}$

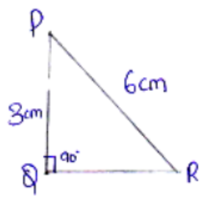
Tan (A + B) = $\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}} = \frac{\frac{5}{6}}{1-\frac{1}{6}} = 1$

Tan (A + B) = Tan 45°

∴ A – B = 45°

39. In ΔPQR, right angled at Q, PQ = 3cm PR = 6cm. Determine ∠P = ? ∠R = ?

Sol:



From above figure

Sin R = $\frac{PQ}{PR}$

Sin R = $\frac{3}{6} = \frac{1}{2}$

∴ Sin R = Sin 30°

R = 30°

We know in Δle ∠P + ∠Q + ∠R = 180°

∠P + 90° + 30° = 180°

∠P = 60°

Exercise 5.3

Evaluate the following:

1. $\frac{Sin\,20°}{\cos 70°}$

Sol:

(i)

$\Rightarrow \frac{Sin\,(90°-70°)}{\cos 70°} \Rightarrow \frac{\cos 70°}{\cos 70°}$ $[\because Sin\,(90° - \theta) = \cos\theta]$

$\Rightarrow \frac{\cos 70°}{\cos 70°} = 1$

(ii)

$\frac{\cos 19°}{\sin 71°}$

$\Rightarrow \frac{\cos(90°-71°)}{\sin 71°} \Rightarrow \frac{\sin 71°}{\sin 71°}$ [$\because \cos(90° - \theta) = \sin\theta$]
= 1

(iii)
$\frac{\sin 21°}{\cos 69°} \Rightarrow \frac{\sin(\cos 69°)}{\cos 69°} = \frac{\cos 69°}{\cos 69°}$ [$\because \sin(90° - \theta) = cos\theta$]
= 1

(iv)
$\frac{Tan\ 10°}{Cot\ 80°} \Rightarrow \frac{\tan(90°-80°)}{\cot 80°} = \frac{\cot 80°}{\cot 80°}$ [$\because \tan(90 - \theta) = \cot\theta$]
$= 1$

(v)
$\frac{\sec 11°}{cosec\ 79°} \Rightarrow \frac{\sec(90°-79°)}{cosec\ 79°} = \frac{cosec\ 79°}{cosec\ 79°}$ [$\because \sec(90 - \theta) \cdot cosec\ \theta$]
= 1

Evaluate the following:

2. (i) $\left[\frac{\sin 49°}{\cos 45}\right]^2 + \left[\frac{\cos 41°}{\sin 49°}\right]$

Sol:

We know that sin(49°) = sin(90° – 41°) = cos 41° similarly cos 41° = sin 49°

$\Rightarrow \left[\frac{\cos 41°}{\cos 41°}\right]^2 + \left[\frac{\sin 49°}{\sin 49°}\right]^2 = 1^2 + 1^2 = 2$

(ii)
Cos 48° - sin 42°

Sol:

Cos 48° = cos (90° − 42°) sin 42°

∴ sin 42° − sin42° = 0

(iii)
$\frac{\cot 40°}{\cos 35°} - \frac{1}{2}\left[\frac{\cos 35°}{\sin 55°}\right]$

Sol:

Cot 40° − cot (90° - 50°) = tan 50°

Cos 35° = cos (90° - 55°) = sin 55°

$\Rightarrow \frac{\tan 50°}{\tan 50°} - \frac{1}{2}\left[\frac{\sin 55°}{\sin 55°}\right]$

$= 1 - \frac{1}{2}[1]$

$= \frac{1}{2}$

(iv)
$\left[\frac{\sin 27°}{\cos 63°}\right] - \left[\frac{\cos 63°}{\sin 27°}\right]^2$

Sol:

Sin 27° = sin (90° - 63°) = cos 63° [∵ sin (90° - θ) = cos θ]

⇒ sin 27° = cos 63°

$\left[\frac{\sin 27°}{\sin 27°}\right]^2 - \left[\frac{\cos 63°}{\cos 63°}\right]^2 = 1 - 1 = 0$

(v)

$\frac{\tan 35°}{\cot 55°} + \frac{\cot 63°}{\cos 63°} - 1$

Sol:

Tan 35° = tan (90° - 55°) = cos 55°

Cot 78° = cot (90° - 12°) = tan 12°

⇒ $\frac{\cot 55°}{\cot 55°} + \frac{\tan 12°}{\tan 12°} - 1$

= tan 1 – 1 = 1

(vi)

$\frac{\sec 70°}{cosec\ 20°} + \frac{\sin 59°}{\cos 31°}$

Sol:

Sec 70° = sec (90° - 20°) = cosec 20° [∵ sec (90 – θ) = cosec θ]

Sin 59° = sin (90° - 31°) = cos 31° [∵ sin (90 - θ) = cos θ]

⇒ $\frac{cosec\ 20}{cosec\ 20} + \frac{\cos 31°}{\cos 31°} = 1 + 1 = 2$

(vii)

Sec 50° Sin 40° + Cos 40° cosec 50°

Sol:

Sec 50° = sec (90° - 40°) = cosec 40°

Cos 40° = cos (90° - 50°) = sin 50°

∴ Sin θ cosec θ = 1

⇒ cosec 40° sin 40° + sin 50° cosec 50°

1 + 1 = 2

3. Express each one of the following in terms of trigonometric ratios of angles lying between 0° and 45°

(i) Sin 59° + cos 56°

Sol:

Sin 59° = sin (90° - 59°) = cos 31°

Cos 56° = cos (65° - 34°) = Sin 34°

⇒ cos 31° + sin 34°

(ii)

Tan 65° + cot 49°

Sol:
Tan 65° = tan (90° - 25°) = cot 25°
Cot 49° = cot (90° - 41°) = tan (41°)
⇒ cot 25° + tan 41°
(iii)
Sec 76° + cosec 52°
Sol:
Sec 76° = sec (90° - 14°) = cosec 14°
Cosec 52° = cosec (90° - 88°) = sec 38°
⇒ Cosec 14° + sec 38°
(iv)
Cos 78° + sec 78°
Sol:
Cos 78° = cos (90° - 12°) = sin 12°
Sec 78° = sec (90° - 12°) = cosec 12°
⇒ sin 12° + cosec 12°
(v)
Cosec 54° + sin 72°
Sol:
Cosec 54° = cosec (90° - 36°) = sec 36°
Sin 72° = sin (90° - 18°) = cos 18°
⇒ sec 36° + cos 18°
(vi)
Cot 85° + cos 75°
Sol:
Cot 85° = cot (90° - 5°) = tan 5°
Cos 75° = cos (90° - 15°) = sin 15°
= tan 5° + sin 15°
(vii)
Sin 67° + cos 75°
Sol:
Sin 67° = Sin (90° − 23°) = cos 23°
Cos 75° = cos (90° −15°) = sin 15°
= cos 23° + sin 15°

4. Express Cos 75° + cot 75° in terms of angles between 0° and 30°.
Sol:
Cot 75° = cos (90° - 15°) = sin 15°
Cot 75° = cot (90° - 15°) = tan 15°
= sin 15° + tan 15°

5. If Sin 3A = cos (A – 26°), where 3A is an acute angle, find the value of A = ?

Sol:

Cos θ = sin (90° - θ)

⇒ Cos (A – 26) = sin (90° −(A− 26°))

⇒ Sin 3A = sin (90° − (A – 26))

Equating angles on both sides

3A = 90° − A + 26°

4A = 116° A = $\frac{116}{4} = 29°$

∴ A = 29°

6. If A, B, C are interior angles of a triangle ABC, prove that (i) tan $\left(\frac{C+A}{2}\right) = \cot\frac{B}{2}$

Sol:

(i) Tan $\left[\frac{c+A}{2}\right] = \cot\frac{B}{2}$

Sol:

Given A + B + C = 180°

C + A = 180° − B

⇒ Tan $\left[\frac{180-B}{2}\right] \Rightarrow Tan\ \left[90° - \frac{B}{2}\right]$

⇒ cot $\frac{B}{2}$ $[\because \tan(90° - \theta) = \cot\theta]$

∴ LHS = RHS

(ii) Sin $\left[\frac{B+C}{2}\right] = \cos\frac{A}{2}$

Sol:

A + B + C = 180°

B + C = 180° - A

LHS = sin $\left[\frac{180°-A}{2}\right] \Rightarrow \sin\left[90° - \frac{A}{2}\right]$

Cos $\frac{A}{2}$ $[\because Sin\ (90° - \theta) \cdot cos\theta]$

∴ LHS =RHS

7. Prove that

(i)

Tan20° Tan 35° tan 45° tan 55° Tan 70° = 1

Sol:

Tan 20° = tan (90° - 70°) = cot 70°

Tan 35° = tan (90° - 70°) = cot 55°

Tan 45° = 1

⇒ cot 70° tan 70° x cot 55° tan 55° x tan 45° · cot θ = tan θ = 1

⇒ 1 × 1 × 1 = 1 Hence proved.

(ii)

Sin 48° sec 42° + cosec 42° = 2

Sol:

Sin 48° = sin (90° - 42°) = cos 42°

Cos (45°) = cos (90° - 42°) = sin 42°

Sec θ · cos θ = 1 · sin θ cosec θ = 1

⇒ cos 42° sec 42° + sin 42° cosec 42°

⇒ 1 + 1 = 2

∴ LHS = RHS

(iii)

$\frac{\sin 70°}{\cos 20°} + \frac{cosec\ 20°}{\sec 70°} - 2\cos 70°\ cosec\ 20° = 0$

Sol:

Sin (70°) = sin (90° - 20°) = cos 20°

Cosec 20° = cosec (90° - 70°) = sec 70°

Cos 70° = cos (90° - 20°) = sin 20°

⇒ $\frac{\cos 20°}{\cos 20°} + \frac{\sec 70°}{\sec 70°} - 2\sin 20\ cosec\ 20°$

1 + 1 – 2(1) = 0

∴ LHS = RHS Hence proved

(iv)

$\frac{\cos 80°}{\sin 10°} + \cos 59°\ cosec\ 31° = 2$

Sol:

Cos 80° = cos (90° - 10°) = sin 10°

Cos 59° = cos (90° - 31°) = sin 31°

⇒ $\frac{\sin 10°}{\sin 10°} + \sin 31°\ cosec\ 31°$

= 1 + 1 = 2 [∵ Sin θ cosec θ = 1]

Hence proved

8. Prove the following:

(i) Sin θ sin (90 - θ) – cos θ cos (90 - θ) = 0

Sol:

Sin (90 – θ) = cos θ

Cos (90 – θ) – cos θ sin θ

= 0

∴ LHS = RHS

Hence proved

(ii) $\frac{\cos(90°-\theta)\sec(90°-\theta)\tan\theta}{cosec\ (90°-\theta)\sin(90°-\theta)\cot(90°-\theta)} + \frac{\tan(90°-\theta)}{\cot\theta} = 2$

Sol:

Cos (90° - θ) = sin A cosec (90 - θ) = sec θ

Sec (90° - θ) = cosec θ sin (90 - θ) = cos θ
Cot (90 - θ) = tan θ
$\Rightarrow \frac{\sin\theta\ cosec\ \theta\ \ \boldsymbol{tan}\,\theta}{\sec\theta.\cos\theta.\tan\theta} = \frac{\sin\theta\ cosec\ \theta}{\sec\theta\cos\theta}$ $[\because \sin\theta\ \ cosec\ \theta = 1]$
= 1 $[\sec\theta\cos\theta = 1]$
$\frac{\tan(90°-\theta)}{\cot\theta} = \frac{\cot\theta}{\cot\theta} = 1$
⇒ 1 + 1 = 2
∴ LHS = RHS
Hence proved

(iii) $\frac{\tan(90-A)\cot A}{cosec^2 A} - \cos^2 A = 0$
Sol:
Tan (90 – A) = cot A
$\Rightarrow \frac{\cot A\,.\cot A}{cosec^2 A} - \cos^2 A$
$\Rightarrow \frac{\cot^2 A}{cosec^2 A} - \cos^2 A$
= $\frac{\cos^2 A}{\sin^2 A} - \cos^2 A \Rightarrow \cos^2 A\cos^2 A = 0$
Hence proved

(iv) $\frac{\cos(90°-A)\sin(90°-A)}{\tan(90°-A)} - \sin^2 A = 0$
Sol:
Cos (90° - A) = sin A   Tan (90° - A) = cot A
Sin (95° A) = cos A
$\frac{\sin A\cos A}{\cot A} - \sin^2 A = 0$
$\frac{\sin A.\cos A.}{\cos A}\sin A - \sin^2 A$
$\sin^2 A - \sin^2 A = 0$
LHS = RHS
Hence Proved

(v) Sin (50° + θ) – cos (40° – θ) + tan 1° tan 10° tan 20° tan 70° tan 80° tan 89° = 1
Sol:
Sin (50 + θ) = cos (90 – (50 + θ)) = cos (40 – θ)
Tan 1 = tan (90° − 89°) · cot 89°
Tan 10° = tan (90° - 80°) = cot 80°
Tan 20° = tan (90° - 70°) = cot 70°
⇒ cos (40° - θ) – cos (40 - θ) = cot 89° tan 89° . cot 80° . cot 70° tan 70°
Cot . tan θ = 1
= 1 · 1 · 1 = 1

LHS = RHS

Hence proved

9. Evaluate:

(i) $\frac{2}{3}\ (\cos^4 30° - \sin^4 45\ °) - 3(\sin^2 60° - \sec^2 45°) + \frac{1}{4}\cot^2 30°$

Sol:

Cos 30° $= \frac{\sqrt{3}}{2}$ sin 60° $= \frac{\sqrt{3}}{2}$ cot 30° $= \sqrt{3}$ sin 45° = $\frac{1}{\sqrt{2}}$ sec 45° = $\frac{1}{\sqrt{2}}$

Substituting above values in (i)

$\frac{2}{3}\left[\left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^4\right] - 3\left[\left(\frac{\sqrt{3}}{2}\right)^2 \cdot \left[\frac{1}{\sqrt{2}}\right]^2\right] + \frac{1}{4}\left(\sqrt{3}\right)^2$

$\frac{2}{3}\left[\frac{9}{16} - \frac{1}{4}\right] - 3\left[\frac{3}{4} - \frac{1}{2}\right]\frac{1-3}{4}$

$\frac{2}{3}\left[\frac{9-4}{16}\right] - 3\left[\frac{3-2}{4}\right] - \frac{3}{4}$

$\Rightarrow \frac{2}{3} \cdot \frac{5}{16} - \frac{3}{4} + \frac{3}{4} \Rightarrow \frac{5}{24}$

(ii) 4 (sin^2 30 + cos^4 60°) $-\frac{2}{3}\ 3\left[\left(\sqrt{\frac{3}{2}}\right)^2 \cdot \left[\frac{1}{\sqrt{2}}\right]^2\right] + \frac{1}{4}\left(\sqrt{3}\right)^2$

Sol:

$Sin\ 30° = \frac{1}{2}\cos 60 = \frac{1}{2}$ $\sin 60° = \frac{\sqrt{3}}{2}$ $\cos 45° = \frac{1}{\sqrt{2}}$ $\tan 60° = \sqrt{3}$

$\Rightarrow 4\left[\left[\frac{1}{2}\right]^4 + \left[\frac{1}{2}\right]^4\right] - \frac{2}{3}\left[\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2\right] + \frac{1}{2}\left(\sqrt{3}\right)^2$

$4\left[2.\frac{1}{16}\right] - \frac{2}{3}\left[\frac{3}{4} - \frac{1}{2}\right] + \frac{3}{2}$

$= \frac{1}{2} - \frac{2}{3} \cdot \frac{1}{4} + \frac{3}{2} = \frac{11}{6}$

(iii) $\frac{\sin 50°}{\cos 40°} + \frac{co\sec 40°}{\sec 50°} - 4\cos 50°\ cosec\ 40°$

Sol:

Sin 50° = sin (90° - 40°) = cos 40°

Cosec 40° = cosec (90° - 50°) = sec 50°

Cos 50° = cos (90° - 40°) = sin 40°

$\Rightarrow \frac{\cos 40°}{\cos 40°} + \frac{\sec 50°}{sec\ 50°} - 4\sin 40°\ cosec\ 40°$

1 + 1 – 4 = − 2 [∵ Sin 40° cosec 40° = 1]

(iv)Tan 35° tan 40° tan 50° tan 55°

Sol:

Tan 35° = tan (90° - 55°) = cot 55°

Tan 40° = tan (90° - 50°) = cot 55°

Tan 65° = 1

Cot 55 tan 55· cot 50 tan 50 · tan 45
1 · 1 · 1 = 1

(v) Cosec (65 + θ) – sec (25 – θ) – tan (55 – θ) + cot (35 + θ)
Sol:
Cosec (65 + θ) = sec (90 – (65 + θ)) = sec (25 – θ)
Tan (55 – θ) = cot (90 – (55 – θ) = cot (35 + θ)
⇒ sec (25 – θ) – sec (25 – θ) tan (55 – θ) + tan (55 – θ) = 0

(vi) Tan 7° tan 23° tan 60° tan 67° tan 83°
Sol:
Tan 7° tan 23° tan 60° tan (90° - 23) tan (90° - 7°)
⇒ tan 7° tan 23° tan 60° cot 23° tan 60°
$1 \cdot 1 \cdot \sqrt{3} = \sqrt{3}$

(vii) $\frac{2 \sin 68}{\cos 22} - \frac{2 \cot 15°}{5 \tan 75°} - \frac{8 \tan 45° \tan 20° \tan 40° \tan 50° \tan 70°}{5}$
Sol:
Sin 68° = sin (90 - 22) = cos 22
Cot 15° = tan (90 - 75) = tan 75
$2 \cdot \frac{\cos 22}{\cos 22} - \frac{2 \tan 75°}{5 \tan 75°} - \frac{3 \tan 45° \tan 20° \tan 40° \cot 40° \cot 20°}{5}$
$= 2 - \frac{2}{5} - \frac{3}{5} = 2 - 1 = 1$

(viii) $\frac{3 \cos 55°}{7 \sin 35°} - \frac{4(\cos 70\ cosec\ 20°)}{7\ (\tan 5° \tan 25° \tan 45° \tan 65° \tan 85°)}$
Sol:
Cos 55° = cos (90° - 35°) = sin 35°
Cos 70° = cos (90 – 20) = sin 20°
Tan 5 = cot 85° tan 25° = cot 65°
$\Rightarrow \frac{3 \sin 35°}{7 \sin 35°} - \frac{4\ (\sin 20°\ cosec\ 20°)}{7(\cot 85° \tan 85° \cot 65° \tan 65° \tan 45°)}$
$= \frac{3}{7} - \frac{4}{7} = -\frac{1}{7}$

(ix) $\frac{\sin 18°}{\cos 72°} + \sqrt{3}\ [\tan 10° \tan 30° \tan 40° \tan 50° \tan 80°]$
Sol:
Sin 18° = sin (90° - 72) = cos 72°
Tan 10° = cot 80° tan 50° = cot 40°
$\Rightarrow \frac{sin\ 18°}{\sin 18°} + \sqrt{3}\ \left[\tan 80 \cos 30 . \tan 40 \cot 40 . \frac{1}{\sqrt{3}}\right]$
$= 1 + \sqrt{3} \cdot \frac{1}{\sqrt{3}} = 2$

(x) $\frac{\cos 58°}{\sin 32°}+\frac{\sin 22°}{\cos 68°}-\frac{\cos 38° \text{ cosec } 52°}{\tan 18° \tan 35° \tan 60° \tan 72° \tan 65°}$

Sol:

Cos 58° = cos (90° - 32°) = sin 32°

Sin 22° = sin (90° - 68°) = cos 68°

Cos 38° = cos (90 – 52) = sin 52°

Tan 18° = cot 72 tan 35° = cot 55°

$\Rightarrow \frac{\sin 32°}{\sin 32°}+\frac{\cos 68°}{\cos 68°}-\frac{\sin 52\ cosec\ 52}{\tan 72 .\cot 72 \tan 55 \cot 55 .\tan 60}$

$= 1 + 1 - \frac{1}{\sqrt{3}} = \frac{2\sqrt{3}-1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6-\sqrt{3}}{3}$

10. If Sin θ = cos (θ – 45°), where θ – 45° are acute angles, find the degree measure of θ.

Sol:

Sin θ = cos (θ – 45°)

Cos θ = cos (90 – θ)

Cos (θ – 45°) = sin (90° - (θ – 45°)) = sin (90 – θ + 45°)

Sin θ = sin (135 – θ)

$\theta = 135 - \theta$

$2\theta = 135$

$\therefore \theta = 135°/2$

11. If A, B, C are the interior angles of a ΔABC, show that:

(i) Sin $\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$ (ii) cos $\left[\frac{B+C}{2}\right] = Sin\frac{A}{2}$

Sol:

A + B + C = 180

B – C = 180 – $\frac{A}{2}$

(i) Sin $\left[90 - \frac{A}{2}\right] = \cos\frac{A}{2}$

∴ LHS = RHS

(ii) Cos $\left[90 - \frac{A}{2}\right] = \sin\frac{A}{2}$

∴ LHS = RHS

12. If 2θ + 45° and 30° − θ are acute angles, find the degree measure of θ satisfying Sin (20 + 45°) = cos (30 - θ°)

Sol:

Here 20 + 45° and 30 – θ° are acute angles:

We know that (90 – θ) = cos θ

Sin (2θ + 45°) = sin (90 – (30 – θ))

Sin (2θ + 45°) = sin (90 – 30 + θ)

Sin (20 + 45°) = sin (60 + θ)
On equating sin of angle of we get
$2\theta + 45 = 60 + \theta$
$2\theta - \theta = 60 - 45$
$\theta = 15°$

13. If θ is a positive acute angle such that sec θ = cosec 60°, find 2 $\cos^2\theta - 1$
Sol:
We know that sec (90 – θ) = $\text{cosec}^2\,\theta$
Sec θ = sec (90 – 60°)
On equating we get
Sec θ = sec 30°
$\theta = 30°$

Find $2\cos^2\theta - 1$
$\Rightarrow 2 \times \cos^2 30° - 1 \qquad \left[\cos 30 = \frac{\sqrt{3}}{2}\right]$
$\Rightarrow 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1$
$\Rightarrow 2 \times \frac{3}{4} - 1$
$\Rightarrow \frac{3}{2} - 1$
$= \frac{1}{2}$

14. If cos 2θ = sin 4θ where 2θ, 4θ are acute angles, find the value of θ.
Sol:
We know that sin (90 – θ) = cos θ
Sin 20 = cos 2θ
Sin 4θ = sin (90 - 2θ)
$4\theta = 90 - 20$
$6\theta = 90$
$\theta = \frac{90}{6}$
$\theta = 15°$

15. If Sin 3θ = cos (θ – 6°) where 3 θ and $\theta - 6°$ are acute angles, find the value of θ.
Sol:
30, θ – 6 are acute angle
We know that sin (90 – θ) = cos θ
Sin 3θ = sin (90 – (θ - 6°))
Sin 3θ = sin(90 – θ + 6°)

Sin $3\theta = \sin (96° - \theta)$
$3\theta = 96° - \theta$
$4\theta = 96°$
$\theta = \frac{96°}{4}$
$\theta = 24°$

16. If Sec 4A = cosec (A – 20°) where 4A is acute angle, find the value of A.
Sol:
Sec 4A = sec $[90 - A - 20]$ $[\because \sec(90 - \theta) = cosec\ \theta]$
Sec 4A = sec (90 – A + 20)
Sec 4A = sec (110 – A)
4A = 110 – A
5A = 110
A = $\frac{110}{5} \Rightarrow A = 22$

17. If Sec 2A = cosec (A – 42°) where 2A is acute angle. Find the value of A.
Sol:
We know that (sec (90 – θ)) = cosec θ
Sec 2A = sec (90 – (A – 42))
Sec 2A = sec (90 – A + 42)
Sec 2A = sec (132 – A)
Now equating both the angles we get
2A = 132 – A
3A = $\frac{132}{3}$
A = 44

Exercise 6.1

Prove the following trigonometric identities:

1. $\left(1-\cos^2 A\right)\cos ec^2 A = 1$

Sol:

We know $\sin^2 A+\cos^2 A=1$

$\sin^2 A = 1-\cos^2 A$

$\Rightarrow \sin^2 A \cdot \cos ec^2 A$

$\Rightarrow \sin^2 A \cdot \frac{1}{\sin^2 A} = 1 \qquad \therefore L.H.S = R.H.S$

2. $\left(1+\cos^2 A\right)\sin^2 A = 1$

Sol:

We know that $\cos ec^2 A - a^2 - A = 1$

$\cos ec^2 A = 1+\cot^2 A$

$\Rightarrow \cos ec^2 A \cdot \sin^2 A = 1$

$\frac{1}{\sin A} \cdot n^2 A \cdot 1$

$1=1 \quad L.H.S = R.H.S$

3. $\tan^2\theta\cos^2\theta = 1-\cos^2\theta$

Sol:

$L.H.S \Rightarrow \frac{\sin^2\theta}{\cos^2\theta}\cdot\cos^2\theta = \sin^2\theta$

$R.H.S \Rightarrow 1-\cos^2\theta \qquad \left[1=\sin\theta+\cos^2\theta\right]$

$\Rightarrow \sin^2\theta \qquad \left[\therefore \sin^2\theta = 1-\cos^2\theta\right]$

$L.H.S = R.H.S$

4. $\cos ec\theta\sqrt{1-\cos^2}\theta = 1$

Sol:

$LHS = \cos ec\theta\sqrt{\sin^2\theta} \qquad \left[\because 1-\cos^2\theta = \sin^2\theta\right]$

$= \cos ec\theta \cdot \sin\theta$

$=1$

$\therefore L.H.S = R.H.S$

5. $\left(\sec^2\theta-1\right)\left(\cos ec^2\theta-1\right)=1$

Sol:

We know that $\sec^2\theta-\tan^2\theta=1$

$\Rightarrow \sec^2\theta=1,\tan\theta$

$\cos ec^2\theta-\cos^2\theta=1$

$\cos ec^2\theta-\cot^2\theta$

$\tan^2\theta\cdot\cot^2\theta=\tan^2\theta\frac{1}{\tan^2\theta}$

6. $\tan\theta\frac{1}{\tan\theta}=\sec\theta\cos ec\theta$

Sol:

$LHS=\tan\theta+\frac{1}{\tan\theta}\Rightarrow\frac{\sin\theta}{\cos\theta}+\frac{1}{\frac{\sin\theta}{\cos\theta}}$

$\Rightarrow\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$

$\Rightarrow\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}=\frac{1}{\sin\theta\cos\theta}$

$\Rightarrow\sec\theta\cos ec\theta$

Hence L.H.S = R.H.S

7. $\frac{\cos\theta}{1-\sin\theta}=\frac{1+\sin\theta}{\cos\theta}$

Sol:

$\cos\theta-\cos 2\cdot\frac{\theta}{2}=\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}$

$\sin\theta=\sin\frac{\theta}{2}\cdot 2\Rightarrow 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$\Rightarrow LHS=\frac{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}{\left[\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}\right]-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\qquad\left[\therefore 1=\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}\right]$

$\Rightarrow\frac{\left[\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right]\left[\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right]}{\left[\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right]}\qquad\left[\because a^2-b^2=\left(a-b\right)\left(a+b\right)\left(a-b\right)^2=a^2+b^2-2ab\right]$

$$\Rightarrow \frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}.$$

$$R.H.S\ \frac{1+\sin\theta}{\cos\theta} \Rightarrow \frac{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}$$

$$\Rightarrow \frac{\left[\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right]}{\left[\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right]}$$

$$\Rightarrow \frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}$$

$\therefore L.H.S = R.H.S$

8. $\dfrac{\cos\theta}{1+\sin\theta} = \dfrac{1-\sin\theta}{\cos\theta}$

Sol:

$$\cos\theta = \cos 2\frac{\theta}{2} = \cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}$$

$$\sin\theta = \sin 2\cdot\frac{\theta}{2} = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$1 = \cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}$$

$$LHS = \frac{\cos\theta}{1+\sin\theta} = \frac{\cos^2\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cot\frac{\theta}{2}}$$

$$\Rightarrow \frac{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}{\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)^2}$$

$$\Rightarrow \frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}$$

$$RHS = \frac{1-\sin\theta}{\cos\theta} = \frac{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}$$

$$\Rightarrow \frac{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)^2}{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}$$

$$\Rightarrow \frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}$$

$\therefore$ LHS = RHS

9. $\cos^2 A + \frac{1}{1+\cot^2 A} = 1$

Sol:

$1+\cot^2 A = \cos ec^2 A$ $\left[\because \cos ec^2 A - \cot^2 A = 1\right]$

$\cos ec^2 A = 1+\cot^2 A.$

$\Rightarrow \cot^2 A + \frac{1}{\cos ec^2 A}$

$\Rightarrow \cos^2 A + \sin^2 A = 1$ $\therefore LHS = RHS$

10. $\sin^2 A + \frac{1}{1+\tan^2 A} = 1$

Sol:

$1+\tan^2 A = \sec^2$ $\left[\because \sec^2 A - \tan^2 A = 1\right]$

$\Rightarrow \sin^2 A + \frac{1}{\sec^2}$ $\left[1+\tan^2 A - \sec^2 A\right]$

$\Rightarrow \sin^2 A + \cos^2 A = 1$

$\therefore LHS = RHS$

11. $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \cos ec\theta - \cot\theta.$

Sol:

$\text{L.H.S} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$ Rationalize numerator with $\sqrt{1-\cos\theta}$

$$\Rightarrow \frac{\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}} \times \frac{\sqrt{1-\cos\theta}}{1-\cos\theta}$$

$$= \frac{\left(\sqrt{1-\cos\theta}\right)^2}{\sqrt{(1-\cos\theta)(1+\cos\theta)}}$$

$$= \frac{1-\cos\theta}{\sqrt{1-\cos^2\theta}} = \frac{1-\cos\theta}{\sqrt{\sin^2\theta}} = \frac{1-\cos\theta}{\sin\theta}$$

$$= \cos ec\theta - \cot\theta$$

12. $\frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$

Sol:

$$1 = \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}$$

$$\cos\theta = \cos 2\cdot\frac{\theta}{2} = \cos^2\frac{\theta}{2} - 8\sin^2\frac{\theta}{2}$$

$$\sin\theta = \sin 2\cdot\frac{\theta}{2} = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$LHS = \frac{1-\cos\theta}{\sin\theta} = \frac{\cos^2\frac{\theta}{2} = \sin^2\frac{\theta}{2} - \left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$\frac{\cos\frac{\theta}{2} + \sin^2\frac{\theta}{2} - \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \tan\frac{\theta}{2}.$$

$$RHS = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \tan\frac{\theta}{2}$$

$$\therefore L.H.S = R.H.S$$

13. $\frac{\sin\theta}{1-\cos\theta} - \cos ec\theta + \cot\theta$

Sol:

$LHS = \frac{\sin\theta}{1-\cos\theta}$

Rationalizer both Nr and Or with $1+\cos\theta$

$\Rightarrow \frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$

$\Rightarrow \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$ $\left[\because (a-b)(a+b) = a^2 - b^2\right]$

$\Rightarrow \frac{\sin\theta + \sin\theta}{\sin^2\theta}$ $\left[\because 1-\cos^2\theta - \sin^2\theta\right]$

$\Rightarrow \frac{\sin\theta}{\sin^2\theta} + \frac{\sin\theta\cos\theta}{\sin^2\theta}$

$\Rightarrow \frac{1}{\sin\theta}\frac{\cos\theta}{\sin\theta} \Rightarrow \cos ec\theta + \cot\theta$

$\therefore LHS = RHS$

14. $\frac{1-\sin\theta}{1+\sin\theta} - (\sec\theta - \tan\theta)^2$

Sol:

$LHS = \frac{1-\sin\theta}{1+\sin\theta}$

Rationalize both Nr and Or with $(1-\sin\theta)$ multiply

$\Rightarrow \frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$

$\Rightarrow \frac{(1-\sin\theta)^2}{\cos^2\theta}$ $\left[\because (1-\sin\theta)(1+\sin\theta) = \cos^2\theta\right]$

$\Rightarrow \left[\frac{1-\sin\theta}{\cos\theta}\right]^2 \Rightarrow \left[\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right]^2$

$\Rightarrow [\sec\theta - \tan\theta]^2$

$= LHS = RHS$ Hence proved

15. $(\cos ec\theta + \sin\theta)(\cos ec\theta - \sin\theta) = \cos^2\theta = \cos^2\theta$

Sol:

$LHS \Rightarrow \cos ec^2\theta - \sin^2\theta$ $\left[(a+b)(a-b) = a^2 - b^2\right]$

$\Rightarrow 1+\cot^2\theta-\left(1-\cos^2\theta\right)$ $\left[\because \cos ec^2\theta=1+\cot^2\theta \sin^2\theta=1-\cos^2\theta\right]$

$\Rightarrow 1+\cot^2-1+\cos^2\theta$

$\Rightarrow \cot^2\theta+\cos^2\theta$

$= LHS = RHS$ Hence proved

16. $\dfrac{\left(1+\cot^2\theta\right)\tan\theta}{\sec^2\theta}=\cot\theta$

Sol:

$LHS=\dfrac{\left(1+\cot^2\theta\right)\tan\theta}{\sec^2\theta}$ $\left[\because \cos ec^2\theta=1+\cot^2\theta\right]$

$\Rightarrow \dfrac{\cos ec^2\theta\cdot\tan\theta}{\sec^2}\Rightarrow \dfrac{1}{\sin^2\theta}\cdot\dfrac{\cos^2\theta}{1}\cdot\dfrac{\sin\theta}{\cos\theta}$

$\Rightarrow \dfrac{\cos\theta}{\sin\theta}=\cot\theta$

$= LHS = RHS$ Hence proved

17. $\left(\sec\theta+\cos\theta\right)\left(\sec\theta-\cos\theta\right)=\tan^2\theta+\sin^2\theta$

Sol:

$LHS=\sec^2\theta-\cos^2\theta$ $\left[\because \left(\sec\theta+\cos\theta\right)\left(\sec\theta-\cos\theta\right)-\sec^2\theta-\cos^2\theta\right]$

$\Rightarrow 1+\tan^2\theta-\left(1-\sin^2\theta\right)$ $\left[\because \sec^2\theta=1+\tan^2\theta \cos^2\theta=1-\tan^2\theta\right]$

$\Rightarrow 1+\tan^2\theta-l+\sin^2\theta$

$\tan^2\theta+\sin^2\theta$

$= LHS = RHS$ Hence proved

18. $\sec A\left(1-\sin A\right)\left(\sec A+\tan A\right)=1$

Sol:

$LHS=\dfrac{1}{\cos+1}=\left(1-\sin A\right)\times\left[\dfrac{1}{\cos A}+\dfrac{\sin A}{\cos A}\right]$ $\left[\because \sec A=\dfrac{1}{\cos A}\ \textit{and}\ \tan A=\dfrac{\sin A}{\cos A}\right]$

$\Rightarrow \dfrac{1}{\cos A}\times\left(1-\sin A\right)\dfrac{\left(1+\sin A\right)}{\cos A}$

$=\dfrac{\cos^2 A}{\cos^2 A}=1$ $\left[\because \left(1-\sin A\right)\left(1+\sin A\right)\cdot\cos^2 A=1-\sin^2 A\right]$

$= LHS = RHS$ Hence proved

19. $\left(\cos ecA-\sin A\right)\left(\sec A-\cos A\right)\left(\tan A+\cot A\right)=1$

Sol:

$$LHS=\left[\frac{1}{\sin A}-\sin A\right]\left[\frac{1}{\cos A}-\cos A\right]\left[\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}\right]$$

$$\Rightarrow \frac{1-\sin^2 A}{\sin A}\times\frac{1-\cos^2 A}{\cos A}\times\frac{\sin^2 A+\cos^2 A}{\sin A\cos A}$$

$$\Rightarrow \frac{\cos^2 A\cdot\sin^2 A\cdot 1}{\sin^2 A\cos^2 A} \qquad \left[\begin{array}{c}\because \cos ec\ A=\frac{1}{\sin}A\\ \sec A=\frac{1}{\cos}A\\ \tan A=\frac{\sin A}{\cos A}\\ \cot A=\frac{\cos A}{\sin A}\end{array}\right]$$

$$=1 \qquad \left[\begin{array}{c}\because 1-\sin^2 A=\cos^2 A\\ 1-\cos^2 A=\sin^2 A\\ \sin^2 A+\cos^2 A=1\end{array}\right]$$

$=LHS=RHS$ Hence proved

20. $\tan^2\theta-\sin^2\theta\ \ \tan^2\theta\sin^2\theta$

Sol:

$$LHS=\tan^2\theta-\sin^2\theta=\frac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta \qquad \left[\because \tan^2\theta=\frac{\sin^2\theta}{\cos^2\theta}\right]$$

$$\Rightarrow \sin^2\theta\left[\frac{1}{\cos^2\theta}-1\right]$$

$$\sin^2\theta\left[\frac{1-\cos^2\theta}{\cos^2\theta}\right]$$

$$\Rightarrow \sin^2\theta\cdot\frac{\sin^2\theta}{\cos^2\theta}=\sin^2\theta\tan^2\theta$$

$=LHS=RHS$ Hence proved

21. $\left(1+\tan^2\theta\right)\left(1-\sin\theta\right)\cdot\left(1+\sin\theta\right)=1$

Sol:

$$LHS=\left(1+\tan^2\theta\right)\left(1-\sin^2\theta\right) \qquad \left[\because \left(a-b\right)\left(a+b\right)=a^2-b^2\right]$$

$$\Rightarrow \quad \sec^2\theta\cdot\cos^2\theta \qquad \left[\because \sec^2\theta=1+\tan^2\theta\right]$$

$=1$
$= LHS = RHS$ Hence proved

22. $\sin^2 A \cot^2 A + \cos^2 A \tan^2 A = 1$

Sol:

$LHS = \sin^2 A \cdot \frac{\cos^2 A}{\sin^2 A} + \cos^2 A \cdot \frac{\sin^2 A}{\cos^2 A}$

$= \cos^2 A + \sin^2 A \qquad \left[\because \cot^2 A = \cos^2 \frac{A}{\sin^2 A} \tan^2 A = \frac{\sin^2 A}{\cos^2 A}\right]$

$= LHS = RHS$ Hence proved

23. (i) $\cos\theta - \tan\theta = \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$

Sol:

$L.H.S = \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$

$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} = \frac{\cos^2\theta}{\sin\theta\cos\theta} \qquad \left[\because \cos^2\theta - \sin^2\theta = \cos\theta\right]$

$\left[\because \cos^2 = 2\cos^2\theta - 1\right]$

$= \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$

$= LHS = RHS$ Hence proved

(ii) $\tan\theta - \cot\theta = \frac{2\sin^2\theta - 1}{\sin\theta\cos\theta}$

Sol:

$LHS = \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$

$\Rightarrow \frac{\sin^2\theta - \cos^2}{\cos\theta\sin\theta}$

$\Rightarrow \frac{\sin^2\theta - \left(1 - \sin^2\theta\right)}{\cos\theta\sin\theta} \qquad \left[\because \cos^2\theta = 1\sin^2\theta\right]$

$\Rightarrow \frac{2\sin^2\theta - 1}{\sin\theta\cos\theta}$

$\therefore LHS = RHS$ Hence proved

24. $\frac{\cos^2\theta}{\sin\theta} - \cos ec\theta + \sin\theta = \theta$

Sol:

$$LHS = \frac{\left(15^2\right) - \sin\theta\cos ec\theta + \sin^2\theta}{\sin\theta}$$

$$\Rightarrow \frac{\cos^2\theta + \sin^2\theta - 1}{\sin\theta} \qquad \left[\because \sin\theta\cos ec\theta = 1\right]$$

$$= 0$$

$\therefore LHS = RHS$ Hence proved

25. $\frac{1}{1+\sin A} + \frac{1}{1-\sin A} = 2\sec^2 A$

Sol:

$$LHS = \frac{1-\sin A + 1 + \sin A}{(1+\sin A)(1-\sin A)}$$

$$\Rightarrow \frac{2}{1-\sin^2 A} \qquad \left[\because (1+\sin A)(1-\sin A) = 1-\sin^2 A\right]$$

$$\Rightarrow \frac{2}{\cos^2 A} \Rightarrow 2\sec^2 A \qquad \left[\because 1-\sin A = \cos A\right]$$

$\therefore LHS = RHS$ Hence proved

26. $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos A}{1+\sin\theta} = 2\sec\theta$

Sol:

$$LHS = \frac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{1+\sin^2\theta + 2\sin\theta + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$\Rightarrow \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)} = 2\sec\theta$$

$\therefore LHS = RHS$ Hence proved

27. $\frac{(1+\sin\theta)^2 + (1-\sin\theta)^2}{2\cos^2\theta} = \frac{1+\sin^2\theta}{1-\sin^2\theta}$

Sol:

$$LHS = \frac{1+\sin^2\theta + 2\sin\theta + 1 + \sin^2\theta - 2\sin\theta}{2\cos\theta}$$

$$\Rightarrow \frac{2\left(1+\sin^2\theta\right)}{2\cos^2\theta} \Rightarrow \frac{1+\sin^2\theta}{1-\sin^2\theta} \qquad \left[\because \cos^2\theta = 1-\sin^2\theta\right]$$

$\therefore LHS = RHS$ Hence proved

28. $\dfrac{1+\tan^2\theta}{1+\cot^2\theta} - \left[\dfrac{1-\tan\theta}{\cot\theta}\right]^2 - \tan^2\theta$

Sol:

$$LHS \Rightarrow \frac{1+\tan^2\theta}{1+\cot^2\theta} = \frac{\sec^2\theta}{\cos ec^2\theta} \qquad \left[\because \tan^2\theta+1=\sec^2\theta \;\; 1+\cot^2\theta = \cos ec^2\theta\right]$$

$$= \frac{1}{\cos^2\theta\cdot 1}\sin^2 = \tan^2\theta$$

$$\Rightarrow \left[\frac{1-\tan\theta}{1-\cot\theta}\right]^2 \Rightarrow \left[\frac{1-\tan\theta}{1-\dfrac{1}{\tan\theta}}\right]^2$$

$$\Rightarrow \left[\frac{1-\tan\theta}{\left(1-\tan\theta\right)}\cdot\tan\theta\right]^2 = \tan^2\theta$$

$\therefore LHS = RHS$ Hence proved

29. $\dfrac{1+\sec\theta}{\sec\theta} = \dfrac{\sin^2\theta}{1-\cos\theta}$

Sol:

$$LHS = \frac{1+\sec\theta}{\sec\theta} = \frac{1+\dfrac{1}{\cos\theta}}{\dfrac{1}{\cos\theta}}$$

$$= \frac{\cos\theta+1}{\cos\theta}\cdot\cos\theta$$

$$= 1+\cos\theta$$

$$RHS = \frac{\sin^2\theta}{1-\cos\theta} \Rightarrow \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$\Rightarrow \frac{\left(1-2\sqrt{5}b\right)+\left(\cos\theta\right)}{1-48} = 1+\cos\theta$$

$\therefore LHS = RHS$ Hence proved

30. $\frac{\tan\theta}{1-\cot\theta}=\frac{\cot\theta}{1-\tan\theta}=1+\tan\theta+\cot\theta.$

Sol:

$LHS=\frac{\tan\theta}{1-\frac{1}{\tan\theta}}+\frac{\cot\theta}{1-\tan\theta}$

$\Rightarrow -\frac{\tan^2\theta}{(1-\tan\theta)}+\frac{\cot\theta}{1-\tan\theta}$

$\frac{1}{1-\tan\theta}\left[\frac{1}{\tan\theta}-\tan^2\theta\right]$

$\frac{1}{1-\tan\theta}\left[\frac{1-\tan^3\theta}{\tan\theta}\right]$

$\Rightarrow \frac{1}{1-\tan\theta}\frac{(1-\tan\theta)(1+\tan\theta+\tan^2\theta)}{\tan\theta}$ $\left[\because a^3-b^3=(a-b)(a^2+ab+b^2)\right]$

$\Rightarrow \frac{1+\tan\theta+\tan^2\theta}{\tan\theta}$

$\Rightarrow \cot\theta+1+\tan\theta$

$\therefore LHS=RHS$ Hence proved

31. $\sec^6\theta=\tan^6\theta+3\tan^2\theta\sec^2\theta+1$

Sol:

We know that $\sec^2\theta-\tan^2\theta:$

Cubing on both sides

$\left(\sec\theta-\tan^2\theta\right)^3=1$

$\sec^6\theta\cdot\tan^6\theta-3\sec^2\theta\left(\sec^2\theta-\tan^2\theta\right)=1$

$\tan^2\theta$ $\left[\because (a-b)^3=a^3-b^3-3ab(a-b)\right]$

$\Rightarrow \sec\theta-\tan^6\theta=3\sec^2\theta\tan^2\theta=1$

$\Rightarrow \sec^6\theta=\tan^6\theta+1+3\tan^2\theta\sec^2\theta$

Hence proved

32. $\cos ec^6\theta=\cot^6\theta+3\cot^2\theta\cos ec^2\theta+1$

Sol:

We know that $\cos ec^2\theta-\cot^2\theta=1$

Cubing on both sides

$\left(\cos ec^2\theta-\cot^2\theta\right)^3=(1)^3$

$\Rightarrow \cos ec^6\theta - \cot^6\theta - 3\cos ec^2\theta\cot^2\theta\left(\cos ec^2\theta - \cot^2\theta\right) = 1$

$\left[\because \left(a-b\right)^3 - a^3 - b^3 - 3ab\left(a-b\right)\right]$

$\Rightarrow \cos ec^6 = 1 + 3\cos ec^2\theta\cot^2\theta + \cot^6\theta$

Hence proved

33. $\dfrac{\left(1+\tan^2\theta\right)\cot\theta}{\cos ec^2\theta} = \tan\theta$

Sol:

$\sec^2\theta = \tan^2\theta = 1$

$\therefore \sec^2\theta = 1 + \tan^2\theta$

$LHS = \dfrac{\sec^2\theta \cdot \cot\theta}{\cos ec^2\theta} \Rightarrow \dfrac{1\cdot\sin^6\theta}{\cos^2\theta}\cdot\dfrac{\cos\theta}{\sin\theta}$

$\left[\because \sec\theta = \dfrac{1}{\cos\theta}, \cos ec\theta = \dfrac{1}{\sin\theta}\cot\theta = \dfrac{\cos\theta}{\sin\theta}\right]$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

$\therefore LHS = RHS$ Hence proved

34. $\dfrac{1+\cos A}{\sin^2 A} = \dfrac{1}{1-\cos A}$

Sol:

We know that $\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$\Rightarrow \sin^2 A = \left(1-\cos A\right)\left(1+\cos A\right)$

$\Rightarrow LHS = \dfrac{\left(1+\cos A\right)}{\left(1-\cos A\right)\left(1+\cos A\right)} = \dfrac{1}{1-\cos A}$

$\therefore L.H.S = R.H.S$ Hence proved

35. $\dfrac{\sec A - \tan A}{\sec A + \tan A} = \dfrac{\cos^2 A}{\left(1+\sin A\right)^2}$

Sol:

$LHS = \dfrac{\sec\theta - \tan\theta}{\sec A + \tan A}$

Rationalizing the denominator y multiply and diving with $\sec A + \tan A$ we get

$$\frac{(\sec A-\tan A)}{(\sec A+\tan A)}\times\frac{(\sec A+\tan A)}{(\sec A+\tan A)}=\frac{\sec^2 A-\tan^2 A}{(\sec A+\tan A)^2}=\frac{1}{(\sec A+\tan A)^2}$$

$\left[\because \sec^2 A-\tan^2 A=1\right]$

$$=\frac{1}{\sec^2 A+\tan^2 A+2\sec A\tan A}=\frac{1}{\frac{1}{\cos^2 A}+\frac{\sin^2 A}{\cos^2 A}+\frac{2\sin A}{\cos^2 A}}$$

$$\Rightarrow\frac{\cos^2}{1+\sin^2 A+2\sin A}=\frac{\cos^2 A}{(1+\sin A)^2}$$

$\therefore L.H.S=R.H.S$ Hence proved

36. $\dfrac{1+\cos A}{\sin A}=\dfrac{\sin A}{1-\cos A}$

Sol:

$$LHS=\frac{1+\cos A}{\sin A}\qquad ...(1)$$

Multiply both Nr and Dr with $(1-\cos A)$ we get

$$\frac{(1+\cos A)(1-\cos A)}{\sin A(1-\cos A)}=\frac{1-\cos^2 A}{\sin A(1-\cos A)}$$

$$=\frac{\sin^2 A}{\sin A(1-\cos A)}=\frac{1-\cos^2 A}{\sin A(1-\cos A)}$$

$$=\frac{\sin^2}{\sin A(1-\cos A)}\qquad\left[\because \cos^2 A=\sin^2 A\right]$$

$$=\frac{\sin A}{1-\cos A}$$

$\therefore L.H.S=R.H.S$ Hence proved

37. $\sqrt{\dfrac{1+\sin A}{1-\sin A}}=\sin A+\tan A$

Sol:

$$LHS=\sqrt{\frac{1+\sin A}{1-\sin A}}.$$

Rationalize the Nr. By multiplying both Nr and Dr with $\sqrt{1+\sin A}.$

$$\Rightarrow\sqrt{\frac{(1+\sin A)(1+\sin A)}{(1+\sin A)(1-\sin A)}}=\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}\qquad\left[\because (1+\sin A)(1-\sin A)=\cos^2 A\right]$$

$=\frac{1+\sin A}{\cos A}=\frac{1}{\cos A}+\frac{\sin A}{\cos A}$

$\sec A+\tan A$

$\therefore L.H.S=R.H.S$ Hence proved

38. $\sqrt{\frac{1-\cos A}{1+\cos A}}=\cos ecA-\cot A$

Sol:

Rationalizing both Nr and Or by multiplying both with $\sqrt{1-\cos A}$ we get

$\Rightarrow\sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}}.$ $\quad\left[\because(1+\cos A)(1-\cos A)=1-\cos^2 A=\sin^2 A\right]$

$\sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$

$=\frac{1-\cos A}{\sin A}$

$=\cos ecA-\cot A.$

$\therefore L.H.S=R.H.S$ Hence proved

39. $(\sec A-\tan A)^2=\frac{1-\sin A}{1+\sin A}$

Sol:

$LHS=(\sec A-\tan A)^2$

$\Rightarrow\left[\frac{1}{\cos A}-\frac{\sin A}{\cos A}\right]^2\Rightarrow\frac{(1-\sin A)^2}{\cos^2 A.}$

$\Rightarrow\frac{(1-\sin A)^2}{1-\sin^2 A}$ $\quad\left[\because 1-\sin^2 A=\cos^2 A\right]$

$\Rightarrow\frac{(1-\sin A)^2}{(1-\sin A)(1+\sin A)}$ $\quad\left[\because a^2-b^2=(a-b)(a+b)\right]$

$=\frac{1-\sin A}{1+\sin A}$

$\therefore L.H.S=R.H.S$ Hence proved

40. $\frac{1-\cos A}{1+\cos A}=(\cot A-\cos ecA)^2$

Sol:

$LHS=\frac{1-\cos A}{1+\cos A}$

Rationalizing Nr by multiplying and dividing with $1-\cos A$.

$=\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}$

$\Rightarrow\frac{(1-\cos A)^2}{1-\cos^2 A}$

$\Rightarrow\frac{(1-\cos A)^2}{\sin^2 A}$ $\left[\because(a+b)(a-b)=a^2-b^2\quad 1-\cos^2 A=\sin^2 A\right]$

$=\left[\frac{1}{\sin A}-\frac{\cos A}{\sin A}\right]^2$ $(\cos ecA-\cot A)^2$

$=(\cot A-\cos ecA)^2$

$\therefore L.H.S=R.H.S$ Hence proved

41. $\frac{1}{\sec A-1}=\frac{1}{\sec A+1}=2\cos ecA\cot A$

Sol:

$LHS=\frac{\sec A+1+\sec A-1}{(\sec A+1)(\sec A-1)}=\frac{2\sec A}{(\sec^2 A-1)}$

$\left[\because(a+b)(a-b)=a^2-b^2\ \sec^2 A-1=\tan^2 A\right]$

$\Rightarrow\frac{2\sec A}{\tan^2 A}=\frac{2\cdot 1\cos^2 A}{\cos A\cdot\sin^2 A}$ $\left[\because\sec A=\frac{1}{\cos A}\tan^2 A=\frac{\sin^2 A}{\cos^2 A}\right]$

$\Rightarrow 2\cos ecA\cot A$

$\therefore L.H.S=R.H.S$ Hence proved

42. $\frac{\cos A}{1-\tan A}+\frac{\sin A}{(1-\cot A)}=\sin A+\cos A$

Sol:

$LHS=\frac{\cos A}{1-\tan A}+\frac{\sin A}{\left(1-\frac{1}{\tan A}\right)}$

$=\frac{\cos A}{1-\tan A}-\frac{\sin A\cdot\tan A}{1-\tan A}$

$$\Rightarrow \frac{\cos A - \sin A \tan A}{(1-\tan A)}$$

$$\Rightarrow \frac{\cos A - \sin A \cdot \dfrac{\sin A}{\cos A}}{1 - \dfrac{\sin A}{\cos A}}$$

$$\Rightarrow \frac{\cos^2 A - \sin^2 A \cos A}{(\cos A - \sin A)\cos A} = \frac{(\cos A - \sin A)(\cos A + \sin A)}{\cos A - \sin A}$$

$\Rightarrow \cos A + \sin A.$

$\therefore L.H.S = R.H.S$ Hence proved

43. $\dfrac{\cos ecA}{\cos ecA - 1} + \dfrac{\cos ecA}{\cos ecA + 1} = 2\sec^2 A.$

Sol:

$$LHS \cos ecA\left[\frac{\cos ec + 1 + \cos ec - 1}{\cos ec^2 A - 1}\right] \qquad \left[\because \cos ec^2 A - 1 = \cot^2 A\right]$$

$$\Rightarrow \cos ecA\left[\frac{2\cos ecA}{\cot^2 A}\right]$$

$$\Rightarrow \frac{2}{\sin^2 A}\frac{\sin^2 A}{\cos^2 A} = 2\sec^2 A.$$

$\therefore LHS = RHS$ Hence proved.

44. $\left(1+\tan^2 A\right) + \left(1 + \dfrac{1}{\tan^2 A}\right) = \dfrac{1}{\sin^2 A - \sin^4 A}$

Sol:

$$LHS = \left[1 + \frac{\sin^2 A}{\cos^2 A}\right] + \left[1 + \frac{\cos^2 A}{\sin^2 A^2}\right]$$

$$\Rightarrow \frac{\cos^2 A + \sin^2 A}{\cos^2 A} + \frac{\sin^2 A + \cos^2 A}{\sin^2 A}$$

$$\Rightarrow \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} \qquad \left[\because \sin^2 A + \cos^2 A = 1\right]$$

$$\Rightarrow \frac{\sin^2 A + \cos^2 A}{\sin^2 A \cos^2 A} = \frac{1}{\sin^2 A(1-\sin A)} \qquad \left[\cos^2 A = 1 - \sin^2 A\right]$$

$$\Rightarrow \frac{1}{\sin^2 A - \sin^2 A}$$

$\therefore LHS = RHS$ Hence proved.

45. $\frac{\tan^2 A}{1+\tan^2 A}+\frac{\cot^2 A}{1+\cot A}$

Sol:

We know that

$\sec^2 A = 1+\tan^2 A$

$\cos ec^2 A = 1+\cot^2 A.$

$\therefore LHS = \frac{\tan^2}{\sec^2}+\frac{\cot^2}{\cos ec^2 A}$

$\Rightarrow \frac{\sin^2 A}{\cos^2 A}\times\frac{\cos^2 A}{1}+\frac{\cos^2 A}{\sin^2 A}=\frac{\sin^2 A}{1}$

$\left[\because \tan A = \sin\frac{A}{\cos A}\sec A = \frac{1}{\cos A}\cot A = \frac{\cos A}{\sin A}\cos ec = \frac{1}{\sin A}\right]$

$\Rightarrow \sin^2 A + \cos^2 A$

$= 1$

$\therefore LHS = RHS$ Hence proved.

46. $\frac{\cot A - \cos A}{\cos A + \cos A} = \frac{\cos ecA - 1}{\cos ecA + 1}$

Sol:

$= \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A} \qquad \left[\because \cot A = \frac{\cos A}{\sin A}\right]$

$= \frac{\cos A\left[\frac{1}{\sin A} - 1\right]}{\cos A\left[\frac{1}{\sin A} + 1\right]}$

$= \frac{\cos ecA - 1}{\cos ecA + 1}$

47. (i) $\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$

(ii) $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}$

(iii) $\frac{\cos\theta-\sin\theta+1}{\cos\theta+\sin\theta-1} = \cos ec + \cot\theta$

Sol:

(i) $\Rightarrow \frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$

Dividing the equation with $\cos\theta$ we get or both Nr and Dr

$$\frac{\frac{1+\cos\theta+\sin\theta}{\cos\theta}}{\frac{1+\cos\theta-\sin\theta}{\cos\theta}}=\frac{\frac{1}{\cos\theta}+\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}-\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}$$

$$=\frac{\sec\theta+1+ta\theta}{\sec\theta+1-\tan\theta}$$

$$=\frac{\sec\theta+\tan\theta+\sec^2\theta-\tan^2\theta}{\sec^2\theta-\tan\theta+1} \qquad \left[\because \sec^2\theta-\tan^2\theta=1\right]$$

Or

$$\frac{\sec\theta+\tan\theta+1}{\sec\theta-\tan\theta+1}$$

$$\frac{\frac{1}{\sec\theta-\tan\theta}+1}{\sec\theta-\tan\theta+1} \qquad \left[\because \sec\theta+\tan\theta=\frac{1}{\sec\theta-\tan\theta}\right]$$

Or

$$\frac{\sec\theta+\tan\theta+1}{\sec\theta-\tan\theta+1}$$

$$\frac{\frac{1}{\sec\theta-\tan\theta}+1}{\sec\theta-\tan\theta+1} \qquad \left[\because \sec\theta+\tan\theta=\frac{1}{\sec\theta-\tan\theta}\right]$$

$$=\frac{1+\sec\theta-\tan\theta}{1+\sec\theta-\tan\theta}\times\frac{1}{\sec\theta-\tan\theta}$$

$$=\frac{1}{\sec\theta-\tan\theta}=\sec\theta+\tan\theta$$

$$=\sec\theta+\tan\theta$$

$$=\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}$$

$$=\frac{1+\cos\theta}{\cos\theta}$$

(ii) $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}$

Divide Nr and Dr with $\cos\theta$, we get

$$\frac{\frac{\sin\theta-\cos\theta+1}{\cos\theta}}{\frac{\sin\theta+\cos\theta-1}{\cos\theta}}=\frac{\tan\theta-1+\sec\theta}{\tan\theta+1-\sec\theta}.$$

$$=\frac{\frac{1}{\sec\theta-\tan\theta}-1}{\tan\theta-\sec\theta+1}$$

$$=\frac{1-\sec\theta+\tan\theta}{1-\sec\theta+\tan\theta}\times\frac{1}{\sec\theta-\tan\theta}$$

$$=\frac{1}{\sec\theta-\tan\theta}$$

(iii) $\frac{\cos\theta-\sin\theta+1}{\cos\theta+\sin\theta-1}=\cos ec+\cot\theta$

Divide both Nr and Dr with $\sin\theta$

$$\frac{\frac{\cos\theta-\sin\theta+1}{\sin\theta}}{\frac{\cos\theta+\sin\theta-1}{\sin\theta}}$$

$$=\frac{\cot\theta-1+\cos ec}{\cot\theta+1-\cos ec}$$

$$=\frac{\cot\theta+\cos ec\theta-\left(\cos ec^2\theta-\cot^2\theta\right)}{\cot\theta-\cos ec\theta+1}$$

$$=\frac{\cot\theta+\cos ec\theta-\left(\cos ec^2\theta+\cot^2\theta\right)}{\cot\theta-\cos ec\theta+1}$$

$$=\frac{\cot\theta+\cos ec\left(1-\left(\cos ec-\cot\theta\right)\right)}{\cot\theta-\cos ec\theta+1}$$

$$=\cot\theta+\cos ec\theta$$

48. $\frac{1}{\sec A+\tan A}-\frac{1}{\cos A}=\frac{1}{\cos A}-\frac{1}{\sec A-\tan A}$

Sol:

$$LHS:\sec A-\tan A\left[\therefore\frac{1}{\sec A+\tan A}=\sec A-\tan A\right]$$

$$=-\tan A$$

$RHS\ \frac{1}{\cos A}-\frac{1}{\sec A-\tan A}$

$\sec A-(\sec A+\tan A)$

$\left[\because \frac{1}{\sec A-\tan A}=\sec A+\tan A\right]$

$=-\tan A$

$LHS = RHS$

49. $\tan^2 A+\cot^2 A=\sec^2 A\cos ec^2 A-2$

Sol:

$\tan^2 A+\cot^2 A=\frac{\sin^2 A}{\cos^2 A}+\frac{\cos^2 A}{\sin^2 A}$

$=\frac{\sin^4 A+\cos^4 A}{\cos^2 A\sin^2 A}$

$=\frac{1-2\sin^2 A\cos^2 A}{\sin^2 A\cos^2 A}$ $\left[\because \sin^4 A+\cos^4 A=1-2\sin^2 A\cos^2 A\right]$

$=\sec^2 A\cos ec^2 A-2$

$\sin^4 A+\cos^4 A$ is in the form of a^4+b^4

$a^4+b^4=\left(a^2+b^2\right)^2-2a^2b^2$

Here $a=\sin A, b=\cos A$

$=\left(\sin^2 A+\cos^2 A\right)^2-2\sin^2 A\cos^2 A$

$=1-2\sin^2 A\cos^2 14$

50. $\frac{1-\tan^2 A}{\cot^2 A-1}=\tan^2 A.$

Sol:

$\frac{1-\frac{\sin^2 A}{\cos^2 A}}{\frac{\cos^2}{\sin^2}-1}=\frac{\frac{\cos^2 A-\sin^2 A}{\cos^2 A}}{\frac{\cos^2 A-\sin^2 A}{\sin^2 A}}$

$=\frac{\sin^2 A}{\cos^2 A}$

$=\tan^2 A.$

51. $1+\frac{\cot^2\theta}{1+\cos ec\theta}=\cos ec\theta$

Sol:

$1+\frac{\cos ec^2\theta-1}{1+\cos ec\theta}$ $\left[\because \cos ec^2\theta-\cot^2\theta=1,\cot^2\theta=\cos ec^2\theta-1\right]$

$1+\frac{(\cos ec\theta-1)(\cos ec\theta+1)}{1+\cos ec\theta}$

$=1+\cos ec\theta-1$ $\left[\because (a+b)(a-b)=a^2-b^2 a=\cos ec\theta, b=1.\right]$

$=\cos ec\theta$

52. $\frac{\cos\theta}{\cos ec\theta+1}+\frac{\cos\theta}{\cos ec\theta-1}=2\tan\theta$

Sol:

$\frac{\cos\theta}{\frac{1}{\sin\theta}+1}+\frac{\cos\theta}{\frac{1}{\sin\theta}-1}$

$\frac{\cos\theta}{\frac{1+\sin\theta}{\sin\theta}}+\frac{\cos\theta}{\frac{1-\sin\theta}{\sin\theta}}$

$\frac{(\cos\theta)(\sin\theta)}{1+\sin\theta}+\frac{(\cos\theta)(\sin\theta)}{1-\sin\theta}$

$\frac{(1-\sin\theta)(\sin\theta\cos\theta)+(\sin\theta\cos\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$\frac{\sin\theta\cos\theta-\sin\theta\cos\theta+\sin\theta\cos\theta+\sin^2\theta\cos^2}{1-\sin^2\theta}$

$=\frac{\sin\theta\cos\theta}{\cos^2\theta}$

$=\frac{2\sin\theta}{\cos\theta}$

$=2\tan\theta$

53. $\dfrac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}=\cot\theta$

Sol:

$\dfrac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}$

$=\dfrac{1-\sin^2\theta+\cos\theta}{\sin\theta(1+\cos\theta)}$

$=\dfrac{\cos^2\theta+\cos\theta}{\sin\theta(1+\cos\theta)}$

$=\dfrac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$=\cot\theta.$

54. $\dfrac{\tan^3\theta}{1+\tan^2\theta}+\dfrac{\cot^3\theta}{1+\cot^2\theta}=\sec\theta\cos ec\theta-2\sin\theta\cos\theta$

Sol:

$\dfrac{\tan^3\theta}{\sec^2\theta}+\dfrac{\cos^2\theta}{\cos ec^2\theta}$ $\left[\because \sec^2\theta-\tan^2\theta=1\cos ec^2\theta-\cot^2\theta=1\right]$

$\cos ec^2\theta=1+\cot^2\theta.$

$\tan\theta+\cos^2\theta+\cot^3\theta\times\sin^3\theta$ $\left[\because \dfrac{1}{\sec^2\theta}=\cos^2\theta, \dfrac{1}{\cos ec^2\theta}=1+\cot^2\theta\right]$

$\dfrac{\sin^3\theta}{\cos^3\theta}\times\cos^2\theta+\dfrac{\cos^3\theta}{\sin^3\theta}\times\sin^2\theta$

$\dfrac{\sin^3\theta}{\cos\theta}+\dfrac{\cos^3\theta}{\sin\theta}$

$=\dfrac{\sin^4\theta+\cos^4\theta}{\sin\theta\cos\theta}$

$\dfrac{1-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$

$\dfrac{1}{\sin\theta\cos\theta}-\dfrac{2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$

$\sec\theta\cos ec\theta-2\sin\theta\cos\theta.$

55. $If\ T_n = \sin^n \theta + \cos^n \theta,\ prove\, that\ \frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}.$

Sol:

$$LHS = \frac{\left(\sin^3 \theta + \cos^3 \theta\right) - \left(\sin^3 \theta + \cos^5 \theta\right)}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^3 \theta\left(1 - \sin^2 \theta\right) + \cos^3 \theta + 1 - \cos^2 \theta}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^3 \theta \times \cos^3 \theta + \cos^3 \theta \times \sin^2 \theta}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^2 \theta + \cos\theta\left(\sin\theta + \cos\theta\right)}{\sin\theta + \cos\theta}$$

$$= \sin^3 \theta \cos^2 \theta$$

$$\frac{T_5 - T_9}{T_3} = \frac{\left(\sin^5 \theta + \cos^5 \theta\right) - \left(\sin^7 \theta + \cos^7 \theta\right)}{\sin^3 \theta + \cos^3 \theta}$$

$$= \frac{\sin^5 \theta\left(1 - \sin^2 \theta\right) + \cos^5 \theta\left(\sin^2 \theta\right)}{\sin^3 \theta + \cos^3 \theta}$$

$$= \frac{\sin^5 \theta + \cos^2 + \cos^5 \theta\left(\sin^2 \theta\right)}{\sin^3 \theta + \cos^3 \theta}$$

$$= \frac{\sin^2 \theta \cos^2 \theta\left(\sin^3 \theta + \cos^3 \theta\right)}{\sin^3 \theta + \cos^3 \theta}$$

$$= \sin^2 \theta \cos^2 \theta$$

$L.H.S = R.H.S$ Hence Proval.

$$= \frac{\sin^2 \theta \cos^2 \theta\left(\sin\theta + \cos\theta\right)}{\sin^2 \theta + \cos\theta}$$

$$= \sin^2 \theta \cos^2 \theta$$

$L.H.S = R.H.S$

56. $\left[\tan\theta + \frac{1}{\cos\theta}\right]^2 + \left[\tan\theta - \frac{1}{\cos\theta}\right]^2 = 2\left(\frac{1 + \sin^2 \theta}{1 - \sin^2 \theta}\right)$

Sol:

$$\Rightarrow \left(\tan\theta + \sec\theta\right)^2 + \left(\tan\theta - \sec\theta\right)^2$$

$$= \tan^2 \theta + \sec^2 \theta + 2\tan\theta\sec\theta + \tan^2 \theta + \sec^2 \theta + 2\tan\theta\sec\theta.$$

$$= 2\tan^2\theta + 2\sec^2\theta$$
$$= 2\left[\tan^2\theta + \sec^2\right]$$
$$= 2\left[\frac{\sin^2\theta}{\cos^2\theta} + \frac{1}{\cos^2\theta}\right]$$
$$= 2\left(\frac{\sin + \sin^2\theta}{\cos^2\theta}\right)$$

57. $\left[\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\cos ec^2\theta - \sin^2\theta}\right]\sin^2\theta\cos^2\theta = \frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}.$

Sol:

$$\Rightarrow \left[\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\cos ec^2\theta - \sin^2\theta}\right]\sin^2\theta\cos^2\theta.$$
$$= \left[\frac{1}{\frac{1-\cos^4\theta}{\cos^2\theta}} + \frac{1}{\frac{1-\sin^4\theta}{\sin^2\theta}}\right]\sin^2\theta\cos^2\theta.$$
$$= \left[\frac{\cos^2\theta}{1-\cos^4\theta} + \frac{\sin^2\theta}{1-\sin^4\theta}\right]\sin^2\theta\cos^2\theta$$
$$= \left[\frac{\cos^2\theta}{\cos^2\theta + \sin^2\theta - \cos^4\theta} + \frac{\sin^2}{\cos^2\theta + \sin^2\theta - \sin^4\theta}\right]\sin^2\theta\cos^2\theta.$$
$$= \left[\frac{\cos^2\theta}{\cos^2\theta\left(1-\cos^2\theta\right) + \sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta\left(1-\sin^2\theta\right) + \cos^2\theta}\right]\sin^2\theta\cos^2\theta.$$
$$= \left[\frac{\cos^2\theta}{\cos^2\theta\sin^2\theta + \sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta\cos^2\theta + \cos^2\theta}\right]\sin^2\theta\cos^2\theta$$
$$= \left[\frac{\cos^2\theta}{\sin^2\theta\left(\cos^2\theta + 1\right)} + \frac{\sin^2\theta}{\cos^2\theta\left(\sin^2\theta + 1\right)}\right]\sin^2\theta\cos^2\theta.$$
$$= \left[\frac{\cos^4\theta\left(1+\sin^2\theta\right) + \sin^4\theta\left(1+\cos^2\theta\right)}{\sin^2\theta\cos^2\theta\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}\right]\sin^2\theta\cos^2\theta$$
$$= \frac{\cos^4\theta\left(1+\sin^2\theta\right) + \sin^4\theta\left(1+\cos^2\theta\right)}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$$

$$=\frac{\cos^4\theta+\cos^4\theta\sin^2\theta+\sin^4\theta+\sin^4\theta\cos^2\theta}{1+\sin^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta}.$$

$$=\frac{1-2\sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta\left(\cos^2\theta+\sin^2\theta\right)}{1+1+\cos^2\theta\sin^2\theta}\quad\left(\because \cos^2\theta+\sin^2\theta=1\right)$$

$$=\frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}.$$

58. $\left[\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right]^2=\frac{1-\cos\theta}{1+\cos\theta}$

Sol:

$$\Rightarrow\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\times\frac{1+\sin\theta-\cos\theta}{1+\sin\theta-\cos\theta}\right)^2$$

$$\Rightarrow\left[\frac{\left(1+\sin\theta-\cos\theta\right)^2}{\left(1+\sin\theta\right)^2-\cos^2\theta}\right]$$

$$=\left[\frac{(1)^2+\sin^2\theta+\cos^2\theta+2\times1\times\sin\theta+2\times\sin\theta\left(-\cos\theta\right)-2\cos\theta}{1-\cos^2\theta+\sin^2\theta+2\sin\theta}\right]$$

(Since, $\sin^2\theta+\cos^2\theta=1$]

$$=\left[\frac{1+1+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{\sin^2\theta-\sin^2\theta+2\sin\theta}\right]^2$$

$$=\left[\frac{2\times2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{2\sin^2\theta+2\sin\theta}\right]^2$$

$$=\left[\frac{2\left(1+\sin\theta\right)-2\cos\theta\left(\sin\theta+1\right)}{2\sin\theta\left(\sin\theta+1\right)}\right]^2$$

$$=\left[\frac{\left(1+\sin\theta\right)\left(2-2\cos\theta\right)}{2\sin\theta\left(\sin\theta+1\right)}\right]^2$$

$$=\left[\frac{2-2\cos\theta}{2\sin\theta}\right]^2$$

$$=\left[\frac{2}{2}-\left(\frac{1-\cos\theta}{\sin\theta}\right)\right]^2$$

$$=\left[\frac{1-\cos\theta}{\sin\theta}\right]^2$$
$$=\frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$
$$=\frac{(1-\cos\theta)\times(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$
$$=\frac{1-\cos\theta}{1+\cos\theta}.$$

59. $(\sec A+\tan A-1)(\sec A-\tan A+1)=2\tan A$

Sol:

$$=\left(\sec A+\tan A-\{\sec^2 A-\tan^2 A\}\right)\left[\sec A-\tan A+\left(\sec^2 A-\tan^2 A\right)\right]$$
$$=(\sec A+\tan A-\sec A+\tan A)(\sec A-\tan A)\left(\sec A-\tan A+(\sec A+\tan A)(\sec A-\tan A)\right)$$
$$=(\sec A+\tan A)\left(1-(\sec A-\tan A)\right)(\sec A-\tan A)(1+\sec A\tan A)$$
$$=(\sec A+\tan A)(1-\sec A+\tan A)(\sec A-\tan A)(1+\sec A\tan A)$$
$$=(\sec A+\tan A)(\sec A-\tan A)(1-\sec A+\tan A)(1+\sec A\tan A)$$
$$=(\sec^2 A-\tan^2 A)(1-\sec A+\tan A)(1-\sec A\tan A)$$
$$=\left[1-\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right]\left[1+\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right]$$
$$=\left(\frac{\cos A-1+\sin A}{\cos A}\right)\left(\frac{\cos A+1+\sin A}{\cos A}\right)$$
$$=\left(\frac{\cos A+\sin A^2-1}{\cos^2 A}\right)$$
$$=\frac{\cos^2 A+\sin^2 A+2\sin A\cos A-1}{\cos^2 A}$$
$$=\frac{1+2\sin A\cos A}{\cos^2 A}-1$$
$$=\frac{2\sin A\cos A}{\cos^2 A} \qquad \left[\because \sin^2 A+\cos^2 A=1\right]$$
$$=2\tan A$$

60. $(1+\cot A - cosec\ A)(1+\tan A+\sec A)=2$

Sol:

$LHS=(1+\cot A-\operatorname{cosec} A)(1+\tan A+\sec A)$

$=\left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$

$=\left(\frac{\sin A+\cos A-1}{\sin}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)$

$=\frac{(\sin A+\cos A)^2-1}{\sin A\cos A}$

$=\frac{1+2\sin A\cos A-1}{\sin A\cos A}$ $\left[\because \sin^2 A+\cos^2 A=1\right]$

$=2.$

61. $(\operatorname{cosec}\theta-\sec\theta)(\cot\theta-\tan\theta)(\operatorname{cosec}\theta+\sec\theta)(\sec\theta\operatorname{cosec}\theta-2)$

Sol:

LHS

$(\operatorname{cosec}\theta-\sec\theta)(\cot\theta-\tan\theta)$

$\left[\frac{1}{\sin\theta}-\frac{1}{\cos\theta}\right]\left[\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}\right]$

$\left[\frac{\cos\theta-\sin\theta}{\sin\theta\cos\theta}\right]\left[\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}\right]$

$\left[\frac{(\cos\theta-\sin\theta)^2(\cos\theta+\sin\theta)}{\cos^2\theta\sin^2\theta}\right]$

RHS

$(\operatorname{cosec}\theta+\sec\theta)(\sec\theta\operatorname{cosec}\theta-2)$

$=\left[\frac{1}{\sin\theta}+\frac{1}{\cos\theta}\right]\left[\frac{1}{\cos\theta}-\frac{1}{\sin\theta}-2\right]$

$=\left[\frac{\sin\theta-\cos\theta}{\sin\theta\cos\theta}\right]\left[\frac{1-2\sin\theta\cos\theta}{\sin\theta\cos\theta}\right]$

$=\left[\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}\right]\left[\frac{\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta}{\sin\theta\cos\theta}\right]$

$=\frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)^2}{\sin^2\theta\cos^2\theta}$ $\left[\because \cos^2\theta+\sin^2\theta=1\right]$

$L.H.S=R.H.S$ Hence proved

62. $(\sec A-\operatorname{cosec}A)(1+\tan A+\cot A)=\tan A\sec A-\cot A\operatorname{cosec}A$

Sol:

$LHS=(\sec A-\operatorname{cosec}A)(1+\tan A+\cot A)$

$=\left[\frac{1}{\cos A}-\frac{1}{\sin A}\right]\left[1+\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}\right]$

$=\left[\frac{\sin A-\cos A}{\sin A\cos A}\right]\left[\frac{\cos A\sin A+\sin^2 A+\cos^2 A}{\sin A\cos A}\right]$

$=\frac{(\sin A-\cos A)(\sin^2 A+\cos A\sin A+\cos^2 A)}{\sin^2 A\cos^2 A}$

$=\frac{(\sin^3 A-\cos^3 A)}{\sin^2 A\cos^2 A}$ $\left[\therefore (a-b)(a^2+ab)+b=(a^3-b^3)\right]$

$RHS=\tan A\sec A-\cot A\operatorname{cosec}A$

$=\frac{\sin A}{\cos A}\times\frac{1}{\cos A}-\frac{\cos A}{\sin A}\times\frac{1}{\cos A}$

$=\frac{\sin A}{\cos^2 A}-\frac{\cos A}{\sin A}$

$=\frac{\sin^3 A-\cos^3 A}{\sin^2 A\cos^2 A}$

$L.H.S=R.H.S$ Hence proved.

63. $\frac{\cos A\operatorname{cosec}A-\sin A\sec A}{\cos A+\sin A}=\operatorname{cosec}A-\sec A$

Sol:

$$LHS \frac{\cos A \cos ecA - \sin A \sec A}{\cos A + \sin A}$$

$$= \frac{\cos A \times \frac{1}{\sin A} - \sin A \times \frac{1}{\cos A}}{\cos A + \sin A}$$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\cos A}}{\cos A + \sin A}$$

$$= \frac{\frac{\cos^2 A - \sin^2 A}{\sin A \cos A}}{\cos A + \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\sin A \cos A} \times \frac{1}{\cos A + \sin A}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\sin A \cos A \times (\cos A + \sin A)}$$

$$= \frac{\cos A - \sin A}{\sin A \cos A}$$

$$= \frac{\cos A}{\sin A \cos A} - \frac{\sin A}{\sin A \cos A}$$

$$= \frac{1}{\sin A} - \frac{1}{\cos A}$$

$$= \cos ecA - \sec A$$

$$= R.H.S$$

Hence proved.

64. $$\frac{\sin A}{\sec A + \tan A - 1} + \frac{\cot A}{\cos ecA + \cot A - 1} = 1$$

Sol:

$$LHS = \frac{\sin A}{\frac{1}{\cos A} + \frac{\sin A}{\cos A} - 1} + \frac{\cos A}{\frac{1}{\sin A} + \frac{\cos A}{\sin A} - 1}$$

$$= \frac{\sin A}{\frac{1 + \sin A - \cos A}{\cos A}} + \frac{\cos A}{\frac{1 + \cos A - \sin A}{\sin A}}$$

$$= \frac{\sin A \cos A}{1 + \sin A - \cos A} + \frac{\sin A \cos A}{1 + \cos A - \sin A}$$

$$= \sin A \cos A \left[\frac{1}{1 + \sin A - \cos A} + \frac{1}{1 + \cos A} - \sin A \right]$$

$$=\sin A\cos A\left[\frac{1+\cos A-\sin A+\cot A\sin A-\cos A}{(1+\sin\theta-\cos\theta)(1+\cos A-\sin A)}\right]$$

$$=\sin A\cos A\left[\frac{2}{\cos A-\sin A+\sin A+\sin A\cos A-\sin^2 A-\cos A-\cos^2 A+\cos A\sin A}\right]$$

$$=\sin A\cos A\left[\frac{2}{1-\sin^2 A-\cos^2 A+2\sin A\cos A}\right]$$

$$=\sin A\cos A\left[\frac{2}{1-\left(\sin^2 A+\cos^2 A\right)+2\sin A\cos A}\right]$$

$$=\sin A\cos A\left[\frac{2}{1-1+2\sin A\cos A}\right]\left(\because \sin^2 A+\cos^2 A=1\right)$$

$$=\sin A\times\cos A\times\frac{2}{2\sin A\cos A}$$

$$=1$$

$L.H.S = R.H.S$

65. $$\frac{\tan A}{\left(1+\tan^2 A\right)}+\frac{\cos A}{\left(1+\cot^2 A\right)^2}=\sin A\cos A$$

Sol:

$$=\frac{\tan A}{\left(\sec^2 A\right)^2}+\frac{\cos A}{\left(\cos ec^2 A\right)^2}\qquad\begin{bmatrix}\because 1+\tan^2 A=\sec^2 A\\ 1+\cot^2 A=\cos ec^2 A\end{bmatrix}$$

$$=\frac{\frac{\sin A}{\cos A}}{\sec^4 A}+\frac{\frac{\cot A}{\sin A}}{\cos ec^4 A}$$

$$=\frac{\frac{\sin A}{\cos A}}{\frac{1}{\cos^4 A}}+\frac{\frac{\cos A}{\sin A}}{\frac{1}{\sin^4 A}}$$

$$=\frac{\sin A}{\cos A}\times\frac{\cos^4 A}{1}+\frac{\cos A}{\sin A}\times\frac{\sin^4}{1}$$

$$=\sin A\times\cos^3 A+\cos A-\sin^3$$

$$=\sin A\cos A\left(\cos^2 A+\sin^2 A\right)$$

$$=\sin A\cos A$$

L.H.S = R.H.S

Hence proved.

66. $\sec^4 A\left(1-\sin^4 A\right)-2\tan^4 A=1$

Sol:

$LHS=\sec^4 A\left(1-\sin^4 A\right)-2\tan^4 A$

$=\sec^4 A-\sec^4 A\times\sin^4 A-2\tan^2 A$

$=\sec^4 A-\dfrac{1}{\cos^4 A}\times\sin^4-2\tan^2 A$

$=\sec^4 A-\tan^4 A-2\tan^2 A$

$=\left(\sec^2 A\right)^2=\tan^4 A-2\tan^2 A$

$=\left(1+\tan^2 A\right)^2-\tan^4 A-2\tan^2 A \qquad \left[\because \sec^2 A-\tan^2 A=1\right]$

$=1+\tan^4 A+2\tan^2 A-\tan^4 A-2\tan^2 A$

$=1=RHS$

Hence proved.

67. $\dfrac{\cot^2 A\left(\sec A-1\right)}{1+\sin A}=\sec^2\left[\dfrac{1-\sin A}{1+\sec A}\right]$

Sol:

$$=\frac{\dfrac{\cos^2 A}{\sin^2 A}\left(\dfrac{1}{\cos A}-1\right)}{1+\sin A}$$

$$=\frac{\dfrac{\cos^2 A}{\sin^2 A}\left(\dfrac{1-\cos A}{\cos A}\right)}{1+\sin A} \qquad \left[\because \sin^2 A+\cos^2 A=1\right]$$

$$=\frac{\dfrac{\left(\cos A\times\cos A\right)}{\left(1-\cos^2 A\right)}\left[\dfrac{1-\cos A}{\cos A}\right]}{1+\sin A}$$

$$=\frac{\left(\cos A\right)\left(1-\cos A\right)}{\left(1+\cos A\right)\left(1-\cos A\right)}=\frac{1}{1+\sin A}$$

$$=\frac{\cos A}{\left(1+\cos A\right)\left(1+\sin A\right)}$$

Solving

$RHS = \sec^2\left[\frac{1-\sin A}{1+\sec A}\right]$

$= \frac{1}{\cos^2 A}\left[\frac{1-\sec A}{1+\sec A}\right]$

$= \frac{1}{\cos^2 A}\left[\frac{1-\sec A}{\cos A+1}\right](\cos A)$

$= \frac{(1-\sin A)}{(\cos A)(\cos A+1)}$

By multiplying Nr and Dr with $(1+\sin A)$

$= \frac{(1-\sin A)}{(\cos A)(1+\cos A)} \times \frac{1+\sin A}{1+\sin A}$

$= \frac{(1)^2 - \sin^2 A}{\cos A(1+\cos A)(1+\sin A)}$

$= \frac{\cos^2 A}{\cos A(1+\cos A)(1+\sin A)}$

$= \frac{\cos^2 A}{(1+\cos A)(1+\sin A)}$

$L.H.S = R.H.S$ hence proved.

68. $(1+\cot A+\tan A)(\sin A-\cos A) = \frac{\sec A}{\cos ec^2 A} - \frac{\cos ecA}{\sec^2 A} == \sin A \tan A - \cos A \cot A$

Sol:

$(1+\cot A+\tan A)(\sin A-\cos A)$

$\sin A - \cos A + \cot A \sin A - \cot A \cos A + \sin A \tan A - \tan A \cos A$

$\sin A - \cos A + \frac{\cos A}{\sin A} \times \sin A - \cot A \cos A + \sin A \tan A - \frac{\sin A}{\cos A} \times \cos A$

$\sin A - \cos + \cos A - \cot A \cos A + \sin A \tan A - \sin A$

$= \sin A \cos A \ \cos A \cot A$

Solving:

$\frac{\sec A}{\cos ec^2 A} - \frac{\cos ecA}{\sec^2 A}$

$$\frac{\frac{1}{\cos A}-\frac{1}{\sin A}}{\frac{1}{\sin^2 A}\quad\frac{1}{\cos^2 A}}$$

$$\frac{\sin^2 A}{\cos A}-\frac{\cos^2 A}{\sin A}$$

$$\frac{\sin^3 A-\cos^3 A}{\sin A\cos A}$$

$$=\sin A\times\frac{\sin A}{\cos A}-\cos A\times\frac{\cos A}{\sin A}$$

$$=\sin A\tan A-\cos A\cot A$$

$$L.H.S=R.H.S$$

69. $\sin^2 A\cos^2 B-\cos^2 A\sin^2 B=\sin^2 A-\sin^2 B$

Sol:

$LHS=\sin^2 A\cos^2 B-\cos^2 A\sin^2 B.$

$=\sin^2 A\left(1-\sin^2 B\right)-\left(1-\sin^2 A\right)\left(\sin^2 A\right)\qquad\left(\because\cos^2 A=1-\sin^2 A\right)$

$=\sin^2 A-\sin^2 A\sin^2 B-\sin^2 B+\sin^2 A\sin^2 B$

$=\sin^2 A-\sin^2 B$

$R.H.S$ Hence Proved.

70. $\frac{\cot A+\tan B}{\cot B+\tan A}=\cot A\tan B$

Sol:

$$LHS=\frac{\cot A+\tan B}{\cot B+\tan A}$$

$$=\frac{\frac{\cos A}{\sin A}+\frac{\sin B}{\cos B}}{\frac{\cos B}{\sin B}+\frac{\cos ecA}{\cos A}}$$

$$=\frac{\frac{\cos A\cos B-\sin A\sin B}{\sin A\cos B}}{\frac{\cos A\cos B+\sin A\sin B}{\cos A\sin B}}$$

$$= \frac{\cos A\cos B + \sin A\sin B}{\sin A\cos B} \times \frac{\cos A\sin B}{\cos A\cos B + \sin A\sin B}$$
$$= \frac{\cos A\sin B}{\sin A\cos B}$$
$$= \cot A\tan B$$
$$= RHS$$
Hence proved

71. $\frac{\tan A + \tan B}{\cot A + \cot B} = \tan A\tan B$

Sol:

$$LHSS = \frac{\tan A + \tan B}{\cot A + \cot B}$$
$$= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}}$$
$$= \frac{\frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B}}{\frac{\cos A\sin B + \cos B\sin A}{\sin A\sin B}}$$
$$= \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B} \times \frac{\sin A\sin B}{\cos A\sin B + \cos B\sin A}$$
$$= \frac{\sin A\sin B}{\cos A\cos B}$$
$$= \tan A + \tan B = RHS$$
Hence proved

72. $\cot^2 A\cos ec^2 B - \cot^2 B\cos ec^2 A = \cot^2 A - \cot^2 B$

Sol:

$$LHS = \cot^2 A\cos ec^2 B - \cot^2 B\cos ec^2 A$$
$$= \cot^2 A\left(1 + \cot^2 B\right) - \cot^2 B\left(1 + \cot^2 B\right) \qquad \left[\because \cos ec^2\theta = 1 + \cot^2\theta\right]$$
$$= \cot^2 A + \cot^2 A\cot^2 B - \cot^2 B - \cot^2 B\cot^2 A$$
$$= \cot^2 A - \cot^2 B.$$
Hence proved

73. $\tan^2 A\sec^2 B - \sec^2 A\tan^2 B = \tan^2 A - \tan^2 B$

Sol:

$LHS = \tan^2 A \sec^2 B - \sec^2 A \tan^2 B$

$= \tan^2 A + \left(1 + \tan^2 B\right) - \sec^2 A\left(\tan^2 A\right)$

$= \tan^2 A + \tan^2 A \tan^2 B - \tan^2 B\left(1 + \tan^2 A\right) \qquad \left(\because \sec^2 A = 4\tan^2 A\right)$

$= \tan^2 + \tan^2 A \tan^2 B - \tan^2 B - \tan^2 B \tan^2 A$

$= \tan^2 A - \tan^2 B$

$= RHS$

74. If x = a sec θ + b tan θ and y $=$ a tan θ + b sec θ, prove that $x^2 - y^2 = a^2 - b^2$

Sol:

$L.H.S = x^2 - y^2$

$= \left(a\sec\theta + b\tan\theta\right)^2 - \left(a\tan\theta + b\sec\theta\right)^2$

$= a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta - a^2\tan^2\theta - b^2\sec\theta - 2ab\sec\theta\tan\theta.$

$= a^2 - \sec^2\theta - b^2\sec^2\theta + b^2\tan^2\theta - a^2\tan^2\theta$

$= \sec^2\theta\left(a^2 - b^2\right) + \tan^2\theta\left(b^2 - a^2\right)$

$= \sec^2\theta\left(a^2 - b^2\right) - \tan^2\theta\left(a^2 - b^2\right)$

$= \left(a^2 - b^2\right)\left(\sec^2\theta - \tan^2\theta\right) \qquad \left[\because \sec^2\theta - \tan^2\theta = 1\right]$

$= a^2 - b^2$

75. If $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ *and* $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$, *prove that* $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

Sol:

$$\left[\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right]^2 + \left[\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right]^2 = (1)^2 + (1)^2$$

$$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta \frac{2xy}{ab}\cos\theta\sin\theta + \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta$$

$$-\frac{2xy}{ab}\sin\theta\cos\theta = 1 + 1$$

$$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + \frac{x^2}{a^2}\sin^2\theta = 2$$

$$\cos^2\theta\left[\frac{x^2}{a^2} + \frac{y^2}{b^2}\right] + \sin^2\theta\left(\frac{x^2}{a^2} + \frac{y^2}{a^2}\right) = 2$$

$$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)\left(\cos^2\theta+\sin^2\theta\right)=2$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}= \qquad \left(\therefore \cos^2\theta+\sin^2\theta=1\right)$$

76. If cosec θ – sin $\theta = a^3, \sec\theta-\cos\theta=b^3, prove\ that\ a^2b^2(a^2+b^2)=1$

Sol:

$$\cos ec\theta-\sin\theta=a^3$$

$$\frac{1}{\sin\theta}-\sin\theta=a^3$$

$$\frac{1-\sin^2}{\sin\theta}=a^3$$

$$\frac{\cos^2\theta}{\sin\theta}=a^3$$

$$a=\frac{\cos^{\frac{1}{3}}\theta}{\sin^{\frac{1}{3}}\theta}$$

$$\Rightarrow a^2=\frac{\cos\frac{4}{3}\theta}{\sin\frac{2}{3}\theta}$$

$$\sec\theta-\cos\theta=b^3$$

$$\frac{1}{\cos\theta}\cos\theta=b^3$$

$$\frac{1-\cos^2\theta}{\cos\theta}=b^3$$

$$\frac{\sin^2\theta}{\cos\theta}=b^3$$

$$b=\frac{\sin^{\frac{2}{3}}\theta}{\cos^{\frac{1}{3}}\theta}$$

Now, $a^2b^2\left(a^2+b^2\right)$

$$=\frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta}\times\frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}\left(\frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta}=\frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}\right)$$

$$= \cos^{\frac{4}{3}-\frac{2}{3}}\theta \times \sin^{\frac{4-2}{3}}\left(\frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta} + \frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}\right)$$

$$= \cos^{\frac{2}{3}}\theta \sin^{\frac{2}{3}}\left(\frac{1}{\sin^{\frac{2}{3}}\theta \cos^{\frac{2}{3}}\theta}\right) \ (\because \cos^2\theta + \sin^2\theta = 1)$$

$$= 1$$

$$L.H.S = R.H.S$$

77. If $a\cos^3\theta + 3a\cos\theta\sin^2\theta = m, a\sin^3\theta + 3\ a\cos^2\theta\sin\theta = n, prove\ that$

$$(m+n)^{\frac{2}{3}} + (m-n)^{\frac{2}{3}}$$

Sol:

$$= \left(a\cos^3\theta + 3a\cos\theta\sin^2\theta + a\sin^3\theta + 3a\cos^2\theta\sin\theta\right)^{\frac{2}{3}}$$

$$+ \left(a\cos^3\theta + 3a\cos\theta\sin^2\theta - a\sin^3\theta - 3a\cos^2\theta\sin\theta\right)^{\frac{2}{3}}$$

$$= a^{\frac{1}{3}}\left(\cos^3\theta + 3\cos\theta\sin^2\theta + \sin^3\theta + 3\cos^2\theta\sin\theta\right)^{\frac{2}{3}}$$

$$+ a^{\frac{2}{3}}\left(\cos^3\theta + 3\cos\theta\sin^2\theta + \sin^3\theta - 3\cos^2\theta\sin\theta\right)^{\frac{2}{3}}$$

$$= a^{\frac{1}{3}}\left[(\cos\theta + \sin\theta)^3\right]^{\frac{2}{3}} + a^{\frac{2}{3}}\left(\cos\theta - \sin\theta)^3\right]^{\frac{2}{3}}$$

$$= a^{\frac{2}{3}}\left[(\cos\theta + \sin\theta)^2\right] + a^{\frac{2}{3}}(\cos\theta - \sin\theta)^2$$

$$= a^{\frac{2}{3}}\left[\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta\right]$$

$$= a^{\frac{2}{3}}\left[\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta\right] + a^{\frac{2}{3}}\left[\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta\right]$$

$$= a^{\frac{2}{3}}\left[1 + 2\sin\theta\cos\theta\right] + a^{\frac{2}{3}}\left[1 - 2\sin\theta\cos\theta\right]$$

$$= a^{\frac{2}{3}}\left[1 + 2\sin\theta\cos\theta + 1 - 2\sin\theta\cos\theta\right]$$

$$= a^{\frac{1}{3}}(1+1) = 2a^{\frac{2}{3}}$$

$$= R.H.S$$

Hence proved.

78. If x = a $\cos^3\theta, y = b\sin^3\theta$, *prove that* $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$

Sol:

$x = a\cos^3\theta : y = b\sin^3\theta$

$\frac{x}{a} = \cos^3\theta : \frac{y}{b} = \sin^3\theta$

$L.H.S = \left[\frac{x}{a}\right]^{\frac{2}{3}} + \left[\frac{y}{b}\right]^{\frac{2}{3}}$

$= \left(\cos^3\theta\right)^{\frac{2}{3}} + \left(\sin^3\theta\right)^{\frac{2}{3}}$

$= \cos^2\theta + \sin^2\theta$ $\quad\left(\therefore \cos^2\theta + \sin^2\theta = 1\right)$

$= 1$

Hence proved

79. If 3 sin $\theta + 5$ cos $\theta = 5$, prove that 5sin θ – 3 cos $\theta = \pm$ 3.

Sol:

Given $3\sin\theta + 5\cos\theta = 5$

$3\sin\theta = 5 - 5\cos\theta$

$3\sin\theta = 5\left(1 - \cos\theta\right)$

$3\sin\theta = \frac{5\left(1-\cos\theta\right)\left(1-\cos\theta\right)}{1+\cos\theta}$

$3\sin\theta = \frac{5\left(1-\cos^2\theta\right)}{\left(1+\cos\theta\right)}$

$3\sin\theta = \frac{5\sin^2\theta}{1+\cos\theta}$

$3 + 3\cos\theta = 5\sin\theta$

$3 = 5\sin - 3\cos\theta$

$= RHS$

Hence proved.

80. If a cos θ + b sin θ = m and a sin θ – b cos θ = n, prove that $a^2 + b^2 = m^2 + n^2$

Sol:

$R.H.S = m^2\sin^2$

$= \left(a\cos\theta + b\sin\theta\right)^2 + \left(a\sin\theta - b\cos\theta\right)^2$

$= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta$

$+ a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta$

$= a^2\cos^2\theta + b^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta$

$= a^2\left(\sin^2\theta+\cos^2\theta\right)+b^2\left(\sin^2\theta+\cos^2\theta\right)$

$= a^2+b^2 \qquad \left(\because \sin^2\theta+\cos^2\theta=1\right)$

81. If $\cos\theta + \cot\theta = \text{m}$ and $\text{cosec}\,\theta - \cot\theta = \text{n}$, prove that m n = 1

Sol:

$LHS = mn$

$=\left(\cos ec\theta+\cot\theta\right)\left(\cos ec\theta-\cot\theta\right)$

$=\cos ec^2\theta-\cot^2\theta$

$=1 \qquad \left[\because (a+b)(a-b)=a^2-b^2 \cos ec^2\theta-\cot^2\theta=1\right]$

$= R.H.S$

82. If cos A + $\cos^2$ A = 1, prove that $\sin^2$ A + $\sin^4$ A = 1

Sol:

$\cos A+\cos^2 A=1$

$\cos A=1-\cos^2 A$

$\cos A=\sin^2 A$

$LHS=\sin^2 A+\sin^4 A$

$=\sin^2 A+\left(\sin^2 A\right)$

$=\sin^2 A+\left(\cos A\right)^2$

$=\sin^2 A+\cos A$

$=1$

83. Prove that:

$(i)\ \sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}=2cosec\ \theta$

$(ii)\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=2\sec\theta$

$(iii)\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}+\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=2cosec\ \theta$

$(iv)\ \frac{\sec\theta-1}{\sec\theta+1}=\left(\frac{\sin\theta}{1+\cos\theta}\right)^2$

Sol:

$$LHS=\sqrt{\frac{\frac{1}{\cos\theta}-1}{\frac{1}{\cos\theta}+1}}+\sqrt{\frac{\frac{1}{\cos\theta}+1}{\frac{1}{\cos\theta}-1}}$$

$$=\sqrt{\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}}+\sqrt{\frac{\frac{1+\cos\theta}{\cos\theta}}{\frac{1-\cos\theta}{\cos\theta}}}$$

$$=\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}+\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$

$$=\sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)}\times\frac{(1-\cos\theta)}{1-\cos\theta}}+\sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}}$$

$$=\sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}+\sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}$$

$$=\frac{1-\cos\theta}{\sin\theta}+\frac{1+\cos\theta}{\sin\theta}$$

$$=\frac{1-\cos\theta+1+\cos\theta}{\sin\theta}$$

$$=\frac{2}{\sin\theta}$$

$$=2\cos ec$$

(2) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}\times\frac{(1+\sin\theta)}{1+\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}}$$

$$\sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}=\sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$=\sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}+\sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$$

$$=\frac{1+\cos\theta}{\sin\theta}+\frac{1-\cos\theta}{\sin\theta}$$

$$=\frac{2}{\sin\theta}=2\cos ec\theta$$

(3) Not given

(4) $\frac{\sec\theta-1}{\sec\theta+1}$

$=\frac{\frac{1}{\cos\theta}-1}{\frac{1}{\cos\theta}+1}$

$=\frac{1-\cos\theta}{1+\cos\theta}$

$=\frac{1-\cos\theta}{1+\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}$

$=\frac{1-\cos^2\theta}{(1+\cos\theta)^2}$

$=\frac{\sin^3}{(1+\cos\theta)^2}$

$=\left[\frac{\sin\theta}{1+\cos\theta}\right]^2$

$=RHS$

Hence proved.

84. If $\cos\theta+\cos^2\theta=1$, prove that

$\sin^{12}\theta+3\sin^{10}\theta+3\sin^8\theta+\sin^6\theta+2\sin^4\theta+2\sin^2\theta-2=1$

Sol:

$\cos\theta+\cos^2\theta=1$

$\cos\theta=1-\cos^2\theta$

$\cos\theta=\sin^2\theta$ .....(1)

Now, $\sin^{12}\theta+3\sin^{10}\theta+3\sin^8\theta+\sin^6\theta+2\sin^4\theta+2\sin^2\theta-2$

$=(\sin^4\theta)^3+3\sin^4\theta\cdot\sin^2\theta(\sin^4\theta+\sin^2\theta)$

$+(\sin^2\theta)^3+2(\sin^2\theta)^2+2\sin^2\theta 2$

Using $(a+b)^3=a^3+b^3+3ab(a+b)$ and also from

(1) $\sin^2\theta\cos\theta$

$(\sin^4\theta+\sin^2\theta)^3+2\cos\theta^2+2\cos\theta-2.$

$\left((\sin^2\theta)^2+\sin^2\theta\right)+2\cos^2\theta+2\cos\theta-2$

$(\cos^2+\sin^2\theta)^3+2\cos^2\theta+2\cos\theta-2$

$\left(\cos^2+\sin\right)^3+2\cos^2\theta+2\sin^2\theta-2 \quad \left[\because \sin^2\theta+\cos^2\theta=1\right]$

$1+2\left(\sin^2\theta+\cos^2\theta\right)-2$

$1+2(1)-2$

$=1$

$L.H.S=R.H.S$

Hence proved.

85. Given that $(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma)=(1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$

Show that one of the values of each member of this equality is sin α sin β sin γ

Sol:

L.H.S

We know that $1+\cos\theta=1+\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}=2\cos^2\frac{\theta}{2}$

$\therefore\Rightarrow 2\cos^2\frac{\alpha}{2}\cdot 2\cos^2\frac{\beta}{2}\cdot 2\cos^2\frac{\gamma}{2} \qquad ....(1)$

Multiply (1) with $\sin\alpha\sin\beta\sin\gamma$ and divide it with same we get

$$\frac{8\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}\cos^2\frac{\gamma}{2}}{\sin\alpha\sin\beta\sin\gamma}\times\sin\alpha\sin\beta\sin\gamma$$

$$\Rightarrow\frac{2\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}\cos^2\frac{\gamma}{2}\times\sin\alpha\sin\beta\sin\gamma}{\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}}$$

$$\Rightarrow\sin\alpha\sin\beta\sin\gamma\times\cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$$

$RHS\,(1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$

We know that $1-\cos\theta=1-\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}=2\sin^2\frac{\theta}{2}$

$\Rightarrow 2\sin^2\frac{\alpha}{2}2\cdot\sin^2\frac{\beta}{2}\cdot 2\sin^2\frac{\gamma}{2}$

Multiply and divide by $\sin\alpha\sin\beta\sin\gamma$ we get

$$\frac{2\sin^2\frac{\alpha}{2}2\sin^2\frac{\beta}{2}2\sin^2\frac{\gamma}{2}\cdot\sin\alpha\sin\beta\sin\gamma}{\sin\alpha\sin\beta\sin\gamma}$$

$$\Rightarrow \frac{2\sin^2\frac{\alpha}{2}\cdot 2\sin^2\frac{\beta}{2}\cdot 2\sin^2\frac{\gamma}{2}\cdot \sin\alpha\sin\beta\sin\gamma}{2\sin\frac{\alpha}{2}\cos\frac{\beta}{2}2\sin\frac{\beta}{2}\cos\frac{\beta}{2}2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}}$$

$$\Rightarrow \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}\sin\alpha\sin\beta\sin\gamma$$

Hence $\sin\alpha\sin\beta\sin\gamma$ is the member of equality.

86. $if\ \sin\theta+\cos\theta=x$ P.T $\sin^6\theta+\cos^6\theta=\frac{4-3\left(x^2-1\right)^2}{4}$

Sol:

$\sin\theta+\cos\theta=x$

Squaring on both sides

$\left(\sin\theta+\cos\theta\right)^2=x^2$

$\Rightarrow \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=x^2$

$\therefore \sin\theta\cos\theta=\frac{x^2-1}{2}$ ......(1)

We know $\sin^2\theta+\cos^2\theta=1$

Cubing on both sides

$\left(\sin^2\theta+\cos^2\theta\right)^3=(1)^3$

$\sin^6\theta+\cos^6\theta+3\sin^2\theta\cos^2\theta\left(\sin^2\theta+\cos^2\theta\right)=1$

$\Rightarrow \sin^6\theta+\cos^6 7=1-3\sin^2\theta\cos^2\theta$

$=1-3\frac{\left(x^2-1\right)^2}{4}$ from (1)

$\therefore \sin^6\theta+\cos^6\theta=\frac{4-3\left(x^2-1\right)^2}{4}$

Hence proved

87. $if\ x=a\sec\theta\cos\phi y=b\sec\theta\sin\phi and\ z=c\tan\theta,\ S.T\ \frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

Sol:

$x^2=a^2\sec^2\theta\cos^2\theta$ .....(i)

$y^2=b^2\sec^2\theta\sin^2\theta$ ....(ii)

$z^2=c^2\tan^2\theta$ ....(iii)

Exercise 6.2

1. *If* $\cos\theta = \frac{4}{5}$, *find all other trigonometric ratios of angle* θ

Sol:

We have $\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{4}{5}\right)^2}$

$= \sqrt{1-\frac{16}{25}}$

$= \sqrt{25-\frac{16}{25}}$

$= \sqrt{\frac{9}{25}} = \frac{3}{5}$

$\therefore \sin\theta = \frac{3}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3/5}{4/5} = \frac{3}{4}\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$

$\cos ec = \frac{1}{\sec\theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3}\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{3}{4}} = \frac{4}{3}.$

2. *If* $\sin\theta = \frac{1}{\sqrt{2}}$, *find all other trigonometric ratios of angle* θ

Sol:

We have $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{1}{2}\right)^2}$

$= \sqrt{\frac{1-1}{2}} = \sqrt{\frac{1}{2}}$

$\therefore \cos\theta = \frac{1}{\sqrt{2}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

$\cos ec\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{1} = 1$$

3. $If\ \tan\theta = \frac{1}{\sqrt{2}}, Find\ the\ value\ of\ \frac{\cos ec^2\theta - \sec^2\theta}{\cos ec^2\theta + \cot^2\theta}$

Sol:

We know that $\sec\theta = \sqrt{1+\tan^2\theta}$

$$= \sqrt{1+\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \sqrt{1+\frac{1}{2}} = \sqrt{\frac{3}{2}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$$\cos ec\theta = \sqrt{1+\cot^2\theta} \Rightarrow \sqrt{1+2} = \sqrt{3}$$

Substituting it in (1) we get

$$\Rightarrow \frac{\left(\sqrt{3}\right)^2 - \left(\sqrt{3}\right)^2}{\left(\sqrt{3}\right)^2 + \left(\sqrt{3}\right)^2} = \frac{3-\frac{3}{2}}{3+2} = \frac{\frac{3}{2}}{5}$$

$$= \frac{3}{10}$$

4. $If\ \tan\theta = \frac{3}{4}, find\ the\ value\ of\ \frac{1-\cos\theta}{1+\cos\theta}$

Sol:

$$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\left(\frac{3}{4}\right)^2} = \sqrt{1+\frac{9}{16}}$$

$$\Rightarrow \sqrt{\frac{16-9}{16}} = \frac{5}{4}$$

$$\therefore \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{5}{4}} = \frac{4}{5} = \cos\theta$$

$$\therefore \text{We get } \frac{1-\frac{4}{5}}{1+\frac{4}{5}} = \frac{\frac{1}{5}}{\frac{9}{5}} = \frac{1}{9}.$$

5. *If* $\tan\theta = \frac{12}{5}$, *find the value of* $\frac{1+\sin\theta}{1-\sin\theta}$

Sol:

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{12}{5}} = \frac{5}{12}$$

$$\cos ec = \sqrt{1+\cot^2\theta} = \sqrt{1+\left[\frac{5}{12}\right]^2} = \sqrt{\frac{144+25}{(12)^2}} = \sqrt{\frac{169}{144}} = \frac{13}{12}.$$

$$\sin\theta = \frac{1}{\cos ec\theta} = \frac{1}{\frac{13}{12}} = \frac{12}{13}.$$

$$\text{We get } \frac{1+\frac{12}{13}}{1-\frac{12}{13}} = \frac{\frac{13+12}{18}}{\frac{13-12}{18}} = \frac{25}{1} = 25$$

6. *If* $\cot\theta = \frac{1}{\sqrt{3}}$, *find the value of* $\frac{1-\cos^2\theta}{2-\sin^2\theta}$

Sol:

$$\cos ec\theta = \sqrt{1+\cot^2}\theta = \sqrt{1+\frac{1}{3}} = \sqrt{\frac{4}{3}}$$

$$\therefore \cos ec\theta = \frac{2}{\sqrt{3}}$$

$$\sin\theta = \frac{1}{\cos ec\theta} = \frac{1}{\frac{2}{\sqrt{3}}} = \frac{\sqrt{3}}{2}$$

$$and\ \frac{1}{\cot\theta} = \frac{\sin\theta}{\cos\theta} = \cos = \frac{\sin\theta}{\frac{1}{\cos\theta}} \Rightarrow \frac{\frac{\sqrt{3}}{2}}{\frac{2}{\sqrt{3}}} = \frac{1}{2}.$$

$\therefore$ on substituting we get

$$\frac{1-\frac{1}{4}}{2-\frac{3}{4}}=\frac{\frac{3}{4}}{\frac{5}{4}}=\frac{3}{5}.$$

7. $If\ \cos ec=\sqrt{2},\ find\ the\ value\ of\ \frac{2\sin^2 A+3\cot^2 A}{4\left(\tan^2 A-\cos^2 A\right)}$

Sol:

We know that $\cot A=\sqrt{\cos ec^2 A-1}$

$=\sqrt{(2)^2-1}=\sqrt{2-1}$

$-1.$

$$\tan A=\frac{1}{\cot A}=\frac{1}{1}=1$$

$$\sin A=\frac{1}{\cos ec}A=\frac{1}{\sqrt{2}}\ \therefore \sin A=\frac{1}{\sqrt{2}}$$

$$\cos A\sqrt{1-\sin^2 A}=\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2}-\sqrt{\frac{1}{\sqrt{2}}}=\frac{1}{\sqrt{2}}.$$

On substituting we get

$$\frac{2\left[\frac{1}{\sqrt{2}}\right]^3+3[1]^2}{4\left[(1)-\left(\frac{1}{\sqrt{2}}\right)^2\right]}=\frac{2=\frac{1}{2}+3}{4\left[1-\frac{1}{2}\right]}$$

$$\Rightarrow \frac{1+3}{4\cdot\frac{1}{2}}=\frac{4}{2}=2.$$

8. $If\ \cot\theta\sqrt{3},\ find\ the\ value\ of\ \frac{\cos ec^2\theta+\cot^2\theta}{\cos ec^2\theta-\cot^2\theta}$

Sol:

$$\cos ec\theta=\sqrt{1+\cot^2\theta}=\sqrt{1+\left(\sqrt{3}\right)^2}=\sqrt{1+3}=2$$

$$\sin\theta=\frac{1}{\cos ec}\theta=\frac{1}{2}\cot\theta=\frac{\cos\theta}{\sin\theta}\qquad \therefore \cos\theta=\cot\theta\cdot\sin\theta$$

$$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{2}{\sqrt{3}}$$

On substituting we get

$$\frac{(2)^2 + \left(\sqrt{3}\right)^2}{(2)^2 - \left(\frac{2}{\sqrt{3}}\right)^2} = \frac{4+3}{\frac{12-4}{3}} = \frac{7}{\frac{8}{3}}$$

$$= \frac{21}{8}.$$

9. *If* $3\cos\theta = 1$, *find the value of* $\dfrac{6\sin^2\theta + \tan^2\theta}{4\cos\theta}$

Sol:

$$\cos\theta = \frac{1}{3} \qquad \sin = \sqrt{1+\cos^2\theta}$$

$$= \sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2\sqrt{2}}{3\cdot\frac{1}{3}} = 2\sqrt{2}$$

On substituting in (1) we get

$$\frac{6\left[\frac{2\sqrt{2}}{3}\right]^2 + \left(2\sqrt{2}\right)^2}{4\cdot\frac{1}{3}} = \frac{6\cdot\frac{3}{5}}{\frac{4}{5}} = \frac{\frac{16+24}{3}}{\frac{4}{3}}$$

$$= \frac{40}{4} = 10$$

10. *If* $\sqrt{3}\tan\theta = \sin\theta$, *find the value of* $\sin^2\theta - \cos^2\theta$

Sol:

$$\sqrt{3}\cdot\frac{\sin\theta}{\cos\theta} = \sin\theta$$

$$\cos\theta = \frac{\sqrt{3}}{3} \Rightarrow \frac{1}{\sqrt{3}}$$

$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{1}{\sqrt{3}}\right)^2}$

$\therefore \sin^2\theta - \cos^2\theta = \left(\sqrt{\frac{2}{3}}\right)^2 - \left[\frac{1}{\sqrt{3}}\right]^2$

$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

11. *If* $\cos ec\theta = \frac{13}{12}$, *find the value of* $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$

Sol:

$\sin\theta = \frac{1}{\cos ec\theta} = \frac{1}{\frac{13}{12}} = \frac{12}{13}$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left[\frac{12}{13}\right]^2} = \sqrt{1-\frac{144}{169}}$

$= \sqrt{\frac{25}{169}} = \frac{5}{13}$

$\Rightarrow \frac{2\cdot\frac{12}{13} - 3\cdot\frac{5}{13}}{4\cdot\frac{12}{13} - 9\cdot\frac{5}{13}} = \frac{\frac{24-15}{13}}{\frac{48-15}{13}} = \frac{9}{3} = 3$

12. *If* $\sin\theta + \cos\theta = \sqrt{2}\cos(90° - \theta)$, *find* $\cot\theta$ find $\cot\theta$

Sol:

$L.H.S \Rightarrow \sin\theta + \cos\theta = \sqrt{2}\sin\theta \qquad [\because \cos(90-\theta) = \sin\theta]$

$\Rightarrow \cos\theta - \sin\theta(\sqrt{2}) - \sin\theta$

$\cos\theta - \sin\theta(\sqrt{2}-1)$

Divide both sides with $\sin\theta$ we get

$\frac{\cos\theta}{\sin\theta} = \frac{\sin\theta}{\sin\theta}(\sqrt{2}-1)$

$= \cot\theta = \sqrt{2}-1$

Exercise – 7.1

1. Calculate the mean for the following distribution:

x:	5	6	7	8	9
f:	4	8	14	11	3

Sol:

x	f	fx
5	4	20
6	8	48
7	14	98
8	11	88
9	3	27
	$N = 40$	$\sum fx = 281$

Mean $= \frac{\sum fx}{N}$

$= \frac{281}{4} = 7 \cdot 025$

2. Find the mean of the following data:

x:	19	21	23	25	27	29	31
f.	13	15	16	18	16	15	13

Sol:

x	f	fx
18	13	247
21	15	315
23	16	368
25	18	450
27	16	432
29	15	435
31	13	403
	$N = 106$	$\sum fx = 2620$

Mean $\left(\bar{x}\right) = \frac{\sum fx}{N} = \frac{2680}{106} = 25.$

3. If the mean of the following data is 20.6. Find the value of p.

x:	10	15	p	25	35
f:	3	10	25	7	5

Sol:

x	F	fx
10	3	30
5	10	150
P	25	25P
25	7	175
35	5	175
	$N=90$	$\sum fx = 530+25P$

Given

$\Rightarrow Mean = 20\cdot 6$

$\Rightarrow \frac{\sum Px}{N} = 20\cdot 6$

$\Rightarrow \frac{530+25P}{50} = 20\cdot 6$

$\Rightarrow 25P = 20\cdot 6(50) - 530$

$\Rightarrow P = \frac{500}{25}$

$\Rightarrow P = 20.$

4. If the mean of the following data is 15, find p.

x:	5	10	15	20	25
f:	6	p	6	10	5

Sol:

x	F	fx
5	6	30
10	P	10P
15	6	90
20	10	200
25	5	125
	$N = P127$	$\sum fx = 10P + 445$

Given

$\Rightarrow Mean = 15$

$\Rightarrow \frac{\sum Px}{N} = 5$

$\Rightarrow \frac{109 + 445}{P + 127} = 15$

$\Rightarrow 10P + 445 = 15P + 405$

$\Rightarrow 15P - 10P = 445 - 405$

$\Rightarrow 5P = 40$

$\Rightarrow P = \frac{40}{5}$

$\Rightarrow P = 8$

5. Find the value of p for the following distribution whose mean is 16.6

x:	8	12	15	p	20	25	30
f.	12	16	20	24	16	8	4

Sol:

x	f	fx
8	12	96
12	16	192
15	20	300
P	24	24P
20	16	220
25	8	200
30	4	420
	$N = 100$	$\sum fx = 24P + 1228$

Given

$\Rightarrow Mean = 16 \cdot 6$

$\Rightarrow \frac{54x}{N} = 16 \cdot 6$

$\Rightarrow \frac{24P + 1228}{100} = 16 \cdot 6$

$\Rightarrow 24P + 1228 = 1660$

$\Rightarrow 24P = 1660 - 1228$

$\Rightarrow P = \frac{432}{24}$

$\Rightarrow P = 18$

6. Find the missing value of p for the following distribution whose mean is 12.58

x:	5	8	10	12	p	20	25
f.	2	5	8	22	7	4	2

Sol:

x	f	fx
5	2	10
8	5	40
10	8	80
12	22	264
P	7	70
20	24	480
25	2	50
	$N = 50$	$\sum fx = 524P + 7P$

Given

$\Rightarrow Mean = 12 = -8$

$\Rightarrow 5\frac{3}{N} = 12 \cdot 58$

$\Rightarrow \frac{528 + 7P}{50} = 12 \cdot 58$

$\Rightarrow 524 + 7P = 629$

$\Rightarrow 7P = 629 - 524$

$\Rightarrow 7P = 105$

$\Rightarrow P = \frac{105}{7}$

$\Rightarrow P = 15$

7. Find the missing frequency (p) for the following distribution whose mean is 7.68.

x:	3	5	7	9	11	13
f:	6	8	15	p	8	4

Sol:

x	f	fx
3	6	18
5	8	40
7	15	105
9	P	9P

11	8	18
13	4	52
	$N = P + 41$	$\sum fx = 9P = 303$

Given

$\Rightarrow Mean = 7 \cdot 68$

$\Rightarrow \frac{\sum fx}{N} = 68$

$\Rightarrow \frac{7P + 303}{P + 41} = 7 \cdot 68$

$\Rightarrow 9P + 303 = P(7 \cdot 68) + 314 \cdot 88$

$\Rightarrow 9P - 7 \cdot 68P = 314 \cdot 88 - 303$

$\Rightarrow 1 \cdot 32P = 11 \cdot 88$

$\Rightarrow P = \frac{11 \cdot 88}{1 \cdot 32}$

$\Rightarrow P = 9.$

8. Find the value of p, if the mean of the following distribution is 20.

x:	15	17	19	20 + p	23
f:	2	3	4	5p	6

Sol:

x	f	fx
15	2	30
17	3	51
19	4	76
20+P	5P	$100P+5P^2$
23	6	138
	$N = 5P + 15$	$\sum fx = 295 + 100P + 2P^2$

$\Rightarrow$ Given Mean $= 2n$

$\Rightarrow \frac{\sum fx}{N} = 20$

$\Rightarrow \frac{295 + 100P - 5P^2}{5 + 15} = 20$

$\Rightarrow 295 + 100P + 5P^2 = 100P + 300$

$\Rightarrow 5P^2 - 5 = 0$

$\Rightarrow 5\left(P^2 - 1\right) = 0$

$\Rightarrow P^2 - 1 = 0 \Rightarrow (P+1)(P-1) = 0$

$\Rightarrow p^2 = 1$

$\Rightarrow p = \pm 1$

If $P + 1 = 0$

$P = -1$ (Reject)

Or $P - 1 = 0$

$P = 1$

9. The following table gives the number of boys of a particular age in a class of 40 students. Calculate the mean age of the students

Age (in years): 15 16 17 18 19 20

No. of students: 3 8 10 10 5 4

Sol:

x	f	fx
15	3	45
16	8	128
17	10	170
18	10	180
19	5	95
20	4	80
	$\sum f = N = 40$	$\sum fx = 498$

Mean age $= \dfrac{\sum fx}{N}$

$= \dfrac{698}{40}$

$= 17 \cdot 45$ *years*

$\therefore$ Mean age $= 17 \cdot 45$ years

10. Candidates of four schools appear in a mathematics test. The data were as follows:

Schools	No. of Candidates	Average Score
I	60	75
II	48	80
III	NA	55
IV	40	50

If the average score of the candidates of all the four schools is 66, find the number of candidates that appeared from school III.

Sol:

Let the number of candidates from school III = P

Schools	No of candidates N_i	Average scores (x_i)
I	60	75
II	48	80
III	P	55
IV	40	50

Given

Average score or all schools $= 66.$

$$\Rightarrow \frac{N_1\overline{x}_1 + N_2\overline{x}_2 + N_3\overline{x}_3 + N_4\overline{x}_4}{N_1 + N_2 + N_3 + N_4} = 66$$

$$\Rightarrow \frac{60\times75 + 48\times80 + P\times55 + 40\times50}{60+48+P+40} = 66$$

$$\Rightarrow \frac{4500+3340+55P+2000}{148+P} = 66$$

$$\Rightarrow 10340 + 55P = 66P + 9768$$

$$\Rightarrow 10340 - 9768 = 66P - 55P$$

$$\Rightarrow P = \frac{572}{11}$$

$$\Rightarrow P = 52.$$

11. Five coins were simultaneously tossed 1000 times and at each toss the number of heads were observed. The number of tosses during which 0, 1, 2, 3, 4 and 5 heads were obtained are shown in the table below. Find the mean number of heads per toss.

No. of heads per toss	No. of tosses
0	38
1	144
2	342
3	287
4	164

5	25
Total	1000

Sol:

No. of heads per toss	No. of tosses
0	38
1	144
2	342
3	287
4	164
5	25

No. of heads per toss	No. of tosses	fx
0	28	0
1	144	144
2	342	684
3	287	861
4	164	656
5	25	125

Mean number of heads per toss $= \frac{\Sigma fx}{N}$

$= \frac{2470}{1000}$

$= 2 \cdot 47$

Mean $= 2 \cdot 47$

12. Find the missing frequencies in the following frequency distribution if it is known that the mean of the distribution is 50.

X: 10 30 50 70 90

f: 17 f_1 32 f_2 19 Total 120.

Sol:

x	f	fx
10	17	170
30	f_1	$30\ f_1$
50	32	1600
70	f_2	$70\ f_2$
90	19	1710
	$N = 120$	$\sum fx = 30 f_1 + 70 f_2 + 3480.$

Given mean

$$\frac{\Sigma fx}{N} = 50$$

$$\frac{30f_1 + 70f_2 + 3480}{120} = 50$$

$30f_1 + 70f_2 + 3480 = 6000$ .....(i)

Also,

$\Sigma f = 120$

$17 + f_1 + 32 + f_2 + 19 = 120$

$f_1 + f_2 = 52$

$f_1 = 52 - f_2$

Substituting value of f_1 in (i)

$30(52 - f_2) + 70f_2 + 3480 = 6000 \Rightarrow 40f_2 = 960$

$\Rightarrow f_2 = 24$

Hence $f_1 = 52 - 24 = 28$ $\therefore f_1 = 28; f_2 = 24$

13. The arithmetic mean of the following data is 14. Find the value of k

x_i:	5	10	15	20	25
f_i:	7	k	8	4	5.

Sol:

x	f	fx
10	17	170
30	f_1	$30f_1$
50	32	1600
70	f_2	$70f_2$
90	19	1710
	$N = 120$	$\sum fx = 30f_1 + 70f_2 + 3480.$

Given mean = 50

$$\frac{\Sigma fx}{N} = 50$$

$$\frac{30f_1 + 70f_2 + 3480}{120} = 50$$

$30f_1 + 70f_2 + 3480 = 6000$ .....(i)

Also,

$\Sigma f = 120$

$17 + f_1 + 32 + f_2 + 19 = 120$

$f_1 + f_2 = 52$

$f_1 = 52 - f_2$

Substituting value of f_1 in (i)

$30(52 - f_2) + 70 f_2 + 3480 = 6000 \Rightarrow 40 f_2 = 960$

$\Rightarrow f_2 = 24$

Hence $f_1 = 52 - 24 = 28$ $\therefore f_1 = 28; f_2 = 24$

14. The arithmetic mean of the following data is 25, find the value of k.

x_i:	5	15	25	35	45
f_i:	3	k	3	6	2

Sol:

x	f	fx
5	3	15
15	K	15k
25	3	75
35	6	210
45	2	90
	$N = k + 14$	$\sum fx = 15k_1 390.$

$\Rightarrow$ Given mean $= 25$

$$\Rightarrow \frac{\Sigma fx}{N} = 25$$

$$\Rightarrow \frac{1 + k + 390}{k + 14} = 25$$

$\Rightarrow 15k + 390 = 25k + 350$

$\Rightarrow 25k - 15k = 40$

$\Rightarrow 10k = 40$

$$\Rightarrow k = \frac{40}{10}$$

$\Rightarrow k = 4.$

15. If the mean of the following data is 18.75. Find the value of p.

x_i:	10	15	p	25	30
f_i:	5	10	7	8	2

Sol:

x	f	fx
10	5	50
15	10	150
P	7	7P
25	8	200
30	2	60
	$N = 32$	$\sum fx = 1P + 460.$

$\Rightarrow$ Given mean $= 18\cdot 75$

$$\Rightarrow \frac{\Sigma fx}{N} = 1\cdot 75$$

$$\Rightarrow \frac{7P + 460}{32} = 18\cdot 75$$

$$\Rightarrow 7P + 460 = 600$$

$$\Rightarrow 7P = -460 + 600$$

$$\Rightarrow 7P = 140$$

$$\Rightarrow P = \frac{140}{7}$$

$$\Rightarrow P = 20$$

Exercise – 7.2

1. The number of telephone calls received at an exchange per interval for 250 successive one-minute intervals are given in the following frequency table:

No. of calls(x):	0	1	2	3	4	5	6
No. of intervals (f):	15	24	29	46	54	43	39

Compute the mean number of calls per interval.

Sol:

Let be assumed mean $(A) = 3$

No. of calls (x_i)	No. of intervals (f_i)	$u_i = x; -A = x_i = 3$	$f_1 4;$
0	15	-3	-45
1	24	-2	-47

2	29	-1	-39
3	46	0	0
4	54		54
5	43	2	43(2) = 86
6	39	3	47
	$N = 250$		$\Sigma f_i u_i = 135$

Mean number of cells $= A + \frac{\Sigma f_i 4;}{N}$

$= 3 + \frac{135}{250}$

$= \frac{750 + 135}{250}$

$= \frac{885}{250}$

$= 3 \cdot 54$

2. Five coins were simultaneously tossed 1000 times, and at each toss the number of heads was observed. The number of tosses during which 0,1,2,3,4 and 5 heads were obtained are shown in the table below. Find the mean number of heads per toss

No. of heads per toss (x):	0	1	2	3	4	5
No. of tosses (f):	38	144	342	287	164	25

Sol:

Let the assumed mean $(A) = 2.$

No. of heads per toss (xi)	No. of intervals (f_i)	$u_i = A; - x$ $= A; - 2$	$f_i 4;$
0	38	-2	-76
1	144	-1	+44
2	342	0	0
3	287	1	287
4	164	2	328
5	25	3	75
	$N = 1000$		$\Sigma f_i u_i = 470$

Mean number of per toss $= A + \frac{\Sigma f_i 4}{N}$

$= 2 + \frac{470}{1000}$

$= 2 + 0 \cdot 47$

$= 2 \cdot 47$

3. The following table gives the number of branches and number of plants in the garden of a school.

No. of branches (x):	2	3	4	5	6
No. of plants (f):	49	43	57	38	13

Calculate the average number of branches per plant.

Sol:

Let the assumed mean $(A) = 4.$

No. of branches (xi)	No. of plants (f_i)	$u_i = x_i - A$ $= v_i - 4$	$f_i u_i$
2	49	-2	-98
3	43	-1	-43
4	57	0	0
5	28 + 10 = 38	1	28
6	13	2	85
	$N = 200$		$\Sigma f_i u_i = -77$

Average number of branches per plant $= A + \frac{\Sigma f_i u_i}{N}$

$= 4 + \frac{-77}{200}$

$= 4 - \frac{77}{200}$

$= \frac{800 - 77}{200}$

$= 3 \cdot 615$

$= 3 \cdot 62 (Approx).$

4. The following table gives the number of children of 150 families in a village

No. of children (x):	0	1	2	3	4	5
No. of families (f):	10	21	55	42	15	7

Find the average number of children per family.

Sol:

Let the assumed mean $(A) = 2$

No. of children (x_i)	No of families (f_i)	$u_i = x_i - A$ $= x_i = 2$	$f_i u_i$
0	10	-2	-20
1	21	-1	-21
3	42	1	42
4	15	2	30
5	7	5	21

	$N = 20$		$\Sigma f_i u_i = 52$

$\therefore$ Average number of children for family $= A + \frac{\Sigma f_i u_i}{N}$

$= 2 + \frac{52}{150}$

$= \frac{300+52}{150}$

$= \frac{352}{150}$

$= 2\cdot 35(approx)$

5. The marks obtained out of 50, by 102 students in a Physics test are given in the frequency table below:

Marks(x):	15	20	22	24	25	30	33	38	45
Frequency (f):	5	8	11	20	23	18	13	3	1

Find the average number of marks.

Sol:

Let the assumed mean $(A) = 25$

Marks (x_i)	Frequency (f_i)	$u_i = x_i - A = x_i - 25$	$f_i u_i$
15	5	-10	-50
20	8	-5	-40
22	8	-3	-33
24	20	-1	-20
25	23	0	0
30	18	5	90
33	13	8	104
38	3	12	39
45	3	20	20
	$N = 122$		$\Sigma f_i u_i = 110$

Average number of marks $= A + \frac{\Sigma f_i u_i}{N}$

$= 25 + \frac{110}{102}$

$= \frac{2550+110}{102}$

$= \frac{2660}{102}$

$= 26\cdot 08(Approx)$

6. The number of students absent in a class were recorded every day for 120 days and the information is given in the following frequency table:

No. of students absent (x):	0	1	2	3	4	5	6	7
No. of days(f):	1	4	10	50	34	15	4	2

Find the mean number of students absent per day.

Sol:

Let the assumed mean $(A) = 3$

No. of students absent x_i	No. of days f_i	$u_i = x_i - A$ $= x_i = 3$	$f_i u_i$
3	1	-3	-3
1	4	-2	-8
2	10	-1	-10
3	50	0	-10
4	34	1	24
5	15	2	30
6	4	3	12
7	2	4	8
	$N = 120$		$\Sigma f_i u_i = 63$

Mean number of students absent per day $= A + \frac{\Sigma f_i u_i}{N}$

$= 3 + \frac{63}{120}$

$= \frac{360 + 63}{120}$

$= \frac{423}{120}$

$= 2 \cdot 525$

$= 3 \cdot 53 (Approx)$

7. In the first proof reading of a book containing 300 pages the following distribution of misprints was obtained:

No. of misprints per page (x):	0	1	2	3	4	5
No. of pages (f):	154	95	36	9	5	1

Find the average number of misprints per page.

Sol:

Let the assumed mean $(A) = 2$

No. of misprints per page (x_i)	No. of days (f_i)	$u_i = x_i - A$ $= x_i = 3$	$f_i u_i$

0	154	-2	-308
1	95	-1	-95
2	36	0	0
3	9	1	9
4	5	2	1
5	1	3	3
	$N = 300$		$\Sigma f_i u_i = -381$

Average number of mis prints per day $= A + \frac{f_i u_i}{N}$

$= 2 + \frac{381}{300}$

$= 2 - \frac{381}{300}$

$= \frac{600 - 381}{300}$

$= \frac{219}{300}$

$= 0 \cdot 73$

8. The following distribution gives the number of accidents met by 160 workers in a factory during a month.

No. of accidents (x):	0	1	2	3	4
No. of workers (f):	70	52	34	3	1

Find the average number of accidents per worker.

Sol:

Let the assumed mean $(A) = 2$

No. of Accidents	No. of workers (f_i)	$u_i = x_i - A$ $= x_i = 3$	$f_i u_i$
0	70	-2	-140
1	52	-1	-52
2	34	0	0
3	3	1	3
4	1	2	2
	$N = 100$		$\Sigma f_i u_i = -100$

Average no of accidents per day workers

$$= A = \frac{f_i u_i}{N}$$

$$= x + \frac{-187}{160}$$

$$= \frac{320 - 187}{160}$$

$$= \frac{133}{160}$$

$$= 0 \cdot 83$$

9. Find the mean from the following frequency distribution of marks at a test in statistics:

Marks(x):	5	10	15	20	25	30	35	40	45	50
No. of students (f):	15	50	80	76	72	45	39	9	8	6

Sol:

Let the assumed mean $(A) = 25$.

Marks (xi)	No. of students (f_i)	$u_i = x_i - A$ $= x_i - 25$	$f_1 u_i$
5	15	-20	-300
10	50	-15	-750
15	80	-10	-800
20	76	-5	-380
25	72	0	0
30	45	5	225
35	39	10	390
40	9	15	135
45	8	20	160
50	6	25	150
	$N = 400$		$\Sigma f_i u_i = -1170$

Mean = $\frac{\Sigma f_i u_i}{N}$

$= 25 + \frac{-1170}{400}$

$= \frac{10000 - 1170}{490}$

$= 22 \cdot 075.$

Exercise – 7.3

1. The following table gives the distribution of total household expenditure (in rupees) of manual workers in a city.

Expenditure (in rupees) (x)	Frequency (f_i)	Expenditure (in rupees) (x_1)	Frequency (f_i)
100 – 150	24	300 – 350	30
150 – 200	40	350 – 400	22
200 – 250	33	400 – 450	16
250 – 300	28	450 – 500	7

Find the average expenditure (in rupees) per household.

Sol:

Let the assumed mean $(A) = 275$.

Class interval	Mid value (x_i)	$d_i = x_i - 275$	$u_i = \frac{x_i - 275}{50}$	Frequency f_i	$f_i u_i$
100-150	125	-150	-3	24	-12
150-200	175	-100	-2	40	-80
200-250	225	-50	-1	33	-33
250-300	275	0	0	28	0
300-350	325	50	1	30	30
350-400	375	100	2	22	44
400-450	425	150	3	16	48
450-500	475	200	4	7	28
				$N = 200$	$\Sigma f_i u_i == -35$

We have

$A = 275, h = 50$

Mean $= A + h \times \frac{\Sigma f_i u_i}{N}$

$= 275 + 50 \times \frac{-35}{200}$

$= 275 - 8 \cdot 75$

$= 266 \cdot 25$

2. A survey was conducted by a group of students as a part of their environment awareness program, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants:	0-2	2-4	4-6	6-8	8-10	10-12	12-14
Number of houses:	1	2	1	5	6	2	3

Which method did you use for finding the mean, and why?

Sol:

Let is find class marks (x_i) for each internal by using the relation

Class mark $(x_i) = \frac{upper\ class\ \lim it + lower\ class\ \lim it}{2}$

Now we may compute x_i *and* $f_i x_i$ as following

Number of plants	Number of house (f_i)	x_i	$f_i x_i$
$0-2$	1	1	$1\times1=1$
$2-4$	2	3	$2\times3=6$
$4-6$	1	5	$1\times5=5$
$6-8$	5	7	$5\times7=35$
$8-10$	6	9	$6\times9=54$
$10-12$	2	11	$2\times11=22$
$12-14$	3	13	$3\times13=39$
Total	20		162

From the table we may observe that

$\Sigma f_i = 20$

$\Sigma f_i x_i = 162$

Mean $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$= \frac{162}{20} = 8\cdot 1$

So mean number of plants per house is $8\cdot 1$

We have used for the direct method values x_i *and* f_i are very small

3. Consider the following distribution of daily wages of 50 workers of a factory

Daily wages (in Rs). 100 - 120 120 - 140 140 - 160 160 - 180 180 - 200

Number of workers: 1 2 14 8 6 10

Find the mean daily wages of the workers of the factory by using an appropriate method.

Sol:

Let the assume mean $(A) = 150$

Class interval	Mid value x_i	$d_i = x_i - 150$	$u_i = \frac{x_i - 150}{20}$	Frequency f_i	$f_i u_i$
100-120	110	-40	-2	12	-24
120-140	130	-20	-1	14	-14
140-160	150	0	0	8	0
160-180	170	20	1	6	6
180-200	190	40	2	10	20
			$N = 50$	$\Sigma f_i u_i = -12$	

We have

$N = 50, h = 20$

Mean $= A + h \times \frac{\Sigma f_i u_i}{N}$

$= 150 + 2qr - \frac{12}{5d}$

$= 150 - \frac{24}{5}$

$= 150 - 4\cdot 8$

$= 145\cdot 2$

4. Thirty women were examined in a hospital by a doctor and the number of heart beats per minute recorded and summarized as follows. Find the mean heart beats per minute for these women, choosing a suitable method.

Number of heat 65 - 68 68 - 71 71 - 74 74 - 77 77 - 80 80 - 83 83 -86

beats per minute:
Number of women: 2 4 3 8 7 4 2

Sol:

We may find class marks of each interval (x_i) by using the relation

$$x_i = \frac{Upper\ class\ \lim it + lower\ class\ \lim it}{2}$$

Class size of this data = 3

Now taking $75 \cdot 5$ as assumed mean (a) we

May calculate, $d_i, u_i, f_i u_i$ as following.

Number of heart beats per minute	Number of women (x_i)	x_i	$d_i = x_i - 75 \cdot 5$	$u_i = \frac{x_i - 755}{h}$	$f_i u_i$
$65 - 68$	2	$66 \cdot 5$	-9	-3	-6
$68 - 71$	9	$69 \cdot 5$	-6	-2	-8
$71 - 74$	3	$72 \cdot 5$	-3	-1	-3
$74 - 77$	8	$75 \cdot 5$	0	0	0
$75 - 80$	7	$78 \cdot 5$	3	1	7
$80 - 83$	4	$81 \cdot 5$	$2 \times 3 \times 6$	2	8
$83 - 86$	2	$84 \cdot 5$	9	3	6
	30				4

Now we may observe from table that $\Sigma f_i = 30; \Sigma f_i u_i = 4$

Mean $(\bar{x}) = 9r\left[\frac{\Sigma f_i u_i}{\Sigma f_i}\right] \times h = 75 \cdot 5 + \left(\frac{4}{30}\right) \times 3$

$= 75 \cdot 5 + 0 \cdot 4 = 75 \cdot 9$

So mean hear beats per minute for those women are $75 \cdot 9$ beats per minute.

Find the mean of each of the following frequency distributions: (5 − 14)

5. Class interval: 0 - 6 6 - 12 12 - 18 18 - 24 24-30
Frequency: 6 8 10 9 7

Sol:

Let a assume mean be 15

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \frac{x_i - 15}{6}$	f_i	$f_i u_i$
$0 - 6$	3	-12	-2	6	-12
$6 - 12$	9	-6	-1	2	-8
$12 - 18$	15	0	0	10	0
$18 - 24$	21	6	1	9	9
$24 - 30$	27	18	2	7	14
				$N = 40$	3

$A = 15, h = 5$

Mean $= A + h\dfrac{\Sigma f_i x_i}{N}$

$= 15 + 6 \times \dfrac{3}{40}$

$= 15 + 0 \cdot 45$

$= 15 + 0 \cdot 45$

$= 15 \cdot 45$

6. Class interval: 50 - 70 70 - 90 90 - 110 110 - 130 130 - 150 150 – 170
Frequency: 18 12 13 27 8 22

Sol:

Let the a assumed mean be 100

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \dfrac{x_i - 15}{6}$	f_i	$f_i u_i$
$50-70$	60	-40	-2	18	-36
$70-90$	80	-20	-1	12	-12
$90-110$	100	0	0	10	0
$110-130$	120	20	1	27	27
$130-150$	140	65	3	22	66
					61

$A = 100, = 20$

Mean $= A + h\dfrac{\Sigma f_i u_i}{n}$

$= 100 + 20 \times \dfrac{61}{100}$

$= 100 + 12 \cdot 2$

$= 112 \cdot 2$

7. Class interval: 0-8 8- 16 16- 24 24-32 32-40
Frequency: 6 7 10 8 9

Sol:

Let the assumed mean $(A) = 20$

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \dfrac{x_i - 15}{6}$	f_i	$f_i u_i$
$0-8$	4	-16	-2	6	-12
$8-16$	12	-8	-1	7	-17
$16-24$	20	0	0	10	0
$24-32$	28	8	8	8	8
$32-40$	36	16	2	9	18
				$N = 40$	$\Sigma f_i u_i = 7$

We have

$A = 20, N = 40$

Mean $A + hx \frac{\Sigma f_i u_i}{N}$

$= 20 + 8v \frac{7}{40}$

$= 20 + 1 \cdot 4$

$= 21 \cdot 4$

8. Class interval: 0-6 6- 12 12- 18 18-24 24-30

Frequency: 7 5 10 12 6

Sol:

Let the assume mean $(A) = 15$

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \frac{x_i - 15}{6}$	Frequency f_i	$f_i u_i$
$0-6$	3	-12	-2	-1	-14
$6-12$	9	-6	-1	5	-5
$12-18$	15	0	0	10	0
$18-24$	21	6	1	12	12
$24-30$	27	12	2	6	12
				$N = 40$	$\Sigma f_i u_i = 5$

We have $A = 90$

$A = 15, h = 6$

Mean, $A + h \times \frac{\Sigma f_i u_i}{N}$

$= 15 + 6 \times \frac{5}{40}$

$= 15 + 0 \cdot 7$

$= 15 \cdot 75$

9. Class interval: 0- 10 10- 20 20-30 30-40 40-50

Frequency: 9 12 15 10 14

Sol:

Let the assumed mean $(A) = 25$

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \frac{x_i - 15}{6}$	Frequency f_i	$f_i u_i$
$0-10$	5	-20	-2	9	-18
$10-20$	15	-10	-1	10	-12

$20-30$	25	0	0	15	0
$30-40$	35	10	1	10	10
$40-50$	45	20	2	14	28
				$N=60$	$\Sigma f_i u_i = 8$

We have $A=25, h=10$

Mean $= A + h\frac{\Sigma f_i u_i}{N}$

$= 25 + 19 \times \frac{8}{60}$

$= 25 + \frac{8}{6}$

$= 25 + \frac{4}{3}$

$= 26 \cdot 333$

10. Class interval: 0-8 8- 16 16-24 24-32 32 -40

Frequency: 5 9 10 8 8

Sol:

Let the assumed mean $(A) = 20$

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \frac{x_i - 15}{6}$	Frequency f_i	$f_i u_i$
$0-8$	4	-16	-2	5	-10
$8-6$	12	-8	-1	9	-9
$16-24$	20	0	0	10	0
$24-32$	28	8	1	8	8
$32-40$	36	16	2	8	16
				$N=40$	$\Sigma f_i u_i = 5$

We have

$A=20, h-18$

Mean $= A + h \times \frac{\Sigma f_i u_i}{N}$

$= 20 + 8 \times \frac{5}{40}$

$= 20 + 1$

$= 21$

11. Class interval: 0-8 8- 16 16- 24 24-32 32-40

Frequency: 5 6 4 3 2

Sol:

Let the assumed $(A) = 20.$

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \frac{x_i - 15}{6}$	Frequency f_i	$f_i u_i$
$0-8$	4	-16	5	-2	-10
$8-16$	12	-8	6	-1	-6
$16-24$	20	0	4	0	0
$24-32$	28	8	3	1	3
$32-40$	36	16	2	8	4
				$N = 20$	$\Sigma f_i u_i = -9$

We have

$A = 20, h - 18$

Mean $= A + h \times \frac{\Sigma f_i u_i}{N}$

$= 20 + 8 \times \frac{-9}{20}$

$= 20 - 3 \cdot 6$

$= 16 \cdot 4$

12. Class interval: 10-30 30-50 50-70 70-90 90-110 110- 130

Frequency: 5 8 12 20 3 2

Sol:

Let the assume mean $(A) = 60$

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \frac{x_i - 15}{6}$	Frequency f_i	$f_i u_i$
$10-30$	20	-40	-2	5	-10
$30-50$	40	-20	-1	8	-8
$50-70$	60	0	0	12	0
$70-90$	80	20	1	20	20
$90-110$	100	40	2	3	6
$110-130$	120	60	3	2	6
				$N = 50$	$\Sigma f_i u_i = 14$

We have

$A = 60, h = 25$

Mean $= A + h \times \frac{\Sigma f_i u_i}{N}$

$= 60 + 20 \times \frac{14}{50}$

$= 60 + 5 \cdot 6$

$= 65 \cdot 6$

13. Class interval: 25-35 35-45 45-55 55 - 65 65 – 75
Frequency: 6 10 8 12 4

Sol:

Let the assume mean $(A) = 50$

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \frac{x_i - 15}{6}$	Frequency f_i	$f_i u_i$
$25-35$	30	-20	-2	6	-12
$35-45$	40	-10	-1	10	-10
$45-55$	50	0	0	8	0
$55-65$	60	10	0	12	12
$65-75$	70	20	0	4	8
				$N = 40$	$\Sigma f_i u_i = -2$

We have

$A = 50, h = 10$

Mean $= A + h\frac{\Sigma f_i u_i}{N}$

$= 50 + 14\left(\frac{-2}{4b}\right)$

$= 50 - 0\cdot 5$

$= 49\cdot 5$

14. Classes: 25 -29 30-34 35-39 40-44 45-49 50-54 55-59
Frequency: 14 22 16 6 5 3 4

Sol:

Let the assume mean $(A) = 42$

Class interval	Mid-value x_i	$d_i = x_i - 15$	$u_i = \frac{x_i - 15}{6}$	Frequency f_i	$f_i u_i$
$25-29$	27	-15	-3	14	-42
$30-34$	32	-10	-2	22	-44
$35-39$	37	-5	-1	16	-16
$40-44$	42	0	0	0	0
$45-49$	47	5	1	5	5
$50-54$	52	10	2	3	6
$55-59$	57	15	3	4	12
				$N = 10$	$\Sigma f_i u_i = -79$

We have

$A - 42, h = 5$

Mean $= A + h \times \frac{\Sigma f_i u_i}{N}$

$= 42 + 5x \frac{-79}{70}$

$= 42 - \frac{5 \times 79}{70}$

$= 42 - \frac{79}{14}$

$= \frac{588 - 79}{14}$

$= 36 \cdot 357$

15. For the following distribution, calculate mean using all suitable methods:

Size of item:	1 -4	4-9	9- 16	16-27
Frequency:	6	12	26	20

Sol:

By direct method

Class interval	Mid-value	Frequency f_i	$f_i u_i$
$1-4$	$2 \cdot 5$	6	15
$4-9$	$6 \cdot 5$	12	18
$9-16$	$12 \cdot 5$	26	325
$16-27$	$21 \cdot 5$	20	430
		$N = 64$	$\Sigma f_i u_i = 848$

Mean $= \frac{\Sigma f_i x_i}{N} + A$

$= \frac{848}{64}$

$= 13 \cdot 25$

By assuming mean method

Let the assumed mean $(A) = 65$

Class interval	Mid-value (x_i)	$l_5 = x_i - A$ $= x_i - 65$	Frequency (f_i)	$f_i u_i$
$1-4$	$2 \cdot 5$	-4	6	-24
$4-9$	$6 \cdot 5$	0	12	0
$9-16$	$12 \cdot 5$	6	26	156
$16-27$	$21 \cdot 5$	15	20	300
			$N = 64$	$\Sigma f_i u_i = 432$

Mean $= A + \frac{\Sigma f_i u_i}{N}$

$= 6 \cdot 5 + \frac{432}{64}$

$=6\cdot 5+\frac{432}{64}$

$=13\cdot 25$

16. The weekly observations on cost of living index in a certain city for the year 2004 - 2005 are given below. Compute the weekly cost of living index.

Cost of living Index	Number of Students	Cost of living Index	Number of Students
1400 – 1500	5	1700 – 1800	9
1500 – 1600	10	1800 – 1900	6
1600 – 1700	20	1900 – 2000	2

Sol:

Let the assume mean $(A)=1650$

Class interval	Mid-value x_i	$d_i=x_i-A$ $=x_i-1650$	$u_i=\frac{x_i-15}{6}$	Frequency f_i	f_iu_i
1400-1500	1450	-200	-2	5	-10
1500-1600	1550	-100	-1	0	-10
1600-1700	1650	0	0	20	0
1700-1800	1750	100	1	9	9
1800-1900	1950	300	3	2	6
				$N=52$	$\Sigma f_iu_i=7$

We have

$A=16, h=100$

Mean $=A+h\times\frac{\Sigma f_iu_i}{N}$

$=1650+100\times\frac{175}{13}$

$=\frac{21450+175}{13}$

$=\frac{21625}{13}$

$=1663\cdot 46$

17. The following table shows the marks scored by 140 students in an examination of a certain paper:

Marks:	0- 10	10-20	20-30	30-40	40-50
Number of students:	20	24	40	36	20

Calculate the average marks by using all the three methods: direct method, assumed mean deviation and shortcut method.

Sol:

Direct method

Class interval	Mid-value	Frequency f_i	$f_i u_i$
0-10	5	20	100
10-20	15	20	350
20-30	25	40	1000
30-40	35	30	1260
40-50	45	20	900
		$N = 140$	8620

$\text{Mean} = \frac{\Sigma f_i x_i}{N}$

$= \frac{3650}{140}$

$= 25 \cdot 857$

Assume mean method : Let the assumed mean $= 25$

$\text{Mean } = A + \frac{\Sigma f_i u_i}{N}$

Class interval	Mid-value	$u; = x_i - A$	f	$f_i u_i$
0-10	5	-20	20	-400
10-20	15	-10	24	-240
20-30	25=A	0	40	0
30-40	35	10	36	360
40-50	45	20	20	400
			$N = 145$	120

$\text{Mean } = A + \frac{\Sigma f_i u_i}{N}$

$= 25 + \frac{120}{145}$

$= 25 + 0 \cdot 867$

$= 25 \cdot 857$

Step deviation method

Let the assumed mean $(A) = 25$

Class interval	Mid-value x_i	$d_i = x_i - A$ $= x_i - 25$	$u_i = \frac{x_i - 25}{10}$	Frequency f_i	$f_i u_i$
0-10	5	-20	-2	20	-40
10-20	15	-10	-1	24	-24
20-30	25	0	0	40	0
30-40	35	10	1	36	36
40-50	45	20	2	20	40

				$N=140$	$\Sigma f_i u_i = 12$

Mean $= A + \frac{\Sigma f_i u_i}{N} \times h$

$= 25 + \frac{120}{140} \times 10 = 25 + 0 \cdot 857$

$= 25 \cdot 857$

18. The mean of the following frequency distribution is 62.8 and the sum of all the frequencies is 50. Compute the missing frequencyf_1 andf_2.

Class:	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 – 120
Frequency:	5	f_1	10	f_2	7	8

Sol:

Class interval	Mid-value	Frequency f_i	$f_i u_i$
0-20	10	5	50
20-40	30	$f_1 20$	30 f_1
40-60	50	10=10	500
60-80	70	f_2	70 f_2
80-100	90	7	630
100-120	110	8	880
		$N=50$	$\Sigma f_i u_i = 30 f_1 + 70 f_2 + 3060$

Given

Sum of frequency $= 50$

$\Rightarrow 5 + f_1 + 50 \cdot f_2 + 7 + 8 = 50$

$\Rightarrow f_1 + f_2 = 50 - 5 - 10 - 7 - 8$

$\Rightarrow f_1 + f_2 = 20$

$\Rightarrow 3 f_1 + 3 f_2 = 60$ ......(1) $[multiply\ it\ by\ '3']$

And mean $= 62 \cdot 8$

$\Rightarrow \frac{\Sigma f_i x_i}{N} = 628$

$\Rightarrow \frac{30 f_1 + 70 f_2 + 2060}{50} = 62 \cdot 8$

$\Rightarrow 30 f_1 + 70 f_2 = 3140 - 2060$

$\Rightarrow 30 f_1 + 70 f_2 = 1080$

$\Rightarrow 3 f_1 + 7 f_2 = 108$ ....(2) $(Divide\ it\ by\ 10)$

Subtract equation (1) from equation (2)

$\Rightarrow 3 f_1 + 7 f_2 - 3 f_1 = 3 f_2 = 108 - 60$

$\Rightarrow 4f_2 = 48$

$\Rightarrow f_2 = 12$

Put value of f_2 in equation (1)

$\Rightarrow 3f_1 + 3 \times 12 = 60$

$\Rightarrow 3f_1 = 60 - 36 = 24$

$\Rightarrow f_1 = \frac{24}{3} = 8$

$f_1 = 8 \text{ and } f_2 = 12$

19. The following distribution shows the daily pocket allowance given to the children of a multistorey building. The average pocket allowance is Rs 18.00. Find out the missing frequency.

Class interval:	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency:	7	6	9	13	-	5	4

Sol:

Given mean = 18, let missing frequency be v

Class interval	Mid-value	Frequency f_i	$f_i u_i$
11-13	12	7	84
13-15	14	6	88
15-17	16	9	144
17-19	18	13	234
19-21	20	x	$20x$
21-23	22	5	110
23-25	14	4	56
		$N = 44 + v$	$752+20x$

Mean $= \frac{\Sigma f_i x_i}{N}$

$1- = \frac{752 + 20x}{44 + x}$

$792 + 18x = 752 + 20x$

$2x = 40$

$x = 20$

20. If the mean of the following distribution is 27, find the value of p.

Class:	0 - 10	10 - 20	20 - 30	30 - 40	40-50
Frequency:	8	p	12	13	10

Sol:

Class interval	Mid-value	Frequency	$f_i u_i$

	(x_i)	f_i	
0-10	5	8	40
10-20	15	P	152
20-30	25	12	300
30-40	35	13	455
40-50	45	16	450
		$N = 43 + P$	$\Sigma f_i x_i = 1245 + 15P$

Given

Mean $= 27$

$$\Rightarrow \frac{\Sigma f_i x_i}{N} = 27$$

$$\Rightarrow \frac{1245 + 15P}{43 + P} = 27$$

$$\Rightarrow 1245 + 15P = 1245 - 161 + 27P$$

$$\Rightarrow 27P - 15P = 1245 - 1161$$

$$\Rightarrow 12P = 84$$

$$\Rightarrow P = \frac{84}{12} = 7$$

21. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes: 50 - 52 53 - 55 56 - 58 59 - 61 62 -64

Number of boxes: 15 110 135 115 25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Sol:

Number of mangoes	Number of boxes (f_i)
50-52	15
53-55	110
56-58	135
59-61	115
62-64	25

We may observe that class internals are not continuous

There is a gap between two class intervals. So we have to add $\frac{1}{2}$ from lower class limit of each interval and class mark (xi) may be obtained by using the relation

$$x_i \frac{Upper\ class\ \lim it + lower\ class\ \lim it}{2}$$

Class size (h) of this data = 3

Now, taking 57 as assumed mean (a) we may calculate

d_i, u_i, f_i, u_i as follows.

Class interval	f_i	x_i	$d_i = 4 - 57$	$u_i = \frac{x;-57}{h}$	$f_i u_i$
$49\cdot5 - 52\cdot5$	15	51	-6	-2	-30
$52\cdot5 - 56\cdot5$	110	54	-3	-1	-110
$55\cdot5 - 58\cdot5$	135	57	0	0	0
$58\cdot5 - 61\cdot5$	115	60	3	1	115
$61\cdot5 - 64\cdot5$	25	63	6	2	50
Total	400				-25

Now, we have

$\Sigma f_i = 400$

$\Sigma f_i u_i = 25$

Mean $= 4 + + = \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$

$= 57 + \left(\frac{45}{400}\right) \times 3$

$= 57 + \frac{3}{16}$

$= 57 + 0\cdot1875$

$= 57 - 1875$

$= 57\cdot19$

Clearly mean number of mangoes kept in packing box is $57\cdot19$

22. The table below shows the daily expenditure on food of 25 households in a locality

Daily expenditure (in Rs):	100 - 150	150 - 200	200 - 250	250 - 300	300 -350
Number of households:	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

Sol:

We may calculate class mark (x_i) for each interval by using the relation

$$x_1 = \frac{Upper\ class\ \lim it + lower\ class\ \lim it}{2}$$

Class size $= 50$

Now, taking 225 as assumed mean can we may calculated d_i, u_i, f_i, u_i as follows

Daily expenditure (in Rs)	f_i	x_i	$d_i = x_i - 225$	$u_i = \frac{x_i - 225}{h}.$	$f_i u_i$
100-150	4	125	-100	-2	-8
150-200	5	175	-50	-1	-5
200-250	12	225	0	0	0

250-300	2	275	50	1	2
300-350	2	325	100	2	4
					-7

Now we may observe that

$\Sigma f_i = 25$

$\Sigma f_i x_i == -7$

Mean $(\bar{x}) = a + \left(\frac{\Sigma f_i u_i}{\Sigma f}\right) \times h$

$= 225 + \left(\frac{-7}{25}\right) \times 50$

$= 225 - 14 = 211$

So, mean daily expenditure on food is RS 211

23. To find out the concentration of SO2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

Concentration of SO_2 (in ppm)	Frequency
0.00-0.04	4
0.04-0.08	9
0.08-0.12	9
0.12-0.16	2
0.16-0.20	4
0.20-0.24	2

Find the mean concentration of SO2 in the air.

Sol:

We may find a class marks for each interval by using the relation

$x = \frac{Upper\ class\ \lim it + Lower\ class\ \lim it}{2}$

Class size of this data $= 0 \cdot 04$

Now, taking $0 \cdot 14$ assumed mean can we use may calculated d,u,fu as following

Concentration SO_2 (in ppm)	Frequency	Class interval (x_i)	$u_i = x_i == -044$	v_i	$f_i u_i$
$0 \cdot 00 - 0 \cdot 04$	4	$0 \cdot 02$	$-0 \cdot 12$	-3	-112
$0 \cdot 04 - 0 \cdot 08$	9	$0 \cdot 06$	$-0 \cdot 08$	-2	-8
$0 \cdot 08 - 0 \cdot 12$	1	$0 \cdot 10$	$-0 \cdot 04$	-1	-9
$0 \cdot 12 - 0 \cdot 12$	2	$0 \cdot 14$	0	0	0
$0 \cdot 16 - 0 \cdot 20$	4	$0 \cdot 18$	$0 \cdot 04$	1	7
$0 \cdot 20 - 0 \cdot 24$	2	$0 \cdot 22$	$0 \cdot 08$	2	4
Total	30				-31

From the table we may observe that

$\Sigma f_i = 30$

$\Sigma f_i u_i = -31$

Mean $\bar{x} = 9 + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$

$= 0 \cdot 14 + \left(\frac{+31}{30}\right)(0 \cdot 04)$

$= 0 \cdot 14 - 0 \cdot 04133$

$= 0 \cdot 099 PPm$

So, mean concentration of SO_2 in the air is $0 \cdot 099 PPm$

24. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days:	0 - 6	6 - 10	10 - 14	14 - 20	20 -28	28 - 38	38 - 40
Number of students:	11	10	7	4	4	3	1

Sol:

We may find class mark of each interval by using the relation

$$x_i = \frac{Upper\ class\ \lim it + Lower\ class\ \lim it}{2}$$

Now, taking 16 as assumed mean (a) we may

Calculate d and $f_i d_i$ as follows

Number of days	Number of students f_i	x_i	$a = x_i + f_i$	$f_i d_i$
$0-6$	11	3	-13	-143
6-10	10	8	-8	-280
10-14	7	12	-4	-28
14-20	7	16	0	0
20-28	8	24	8	32
28-36	3	33	17	51
30-40	1	39	23	23
Total	70			-145

Now we may observe that

$\Sigma f_i = 40$

$\Sigma f_i d_i = -145$

Mean $(\bar{x}) = a + \left(\frac{\Sigma f_i d_i}{\Sigma f_i}\right)$

$= 16 + \left(\frac{-145}{40}\right) = 16 - 3 \cdot 623$

$= 12 \cdot 38$

So, mean number of days is 12, 38 days for which student was absent

25. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %): 45 - 55 55 - 65 65 - 75 75 - 85 85 -95

Number of cities: 3 10 11 8 3

Sol:

We may find class marks by using the relation

$$x_i = \frac{Upper\ class\ \lim it + Lower\ class\ \lim it}{2}$$

Class size (h) for this data = 10

Now taking 70 as assumed mean (a) wrong

Calculate d_i, u_i and f_i, u_i as follows

Library rate $(in\, r_i)$	Number of cities (f_i)	x_i	$d_i = x_i 30$	$x_i = \frac{d_i}{10}$	$f_i u_i$
45-55	3	10	-20	-2	-6
55-65	10	60	-10	-1	-10
65-75	11	70	0	0	0
75-85	8	80	10	1	8
85-95	3	90	20	2	6
Total	35				-2

Now we may observe that

$\Sigma f_i = 35$

$\Sigma f_i u_i = -2$

Mean $(\bar{x}) = a + \left(\frac{\Sigma f_i u_i}{\Sigma u_i}\right) \times h$

$= 70 + \left(\frac{-2}{35}\right)10$

$= 70 \frac{-4}{7}$

$= 70 - 0 \cdot 57 = 69 \cdot 43$

So, mean literacy rate is $69 \cdot 437$.

Exercise – 7.4

1. Following are the lives in hours of 15 pieces of the components of aircraft engine. Find the median:

715, 724, 725, 710, 729, 745, 694, 699, 696, 712, 734, 728, 716, 705, 719.

Sol:

Lives in hours of is pieces are

= 715, 724, 725, 710, 729, 745, 694, 699, 696, 712, 734, 728, 719, 705, 705, 719.

Arrange the above data in a sending order

694, 696, 699, 705, 710, 712, 715, 716, 719, 721, 725, 728, 729, 734, 745

$N = 15(odd)$

Median $= \left(\frac{N+1}{2}\right)^{th}$ *term*

$= \left(\frac{15+1}{2}\right)^{th}$ *term*

$= 8^{th}$ *term*

$= 716$

2. The following is the distribution of height of students of a certain class in a certain city:

Height (in cm):	160 - 162	163 - 165	166 - 168	169 - 171	172 -174
No. of students:	15	118	142	127	18

Find the median height.

Sol:

Class interval (inclusive)	Class interval (inclusive)	Class interval Frequency	Cumulative frequency
160-162	$159.2-162\cdot5$	15	15
163-164	$162\cdot5-165\cdot5$	118	133 (F)
166-168	$165\cdot5-168\cdot5$	142(f)	275
169-171	$168\cdot5-168\cdot5$	127	402
$172-174$	$171\cdot5-174\cdot5$	18	420
		$N = 420$	

We have

$N = 420$

$\frac{N}{2} = \frac{420}{2} = 210$

The cumulative frequency just greater than $\frac{N}{2}$ is 275 then $165\cdot5-168\cdot5$ is the median class such, that

$l = 165\cdot5,\ f = 142, F = 133\ and\ h = 168\cdot5 - 105\cdot5 = 3$

Mean $= l + \dfrac{\dfrac{N}{2} - F}{f} \times h$

$= 165\cdot5 + \dfrac{10\times2}{142} = 10$

$= 165\cdot7 + \dfrac{17\times4}{142}$

$= 65\cdot5 + 1\cdot63$

$= 168\cdot13$

3. Following is the distribution of I.Q. of loo students. Find the median I.Q.

I.Q.:	55-64	65-74	75-84	85-94	95-104	105-114	115-124	125-134	135-144
No of Students:	1	2	9	22	33	22	8	2	1

Sol:

Class interval (inclusive)	Class interval (exclusive)	Frequency	Cumulative frequency
55-64	$54\cdot5 - 64\cdot5$	1	1
65-74	$64\cdot5 - 74\cdot5$	2	3
75-84	$74\cdot5 - 84\cdot5$	9	12
85-94	$84\cdot5 - 94\cdot5$	22	34(f)
95-104	$94\cdot5 - 104\cdot5$	33(f)	37
105-114	$104\cdot5 - 114\cdot5$	22	89
115-124	$114\cdot5 - 124\cdot5$	8	97
125-134	$124\cdot5 - 134\cdot5$	2	99
135-144	$134\cdot5 - 1343$	1	100
		$N = 100$	

We have

$N = 100$

$\dfrac{N}{2} = \dfrac{100}{2} = 50$

The cumulative frequency just greater than $\dfrac{N}{2}$ is 67 then the median class is

$94\cdot5 - 104\cdot5 - 94\cdot5 = 10$

Mean $= l + \dfrac{\dfrac{N}{2} - F}{f} \times h$

$= 94\cdot5 + \dfrac{50 - 34}{33} \times 10$

$$= 94\cdot 5 + \frac{16\times 10}{33} = 94\cdot 5 + 4\cdot 88 = 99\cdot 35$$

4. Calculate the median from the following data:

Rent (in Rs.): 15-25 25-35 35-45 45-55 55-65 65-75 75-85 85-95

No. of Houses: 8 10 15 25 40 20 15 7

Sol:

Class interval	Frequency	Cumulative frequency
15-25	8	8
25-35	10	18
35-45	15	33(f)
45-55	25	58(f)
55-65	40(f)	28
65-75	20	38
75-85	15	183
85-95	9	140
	$N = 110$	

We have $N = 140$

$$\frac{N}{2} = \frac{140}{2} = 3$$

The cumulative frequency just greater than Σ is 98 then media class is 55-65 such that

$l = 55, f = 40, f = 58, h = 65 - 55 = 10$

$$\text{Median} = l + \frac{\frac{N}{2} - f}{f} \times h$$

$$= 55 + \frac{\frac{70-78}{40}}{1} \times 10$$

$$= 55 + \frac{12\times 10}{40}$$

$$= 55 + 3$$

$$= 58$$

$\therefore$ Median = 58

5. Calculate the median from the following data:

Marks below: 10 20 30 40 50 60 70 80

No. of students: 15 35 60 84 96 127 198 250

Sol:

Marks below	No of students	Class interval	Frequency	Cumulative frequency

10	15	0-10	15	15
20	35	10-20	20	35
30	60	20-30	25	60
40	84	30-40	24	84
50	96	40-50	12	96(f)
60	127	50-60	37(f)	127
70	198	60-70	71	198
80	250	70-8	52	250
			$N = 250$	

We have $N = 250$

$$\frac{N}{2} = \frac{250}{2} = 12$$

The cumulative frequency just greater than $\frac{N}{2}$ is 127 then median class is 50-60 such that

$l = 50, f = 31, F = 96, h = 60 - 50 - 10$

$$\text{Median } = L + \frac{\frac{N}{2} - F}{f} \times h$$

$$= 50 + \frac{125 - 96}{31} \times 10$$

$$= 50 + \frac{29 \times 10}{31}$$

$$= \frac{155 + 290}{31}$$

$$= \frac{445}{31}$$

$$= 59 \cdot 35$$

6. An incomplete distribution is given as follows:

Variable: 0 - 10 10 - 20 20 - 30 30 - 40 40 – 50 50 - 60 60 - 70

Frequency: 10 20 ? 40 ? 25 15

You are given that the median value is 35 and the sum of all the frequencies is 170. Using the median formula, fill up the missing frequencies.

Sol:

Class interval	Frequency	Cumulative frequency
0-10	10	10
10-20	20	30
20-30	f_1	30+ $f_i(F)$

30-40	40(F)	70+ f_1
40-50	f_2	70+ $f_1 + f_2$
50-60	25	$95 + f_1 + f_2$
60-70	15	$40 + f_1 + f_2$
	$N = 170$	

Given median $= 35$

The median class $= 30 - 40$

$l = 30, h = 40 - 30 = 10, f = 40, F = 30 + f_1$

Median $l + \frac{\frac{N}{2} - F}{F} \times h$

$$35 = 30 + \frac{85 - (30 + f_1)}{40} \times 10$$

$$\Rightarrow 5 = \frac{55 - f_1}{4}$$

$$\Rightarrow F_1 = 55 - 20 = 25$$

Given

Sum of frequencies = 170

$$\Rightarrow 10 + 20 + f_1 + 40 + f_2 + 25 + 15 = 170$$

$$\Rightarrow 10 + 20 + 35 + 40 + f_2 + 25 + 15 = 170$$

$$\Rightarrow f_2 = 170 - 145$$

$$\Rightarrow f_2 = 25$$

$$\therefore f_1 = 35 \text{ and } f_2 = 25$$

7. Calculate the missing frequency from the following distribution, it being given that the median of the distribution is 24.

Age in years:	0 - 10	10 - 20	20 - 30	30 - 40	40-50
No. of persons:	5	25	?	18	7

Sol:

Class interval	Frequency	Cumulative frequency
0-10	5	5
10-20	25	30(F)
20-30	x(f)	30 + x
30-40	18	48 + x
40-50	7	55 + x
	$N = 170$	

Given

Median = 24

Then median class $= 20 - 30$

$l = 20, h = 30 - 20, F = 30$

Median $= l + \dfrac{\frac{N}{2} - f}{f} h$

$\Rightarrow 24 \cdot 20 + \dfrac{\frac{55+x}{2} - 30}{x} \times 30$

$\Rightarrow 4x = 20\left(\dfrac{55+x}{2} - 30\right) \times 10$

$\Rightarrow 4x = 275 + 5v - 300$

$\Rightarrow 4x - 5x = -25$

$\Rightarrow -x = -25$

$\Rightarrow x - 25$

$\therefore$ Missing frequency $= 25$

8. Find the missing frequencies and the median for the following distribution if the mean is 1.46.

No. of accidents:	0	1	2	3	4	5	Total
Frequency (No. of days):	46	?	?	25	10	5	200

Sol:

No. of accidents (x)	No. of days (f)	fx
0	46	0
1	x	x
2	y	$2y$
3	2s	75
4	10	40
5	5	25
	$N = 200$	$\Sigma f_i x_i = x + 2y + 140$

Given, $N = 200$

$\Rightarrow 46 + x + y + 25 + 10 + 5 + 5 = 2n$

$\Rightarrow x + y = 200 - 46 - 25 - 10 - 0$

$\Rightarrow x + y = 114 \qquad ....(i)$

And mean $= 1 \cdot 46$

$\Rightarrow \frac{\Sigma fx}{N} = 1.46$

$\Rightarrow \frac{x+2y+140}{200} = 1 \cdot 46$

$\Rightarrow x+2y+140 = 292$

$\Rightarrow x+2y = 292+40$

$\Rightarrow x+2y = 152 \qquad ....(2)$

Subtract equation (1) from equation (2)

$\Rightarrow x+2y-x-y = 152--114$

$\Rightarrow y = 38$

Put the value of y in (1), we have $x = 114-38 = 76$

No. of accidents	No. of days	Cumulative frequency
0	46	46
1	76	122
2	38	160
3	25	185
4	10	195
5	5	200
	$N = 200$	

We have

$N = 200$

$\frac{N}{2} = \frac{200}{2} = 100$

The cumulative frequency just more than $\frac{N}{2}$ is 122 then the median is 1.

9. An incomplete distribution is given below:

Variable:	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency:	12	30	-	65	-	25	18

You are given that the median value is 46 and the total number of items is 230.

(i) Using the median formula fill up missing frequencies.

(ii) Calculate the AM of the completed distribution.

Sol:

(i)

Class interval	Frequency	Cumulative frequency
10-20	12	12

20-30	30	42
30-40	x	$42+x(F)$
40-50	65(f)	$1107+x$
50-60	y	$107+x+y$
60-70	25	$132x+x+y$
70-80	18	$150+x+y$
	$N=200$	

Given median $=46$

Then, median as $=40-50$

$\therefore l=40, h=50-40=10, f=65, F=42+x$

$$\therefore \text{median } = l+\frac{\frac{N}{2}-F}{f}\times h$$

$$\Rightarrow 46=40+\frac{115-(42+x)}{65}\times 10$$

$$\Rightarrow \frac{6\times 65}{10}=73-x$$

$\Rightarrow 39=73-x$

$\Rightarrow x=73-39$

$\Rightarrow x=34$

Given $N=230$

$=12+30+34+65+y+25+18=230$

$\Rightarrow 184+y=230$

$\Rightarrow y=230-184=46$

(ii)

Class interval	Mid value	Frequency	fx
10-20	15	12	180
20-30	25	30	750
30-40	*35*	*34*	1190
40-50	45	65	2925
50-60	*55*	*46*	2530
60-70	65	25	1625
70-80	75	18	1650
		$N=270$	$\Sigma fx=10550$

Mean $=\frac{\Sigma fx}{N}$

$=\frac{10550}{250}$

$\therefore 4=87$

10. The following table gives the frequency distribution of married women by age at marriage:

Age (in years)	Frequency	Age (in years)	Frequency
15-19	53	40-44	9
20-24	140	45-49	5
25-29	98	50-54	3
30-34	32	55-59	3
35-39	12	60 and above	2

Calculate the median and interpret the results

Sol:

Class interval (exclusive)	Class interval (inclusive)	Frequency	Cumulative frequency
15-19	$14\cdot5-19\cdot5$	53	53(F)
20-24	$19\cdot5-24\cdot5$	140(f)	193
25-29	$24\cdot5-29\cdot5$	98	291
30-34	$29\cdot5-34\cdot5$	32	393
35-39	$34\cdot5-39\cdot5$	12	335
40-44	$39\cdot5-44\cdot5$	9	344
45-49	$44\cdot5-49\cdot5$	5	349
50-54	$49\cdot5-54\cdot5$	3	352
54-59	$554\cdot5-59\cdot5$	3	355
60 and above	$59\cdot5$ and above	2	357
		$N=357$	

$N=357$

$$\frac{N}{2}=\frac{35}{2}=178\cdot5$$

The cumulative frequency just greater than $\frac{N}{2}$ is 193, then the median class is 19.5-24.5 such that

$l=19\cdot5,\ f=140.f=53, h=24\cdot5-19\cdot5=5$

$$\text{Median }=l+\frac{\frac{N}{2}-F}{f}\times h$$

$$\text{Median }=19\cdot5+\frac{178\cdot5-53}{140}\times5=23\cdot98$$

Nearly half the a women were married between ages 15 and 25.

11. If the median of the following frequency distribution is 28.5 find the missing frequencies:

Class interval:	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency:	5	f_1	20	15	f_2	5	60

Sol:

Class interval	Frequency	Cumulative frequency
0-10	5	5
10-20	f_1	$5+f_1(F)$
20-30	$20(F)$	$25+F_1$
30-40	15	$40+f_1$
40-50	f_2	$40+f_1+f_2$
50-60	5	$45+f_1+f_2$
	$N=60$	

Given

Median $=28\cdot5$

Then, median class $=20+30$

$l=20, f=20, F=5+fx, h=30-20=10$

Median $=l+\dfrac{\frac{N}{2}-F}{f}\times h$

$\Rightarrow 28\cdot5-201+\dfrac{30-(5+f_1)}{20}\times10$

$\Rightarrow 28\cdot5-20=\dfrac{30-5-f_1}{20}\times10$

$\Rightarrow 8\cdot5=\dfrac{25-f_1}{2}$

$\Rightarrow f_1=25-17$

$\Rightarrow f_1=8$

Given sum of frequency = 60

$\Rightarrow 5+f_1+20+15+f_2+5=60$

$\Rightarrow 5+8+20+15+f_2+5=60$

$\Rightarrow f_2=7$

$f_1=8; f_2=7$

12. The median of the following data is 525. Find the missing frequency, if it is given that there are 100 observations in the data:

Class interval	Frequency	Class interval	Frequency
0-100	2	500-600	20
100-200	5	600-700	f_2
200-300	f_1	700-800	*9*
300-400	*12*	800-900	*7*

400-500	*17*	900-1000	*4*

Sol:

Class interval	Frequency	Cumulative frequency
0-100	2	2
100-200	5	7
200-300	f_1	$7+f_1$
300-400	12	$19+f_1$
400-500	17	$36+f_1(F)$
500-600	20(f)	$56+f_1$
600-700	f_2	$56+f_1+f_2$
700-800	9	$65+f_1+f_2$
800-900	7	$75+f_1+f_2$
900-1000	4	$76+f_1+f_2$
	$N=100$	

Given media $=525$

Then media class $=500-600$

$l=500,\ f=20,\ f=36+f_1, h=600-500=100$

Median $=l+\dfrac{\frac{N}{2}-F}{f}\times h$

$\Rightarrow 525=500+\dfrac{50-(36+f_1)}{20}\times 100$

$\Rightarrow 525-500=\dfrac{50-36-f_1}{20}\times 100$

$\Rightarrow 25=(14-f_1)5$

$\Rightarrow 5f_1=45 \Rightarrow f_1=9$

Given sum of frequencies $=100$

$\Rightarrow 2+5+f_1+12+17+20+f_2+9+7+4=100$

$\Rightarrow 2+5+9+12+17+20+f_2+9+17+4=100$

$\Rightarrow 86+f_2=100 \Rightarrow f_2=15$

$\therefore f_1=9;\ f_2=15$

13. If the median of the following data is 32.5, find the missing frequencies.

Class interval:	0- 10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequency:	f_1	5	9	12	f_2	3	2	40

Sol:

Class interval	Frequency	Cumulative frequency
0-10	f_1	f_1
10-20	5	$5+f_1$
20-30	9	$14+f_1(f)$
30-40	12(f)	$26+f_1$
40-50	f_2	$26+f_1+f_2$
50-60	3	$29+f_1+f_2$
60-70	2	$31+f_1+f_2$
	$N=40$	

Given

Median $=32\cdot5$

The median class $=90-40$

$l=30 \therefore 40-30=10,\ f=12, F=14+f_1$

Median $=1+\dfrac{\frac{N}{2}-F}{f}\times h$

$\Rightarrow 32\cdot5=30+\dfrac{20-(14+f_1)}{12}\times10$

$\Rightarrow 2\cdot5=\dfrac{6-f_1}{6}\times5 \qquad \Rightarrow 15=(6-8_1)5$

$\Rightarrow 3=6-f_1 \qquad \Rightarrow \dfrac{15}{5}=6-f_1$

$\Rightarrow f_1=3$

Given sum of frequencies = 40

$\Rightarrow 3+5+9+12+f_2+3+2=40$

$\Rightarrow 34+f_2=40$

$\Rightarrow f_2=6$

$\therefore f_1=3; f_2=6$

14. A survey regarding the height (in cm) of 51 girls of X of a school was conducted and the following data was obtained:

(i) Marks	No. of students	(ii) Marks	No. of students
Less than 10	0	More than 150	0
Less than 30	10	More than 140	12
Less than 50	25	More than 130	27
Less than 70	43	More than 120	60
Less than 90	65	More than 110	105

Less than 110	87	More than 100	124
Less than 130	96	Morethan90	141
Less than 150	100	More than 80	150

Sol:

(i)

Marks	No. of students	Class internal	Frequency	Cumulative frequency
Less than 10	0	0-10	0	0
Less than 30	10	10-30	10	10
Less than 50	25	30-50	15	25
Less than 70	43	50-70	18	43(F)
Less than 90	65	70-090	22(f)	65
Less than 110	87	90-110	22	87
Less than 130	96	110-130	9	96
Less than 150	100	130-150	8	100
			$N=100$	

We have $N=100$

$\frac{N}{2}=\frac{100}{2}=60$

The commutative frequencies just greater than $\frac{N}{2}$ is 65 then median class is 70-90 such that $l=90, f=22, f=43, h=90-70=20$

Median $=l+\frac{\frac{N}{2}-F}{f}\times h$

$=70+\frac{50-43}{22}\times 20$

$=70+\frac{7\times 20}{22}$

$=70+\frac{50-43}{22}\times 20$

$=70+\frac{7\times 20}{22}$

$=70+6\cdot 36$

$=76\cdot 36$

(ii)

Marks	No. of students	Class internal	Frequency	Cumulative frequency
Less than 80	150	80-90	9	9
Less than 90	141	90-100	17	26
Less than 100	124	100-110	19	45(F)
Less than 110	105	110-120	45(f)	90
Less than 120	60	120-130	33	123

Less than 130	27	130-140	45	138
Less than 140	12	150-160	0	150
Less than 150	0	150-160	0	150
			$N=150$	

We have $N=150$

$\frac{N}{2}=\frac{150}{2}=7$

The commutative frequencies just greater than $\frac{N}{2}$ is 90 then median class is 110-120 such that $l=110, f=45, F=45, h=120-110-10$

Median $=l+\frac{\frac{N}{2}-F}{f}\times h$

$=110+\frac{75-45}{45}\times 10$

$=110+\frac{30\times 10}{45}$

$=110+6+67$

$=116\cdot 67.$

15. A survey regarding the height (in cm) of 51 girls of class X of a school was conducted and the following data was obtained:

Height in cm	Number of Girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than l65	51

Find the median height.

Sol:

To calculate the median height, we need to find the class intervals and their corresponding frequencies

The given distribution being of thee less than type 140, 145, 150,…..,165 give the upper limits of the corresponding class intervals. So, the classes should be below 140, 145, 150,….., 160, 165 observe that from the given distribution, we find that there are 4-girls with height less than 140 is 4. Now there are 4 girls with heights less than 140. Therefore, the number of girls with height in the interval 140, 145 is 11- 4=7, similarly. The frequencies of 145 150 is 29-11=18, for 150-155 it is 40-29=11, and so on so our frequencies distribution becomes.

Class interval	Frequency	Cumulative frequency
below 140	4	4
140-145	7	11
145-150	18	29
150-155	11	40
155-160	6	46
160-165	5	51

Now $n = 51$, So, $\frac{n}{2} = \frac{51}{2} = 25 \cdot 5.$ This observation lies in the class $145 - 150$.

Then,
The lower limit = 145
CFC The cumulative frequency of the class
Preceding $145 - 150 = 11$
F (The frequency of the median as 145+800=18,
h(class limit) = 5

Median $= 145 + \left(\frac{25 \cdot 5 - 11}{18}\right) \times 5$

$= 145 + \frac{725}{18}$

$= 145 \cdot 03$

So, the median height of the girls is $149 \cdot 03 cm$. This means what the height of be about 50% of the girls in less than this height, and 50% are taller than this height,

16. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are only given to persons having age 18 years onwards but less than 60 years.

Age in years	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

Sol:

Here class width is not same. There is no need to adjust the frequencies according to class intervals. Now given frequencies table is of less than type represented with upper class limits. As policies were given only to persons having age 18 years onwards but less than 60 years we can definite class intervals with their respective cumulative frequencies as below

Age (in years)	No of policy planers	Cumulative frequency (cr)
18-20	2	2
20-25	6-2=4	5
25-30	24-6=18	24
30-35	45-24=21	45
35-40	78-45=33	78
40-45	89-78=11	89
45-50	92-89=3	92
50-55	98-92=6	92
55-60	100-98=2	100

Total (n)

Now from the table we may observe that n=100 cumulative frequencies (F) just greater than $\frac{n}{2}\left(i.e., \frac{100}{2}=50\right)$ is 78 belonging to interval 35-40

So median class = 35-40

Lower limit (1) o median class = 35

Class size (h) = 5

Frequencies (f) of median class = 33

Cumulative frequency (f) off class preceding median class = 45

Median $=\frac{\left(\frac{n}{2}-cf\right)}{f}\times h$

$=35+\left(\frac{50-15}{33}\right)x$

$=35+\frac{2}{33}$

$=35\cdot 76$

So, median age is $35\cdot 76$ years

17. The lengths of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table:

Length (in mm):	118-126	127-135	136-144	145-153	154-162	163-171	172-180
No. of leaves:	3	5	9	12	5	4	2

Find the mean length of life.

Sol:

The given data is not having continuous class intervals is 1. So, we have to add and subtract $\frac{1}{2}=0\cdot 5$ o upper class limits and lower class limits

Now continuous class intervals with respective cumulative frequencies can be presented as below

Length (in mm)	Number of leaves f_i	Cumulative frequencies
$117\cdot5-126\cdot5$	3	3
$126\cdot5-135\cdot5$	5	$8=3+5$
$13\cdot5\cdot5-144\cdot5$	9	$17=8+9$
$14\cdot5-53\cdot5$	12	$29=17+12$
$153\cdot5-162\cdot5$	5	$37=29+5$
$162\cdot5-171\cdot5$	4	$34+4=38$
$171\cdot5-180\cdot5$	2	$38+2=40$

From the table we may observe that cumulative frequencies just greater than

$\frac{n}{2}\left(i.e., \frac{40}{2}=20\right)$ is 29 belonging class interval $144\cdot5-153\cdot5$

Median class $=144\cdot5-153\cdot5$

Lower limit (L) of median class $=144\cdot5$

Class size $(h)=9$

Frequencies (f) of median class = 12

Cumulative frequencies (f) of class preceding median class = 17

$$\text{Median } = l+\frac{\left(\frac{n}{2}-cf\right)}{f}\times h$$

$$=144\cdot5+\left(\frac{20-27}{112}\right)\times h$$

$$=144\cdot5+\frac{9}{4}$$

$$=146\cdot75$$

So, median length is $146\cdot75\ mm$

18. The following table gives the distribution of the life time of 400 neon lamps:

Lite time: (in hours) Number of lamps1500-2000 14

2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

Find the median life.

Sol:

We can find cumulative frequencies with their respective class intervals as below

Life time	Number of lams (f_i)	Cumulative frequencies
1500-2000	14	14
2000-2500	56	14+56=70
2500-3000	60	70+50=130
3000-3500	86	130+86=216
3500-4000	74	216+74=290
4000-4500	62	290+62=352
4500-5000	48	352+48=400
Total	420	

Now we may observe that cumulative frequencies just greater $430x\frac{n}{2}\left(i.e., \frac{400}{2}=200\right)$ is

216 belonging to class interval $3000-3500$

Median class $3000-3500$

Lower limit (1) of median class = 3000

Frequencies (f) of median class = 86

Cumulative frequencies (cf) of class preceding

Median class $=130$

Class size = 500

$$\text{Median } = l+\left(\frac{\frac{N}{2}-c.f}{f}\right)\times h = 3000+\left(\frac{200-130}{86}\right)\times 500$$

$$=3000+\frac{70\times 500}{86}=3406\cdot 98 \ hours$$

So, median life time is $3406\cdot 98 \ hours$

19. The distribution below gives the weight of 30 students in a class. Find the median weight of students:

Weight (in kg):	40-45	45-50	50-55	55-60	60-65	65-70	70-75
No. of students:	2	3	8	6	6	3	2

Sol:

We may find cumulative frequencies with their respective class internals as below

Weight in (kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Number of students (f)	2	3	8	6	6	3	2
cf	2	5	13	19	25	28	30

Cumulative frequencies just great class interval er than $\frac{n}{2}\left(i.e., \frac{30}{2}=15\right)$ is 19, belonging to

class interval 55-60

Median class $= 55 - 60$
Lower limit (1) of media class = 55
Frequency of median class = 6
Cumulative frequencies y(f) of median class = 13
Class $55(h) = 5$

Median $= 1 + \left(\frac{n}{2} - \frac{cf}{f}\right) \times h$

$= 55 + \left(\frac{15-13}{6}\right) \times 5$

$= 55 + \frac{10}{6}$

$= 56 \cdot 666$

So, median weight is $56 \cdot 67kg$

Exercise – 7.5

1. Find the mode of the following data:
(i) 3,5,7,4,5,3,5,6,8,9,5,3,5,3,6,9,7,4
(ii) 3, 3, 7, 4, 5, 3, 5, 6, 8, 9, 5, 3, 5, 3, 6, 9, 7,4
(iii) 15, 8, 26, 25, 24, 15, 18, 20, 24, 15, 19, 15
Sol:
(i)

Value (x)	3	4	5	6	7	8	9
Frequency (f)	4	2	5	2	2	1	2

Mode = 5 because it occurs maximum number of times
(ii)

Value (x)	3	4	5	6	7	8	9
Frequency (f)	5	2	4	2	2	1	2

Mode = 3 because it occurs maximum number of times
(iii)

Value (x)	3	4	5	6	7	8	9
Frequency (f)	1	4	1	1	2	1	1

Mode = 15 because it occurs maximum number of times

2. The shirt sizes worn by a group of 200 persons, who bought the shirt from a store, are as follows:

Shirt size:	37	38	39	40	41	42	43	44
Number of persons:	15	25	39	41	36	17	15	12

Find the model shirt size worn by the group.

Sol:

Shirt size	37	38	39	40	41	42	43	44
Frequency (f)	15	25	39	41	36	17	15	12

Model shirt size = 40 because it occurs maximum number of times

3. Find the mode of the following distribution.

(i) Class-interval: 0-10 10-20 20-30 30-40 40-50 50-60 60-70 70-80

Frequency: 5 8 7 12 28 20 10 10

(ii) Class-interval: 10-15 15-20 20-25 25-30 30-35 35-40

Frequency: 30 45 75 35 25 15

(iii) Class-interval: 25-30 30-35 35-40 40-45 45-50 50-60

Frequency: 25 34 50 42 38 14

Sol:

(i)

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of persons	5	8	7	12	18	20	10	10

Here the maximum frequency is 28 then the corresponding class 40 – 50 is the model class

L = 40, h = 50 – 40 = 10, f = 28, f_1 = 12, f_2 = 20

Mode = $L + \frac{f-f_1}{2f-f_1-f_2} \times h$

= 40 + $\frac{28-12}{2\times28-12} \times 10$

= 40 + $\frac{16\times10}{24}$

= 40 + 160 = 46.67

(ii)

Class interval	10-15	15-20	20-25	25-30	30-35	35-40
No. of persons	30	45	75	35	25	15

Here the maximum frequency is 75 then the corresponding class 20 – 25 is the model class

L = 25, h = 25 – 20 = 5, f = 75, f_1 = 45, f_2 = 35

Mode = $L + \frac{f-f_1}{2f-f_1-f_2} \times h$

= $20 + \frac{75-45}{2\times75-45-35} \times 5$

= $20 + \frac{30\times5}{70}$

= $20 + 2.14$

= 22.14

(iii)

Class interval	25-30	30-35	35-40	40-45	45-50	50-55
No. of persons	25	34	50	42	38	14

Here the maximum frequency is 28 then the corresponding class 40 – 50 is the model class

L = 35, h = 40 – 35 = 5, f = 50, f_1 = 34, f_2 = 42

Mode = $L + \frac{f-f_1}{2f-f_1-f_2} \times h$

$= 35 + \frac{50-34}{2(50)-34-42} \times 5$

= 35 + $\frac{16\times5}{24}$

= 35 + 3.33

= 38.33

4. Compare the modal ages of two groups of students appearing for an entrance test:

Age (in years):	16-18	18-20	20-22	22-24	24-26
Group A:	50	78	46	28	23
Group B:	54	89	40	25	17

Sol:

Age in years	16 – 18	18 – 20	20 – 22	22 – 24	24 – 26
Group A	50	78	46	28	23
Group B	54	89	40	25	17

For Group A

Here the maximum frequency is 78, then the corresponding class 18 – 20 is model class

L = 18, h = 20 – 18 = 2, f = 78, f_1 = 50, f_2 = 46

Mode = $L + \frac{f-f_1}{2f-f_1-f_2} \times h$

$= 18 + \frac{78-54}{156-50-46} \times 2$

$= 18 + \frac{56}{60} = 18 + 0.93$

= 18.93 years

For group B

Here the maximum frequency is 89, then the corresponding class 18 – 20 is model class

L = 18, h = 18 +20 = 2, f = 89, f_1 = 54, f_2 = 40

Mode = $L + \frac{f-f_1}{2f-f_1-f_2} \times h$

$= 18 + \frac{78-54}{156-54-40} \times 5$

$= 18 + \frac{70}{84}$

= 18 + 0.83

= 18.83
Hence the mode of age for the group A is higher than group B

5. The marks in science of 80 students of class X are given below: Find the mode of the marks obtained by the students in science.
Marks: 0-10 10-20 20-30 30-40 40-50 50-60 60-70 70-80 80-90 90-100
Frequency: 3 5 16 12 13 20 5 4 1 1
Sol:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	5	16	12	13	20	5	4	1	1

Here the maximum frequency is 20, then the corresponding class 50 – 60 is model class
L = 50, h = 60 – 50 = 10, f = 20, f_1 = 13, f_2 = 5

Mode = $L + \frac{f-f_1}{2f-f_1-f_2} \times h$

$= 50 + \frac{20-13}{40-13-5} \times 10$

$= 50 + \frac{7\times10}{22}$

= 50 +3.18

= 53.18

6. The following is the distribution of height of students of a certain class in a certain city:
Height (in cm): 160-162 163-165 166-168 169-171 172-174
No. of students: 15 118 142 127 18
Find the average height of maximum number of students.
Sol:

Height(exclusive)	160-162	163-165	166-168	169-171	172-174
Height(inclusive)	159.5-162.5	162.5-165.5	165.5-168.5	168.5-171-.5	171.5-174.5
No. of students	15	118	142	127	18

Here the maximum frequency is 142, then the corresponding class 165.5 – 168.5 is modal class
L = 165.5, h = 168.5 – 165.5 = 3, f = 142, f_1 = 118, f_2 = 127

Mode = $L + \frac{f-f_1}{2f-f_1-f_2} \times h$

$= 165.5 + \frac{142-118}{2\times142-118-127} \times 3$

$= 165.5 + \frac{24\times3}{39}$

$= 165.5 + 1.85$

$= 167.35cm$

7. The following table shows the ages of the patients admitted in a hospital during a year:
Age (in years): 5-15 15-25 25-35 35-45 45-55 55-65

No. of students: 6 11 21 23 14 5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Sol:

We may observe compute class marks (xi) as per the relation

$x_i = \frac{upper\ class\ limit + lower\ class\ limit}{2}$

$Now\ taking\ 30\ as\ assumed\ mean\ (a) we\ may\ calculated\ and\ f_1 d_1 as\ follows$

Age (in yrs)	No. of patients (f_i)	Class Mark x_i	$d_i = x_i - 30$	$f_i d_i$
5-15	6	10	-20	-120
15-25	11	20	-10	-110
25-35	21	30	0	0
35-45	23	40	10	230
45-55	14	50	20	280
55-65	5	60	60	150
Total	80			430

From the table we may observe that $\sum f_i = 80$

$\sum f_i\, d_i = 430$

Mean = $a + \frac{\sum f_i d_i}{\sum f_i}$

= $30 + \left(\frac{30}{80}\right)$

= $30 + 5.375$

= 35.38

Clearly mean of this data is 35.38. It represents that on an average the age of patient admitted to hospital was 35.58 years. As we may observe that maximum class frequency 23 belonging to class interval 35 – 45

So, modal class = 35 – 45

Lower limit (L) of modal class = 35

Frequency (f_1) of modal class = 23

Class size (h) = 10

Frequency (f_0) of class preceding the modal = 21

Frequency (f_2) of class succeeding the modal = 14

Now mode = L + $\left(\frac{f - f_0}{2f - f_0 - f_2}\right) h$

= $35 + \left[\frac{23-21}{2(23)-21-14}\right] \times 6$

= 35 + $\frac{20}{11}$

= 35.81

= 36.8

8. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

Lifetimes (in hours):	0-20	20-40	40-60	60-80	80-100	100-120
No. of components:	10	35	52	61	38	29

Determine the modal lifetimes of the components.

Sol:

From data as given above we may observe that maximum class frequency 61 belonging to class interval 60 – 80.

So, modal class 60 – 80

L = 60, h = 20, f_0 = 52, f_1 = 61, f_2 = 38

Mode = $L + \left(\frac{f_1 - f_0}{2f - f_0 - f_2}\right) h$

$= 60 + \left(\frac{61-52}{2(61)-52-37}\right) 20$

$= 60 + \frac{9\times 20}{32} = 60 + \frac{90}{16} = 60 + 5.625$

= 65.625

9. The following data gives the distribution of total monthly houshold expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure:

Expenditure (in Rs.)	Frequency	Expenditure (in Rs.)	Frequency
1000-1500	24	3000-3500	30
1500-2000	40	3500-4000	22
2000-2500	33	4000-4500	16
2500-3000	28	4500-5000	7

Sol:

We may observe that the given data the maximum class frequency is 40 belonging to 1500 – 2000 interval. So modal class = 1500 – 2000

L.L (L) = 1500, f. of M.C (f_1) = 40

Frequency of class preceeding modal class $f_0 = 24$

Frequency of class succeeding modal class $f_2 = 33$

Class size (h) = 50

Mode = L +$\left(\frac{f_1 - f_0}{2f - f_0 - f_2}\right) h$

= 1500 + $\left[\frac{40-24}{2(40)-24-33}\right] \times 500$

= 1500 + $\left[\frac{16}{80-67} \times 500\right]$

= 1500 + $\frac{8000}{23}$

$= 1500 + 347.826$

1847.826 = 1847.83

So modal class monthly expenditure was Rs. 1847.83

Now we may find class mark as

Class mark = $\frac{upper\ class\ limit + lower\ class\ limit}{2}$

Class size (h) of given data = 500

Now taking 2750 as assumed mean(a) we may calculate d, 4 and f, 4 as follows.

Expenditure In Rs.	No. of families f_i	x_i	$d_i = x_i - 2\pi0$	4i	$f_i 4i$
1000-1500	24	1250	-1500	-3	-72
1500-2000	40	1750	-1000	-2	-80
2000-2500	33	2250	-500	-1	-33
2500-3000	28	2750	0	0	0
3000-3500	30	3250	500	1	30
3500-4000	22	3750	1000	2	44
4000-4500	16	4250	1500	3	48
4500-5000	7	4750	2000	4	28
Total	200				-35

Now from table we may observe that

$\sum f_i 4_i = 200$

$\sum f_i 4_i = -35$

$(\bar{x})\ mean = a + \left(\frac{\sum f_i 4_i}{\sum f_i}\right) \times h$

$(\bar{x}) = 2750 + \left(\frac{-35}{200}\right) \times 500$

= 2750 – 87.5

= 2662.5

So mean, monthly expenditure was Rs. 2662.50 ps.

We may observe them the given data the maximum class frequency is 10 belonging to class interval 30 – 35

So modal class 30 – 35

Class size (h) = 5

Lower limit (L) of modal class = 30

Frequency (f_1) of modal class 10

Frequency (f_0) of class preceeding modal class = 9

Frequency (f_0) of class succeeding modal class = 3

Mode = L +$\left(\frac{f_1 - f_0}{2f - f_0 - f_2}\right) h$

= 30 + $\left(\frac{10-9}{30-9-3}\right) 5$

= 30 + $\frac{5}{8}$

= 30.625

Mode = 30.6

It represents that most of states 1 UT have a teacher – student ratio as 30.6
Now we may find class marks by using the relation
Class mark = $\frac{upper\ class\ limit\ lower\ class\ limit}{2}$
Now taking 325 as assumed mean (a) we may calculated $d_i 4_i$ and $f_i 4_i$ as following.

10. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret, the two measures:

Number of students per teacher	Number of states / U.T.	Number of students per teacher	Number of states / U.T.
15-20	3	35-40	3
20-25	8	40-45	0
25-30	9	45-50	0
30-35	10	50-55	2

Sol:

No. of Students / teacher	No. of states / U.T. (f_i)	x_i	$d_i = x_i - 328$	4i	$f_i 4i$
15 – 20	3	17.5	-15	-3	-9
20 – 25	8	22.5	-10	-2	-16
25 – 30	9	27.5	-5	-1	-9
30 – 35	10	32.5	0	0	0
35 – 40	3	37.5	5	1	3
40 – 45	0	42.5	10	2	0
45 – 50	0	47.5	15	3	0
50 – 55	2	52.5	25	4	8
Total	35				-23

Now mean ($\bar{x}$) = a + $\left(\frac{\Sigma f_i 4_i}{\Sigma f_i}\right) h$
= $32.5 + \left(\frac{-23}{35} \times 5\right)$
= $32.5 - \frac{23}{7}$
= 32.5 – 3.28
= 29.22
So mean of data is 29.2
It represents that on an average teacher.
Student ratio was 29.2

11. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Runs scored	No. of batsman	Runs scored	No. of batsman
3000-4000	4	7000-8000	6
4000-5000	18	8000-9000	3
5000-6000	9	9000-10000	1
6000-7000	7	10000-11000	1

Find the mode of the data.

Sol:

From the given data we may observe that maximum class frequently is 18 belonging to class interval 4000 – 5000

So modal class 4000 – 5000

Lower limit (1) of model class = 4000

Frequently $\left(f_1\right)$ of class preceding modal class= 4

Frequently $\left(f_2\right)$ of class succeeding modal class = 9

Frequently of modal case $\left(f_i\right) = 18$

Class size = 1000

Now mode $= 1 + \left(\frac{1 - fn}{2 - f_0 - f_2}\right) \times h$

$= 4000\left(\frac{18 - 4}{2(18) - 4 - 9}\right) \times 1000$

$4000 + \frac{14000}{23} = 4608 \cdot 695$

So, mode of given data is $4608 \cdot 7$ *Runs*

12. A student noted the number of cars passing through a spot on a road for loo periods each of 3 minutes and summarized it in the table given below. Find the mode of the data:

Sol:

From the given data we may observe that maximum class internal frequency is 200 belonging to modal class $40 - 50$

$l = 40,\ f_1 = 20,\ f_0 = 12,\ f_2 = 11, h = 10$

Mode = $1 + \left(\frac{f - f_0}{2f - f_0 - f_2}\right) h$

$= 40 + \left[\frac{20 - 12}{40 - 12 - 11}\right] \times 10$

$$= 40 + \frac{180}{17} = 40 + 4 \cdot 7 = 44 \cdot 7$$

13. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption - 65-85 85-105 105-125 125-145 145-165 165-185 185-205
(in units)
No. of consumers: 4 5 13 20 14 8 4

Sol:

Class interval	Mid value x	Frequency f	fx	Cumulative frequency
65-75	75	4	300	4
85-105	95	5	475	9
105-125	115	13	1495	22
125-145	135	20	2700	42
145-165	155	17	2170	56
165-185	175	8	1400	64
185-205	195	4	78	68
Total		$N = 68$	$\Sigma fx = 9320$	

Mean $= \frac{\Sigma fx}{N} = \frac{9320}{68} = 137 \cdot 08$

We have $N = 68$

$\frac{N}{2} = \frac{68}{2} = 34$

Thee cumulative frequency just $> \frac{N}{2}$ is 42 then the median mass $125 - 145$ such that

$l = 125,\ f = 20\ F = 22, h = 20$

Median $= l + \frac{\frac{N}{2} - f}{f} \times h = 125 + \frac{34 - 22}{20} \times 20 = 137$

Here the maximum frequently is 20, then the corresponding class 125-1145 is the modal class $l = 125, h = 20\, f = 20,\ f_1 = 13,\ f_2 = 14$

Mode $= l + \frac{f - f_1}{2f - f_1 f_2} \times h = 125 + \frac{20 - 13}{40 - 13 - 14} \times 20$

$= 125 + \frac{7 \times 20}{13} = 135 \cdot 77$

14. 100 surnames were randomly picked up from a local telephone directly and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

Number of letters: l-4 4-7 7-10 10-13 13-16 16-19
Number surnames: 6 30 40 16 4 4
Determine the median number of letters in the surnames. Find the mean number of letters in the surnames. Also, find the modal size of the surnames.

Sol:

Class interval	Mid value x	Frequency f	fx	Cumulative frequency
1-4	$2\cdot5$	6	15	6
4-7	$5\cdot5$	30	165	36
7-10	$8\cdot5$	40	340	76
10-13	$11\cdot5$	16	185	92
13-16	$14\cdot5$	4	58	96
16-19	$17\cdot5$	4	70	100
		$N=100$	$\Sigma fx=832$	

Mean $=\frac{\Sigma fx}{N}=\frac{832}{100}=8-32$

$N=100 \Rightarrow N=50$

The cumulative frequency $>\frac{N}{2}$ is 76, median class $7-10$

$l=7, h=3, f=40, f=36.$

Median $=1+\frac{\frac{N}{2}-F}{f}\times h=7+\frac{50-36}{40}\times 3$

$=7+\frac{14\times 3}{40}=8\cdot 05$

Here the maximum frequency is 40, then the corresponding class 7-10 is thee modal class

$l=7, h=10-7=3, f=40, f_1=30, f_2=36$

Mode $=1+\frac{f-f_1}{28-f_1-f_2}\times h=7+\frac{40-30}{2\times 40-30+6}\times 3$

$=7+\frac{10\times 3}{34}=7\cdot 88$

15. Find the mean, median and mode of the following data:
Classes: 0-20 20-40 40-60 60-80 80-100 100-120 120-140
Frequency: 6 8 10 12 6 5 3

Sol:

Class interval	Mid value x	Frequency f	fx	Cumulative frequency
0-20	10	6	60	6
20-30	30	8	240	17

40-60	50	10	500	24
60-080	70	12	840	36
80-100	90	6	540	42
100-120	110	5	550	47
120-140	130	3	390	50
		$N = 60$	$\Sigma fx = 3120$	

Mean $= \frac{\Sigma fx}{N} = \frac{320}{50} = 62 \cdot 4$

We have $N = 60$

Then, $\frac{1}{2} = \frac{50}{2} = 25$

$c, > \frac{N}{2}$ is 36 then median class 60-80 such that

$l = 60, h = 20, f = 12, F = 24$

Median $= l + \frac{\frac{N}{2} - F}{f} \times h = 60 + \frac{25 \cdot 24}{12} \times 20 = 60 + 1 \cdot 67$

Modal class $l = 60, h = 20, f = 12, f = 10, f_2 = 6$

Mode $= l + \left[\frac{f - f_1}{2f - f_1 - f_2}\right] h = 60 + \left[\frac{12 - 10}{24 - 10 - 6}\right] 20$

$= 60 + \frac{40}{8} = 65$

Mode $= 65$

16. Find the mean, median and mode of the following data:

Classes: 0-50 50-100 100-150 150-200 200-250 250-300 300-350

Frequency: 2 3 5 6 5 3 1

Sol:

Class interval	Mid value x	Frequency f	fx	Cumulative frequency
0-50	25	2	50	2
50-100	75	3	225	5
100-150	125	5	625	10
150-200	175	6	1050	16
200-250	225	5	1127	21
250-300	275	3	825	24
300-350	325	1	325	25
		$N = 25$	$\Sigma fx = 4225$	

Mean $= \frac{\Sigma fx}{25} = \frac{4225}{25} = 169$

We have $N = 25$ then $\frac{N}{2} = 12\cdot5$

$c.f > \frac{N}{2} 15 \;\; 16,$ median class $150-200$ such that

$l = 150, h = 4200-150 = 50,\ f = 6, F = 10$

Median $= l + \frac{\frac{N}{2}-F}{f} \times h = 150 + \frac{12\cdot5-10}{6} \times 50$

$= 150 + 20\cdot83 = 170\cdot83$

Here the maximum frequency is 6 then the corresponding class 150-200 is the modal class

$l = 150, h = 200-150 = 50,\ f = 6,\ f_1 = 5,\ f_2 = 5$

Mode $= t + \frac{F - t_1}{2f - f_1 - f_2} \times h = 150 + \frac{6-5}{12-5-5} \times 50$

$= 150 + \frac{50}{2} = 175.$

17. The following table gives the daily income of 50 workers of a factory:

Daily income (in Rs)	100 - 120	120 - 140	140 - 160	160 -180	180 – 200
Number of workers:	12	14	8	6	10

Find the mean, mode and median of the above data.

Sol:

Class interval	Mid value x	Frequency f	fx	Cumulative frequency
100-200	110	12	1320	12
120-140	130	14	1820	26
140-160	150	8	1200	34
160-180	170	6	1000	40
180-200	190	10	1900	50
		$N = 50$	$\Sigma fx = 7260$	

Mean $= \frac{\Sigma fx}{N} = \frac{7250}{50}$

$= 145\cdot2$

We have

$N = 50$

Then $\frac{N}{2} = \frac{50}{2} = 25$

The cumulative frequency is $> \frac{N}{2}$ is 26 corresponding class median class 120-140 such that

$l = 120, h = 140-120 = 20,\ f = 14, F = 12$

Median $= l + \dfrac{\frac{N}{2} - F}{f} \times h$

$= 120 + \dfrac{25 - 12}{14} \times 20$

$= 120 + 18 \cdot 57$

$= 138 \cdot 57$

Here the maximum frequency is 14, then the corresponding class 120-140 is thee modal class

$l = 120, h = 140 - 120 = 20,\ f = 14,\ f_1 = 12,\ f_2 = 8$

Mode $= 1 + \dfrac{f - f_1}{2f - f_1 - f_2} \times h$

$= 120 + \dfrac{14 - 12}{2 \times 4 - 12 - 8} \times 25$

$= 120 + \dfrac{2 \times 25}{5}$

$\Rightarrow 120 + 5$

$= 125$

Mode $= 125$

Exercise – 7.6

1. Draw an given by less than method for the following data:

No.of rooms: 1 2 3 4 5 6 7 8 9 10

No. of houses: 4 9 22 28 24 12 8 6 5 2

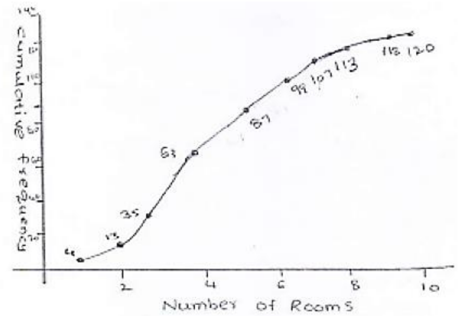
Sol:

We first prepare the cumulative frequency distribution table by less than method as given be

No. of Rooms	No. of houses	Cumulative frequency
Less than or equal to 1	4	4
Less than or equal to 2	9	13
Less than or equal to 3	22	35
Less than or equal to 4	28	63
Less than or equal to 5	24	87
Less than or equal to 6	12	99
Less than or equal to 7	8	107
Less than or equal to 8	6	113
Less than or equal to 9	8	118
Less than or equal to 10	5	120

Now, we mark the upper class limits along x-axis and cumulative frequency along y-axis. Thus we plot the point (1, 4), (2, 3) (3, 35), (4, 63), (5, 87), (6, 99), (7, 107), (8, 113), (9, 118), (10, 120).

Cumulative frequency



2. The marks scored by 750 students in an examination are given in the form of a frequency distribution table:

Marks	No. of students	Marks	No. of students
600 – 640	16	760 – 800	172
640 – 680	45	800 – 840	59
680 – 720	156	840 – 800	18
720 – 760	284		

Prepare a cumulative frequency table by less than method and draw an ogive.

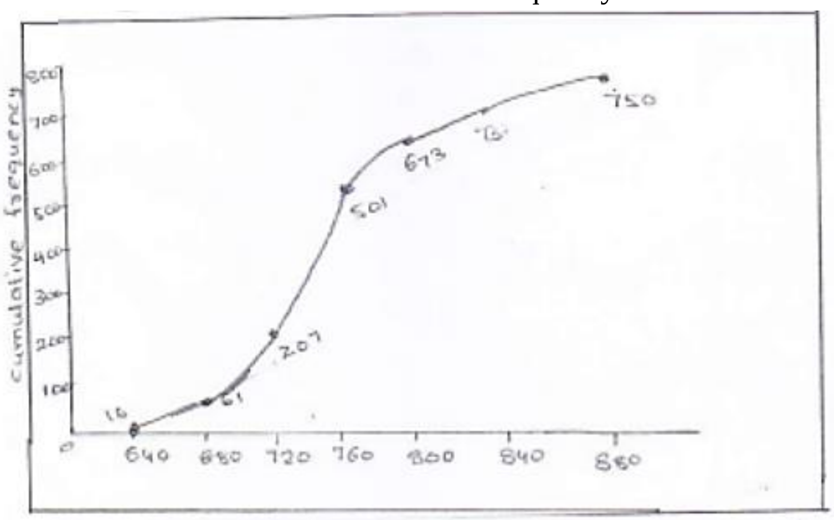
Sol:

We first prepare the cumulative frequency table by less than method as given below

Marks	No. of students	Marks less than	Cumulative frequency
600 – 640	16	640	16
640 – 680	45	680	61
680 – 720	156	720	217
720 – 760	284	760	501
760 – 800	172	800	693
800 – 840	59	840	732
840 – 880	18	880	750

Now, we mark the upper class limits along x-axis and cumulative frequency along y-axis on a suitable gear.
Thus, we plot the points (640, 16) (680, 61), (720, 217), (760, 501), (600, 673), (840, 732) and (880, 750)

Cumulative frequency



3. Draw an ogive to represent the following frequency distribution:

Class-interval: 0 - 4 5 - 9 10 - 14 15 - 19 20-24

No. of students: 2 6 10 5 3

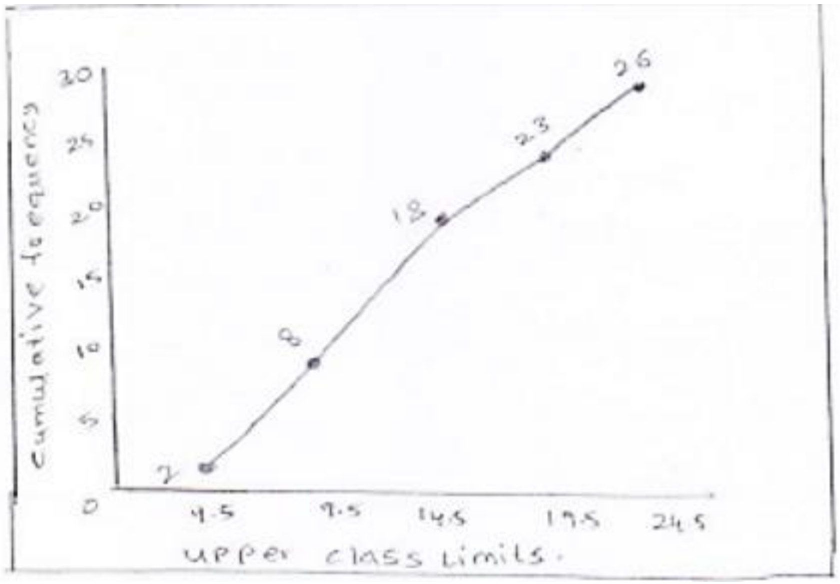
Sol:

The given frequency of distribution is not continuous so we first make it continuous and prepare the cumulative frequency distribution as under

Class Interval	No. of Students	Less than	Cumulative frequency
0.5 – 4.5	2	4.5	2
4.5 – 9.5	6	9.5	8
9.5 – 14.5	10	14.5	18
14.5 – 19.5	5	19.5	23
19.5 – 24.5	3	24.5	26

Now, we mark the upper class limits along x-axis and cumulative frequency along y-axis.
Thus we plot the points (4, 5, 2), (9, 5, 8), (14, 5, 08), (19, 5, 23) and (24, 5, 26)

Cumulative frequency



4. The monthly profits (in Rs.) of 100 shops are distributed as follows:

Profits per shop: 0 - 50 50 - 100 100 - 150 150 - 200 200 - 250 250 – 300

No. of shops: 12 18 27 20 17 6

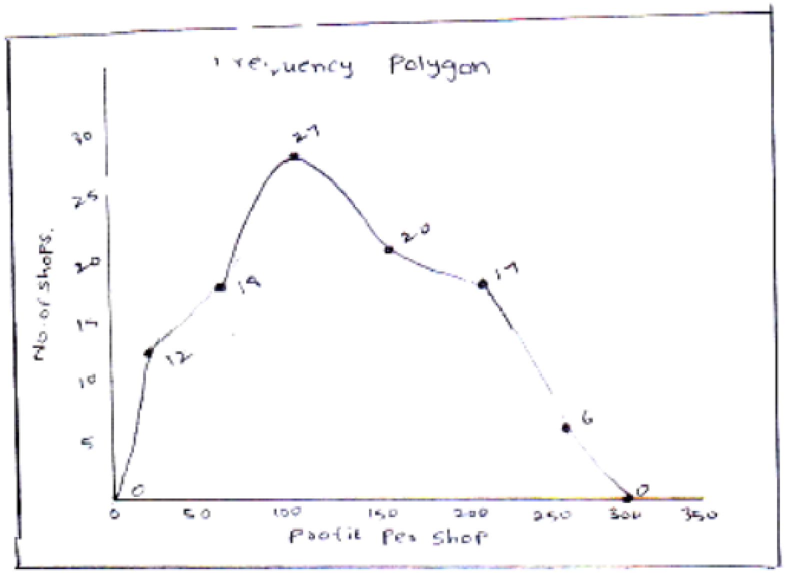
Draw the frequency polygon for it.

Sol:

We have,

Profit per shop	Mid value	No. of shops
Less than 0	0	0
0 – 50	25	12
50 – 100	75	18
100 – 150	125	27
150 – 200	175	20
200 – 250	225	17
250 – 300	275	6

Above 300	300	0



5. The following table gives the height of trees:

Height	No. of trees
Less than 7	26
Less than 14	57
Less than 21	92
Less than 28	134
Less than 35	216
Less than 42	287
Less than 49	341
Less than 56	360

Draw 'less than' ogive and 'more than' ogive.

Sol:

Less than method,

It is given that,

Height	No of trees
Less than 7	26
Less than 14	57
Less than 21	92
Less than 28	134
Less than 35	216
Less than 42	287
Less than 49	341
Less than 56	360

Now, we mark the upper class limits along x-axis and cumulative frequency along y-axis.

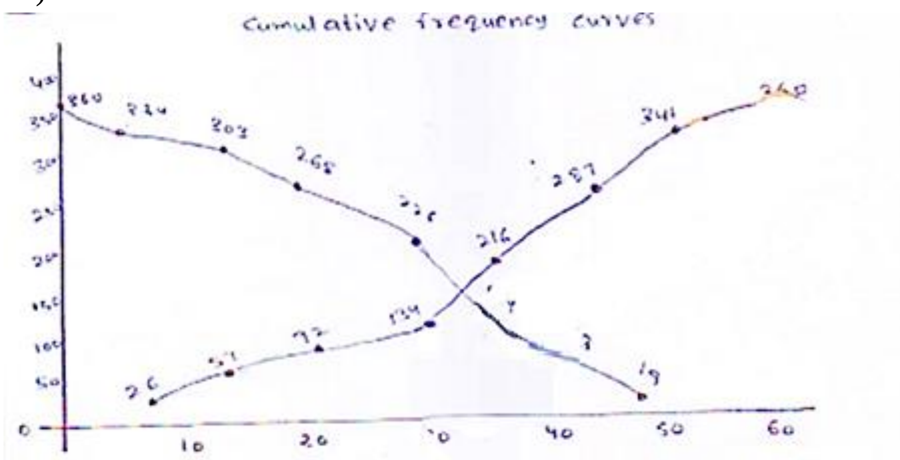
Thus we plot the points (7, 26) (14, 57) (21, 92) (28, 134) (35, 216) (42, 287) (49, 341) (56, 360)

More than method: we prepare the cf table by more than method as given below:

Height	Frequency	Height more than	Cumulative frequency
0 – 7	26	0	360
7 – 14	31	7	334
14 – 21	42	21	268
21 – 28	82	28	226
28 – 35	71	35	144
35 – 42	54	42	73
49 – 56	19	49	19

Now, we mark on x –axis lower class limits, y-axis cumulative frequency

Thus, we plot graph at (0, 360) (7, 334) (14, 303) (21, 268) (28, 226) (35, 144) (42, 73) (49, 19)



6. The annual profits earned by 30 shops of a shopping complex in a locality give rise to the following distribution:

Profit (in lakhs in Rs)	Number of shops (frequency)
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3

Draw both ogives for the above data and hence obtain the median.

Sol:

More than method

Profit (in lakhs in Rs)	No. of shops (frequency)
≥ 5	30

≥ 10	28
≥ 15	16
≥ 20	14
≥ 25	10
≥ 30	7
≥ 35	3

Now, we mark on x- axis lower class limits, y- axis cumulative frequency

Thus, we plot the points (5, 30) (10, 28) (15, 16) (20, 14) (25, 10) (30, 7) and (35, 3)

Less than method

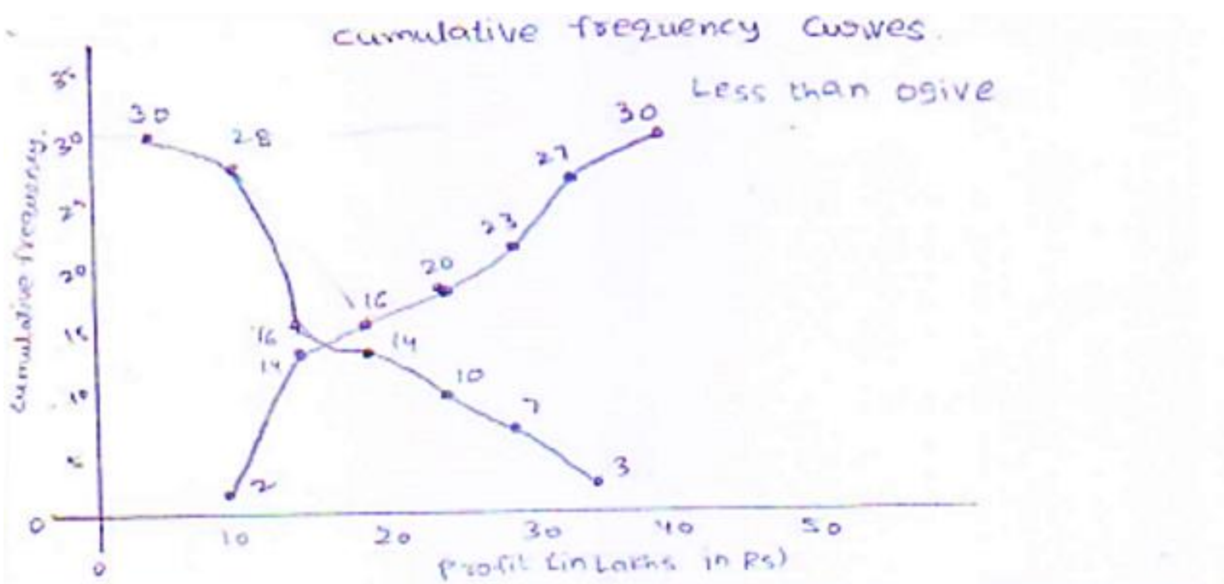
Profit (in lakhs in Rs)	No. of shops (frequency)	Profit less than	Cumulative frequency
0 – 10	2	10	2
10 – 15	12	15	14
15 – 20	2	20	16
20 – 25	4	25	20
25 – 30	3	30	23
30 – 35	4	35	27
35 – 40	3	40	30

Now, we mark the upper class limits along x-axis and cumulative frequency along y-axis.

Thus we plot the points. (10, 2) (15, 14) (20, 16) (25, 20) (30, 23) (35, 27) (40, 30)

We find that the two types of curves intersect of point P from point L it is drawn on x-axis.

The value of a profit corresponding to M is 17.5 lakh, Hence median is 17.5 Lakh



7. The following distribution gives the daily income of 50 workers of a factory:

Daily income (in Rs): 100 - 120 120 - 140 140 - 160 160 - 180 180 – 200
Number of workers: 12 14 8 6 10

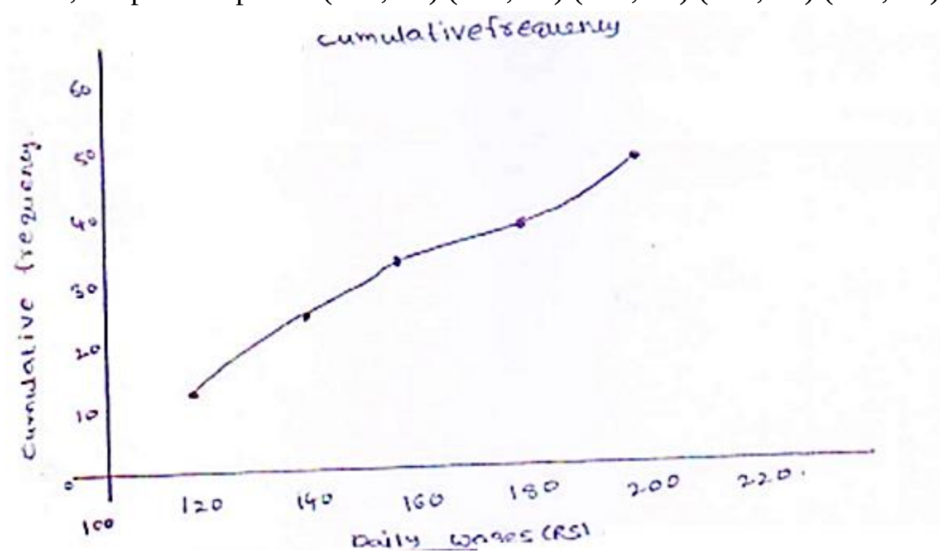
Convert the above distribution to a less than type cumula five frequency distribution and draw its ogive.

Sol:

We first prepare the cumulative frequency table by less than method as given below.

Daily income (in Rs.)	Cumulative frequency
< 120	12
< 140	26
< 160	34
< 180	40
< 200	50

Now, we mark on x – axis upper class limit, y – axis cumulative frequencies.
Thus, we plot the points (120, 12) (140, 26) (160, 34) (180, 40) (200, 50)



8. The following table gives production yield per hectare of wheat of 100 farms of a village:

Production yield 50 - 55 55 - 60 60 -65 65 - 70 70 - 75 75 - 80 in kg per hectare:
Number of farms: 2 8 12 24 38 16

Draw 'less than' ogive and 'more than' ogive.

Sol:

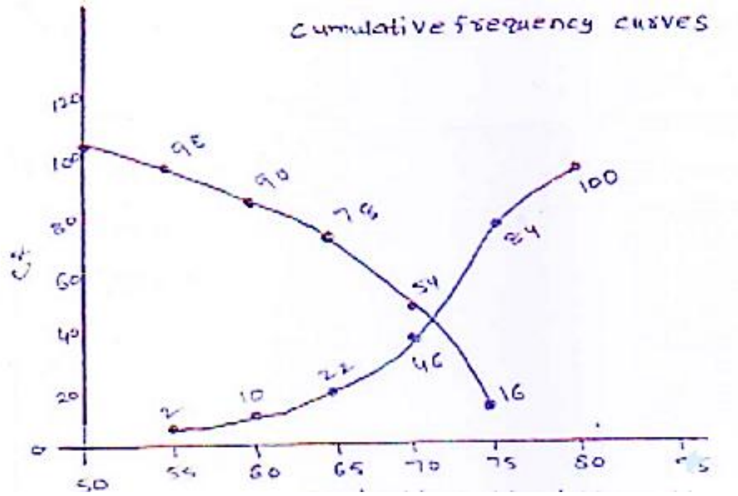
Less than method:

Cumulative frequency table by less than method.

Production yield (integer)	Number of farms	Production yield more than	Cumulative frequency
50 – 55	2	50	100
55 – 60	8	55	98
60 – 65	12	60	90

65 – 70	24	65	78
70 – 75	38	70	54
75 – 80	16	75	16

Now, we mark on x – axis upper class limit, y – axis cumulative frequencies.
We plot the points (50, 100) (55, 98) (60, 90) (65, 78) (70, 54) (75, 16)



9. During the medical check-up of 35 students of a class, their weights were recorded as follows:

Weight (in kg)	No. of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Draw a less than type ogive for the given data. Hence, obtain the median weight from the graph and verify the result by using the formula

Sol:

Less than method

It is given that

On x- axis upper class limits. Y- axis cf.

We plot the points (38, 0) (40, 3) (42, 5) (44, 9) (46, 4) (48, 28) (50, 32) (52, 35)

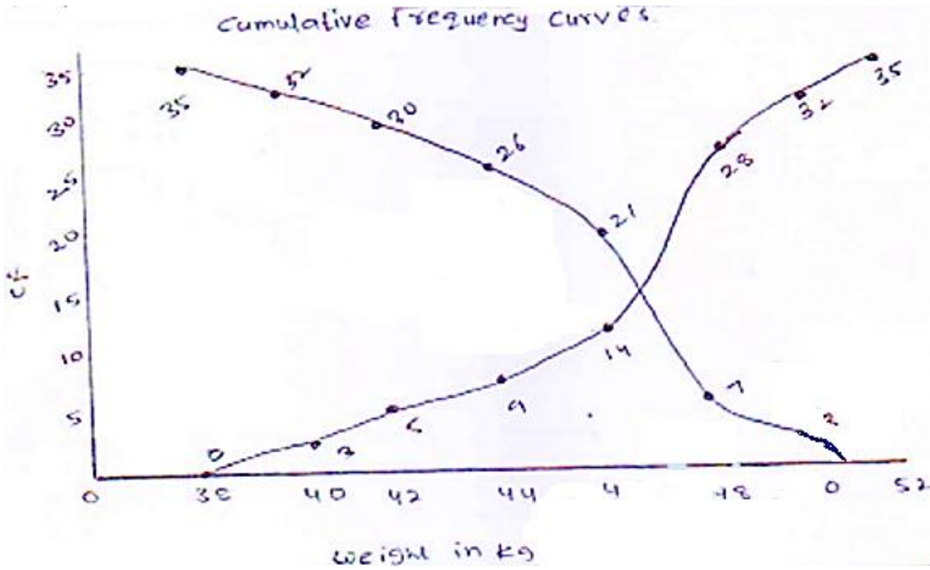
More than method: Cf table

Weight (in kg)	No. of students	Weight more than	Cumulative frequency
38 – 40	3	38	34
40 – 42	2	40	32
42 – 44	4	42	30

44 – 46	5	44	26
46 – 48	14	46	21
48 – 50	4	48	7
50 – 52	3	50	3

x- axis lower class limits on y-axis – cf

We plot the points (38, 35) (40, 32) (42, 30) (44, 26) (46, 21) (48, 7) (50, 3)



We find the two types of curves intersect at a point P. From point P. from P perpendicular PM is draw on x-axis.

The verification,

We have

Weight (in kg)	No. of students	Cumulative frequency
36 – 38	0	0
38 – 40	3	3
40 – 42	2	5
42 – 44	4	9
44 – 46	5	28
46 – 48	14	32
48 – 50	4	32
50 – 0	3	35

Now, N = 35

$\Rightarrow \frac{N}{2} = \frac{35}{2} = 17.5$

The cumulative frequency just greater than $\frac{N}{2}$ is 28 and the corresponding class is 46 – 48

Thus 46 – 48 is the median class such that

L = 46, f = 14, c_1 = 14 and h = 2

$\therefore$ Median = L + $\frac{\frac{N}{2}-c_1}{f} \times h$

$= 46 + \frac{17.5-14}{14} \times 2$

$= 46 + \frac{7}{14}$

$= 46.5$

∴ Median = 46.5 kg

∴ Hence verify.

Exercise 8.1

1. Which of the following are quadratic equations?

(i) $x^2 + 6x - 4 = 0$

(ii) $\sqrt{3x^2} - 2x + \frac{1}{2} = 0$

(iii) $x^2 + \frac{1}{x^2} = 5$

(iv) $x - \frac{3}{x} = x^2$

(v) $2x^2 - \sqrt{3x} + 9 = 0$

(vi) $x^2 - 2x - \sqrt{x} - 5 = 0$

(vii) $3x^2 - 5x + 9 = x^2 - 7x + 3$

(viii) $x + \frac{1}{x} = 1$

(ix) $x^2 - 3x = 0$

(x) $\left(x + \frac{1}{x}\right)^2 = 3\left(1 + \frac{1}{x}\right) + 4$

(xi) $(2x + 1)(3x + 2) = 6(x - 1)(x - 2)$

(xii) $x + \frac{1}{x} = x^2, x \neq 0$

(xiii) $16x^2 - 3 = (2x + 5)(5x - 3)$

(xiv) $(x + 2)^3 = x^3 - 4$

(xv) $x(x + 1) + 8 = (x + 2)(x - 2)$

Sol:

(i) $x^2 + 6x - 4 = 0$

(ii) $\sqrt{3x^2} - 2x + \frac{1}{2} = 0$

(iii) $3x^2 - 5x + 9 = x^2 - 7x + 3$

(iv) $x + \frac{1}{x} = 1$

(v) $(2x + 1)(3x + 2) = 6(x - 1)(x - 2)$

(vi) $16x^2 - 3 = (2x + 5)(5x - 3)$

(vii) $(x + 2)^3 = x^3 - 4$

These are all quadratic equations

2. In each of the following, determine whether the given values are solutions of the given equation or not:

(i) $x^2 - 3x + 2 = 0, x = 2, x = -1$

(ii) $ax^2 - 3abx + 2b^2 = 0, x = \frac{a}{b}$ and $x = \frac{b}{a}$

(iii) $x^2 - \sqrt{2}x - 4 = 0, x = -\sqrt{2}$ and $x = -8\sqrt{2}$

(iv) $2x^2 - x + 9 = x^2 + 4x + 3, x = 2$ and $x = 3$

(v) $x+\frac{1}{2}=\frac{13}{6}=x=\frac{5}{6}, x=\frac{4}{3}$

(vi) $x^2-3\sqrt{3x}+6=0$, x = $\sqrt{3}$, x = $-8\sqrt{3}$

(vii) $x^2+x+1=0$, x = 0, x = 1

Sol:

(i) $x^2-3x+2=0, x=2, x=-1$

Here LHS = x^2-3x+2

and RHS = 0

Now, substitute x = 2 in LHS

We get $(2)^2-3(2)+2$ = 4 – 6 + 2

= 6 – 6

= 0

⟹ RHS

$Since, LHS = RHS$

x = 2 is a solution for the given equation.

Similarly,

Now substitute x = - 1 in LHS

We get $(-1)^2$ -3(-1) +2

⟹ 1 + 3 + 2 = 6 ≠ RHS

Since LHS ≠ RHS

x = -1 is not a solution for the given equation

(ii) $x^2+x+1=0$, x = 0, x = 1

$Here\ LHS = x^2+x+1\ and\ RHS = 0$

Now substitute x = 0 and x = 1 in LHS

⟹ 0^2+0+1 and $(1)^2+(1)+1$

⟹ 1 and 1 + 1 + 1 = 3

≠ RHS ≠ RHS

∴ x = 0, x = 1 are not solutions of the given equation

(iii) $x^2-3\sqrt{3x}+6=0$, x = $\sqrt{3}$, x = $-8\sqrt{3}$

Here LHS $=x^2-3\sqrt{3}x+6$ and RHS $=0$

Substitute $x=\sqrt{3}$ and $x=-2\sqrt{3}$ in LHS

$\Rightarrow\left(\sqrt{3}\right)^2-3\sqrt{3}\left(\sqrt{3}\right)+6$ and $\left(-2\sqrt{3}\right)^2-3\sqrt{3}\left(-2\sqrt{3}\right)+6$

$\Rightarrow 3-9+6$ and $18+18+6$

$\Rightarrow 0$ and 36

⟹ RHS ≠ RHS

$\therefore x=\sqrt{3}$ is a solution and $x=-2\sqrt{3}$ is not a solution for the given equation

(iv) $x+\frac{1}{2}=\frac{13}{6}=x=\frac{5}{6}, x=\frac{4}{3}$

Here LHS $= x + \frac{1}{x}$ and RHS $= \frac{13}{6}$

Substitute $x = \frac{5}{6}$ and $x = \frac{4}{3}$ in the LHS

$\Rightarrow \frac{5}{6} + \frac{1}{\left(\frac{5}{6}\right)}$ and $\frac{4}{3} + \frac{1}{\left(\frac{4}{9}\right)}$

$\Rightarrow \frac{5}{6} + \frac{6}{5}$ and $\frac{4}{3} + \frac{3}{4}$

$\Rightarrow \frac{85+36}{30}$ and $\frac{16+9}{18}$

$\Rightarrow \frac{61}{30}$ and $\frac{85}{18}$

$\neq$ RHS $\neq$ RHS

$\therefore x = \frac{5}{6}$ and $x = \frac{4}{3}$ are not solutions of the given equation

(v) $2x^2 - x + 9 = x^2 + 4x + 3, x = 2$ and $x = 3$

$\Rightarrow 2x^2 - x^2 - x - 4x + 9 - 3 = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

Here, LHS $= x^2 - 5x + 6$ and RHS $= 0$

Substitute $x = 2$ and $x = 3$ in LHS

$\Rightarrow (2)^2 - 5(2) + 6$ and $(3)^2 - 5(3) + 6$

$\Rightarrow 4 - 10 + 6$ and $9 - 15 + 6$

$\Rightarrow 10 - 10$ and $15 - 15$

$\Rightarrow 0$ and $\Rightarrow 0$

= RHS = RHS

$x = 3$ and $x = 2$ are solutions of the given equation.

(vi) $x^2 - \sqrt{2}x - 4 = 0, x = -\sqrt{2}$ and $x = -8\sqrt{2}$

Here, LHS $= x^2 - \sqrt{2}x - 4$ and RHS $= 0$

Substitute $x = -\sqrt{2}$ and $x = -2\sqrt{2}$ in LHS

$\Rightarrow \left(-\sqrt{2}\right)^2 - \sqrt{2}\left(\sqrt{2}\right) - 4$ and $\left(-2\sqrt{2}\right)^2 - \sqrt{2}\left(-2\sqrt{2}\right) - 4$

$\Rightarrow 2 + 2 - 4$ and $8 + 4 - 4$

$\Rightarrow 4 - 4$ and $8 - 4$

$\Rightarrow 0$ and 8

= RHS $\neq$ RHS

$\therefore x = -\sqrt{2}$ is a solution and $x = -2\sqrt{2}$ is not a solution is the given equation.

(vii) $ax^2 - 3abx + 2b^2 = 0, x = \frac{a}{b}$ and $x = \frac{b}{a}$

Here, LHS $= ax^2 - 3abx + 2b^2$ and RHS $= 0$

Substitute $x = \frac{a}{b}$ and $x = \frac{b}{a}$ in LHS

$\Rightarrow a^2\left(\frac{a}{b}\right)^2 - 3ab\left(\frac{a}{b}\right) + 2b^2$ and $a^2\left(\frac{b}{a}\right)^2 - 3ab\left(\frac{b}{a}\right) + 2b^2$

$\Rightarrow a^2\left(\frac{a^2}{b^2}\right) - 3a \times a + 2b^2$ and $a^2 \times \frac{b^2}{a^2} - 3b \times b + 2b^2$

$\Rightarrow \frac{a^2}{b^2} - 3a^2 + 2b^2$ and $b^2 - 3b^2 + 2b^2$

$\Rightarrow \frac{a^4}{b^2} - 3a^2 + 2b^2$ and $3b^2 - 3b^2 = 0$

$\Rightarrow \neq$ RHS $=$ RHS

$\therefore x = \frac{b}{a}$ is a solution and $x = \frac{a}{b}$ is not a solution for the given equation.

3. In each of the following, find the value of k for which the given value is a solution of the given equation:

(i) $7x^2 + kx - 3 = 0, x = \frac{2}{3}$

(ii) $x^2 - x(a + b) + k = 0, x = a$

(iii) $kx^2 + \sqrt{2}x - 4 = 0, x = \sqrt{2}$

(iv) $x^2 + 3ax + k = 0, x = -a$

Sol:

(i) Given that $x = \frac{2}{3}$ is a root of the given equation

$\Rightarrow x = \frac{2}{3}$ satisfies the equation

$i \cdot e \quad 7\left(\frac{2}{3}\right)^2 + k\left(\frac{2}{3}\right) - 3 = 0$

$\Rightarrow 7 \times \frac{4}{9} + 2\frac{k}{3} - 3 = 0$

$\Rightarrow 2\frac{k}{3} = 3 - \frac{28}{9}$

$\Rightarrow 2\frac{k}{3} = \frac{27 - 28}{9}$

$\Rightarrow 2\frac{k}{\not{3}} = -\frac{1}{\not{2}_3} \Rightarrow \boxed{k = \frac{-1}{6}}$

(ii) Given that $x = a$ is a root of the given equation

$x^2 - x(a+b) + k = 0$

$\Rightarrow x = a$ Satisfies the equation

$i \cdot e\, (a)^2 - a(a+b) + k = 0$

$\Rightarrow a^2 - a^2 - ab + k = 0 \Rightarrow -ab + k = 0$

$\Rightarrow \boxed{k = ab}$

(iii) Given that $x = \sqrt{2}$ is a root at the given equation

$kx^2 + \sqrt{2}x - 4 = 0$

$\Rightarrow x = \sqrt{2}$ Satisfies the equation

$i \cdot e\;\; k\left(\sqrt{2}\right)^2 + \sqrt{2}\left(\sqrt{2}\right) - 4 = 0$

$\Rightarrow 2k + 2 - 4 = 0$

$\Rightarrow 2k - 2 = 0 \Rightarrow 2k = 2$

$\Rightarrow \boxed{k = 1}$

(iv) Given that $x = -0$ is a root of the given equation $x^2 + 3ax + k = 0$

$\Rightarrow x = -a$ Satisfies the equation

$i \cdot e\, (-a)^2 + 3a(-a) + k = 0$

$\Rightarrow a^2 - 3a^2 + k = 0 \Rightarrow -2a^2 + k = 0$

$\Rightarrow \boxed{k = 2a^2}$

4. If x = $\frac{2}{3}$ and x = - 3 are the roots of the equation $ax^2 + 7x + b = 0$, find the values of a and b.

Sol:

a = 3, b = $-$ 6

5. Determine if, 3 is a root of the equation given below:

$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 16}$

Sol:

Given to check whether 3 is a root of the equation

$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 16}$

Here LHS $= \sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9}$ and RHS $= \sqrt{4x^2 - 14x + 16}$

Substitute $x = 3$ in LHS

$\Rightarrow \sqrt{3^2-4(3)+3}+\sqrt{(3)^2=9}$

$\Rightarrow \sqrt{9-18+3}+\sqrt{9-9}$

$\Rightarrow \sqrt{0}+\sqrt{0}\Rightarrow 0$ $\quad\therefore$ LHS $=0$

Similarly, substitute $x=3$ in RHS.

$\Rightarrow \sqrt{4(3)^2-14(3)+16}$

$\Rightarrow \sqrt{4\times 9-42+16}\Rightarrow \sqrt{36+42+16}$

$\Rightarrow \sqrt{52-42}\Rightarrow \sqrt{10}$

$\therefore$ RHS $=\sqrt{10}$

Now, we can observe that

LHS $\neq$ RHS

$\therefore x=3$ is not a solution or root for the equation

$\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+16}$

Exercise 8.2

1. The product of two consecutive positive integers is 306. Form the quadratic equation to find the integers, if x denotes the smaller integer.

Sol:

Given that the smallest integer of 2 consecutive integer is denoted by x

$\Rightarrow$ The two integer will be x and $(x+1)$

Product of two integers $\Rightarrow x(x+1)$

Given that the product is 306

$\therefore x(x+1)=306$

$\Rightarrow x^2+x=306\Rightarrow x^2+x-306=0$

$\therefore$ The required quadratic equation is $x^2+x-306=0$

2. John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 128. Form the quadratic equation to find how many marbles they had to start with, if John had x marbles.

Sol:

Given that John and Jivani together have 45 marbles and John has x marbles

$\Rightarrow$ Jivani had $(45-x)$ marbles

No. of marbles John had after loosing 5 marbles $=x-5$

No. of marbles Jivani had after loosing 5 marbles $=(45-x)-5$

$= 45 - 5 - x$

$= 40 - x$

Given that product of the no of marbles they now have $= 128$

$\Rightarrow (x-5)(40-x) = 128$

$\Rightarrow 40x - x^2 - 40 \times 5 + 5x = 128$

$\Rightarrow 45x - x^2 - 200 = 128 \Rightarrow x^2 - 45x + 128 + 200 = 0$

$\Rightarrow x^2 - 45x + 328 = 0$

$\therefore$ The required quadratic equation is $x^2 - 45x + 328 = 0$

3. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of articles produced in a day. On a particular day, the total cost of production was Rs. 750. If x denotes the number of toys produced that day, form the quadratic equation fo find x.

Sol:

Given that x denotes the no of toys product in a day

$\Rightarrow$ The cost of production of each by $= 55 -$ no. of toys produced in a day

$= (55 - x)$

Total cost of production is nothing but product of no. of toys produced in a day and cost of production of each toy

$\Rightarrow x(55 - x)$

But total cost of production = Rs 750

$\Rightarrow x(55 - x) = 750$

$\Rightarrow 55x - x^2 = 750$

$\Rightarrow x^2 - 55x + 750 = 0$

$\therefore$ The required quadratic from of the given data is $x^2 - 55x + 750 = 0$

4. The height of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, form the quadratic equation to find the base of the triangle.

Sol:

Given that in a right triangle is 7cm less than its base

Let base of the triangle be denoted by x

$\Rightarrow$ Height of the triangle $= (x - 7)$cm

We have hypotenuse of the triangle $= 13$cm

We know that, in a right triangle

$(\text{base})^2 + (\text{Height})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (x)^2 + (x-2)^2 = (13)^2$

$\Rightarrow x^2 + x^2 - 14x + 49 = 169$

$\Rightarrow 2x^2 - 14x + 49 - 169 = 0$

$\Rightarrow 2x^2 - 14x - 120 = 0$

$\Rightarrow 2\left(x^2 - 7x - 60\right) = 0$

$\Rightarrow x^2 - 7x - 60 = 0$

$\therefore$ The required quadratic equation is $x^2 - 7x - 60 = 0$

5. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore. If the average speed of the express train is 1 1 km/hr more than that of the passenger train, form the quadratic equation to find the average speed of express train.

Sol:

Let the arrange speed of express train be denoted by $x\ km/hr$

Given that average speed of express train is $11\ km/hr$ more than that of the passenger train

$\Rightarrow$ Average speed of passenger train $= \left(x = 11\right) km/hr$

Total distance travelled by the train $= 132\ km$

We know that,

Time taken to travel $= \frac{\text{Distance travelled}}{\text{Average speed}}$

$\Rightarrow$ Time taken by express train $= \frac{\text{Distance travelled}}{\text{Average speed of express train}}$

$= \frac{132}{x}\text{hr}$

$\Rightarrow$ Time taken by express train $= \frac{132}{\left(x - 11\right)}\text{hr}$

Given that time taken by express train is 1 hour less than that of passenger train.

$\Rightarrow$ Time taken by passenger train _ Time taken by express train 1 hour

$\Rightarrow \frac{132}{x - 11} - \frac{132}{x} = 1$

$\Rightarrow 132\left(\frac{1}{x - 11} - \frac{1}{x}\right) = 1$

$\Rightarrow 132\left(\frac{x - \left(x - 11\right)}{x\left(x - 11\right)}\right) = 1$

$\Rightarrow 132\left(x - 2 + 11\right) = x\left(x - 11\right)$

$\Rightarrow 132\left(11\right) = x^2 - 11x$

$\Rightarrow x^2 - 11x = 1452$

$\Rightarrow x^2 - 11x - 1452 = 0$

The required quadratic is $x^2 - 11x - 1452 = 0$

6. A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Form the quadratic eqiation to find the speed of the train.

Sol:

Let Speed of train be $x\ km/hr$

Distance travelled by train $= 360\,km$

We know that

Time of total $= \dfrac{\text{Distance travelled}}{\text{Speed of the train}} = \dfrac{360}{x} hr$

If speed had been 5 km/hr more $\Rightarrow (x+5) km/hr$

Time of travel $= \dfrac{\text{Distance travelled}}{\text{Speed of the train}} = \dfrac{360}{x+5} hr$

Give that,

Time of travel when speed is increased is 1 hour less than of the actual time of travel

$\Rightarrow \dfrac{360}{x} - \dfrac{360}{x+5} = 1$

$\Rightarrow 360\left(\dfrac{1}{x} - \dfrac{1}{x+5}\right) = 1$

$\Rightarrow 360\left(\dfrac{x+5-x}{x(x+5)}\right) = 1$

$\Rightarrow 360(5) = x(x+5)$

$\Rightarrow x^2 + 5x = 1800$

$\Rightarrow x^2 + 5x + 1800 = 0$

$\therefore$ The required quadratic equation to find the speed of the train is $x^2 + 5x - 1800 = 0$

Exercise 8.3

Solve the following quadratic equations by factorization:

1. $(x-4)(x+2) = 0$

Sol:

We have

$(x-4)(x+2) = 0$

$\Rightarrow$ either $(x-4)=0$ or $(x+2)=0$

$\Rightarrow x=4$ or $x=-2$

Thus, $x=4$ and $x=-2$ are two roots of the equation $(x-4)(x+2)=0$

2. $(2x+3)(3x-7)=0$

Sol:

We have,

$(2x+3)(3x-7)=0$

$\Rightarrow (2x+3)=0$ or $(3x-7)=0$

$\Rightarrow 2x=-3$ or $3x=7$

$\Rightarrow x=\frac{-3}{2}$ or $x=\frac{7}{3}$

Thus, $x=\frac{-3}{2}$ and $x=\frac{7}{3}$ are two roots of the equation $(2x+3)(3x-7)=0$

3. $4x^2+5x=0$

Sol:

We have $4x^2+5x=0$

$\Rightarrow x(4x+5)=0$

$\Rightarrow$ either $x=0$ or $4x+5=0$

$\Rightarrow x=0$ or $4x=-5$

$\Rightarrow x=0$ or $x=\frac{-5}{4}$

Thus, $x=0$ and $x=\frac{-5}{4}$ are two roots of equation $4x^2+5x=0$

4. $9x^2-3x-2=0$

Sol:

We have $9x^2-3x-2=0$

$\Rightarrow 9x^2-6x+3x-2=0$

$\Rightarrow 3x(3x-2)+1(3x-2)=0$

$\Rightarrow (3x-2)(3x+1)=0$

$\Rightarrow$ either $3x-2=0$ or $3x+1=0$

$\Rightarrow 3x=2$ or $3x=-1$

$\Rightarrow x=\frac{2}{3}$ or $x=-\frac{1}{3}$

Thus, $x=\frac{2}{3}$ and $x=-\frac{1}{3}$ are two roots of the equation $9x^2-3x-2=0$

5. $6x^2-x-2=0$

Sol:

We have $6x^2-x-2=0$

$\Rightarrow 6x^2+3x-4x-2=0$

$\Rightarrow 3x(2x+1)-2(2x+1)=0$

$\Rightarrow (2x+1)(3x-2)=0$

$\Rightarrow$ either $2x+1=0$ or $3x-2=0$

$\Rightarrow 2x=-1$ or $3x=2$

$\Rightarrow x=-\frac{1}{2}$ or $x=\frac{2}{3}$

Thus, $x=-\frac{1}{2}$ and $x=\frac{2}{3}$ are two roots of the equation $6x^2-x-2=0$

6. $6x^2+11x+3=0$

Sol:

We have

$6x^2+11x+3=0$

$\Rightarrow 6x^2+9x+2x+3=0$

$\Rightarrow 3x(2x+3)+1(2x+3)=0$

$\Rightarrow (2x+3)(3x+1)=0$

$\Rightarrow 2x+3=0$ or $x=-\frac{1}{3}$

Thus, $x=-\frac{3}{2}$ and $x=-\frac{1}{3}$ are the two roots of the given equation.

7. $5x^2-3x-2=0$

Sol:

We have,

$5x^2-3x-2=0$

$\Rightarrow 5x^2-5x+2(x-1)=0$

$\Rightarrow 5x(x-1)+2(x-1)=0$

$\Rightarrow (x-1)(5x+2)=0$

$\Rightarrow (x-1)=0$ or $5x+2=0$

$\Rightarrow x=1$ or $x=-\frac{2}{5}$

$\therefore x=1$ and $x=-\frac{2}{5}$ are the two roots of the given equation.

8. $48x^2-13x-1=0$

Sol:

We have

$48x^2-13x-1=0$

$\Rightarrow 48x^2-16x+3x-1=0$

$\Rightarrow 16x(3x-1)+1(3x-1)=0$

$\Rightarrow (3x-1)(16x+1)=0$

$\Rightarrow 3x-1=0$ or $16x+1=0$

$\Rightarrow x=\frac{1}{3}$ or $x=-\frac{1}{16}$

$\therefore\ x=-\frac{1}{16}$ and $x=\frac{1}{3}$ are the two roots of the given equation.

9. $3x^2=-11x-10$

Sol:

We have

$3x^2=-11x-10$

$\Rightarrow 3x^2+11x+10=0$

$\Rightarrow 3x^2+6x+5x+10=0$

$\Rightarrow 3x(x+2)+5(x+2)=0$

$\Rightarrow (x+2)(3x+5)=0$

$\Rightarrow (x+2)=0$ or $x=-\frac{5}{3}$ $\therefore x=2$ and $x=-\frac{5}{3}$ are the two roots at the quadratic equation $3x^2=-11x-10$

10. $25x(x+1)=-4$

Sol:

We have

$(x+1)=-4$

$\Rightarrow (25x)\times x+(25x)\times 10-4$

$\Rightarrow 25x^2+25x+4=0$ $\quad [25\times 4=100\Rightarrow 25=20+5\Rightarrow 100=20\times 5]$

$\Rightarrow 25x^2+20x+5x+4=0$

$\Rightarrow 5x(5x+4)+1(5x+4)=0$

$\Rightarrow (5x+4)(5x+1)=0$

$\Rightarrow 5x+4=0$ or $5x+1=0$

$\Rightarrow x=-\frac{4}{3}$ or $x=-\frac{1}{3}$

$\therefore x=-\frac{4}{3}$ and $x=-\frac{1}{5}$ are the two solutions of the quadratic equation $25x(x+1)=-4$

11. $10x-\frac{1}{x}=3$

Sol:

We have

$10x-\frac{1}{x}=3$

$\Rightarrow \frac{10x^2-1}{x}=3$

$\Rightarrow 10x^2-1=3x$

$\Rightarrow 10x^2-3x-1=0$ $\quad [10x-1=-10 \Rightarrow -10=-5\times 2$ and $-3=-5+2]$

$\Rightarrow 10x^2-5x^2+2x-1=0$

$\Rightarrow 5x(2x-1)+1(2x-1)=0$

$\Rightarrow (2x-1)(5x+1)=0$

$\Rightarrow 2x-1=0$ or $5x+1=0$

$\Rightarrow x=\frac{1}{2}$ or $x=-\frac{1}{5}$

$\therefore x=\frac{1}{2}$ and $x=-\frac{1}{5}$ are the two roots of the given equation

12. $\frac{2}{2^2}-\frac{5}{x}+2=0$

Sol:

We have,

$\frac{2}{2^2}-\frac{5}{x}+2=0$

$\Rightarrow \frac{2-5x+2x^2}{x^2}=0$

$\Rightarrow 2x^2-5x+2=0$

$\left[2\times2=4\Rightarrow4=-4\times-1\Rightarrow-5=-4=1\right]$

$\Rightarrow 2x^2-4x-x+8=0$

$\Rightarrow 2x(x-2)-1(x-2)=0$

$\Rightarrow (x-2)(2x-1)=0$

$\Rightarrow x-2=0$ or $2x-1=0$

$\Rightarrow x=2$ or $x=\frac{1}{2}$

$\therefore x=2$ and $x=\frac{1}{2}$ are the two roots at the given quadratic equation

13. $4\sqrt{3}x^2+5x-2\sqrt{3}=0$

Sol:

We have,

$4\sqrt{3}x^2+5x-2\sqrt{3}=0$

$\left[4\sqrt{3}\times2\sqrt{3}=-8\times3=-24\Rightarrow-24=-8\times3=-3\times8\Rightarrow5=-3+8\right]$

$\Rightarrow 4\sqrt{3}x^2+8x-3x-2\sqrt{3}=0$

$\Rightarrow 4x\left(\sqrt{3}x+2\right)-\sqrt{3}\left(\sqrt{3}x+2\right)=0$

$\Rightarrow \left(4x-\sqrt{3}\right)\left(\sqrt{3}x+2\right)=0$

$\Rightarrow 4x-\sqrt{3}=0$ or $\sqrt{3}x=-2$

$\Rightarrow x=\frac{\sqrt{3}}{4}$ or $x=-\frac{2}{\sqrt{3}}$

$\therefore x=\frac{\sqrt{3}}{4}$ and $x=-\frac{2}{\sqrt{3}}$ are the two roots of the given quadratic equation

14. $\sqrt{2}x^2-3x-2\sqrt{2}=0$

Sol:

We have,

$\sqrt{2}x^2-3x-2\sqrt{2}=0$

$\left[\sqrt{2}\times-2\sqrt{2}=-2\times2=-4\Rightarrow-4=-4\times1\Rightarrow-3=-4+1\right]$

$\Rightarrow \sqrt{2}x^2-4x+x-2\sqrt{2}=0$

$\Rightarrow \sqrt{2}x^2-\left(2\sqrt{2x}\sqrt{2}\right)x+x-2\sqrt{2}=0$

$\Rightarrow \sqrt{2}x-\left(x-2\sqrt{2}\right)+1\left(x-2\sqrt{2}\right)=0$

$\Rightarrow \left(x-2\sqrt{2}\right)\left(\sqrt{2}x+1\right)=0$

$\Rightarrow x-2\sqrt{2}=0$ or $\sqrt{2}x+=0$

$\Rightarrow x=2\sqrt{2}$ or $x=\frac{-1}{\sqrt{2}}$

$\therefore x=-\frac{1}{\sqrt{2}}$ and $x=2\sqrt{2}$ are the two roots of the given quadratic equation.

15. $a^2x^2-30bx+2b^2=0$

Sol:

We have,

$a^2x^2-30bx+2b^2=0$

$\Rightarrow a^2x^2-abx-2abx+2b^2=0$

$\left[a^2\times 2b^2=2a^2b^2 \Rightarrow 2a^2b^2=2ab\times ab=-2ab\times -ab \Rightarrow -3ab=-2ab-ab\right]$

$\Rightarrow ax(ax-b)-2b(ax-b)=0$

$\Rightarrow (ax-2b)(ax-b)=0$

$\Rightarrow ax-2b=0$ or $ax-b=0$

$\Rightarrow ax=2b$ or $ax=b$

$\Rightarrow x=\frac{2b}{a}$ or $x=\frac{b}{a}$

$\therefore x=\frac{b}{a}$ and $x=\frac{2b}{a}$ are the two roots of the given quadratic equation

16. $x^2-(\sqrt{2}+1)x+\sqrt{2}=0$

Sol:

We have,

$x^2-(\sqrt{2}+1)x+\sqrt{2}=0$

$\Rightarrow x^2-\sqrt{2}x-1\times x+\sqrt{2}=0$

$\left[1\times\sqrt{2}=\sqrt{2} \Rightarrow \sqrt{2}=-\sqrt{2}\times -1\right]$

$\Rightarrow x^2-\sqrt{2}x-x+\sqrt{2}=0$

$\Rightarrow x(x-\sqrt{2})-1(x-\sqrt{2})=0$

$\Rightarrow (x-\sqrt{2})(x-1)=0$

$\Rightarrow x=\sqrt{2}=0$ or $x-1=0$

$\Rightarrow x=\sqrt{2}$ or $x=1$

$\therefore x=1$ and $x=\sqrt{2}$ are the roots of the given quadratic equation

17. $x^2-\left(\sqrt{3}+1\right)x+\sqrt{3}=0$

Sol:

We have,

$x^2-\left(\sqrt{3}+1\right)x+\sqrt{3}=0$

$\Rightarrow x^2-\sqrt{3}x-1\times x+\sqrt{3}=0$

$\left[\sqrt{3}\times 1=\sqrt{3}\Rightarrow\sqrt{3}=-\sqrt{3}\times -1\Rightarrow\left(\sqrt{3}+1\right)=-\sqrt{3}-1\right]$

$\Rightarrow x\left(x-\sqrt{3}\right)-1\left(x-\sqrt{3}\right)=0$

$\Rightarrow\left(x-\sqrt{3}\right)\left(x-1\right)=0$

$\Rightarrow x-\sqrt{3}=0$ or $x-1=0$

$\Rightarrow x=\sqrt{3}$ or $x=1$

$\therefore$ $x=1$ and $x=\sqrt{3}$ are the two roots of the given quadratic equation

18. $4x^2+4bx-\left(a^2-b^2\right)=0$

Sol:

We have,

$4x^2+4bx-\left(a^2-b^2\right)=0$

$\left[4x-\left(a^2-b^2\right)=4x-\left(a-b\right)\left(a+b\right)=-2\left(a-b\right)\times 2\left(a+b\right)=2\left(b-a\right)\times 2\left(a+b\right)\Rightarrow 4b=2b+2b=8\left(b-a\right)+\right.$

$\Rightarrow 4x^2+\left(2\left(b-a\right)+2\left(a+b\right)x-\left(a-b\right)\left(a+b\right)=0\right)$

$\Rightarrow 2x^2+\left(2x+b-a\right)+\left(a+b\right)\left(2x+\left(b-a\right)\right)=0$

$\Rightarrow\left(2x+b-a\right)$ or $2x+a+b=0$

$\Rightarrow 2x=a-b$ or $2x=-a-b$

$\Rightarrow x=\frac{a-b}{2}$ or $2x=-\left(a+b\right)\Rightarrow x=-\frac{\left(a+b\right)}{2}$

$\therefore x=-\frac{\left(a+b\right)}{2}$ and $x=\frac{a-b}{2}$ are the two roots of the given quadratic equation

19. $ax^2+\left(4a^2-3b\right)x-12ab=0$

Sol:

We have,

$ax^2+\left(4a^2-3b\right)x-12ab=0$

$\left[a \times 12ab = -12a^2b^2 = 4a^2 \times -3b\right]$

$\Rightarrow ax^2 + 4a^2x - 3bx + \left(4a \times (-3b)\right) = 0$

$\Rightarrow ax(x + 4a) - 3b(x + 4a) = 0$

$\Rightarrow (a + 4a)(ax - 3b) = 0$

$\Rightarrow (x + 4a) = 0$ or $(ax - 3b) = 0$

$\Rightarrow x = -4a$ or $x = \frac{3b}{a}$

$\therefore x = \frac{3b}{a}$ and $x = -4a$ are the two roots of the given equations

20. $\left(x - \frac{1}{2}\right)^2 = 4$

Sol:

We have,

$\left(x - \frac{1}{2}\right)^2 = 4$

$\Rightarrow \left(x - \frac{1}{2}\right)^2 - 4 = 0$

$\Rightarrow \left(x - \frac{1}{2}\right)^2 - (2)^2 = 0$

$\Rightarrow \left[\left(x - \frac{1}{2}\right) + 2\right]\left[\left(x - \frac{1}{2}\right) = 2\right] = 0 \quad \left[\because a^2 - b^2 = (a + b)(a - b)\right]$

$\Rightarrow \left(x - \frac{1}{2} + 2\right) = 0$ or $\left(x - \frac{1}{x} - 2\right) = 0$

$\Rightarrow x = 2 - \frac{1}{2}$ or $x = 2 - \frac{1}{2}$

$\Rightarrow x = \frac{4 - 1}{2}$ or $x = \frac{4 + 1}{2}$

$\Rightarrow x = \frac{3}{2}$ or $x = \frac{5}{2}$

$\Rightarrow x = \frac{3}{2}$ and $x = \frac{5}{2}$ are the two rots at the given equations

21. $x^2-4\sqrt{2}x+6=0$

Sol:

We have,

$x^2-4\sqrt{2}x+6=0$

$\left[1\times 6=6\Rightarrow 6=-3\sqrt{2}x-\sqrt{2} \text{ and } -4\sqrt{2}=-3\sqrt{2}-\sqrt{2}\right]$

$\Rightarrow x^2-3\sqrt{2}x-\sqrt{2}x+\left(-3\sqrt{2}\times -2\right)=0$

$\Rightarrow x\left(x-3\sqrt{2}\right)\sqrt{2}\left(x-3\sqrt{2}\right)=0$

$\Rightarrow \left(x-3\sqrt{2}\right)\left(x-\sqrt{2}\right)=0$

$\Rightarrow x=3\sqrt{2}=0$ or $x-\sqrt{2}=0$

$\Rightarrow x=3\sqrt{2}$ or $x=\sqrt{2}$

$\therefore x=3\sqrt{2}$ and $x=\sqrt{2}$ are the two roots of the given equation.

22. $\frac{x+3}{x+2}=\frac{3x-7}{2x-3}$

Sol:

We have, $\frac{x+3}{x+2}=\frac{3x-7}{2x-3}$

$\Rightarrow (x+3)(2x-3)=(x+2)(3x-7)$

$\Rightarrow 2x^2-3x+6x-9=3x^2-x-14$

$\Rightarrow 2x^2+3x-9=3x^2-x-14$

$\Rightarrow x^2-3x-x-14+9=0$

$\Rightarrow x^2-4x-5=0$

$[1x-5=-5-4=-5+1]$

$\Rightarrow x^2-5x+x-5=0$

$\Rightarrow x(x-5)+1(x-5)=0$

$\Rightarrow (x-5)(x+1)=0$

$\Rightarrow x-5=0$ or $x+1=0$

$\Rightarrow x=5$ or $x=-1$

$\therefore x=5$ and $x=-1$ are the two roots of the given quadratic equation.

23. $\frac{2x}{x-4}+\frac{2x-5}{x-3}=\frac{25}{3}$

Sol:

We have, $\frac{2x}{x-4}+\frac{2x-5}{x-3}=\frac{25}{3}$

$\Rightarrow \frac{2x(x-3)7(x-4)(2x-5)}{(x-4)(x-3)}=\frac{25}{3}$

$\Rightarrow \frac{2x^2-6x+2x^2-5x-5x+20}{x^2-4x-3x+12}=\frac{25}{3}$

$\Rightarrow \frac{4x^2-19x+20}{x^2-7x+12}=\frac{25}{3}$

$\Rightarrow 3(4x^2-19x+20)=25(x^2-7x+12)$

$\Rightarrow 12x^2-57x+60=25x^2-175x+300$

$\Rightarrow 25x^2-12x^2-175x+57x+300-60=0$

$\Rightarrow 13x^2-118x+240=0$

$\Rightarrow 13x^2-78x-40x+240=0$

$[\because 13\times 240=3120\Rightarrow 3180=-78\times 40 \text{ and } -118=-78-40]$

$\Rightarrow 13x(x-6)-40(x-6)=0$

$\Rightarrow (x-6)(13x-40)=0$

$\Rightarrow x-6=0 \text{ or } 13x-40=0$

$\Rightarrow x=6 \text{ or } x=\frac{40}{13}$

$\therefore x=6$ and $x=\frac{40}{13}$ are the two roots of the given equation.

24. $\frac{x+3}{x-2}-\frac{1-x}{x}=\frac{17}{4}$

Sol:

We have,

$\frac{x+3}{x-2}-\frac{1-x}{x}=\frac{17}{4}$

$\Rightarrow \frac{x(x+3)-(x-2)(1-x)}{x(x-2)}=\frac{17}{4}$

$\Rightarrow \frac{x^2+3x-(x-x^2-2+2x)}{x^2-2x}=\frac{17}{4}$

$\Rightarrow \frac{x^2+3x-x+x^2+2-2x}{x^2-2x}=\frac{17}{4}$

$\Rightarrow \frac{2x^2+2}{x^2-2x}=\frac{17}{4}$

$\Rightarrow 4\left(2x^2+2\right)=17\left(x^2-2x\right)$

$\Rightarrow 8x^2+8=17x^2-34x$

$\Rightarrow 8x^2+8=17x^2-34x$

$\Rightarrow \left(17-8\right)x^2-34x-8=0$

$\Rightarrow 9x^2-34x-8=0$

$\left[9\times -8=-72\Rightarrow -72=-36\times 2 \text{ and } -34=-36+2\right]$

$\Rightarrow 9x^2-36x+2x-8=0$

$\Rightarrow 9x\left(x-4\right)+2\left(x-4\right)=0$

$\Rightarrow \left(x-4\right)\left(9x+2\right)=0$

$\Rightarrow \left(x-4\right)=0$ or $9x+2=0$

$\Rightarrow x=4$ or $x=-\frac{8}{9}$

$\therefore x=4$ and $x=-\frac{8}{9}$ are the two roots of the given equations

25. $\frac{x-3}{x+3}-\frac{x+3}{x-3}=\frac{48}{7}, x\neq 3, x\neq -3$

Sol:

$-4, \frac{9}{4}$

26. $\frac{1}{x-2}+\frac{2}{x-1}=\frac{6}{x}, x\neq 0$

Sol:

We have, $\frac{1}{x-2}+\frac{2}{x-1}=\frac{6}{x}, x\neq 0$

$\Rightarrow \frac{\left(x+1\right)+1\left(x-2\right)}{\left(x-2\right)\left(x-1\right)}=\frac{6}{x}$

$\Rightarrow \frac{x-1+2x-4}{x^2-2x-x+2}=\frac{6}{x}$

$\Rightarrow \frac{3x-5}{x^2-3x+2}=\frac{6}{x}$

$\Rightarrow x\left(3x-5\right)=6\left(x^2-3x+2\right)$

$\Rightarrow 3x^2-5x=6x^2-16x+12$

$\Rightarrow 3x^2-18x+5x+18=0$

$\Rightarrow 3x^2-13x+18=0$

$[\because 3\times 18=36 \Rightarrow -9x-4 \text{ and } -13=-9-4]$

$\Rightarrow 3x^2-9x-4x+12=0$

$\Rightarrow 3x(x-3)-4(x-3)=0$

$\Rightarrow (x-3)(3x-4)=0$

$\Rightarrow x-3=0 \text{ or } 3x-4=0$

$\Rightarrow x=3 \text{ or } x=\frac{4}{3}$

$\therefore x=3$ and $x=\frac{4}{3}$ are the two roots of the given equation

27. $\frac{x+1}{x-1}-\frac{x-1}{x+1}=\frac{5}{6}, x\neq 1$ and $x\pm -1$

Sol:

We have

$\frac{x+1}{x-1}-\frac{x-1}{x+1}=\frac{5}{6}, x\neq 1$ and $x\pm -1$

$\Rightarrow \frac{(x+1)(x+1)-(x-1)(x-1)}{(x-1)(x+1)}=\frac{5}{6}$

$\Rightarrow \frac{(x+1)^2-(x-1)^2}{x^2-1^2}=\frac{5}{6}$

$\Rightarrow \frac{4\times x\times 1}{x^2-1}=\frac{5}{6}$ $\quad [\because (x+b)^2-(a-b)^2=4ab \text{ and } (a-b)(a+b)=a^2-b^2]$

$\Rightarrow 6(4x)=5(x^2-1)$

$\Rightarrow 24x=5x^2-5$

$\Rightarrow 5x^2-5-24x=0$

$\Rightarrow 5x^2-24x-5=0$ $\quad [\because 5x-5=-25 \Rightarrow -25=-25\times 1-24=-25+1]$

$\Rightarrow 5x^2-25x+x-5=0$

$\Rightarrow 5x(x-5)+1(x-5)=0$

$\Rightarrow (x-5)(5x+1)=0$

$\Rightarrow x-5=0 \text{ or } 5x+1=0$

$\Rightarrow x=5 \text{ or } x=-\frac{1}{5}$

$\therefore x=5$ and $x=-\frac{1}{5}$ are the two roots of the given equation.

28. $\frac{x-1}{2x+1}-\frac{2x+1}{x-1}=\frac{5}{2}, x\neq-\frac{1}{2},1$

Sol:

We have

$\frac{x-1}{2x+1}-\frac{2x+1}{x-1}=\frac{5}{2}, x\neq-\frac{1}{2},1$

$\Rightarrow \frac{(x-1)(x-1)-(2x+1)(2x+1)}{(2x+1)(x+1)}=\frac{5}{2}$

$\Rightarrow \frac{(x-1)^2+(2x+1)^2}{2x^2-2x+x-1}=\frac{5}{2}$

$\Rightarrow \frac{x^2-2x+1+4x^2+4x+1}{2x^2-x-1}=-\frac{5}{2}$ $\left[\because (a+b)^2=a^2+b^2+2ab,(a-b)^2=a^2+b^2-2ab\right]$

$\Rightarrow \frac{5x^2+2x+2}{2x^2-x-1}=\frac{5}{2}$

$\Rightarrow 2(5x^2+2x+2)=5(2x^2-x-1)$

$\Rightarrow 10x^2+4x+4=10x^2-5x-5$

$\Rightarrow 4x+5x+4+5=0$

$\Rightarrow 9x+9=0$

$\Rightarrow 9x=-9$

$\Rightarrow \boxed{x=-1}$

$\therefore x=-1$ is the only root for the given equation

29. $3x^2-14x-5=0$

Sol:

We have, $3x^2-14x-5=0$

$\Rightarrow 3x^2-15x+x-5=0$

$\Rightarrow 3x(x-5)+1(x-5)=0$ $[\therefore 3\times-5=-15\Rightarrow-15=-15\times1 \ and \ +4=+5+1]$

$\Rightarrow (x-5)(3x+1)=0$

$\Rightarrow x-5=0$ or $3x+1=0$

$\Rightarrow x=5$ or $x=-\frac{1}{3}$

$\therefore$ $x=5$ and $x=-\frac{1}{3}$ are the two roots of the given quadratic equation

30. $\frac{m}{n}x^2+\frac{n}{m}=1-2x$

Sol:

We have given,

$\frac{m}{n}x^2+\frac{n}{m}=1-2x$

$\Rightarrow \frac{m^2x^2+n^2}{mn}=1-2x$

$\Rightarrow m^2x^2+2mnx+\left(n^2-mn\right)=0$

Now we solve the above quadratic equation using factorization method.

Therefore,

$\left[m^2x^2+mnx+m\sqrt{mnx}\right]+\left[mnx-m\sqrt{mnx}+\left(n+\sqrt{mn}\right)\left(n-\sqrt{mn}\right)\right]=0$

$\Rightarrow\left[m^2x^2+mnx+m\sqrt{mnx}\right]+\left[(mx)\left(n-\sqrt{mn}\right)+\left(n+\sqrt{mn}\right)\left(n-\sqrt{mn}\right)\right]=0$

$\Rightarrow(mx)\left(mx+n+\sqrt{mn}\right)+\left(n-\sqrt{mn}\right)\left(mx+n+\sqrt{mn}\right)=0$

$\Rightarrow\left(mx+n+\sqrt{mn}\right)\left(mx+n-\sqrt{mn}\right)=0$

Now, one of the products must be equal to zero for the whole product to be zero. Hence we equate both the product to zero. In order to find the value of x. Therefore,

$mx+n+\sqrt{mn}=0$

$\Rightarrow mx=-n-\sqrt{mn}$

$\Rightarrow x=\frac{-n-\sqrt{mn}}{m}$

Or

$mx+n-\sqrt{mn}=0$

$\Rightarrow mx=-n+\sqrt{mn}$

$\Rightarrow x=\frac{-n+\sqrt{mn}}{m}$

Hence $x=\frac{-n-\sqrt{mn}}{m}$ or $n=\frac{-n+\sqrt{mn}}{m}$.

31. $\frac{x-a}{x-b}+\frac{x-b}{x-a}=\frac{a}{b}+\frac{b}{a}$

Sol:

We have,

$\frac{x-a}{x-b}+\frac{x-b}{x-a}=\frac{a}{b}+\frac{b}{a}$

$$\Rightarrow \frac{(x-a)(x-a)+(x-b)(x-b)}{(x-b)(x-a)}=\frac{a^2+b^2}{ab}$$

$$\Rightarrow \frac{(x-a)^2+(x-b)^2}{x^2-ax-bx+ab}=\frac{a^2+b^2}{ab}$$

$$\Rightarrow \frac{x^2-2ax+a^2+x^2-2bx+b^2}{x^2-(a+b)x+ab}=\frac{a^2+b^2}{ab}$$

$$\Rightarrow \left(2x^2-2x(a+b)+a^2+b^2\right)ab=\left(a^2+b^2\right)\left(x^2-(a+b)x+ab\right)$$

$$\Rightarrow 2abx^2-2abx(a+b)+ab\left(a^2+b^2\right)=\left(a^2+b^2\right)x^2-\left(a^2+b^2\right)(a+b)x+ab\left(a^2+b^2\right)$$

$$\Rightarrow \left(a^2+b^2-2ab\right)x^2-(a+b)\left(a^2+b^2-2ab\right)x=0$$

$$\Rightarrow (a-b)^2x^2-(a+b)(a-b)^2x=0$$

$$\Rightarrow (a-b)^2\left(x-(a+b)\right)=0$$

$$\Rightarrow x\left(x-(a+b)\right)=0$$

$\Rightarrow x=0$ or $x-(a+b)=0 \Rightarrow x=a+b$

$\therefore x=0$ and $x=(a+b)$ are the two roots of the equation

32. $$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}+\frac{1}{(x-3)(x-4)}=\frac{1}{6}$$

Sol:

We have,

$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}+\frac{1}{(x-3)(x-4)}=\frac{1}{6}$$

$$\Rightarrow \frac{(x-3)(x-4)+(x-1)(x-4)+(x-1)(x-8)}{(x-1)(x-2)(x-3)(x-4)}=\frac{1}{6}$$

$$\Rightarrow \frac{(x-3)(x-4)+(x-1)\left[(x-4)+(x-8)\right]}{(x-1)(x-2)(x-3)(x-4)}=\frac{1}{6}$$

$$\Rightarrow \frac{(x-3)(x-4)+(x-1)(2x-6)}{(x-1)(x-2)(x-3)(x-4)}=\frac{1}{6}$$

$$\Rightarrow \frac{(x-3)(x-4)+(x-1)\times 2(x-3)}{(x-1)(x-2)(x-3)(x-4)}=\frac{1}{6}$$

$$\Rightarrow \frac{(x-3)\left[x-4+2x-2\right]}{(x-1)(x-2)(x-3)(x-4)}=\frac{1}{6}$$

$\Rightarrow \frac{(x-3)(3x-6)}{(x-1)(x-2)(x-3)(x-4)} = \frac{1}{6}$

$\Rightarrow \frac{(x-3)(x-2)\times 3}{(x-1)(x-2)(x-3)(x-4)} = \frac{1}{6}$

$\Rightarrow \frac{3}{(x-1)(x-4)} = \frac{1}{6}$

$\Rightarrow (x-1)(x-4) = 3\times 6$

$\Rightarrow x^2 - 4x - x + 4 = 18$

$\Rightarrow x^2 - 5x - 14 = 0$ $[\because -14 = -7\times 8 \text{ and } -5 = -7+8]$

$\Rightarrow x^2 - 7x + 8x - 14 = 0$

$\Rightarrow x(x-7) + 8(x-7) = 0$

$\Rightarrow (x-7)(x+8) = 0$

$\Rightarrow x-7=0$ or $x+2=0$

$\Rightarrow x=7$ or $x=-2$

$\therefore x=7$ and $x=-8$ are the two roots of the given equation.

33. $(x-5)(x-6) = \frac{25}{(24)^2}$

Sol:

We have,

$(x-5)(x-6) = \frac{25}{(24)^2}$

$\Rightarrow x^2 - 5x - 6x + 30 - \frac{25}{(24)^2} = 0$

$\Rightarrow x^2 - 11x + \frac{30(24)^2 - 25}{(24)^2} = 0$

$\Rightarrow x^2 - 11x + \frac{17280 - 25}{(2)^2} = 0$

$\Rightarrow x^2 - 11x + \frac{17255 - 25}{(24)^2} = 0$

$\Rightarrow x^2 - 11x + \frac{119}{24}\times\frac{145}{24} = 0$ $[\because 17255 = 145\times 119]$

$\Rightarrow x^2 - \frac{264}{24}x + \frac{119}{24}\times\frac{145}{24} = 0$ $[\because 11\times 24 = 264]$

$\Rightarrow x^2 - \left(\frac{119+145}{24}\right)x + \frac{119}{24} \times \frac{145}{24} = 0$

$\Rightarrow x^2 - \frac{119}{24}x - \frac{145}{24}x + \frac{119}{24} \times \frac{145}{24} = 0$

$\Rightarrow x\left(x - \frac{119}{24}\right) - \frac{145}{24}\left(x - \frac{119}{24}\right) = 0$

$\Rightarrow \left(x - \frac{119}{24}\right)\left(x - \frac{145}{24}\right) = 0$

$\Rightarrow x - \frac{119}{24} = 0$ and $x - \frac{145}{25} = 0$

$\Rightarrow x = \frac{119}{24}$ or $x = \frac{145}{24}$

$\Rightarrow x = 4\frac{23}{24}$ or $x = 6\frac{1}{24}$

$\therefore x = 4\frac{23}{24}$ and $x = 6\frac{1}{24}$ are the two roots of the given equation.

34. $7x + \frac{3}{x} = 35\frac{3}{5}$

Sol:

We have, $7x + \frac{3}{x} = 35\frac{3}{5}$

$\Rightarrow \frac{7x^2 + 3}{x} = 35 + \frac{3}{5}$

$\Rightarrow 7x^2 + 3 = \left(35 + \frac{3}{5}\right)x$

$\Rightarrow 7x^2 - \left(35 + \frac{3}{5}\right)x + 3 = 0$

$\Rightarrow 7x^2 - 35x - \frac{3}{5}x + 3 = 0$

$\Rightarrow 7x^2 - 35x - \frac{1}{5}(3x - 3 \times 5) = 0$

$\Rightarrow 7x(x-5) - \frac{3}{5}(x-5) = 0$

$\Rightarrow (x-5)\left(7x - \frac{3}{5}\right) = 0$

$\Rightarrow (x-5) = 0$ or $7x - \frac{3}{5} = 0$

$\Rightarrow x=5$ or $7x=\frac{3}{5} \Rightarrow x=\frac{3}{35}$

$\therefore x=5$ and $x=\frac{3}{35}$ are the two roots of the given equation.

35. $\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}$

Sol:

We have,

$\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}$

$\Rightarrow \frac{a(x-b)+b(x-a)}{(x-a)(x-b)}=\frac{2c}{x-c}$

$\Rightarrow \frac{ax-ab+bx-ab}{x^2-ax-bx+ab}=\frac{2c}{x-c}$

$\Rightarrow (x-c)((a+b)x-2ab)=2c(x^2-(a+b)x+ab)$

$\Rightarrow (a+b)x^2-2abx-(a+b)c\ x+2abc=2cx^2-2c(a+b)x+2abc$

$\Rightarrow (a+b-8c)x^2-2abx-(a+b)x\,c+8c(a+b)x=0$

$\Rightarrow (a+b-8c)x^2+x(-8ab-ac-bc+8ac+8bc)=0$

$\Rightarrow (a+b-8c)x^2+x(-2ab+ac+bc)=0$

$\Rightarrow x\left[x(a+b-2c)+(ac+bc-2ab)\right]=0$

$\Rightarrow x=0$ or $x(a+b-2c)+(ac+bc-8ab)=0$

$\Rightarrow x=0$ or $x=-\frac{(ac+bc-8ab)}{a+b-8c}$

$\Rightarrow x=\frac{8ab-ac-bc}{a+b-2c}$

$\therefore x=0$ and $x=\frac{2ab-ac-bc}{a+b-8c}$ are the two roots of the given equation.

$\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}$

$\Rightarrow \frac{a(x-b)+b(x-a)}{(x-a)(x-b)}=\frac{2c}{x-c}$

$\Rightarrow \frac{ax-ab+bx-ab}{x^2-ax-bx+ab}=\frac{2c}{x-c}$

$\Rightarrow (x-c)((a+b)x-2ab)=2c(x^2-(a+b)x+ab)$

$\Rightarrow (a+b)x^2 - 2abx - (a+b)c\ x + 2abc = 2cx^2 - 2c(a+b)x + 2abc$

36. $x^2 + 2ab = (2a+b)x$

Sol:

We have

$x^2 + 2ab = (2a+b)x$

$\Rightarrow x^2 - (2a+b)x + 2ab = 0 \qquad [\because 2ab = -8a \times -b \Rightarrow -(8a+b) = -8a-b]$

$\Rightarrow x^2 - 2ax - bx + 2ab = 0$

$\Rightarrow x - (x-8a) - b(x-2a) = 0$

$\Rightarrow (x-8a)(x-b) = 0$

$\Rightarrow x - 8a = 0$ or $x - b = 0$

$\Rightarrow x = 8a$ or $x = b$

$\therefore x = 8a$ and $x = b$ are the two roots of the given equation .

37. $(a+b)^2 x^2 - (4ab)x - (a-b)^2 = 0$

Sol:

We have,

$(a+b)^2 x^2 - (4ab)x - (a-b)^2 = 0$

$\Rightarrow (a+b)^2 x^2 - \left((a+b)^2 - (a-b)^2\right)x - (a-b)^2 = 0 \qquad \left[\because (a+b)^2 - (a-b)^2 = 4ab\right]$

$\Rightarrow (a+b)^2 x^2 - (a+b)^2 x + (a-b)^2 x - (a-b)^2 = 0$

$\Rightarrow (a+b)^2 x(x-1) + (a-b)^2 (x-1) = 0$

$\Rightarrow (x-1)\left((a+b)^2 x + (a-b)^2\right) = 0$

$\Rightarrow x - 1 = 0$ or $(a+b)^2 x + (a-b)^2 = 0$

$\Rightarrow x = 1$ or $x = -\frac{(a-b)^2}{(a+b)^2} = -\left[\frac{a-b}{a+b}\right]^2$

$\therefore x = 1$ and $x = -\left[\frac{a-b}{a+b}\right]^2$ are the two roots of the given equation

38. $a(x^2+1) - x(a^2+1) = 0$

Sol:

We have

$a(x^2+1) - x(a^2+1) = 0$

$\Rightarrow ax^2 - a^2x - x + a = 0$ $\quad \left[\because a \times a = a^2 \Rightarrow a^2 = -a^2 \times -1 - \left(a^2+1\right) = a^2 - 1\right]$

$\Rightarrow a \times (x - a) - 1(x - a) = 0$

$\Rightarrow (x - a)(ax - 1) = 0$

$\Rightarrow x - a = 0$ or $ax - 1 = 0$

$\Rightarrow x = a$ or $x = \frac{1}{a}$

$\therefore x = a$ and $x = \frac{1}{a}$ are the two roots of the given equation

39. $x^2 - x - x(a+1) = 0$

Sol:

We have,

$x^2 - x - x(a+1) = 0$

$\Rightarrow x^2 - (a+1-a)x - a(a+1) = 0$ $\quad \left[\because -a(a+1) = -(a+1) \times a - 1 = a - (a+1)\right]$

$\Rightarrow x^2 - (a+1)x + ax + ax\left(-(a+1)\right) = 0$

$\Rightarrow x\left(x - (a+1)\right) + a\left(x - (a+1)\right) = 0$

$\Rightarrow \left(x - (a+1)\right)(x + a) = 0$

$\Rightarrow x - (a+1) = 0$ or $x + a = 0$

$\Rightarrow x = a + 1$ or $x = -a$

$\therefore x = (a+1)$ and $x = -a$ are the two roots of the given equation.

40. $x^2 + \left(a + \frac{1}{a}\right)x + 1 = 0$

Sol:

We have,

$x^2 + \left(a + \frac{1}{a}\right)x + 1 = 0$

$\Rightarrow x^2 + ax + \frac{1}{a}x + a \times \frac{1}{a} = 0$ $\quad \left[\because 1 = a \times \frac{1}{a}\left(a + \frac{1}{a}\right)x = ax + \frac{1}{a}x\right]$

$\Rightarrow x(x + a) + \frac{1}{a}(x + a) = 0$

$\Rightarrow (x + a)\left(x + \frac{1}{a}\right) = 0$

$\Rightarrow x+a=0$ or $x+\frac{1}{a}=0$

$\Rightarrow x=-a$ or $x=-\frac{1}{a}$

$\therefore x=a$ and $x=-\frac{1}{a}$ are the two roots of the given equation.

41. $abx^2+\left(b^2-ac\right)x-bc=0$

Sol:

We have,

$abx^2+\left(b^2-ac\right)x-bc=0$

$\left[abx-bc=-ab^2c\Rightarrow -ab^2c=b^2\times -ac \text{ and } b^2-ac=b^2+(-ac)\right]$

$\Rightarrow abx^2+b^2x-acx-bc=0$

$\Rightarrow bx(ax+b)-c(ax+b)=0$

$\Rightarrow (ax+b)(bx-c)=0$

$\Rightarrow ax+b=0$ or $bx-c=0$

$\Rightarrow x=-\frac{b}{a}$ or $x=\frac{c}{b}$

$\therefore x=-\frac{b}{a}$ and $x=\frac{c}{b}$ are the two roots of the given equation

42. $a^2b^2x^2+b^2x-a^2x-1=0$

Sol:

We have, $a^2b^2x^2+b^2x-a^2x-1=0$

$\left[-1\times a^2b^2=-a^2b^2\Rightarrow -a^2b^2=-a^2\times b^2\right]$

$\Rightarrow a^2b^2x^2+b^2x-a^2x-1=0$

$\Rightarrow b^2\times\left(a^2x+1\right)-1\left(a^2x-1\right)=0$

$\Rightarrow \left(a^2x+1\right)\left(b^2x-1\right)=0$

$\Rightarrow a^2x+1=0$ or $b^2x-1=0$

$\Rightarrow x=-\frac{1}{a^2}$ and $x=\frac{1}{b^2}$ are the two root of te given equation

43. $\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}, x\neq 2$ and $x\neq 4$

Sol:

We have,

$\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}, x\neq 2$ and $x\neq 4$

$\Rightarrow \frac{(x-1)(x-4)+(x-3)(x-2)}{(x-2)(x-4)}=\frac{10}{3}$

$\Rightarrow \frac{x^2-x-4x+4+x^2-3x-2x+6}{x^2-2x-4x+8}=\frac{10}{3}$

$\Rightarrow \frac{2x^2-10x+10}{x^2-6x+8}=\frac{10}{3}$

$\Rightarrow 2(x^2-5x+5)\times 3=5(x^2-6x\times 8)$

$\Rightarrow 3x^2-15x+15=5x^2-30x+40$

$\Rightarrow 2x^2-30x+15x+40-15=0$

$\Rightarrow 2x^2-15x+25=0$

$\Rightarrow 2x^2-10x-5x+25=0$

$\Rightarrow 2x(x-5)-5(x-5)=0$

$\Rightarrow (x-5)(2x-5)=0$

$\Rightarrow (x-5)=0$ or $2x-5=0$

$\Rightarrow x=5$ or $x=\frac{5}{2}$

$\therefore x=5$ and $x=\frac{5}{2}$ are the two roots of the given equation

44. $3x^2-2\sqrt{6}x+2=0$

Sol:

We have $3x^2-2\sqrt{6}x+2=0$ Now we solve the above quadratic equation using factorization method.

Therefore

$3x^2-\sqrt{6}x-\sqrt{6}x+2=0$

$\Rightarrow \sqrt{3}x(\sqrt{3}x-\sqrt{2})-\sqrt{2}(\sqrt{3}x-\sqrt{2})=0$

$\Rightarrow (\sqrt{3}-\sqrt{2})(\sqrt{3}x-\sqrt{2})=0$

$\Rightarrow (\sqrt{3}x-\sqrt{2})=0$ or $(\sqrt{3}x-\sqrt{2})=0$

$\Rightarrow \sqrt{3}x = \sqrt{2}$ or $\sqrt{3}x = \sqrt{2}$

$\Rightarrow x = \frac{\sqrt{2}}{\sqrt{3}}$ or $x = \frac{\sqrt{2}}{\sqrt{3}}$

$\Rightarrow x = \sqrt{\frac{2}{3}}$ or $x = \sqrt{\frac{2}{3}}$

Hence $x = \sqrt{\frac{2}{3}}$ or $x = \sqrt{\frac{2}{3}}$

45. $\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}, x \neq 1, -5$

Sol:

We have

$\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}, x \neq 1, -5$

$\Rightarrow \frac{x+5-(x-1)}{(x-1)(x+5)} = \frac{6}{7}$

$\Rightarrow \frac{x-5-x+1}{x^2+5x-x-5} = \frac{6}{7}$

$\Rightarrow \frac{6}{x^2+4x-5} = \frac{6}{7}$

$\Rightarrow x^2+4x-5 = 7$

$\Rightarrow x^2+4x-5-7 = 0$

$\Rightarrow x^2+4x-18 = 0$

$\Rightarrow x^2+6x-2x-12 = 0$

$\Rightarrow x(x+6)-2(x+6) = 0$

$\Rightarrow x+6 = 0$ or $x-8 = 0$

$\Rightarrow x = -6$ or $x = 8$

$\therefore x = -6$ and $x = 8$ are the two roots of the given equation.

46. $\frac{1}{x} - \frac{1}{x-2} = 3, x \neq 0, 2$

Sol:

We have,

$\frac{1}{x} - \frac{1}{x-2} = 3, x \neq 0, 2$

$\Rightarrow \frac{x-2-x}{x(x-2)} = 3$

$\Rightarrow -2 = 3\left(x^2 - 2x\right)$

$\Rightarrow 3x^2 - 6x + 2 = 0$

$\Rightarrow 3x^2 - (3+3)x + 2 = 0$

$\Rightarrow 3x^2 - \left(3 + \sqrt{3} + 3\sqrt{3}\right)x + (3-1) = 0$

$\Rightarrow 3x^2 - \sqrt{3}\left(\sqrt{3}+1\right)x - \sqrt{3}\left(\sqrt{3}-1\right)x + \left[\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)\right] = 0$

$\left[\because a^2 - b^2 = (a+b)(a-b)\right]$

$\Rightarrow \left(\sqrt{3}\right)^2 x^2 - \sqrt{3}\left(\sqrt{3}-1\right)x + \left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right) = 0$

$\Rightarrow \sqrt{3}\left(\sqrt{3}+1\right) - \left(\sqrt{3}-1\right)\left(\sqrt{3}x - \left(\sqrt{3}+1\right)\right) = 0$

$\Rightarrow \left[\sqrt{3}x\left(\sqrt{3}+1\right)\right]\left[\sqrt{3}x - \left(\sqrt{3}-1\right)\right] = 0$

$\Rightarrow \sqrt{3}x - \left(\sqrt{3}+1\right) = 0$ or $\sqrt{3}x - \left(\sqrt{3}-1\right) = 0$

47. $x - \dfrac{1}{x} = 3, x \neq 0$

Sol:

We have,

$x - \dfrac{1}{x} = 3, x \neq 0$

$\Rightarrow \dfrac{x^2 - 1}{x} = 3$

$\Rightarrow x^2 - 1 = 3x$

$\Rightarrow x^2 - 3x - 1 = 0$

$\Rightarrow x^2 - \left(\dfrac{3}{2} + \dfrac{3}{2}\right)x - 1 = 0$

$\Rightarrow x^2 - \left(\dfrac{3+\sqrt{13}}{2} + \dfrac{3-\sqrt{13}}{2}\right)x + (-1) = 0$

$\Rightarrow x^2 - \left(\dfrac{3+\sqrt{13}}{2}\right)x - \left(\dfrac{3-\sqrt{13}}{2}\right)x + (-1) = 0$

$\Rightarrow x^2 - \left(\dfrac{3+\sqrt{13}}{2}\right)x - \left(\dfrac{3-\sqrt{13}}{2}\right)x + \left(\dfrac{-4}{4}\right) = 0$

$\Rightarrow x^2 - \left(\dfrac{3+\sqrt{13}}{2}\right)x - \left(\dfrac{3-\sqrt{13}}{2}\right)x + \left(\dfrac{9-13}{4}\right) = 0$

$$\Rightarrow x^2-\left(\frac{3+\sqrt{13}}{2}\right)x-\left(\frac{3-\sqrt{13}}{2}\right)x+\left(\frac{3^2-\left(\sqrt{13}\right)^2}{2^2}\right)=0$$

$$\Rightarrow x^2-\left(\frac{3+\sqrt{13}}{2}\right)x-\left(\frac{3-\sqrt{13}}{2}\right)x+\frac{\left(3+\sqrt{13}\right)}{2}+\frac{\left(3-\sqrt{13}\right)}{2}=0$$

$$\Rightarrow x\left(x-\left(\frac{3+\sqrt{13}}{2}\right)\right)-\left(\frac{3-\sqrt{13}}{2}\right)\left(x-\left(\frac{3+\sqrt{13}}{2}\right)\right)=0$$

$$\Rightarrow\left(x-\frac{3+\sqrt{13}}{2}\right)\left(x-\frac{3-\sqrt{13}}{2}\right)=0$$

$$\Rightarrow x-\left(\frac{3+\sqrt{13}}{2}\right)=0 \text{ or } x-\left(\frac{3-\sqrt{13}}{2}\right)=0$$

$$\Rightarrow x=\frac{3+\sqrt{13}}{2} \text{ or } x=\frac{3-\sqrt{13}}{2}$$

$\therefore x=\frac{3+\sqrt{13}}{2}$ and $x=\frac{3-\sqrt{13}}{2}$ are the two roots of the given equation.

48. $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}, x\neq 4,7$

Sol:

We have,

$$\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}, x\neq 4,7$$

$$\Rightarrow\frac{x-7-(x+4)}{(x+4)(x+7)}=\frac{11}{30}$$

$$\Rightarrow\frac{x-7-x-4}{x^2-7x+4x-28}=\frac{11}{30}$$

$$\Rightarrow\frac{-11}{x^2-3x-28}=\frac{11}{30}$$

$$\Rightarrow(-1)\times 30=1\times\left(x^2-3x-28\right)$$

$$\Rightarrow -30=x^2-3x-28$$

$$\Rightarrow x^2-3x-28+30=0$$

$$\Rightarrow x^2-3x+2=0 \qquad [\because 2=-2\times -1-3=-2-1]$$

$$\Rightarrow x^2-2x-x+2=0$$

$\Rightarrow x(x-8)-1(x-8)=0$

$\Rightarrow (x-8)(x-1)=0$

$\Rightarrow x-8=0$ or $x-1=0$

$\Rightarrow x=8$ or $x=1$

$\therefore x=2$ and $x=1$ are the two roots of the given equation.

Exercise 8.4

Find the roots of the following quadratic equations (if they exist) by the method of completing the square.

1. $x^2-4\sqrt{2}\ x+6=0$

Sol:

We have,

$x^2-4\sqrt{2}\ x+6=0$

$\Rightarrow x^2-2\times x\times 2\sqrt{2}+\left(2\sqrt{2}\right)^2-\left(2\sqrt{2}\right)^2+6=0$

$\Rightarrow \left(x-2\sqrt{2}\right)^2=\left(2\sqrt{2}\right)^2-6$

$\Rightarrow \left(x-2\sqrt{2}\right)^2=(4\times 2)-6$

$\Rightarrow \left(x-2\sqrt{2}\right)^2=8-6$

$\Rightarrow \left(x-2\sqrt{2}\right)^2=2$

$\Rightarrow x-2\sqrt{2}=\pm\sqrt{2}$

$\Rightarrow x-2\sqrt{2}=\sqrt{2}$ or $x-2\sqrt{2}=-\sqrt{2}$

$\Rightarrow x=3\sqrt{2}$ or $x=\sqrt{2}$

$\therefore x=\sqrt{2}$ and $x=3\sqrt{2}$ are the roots of the given equation.

2. $2x^2-7x+3=0$

Sol:

We have,

$2x^2-7x+3=0$

$2\left(x^2-\frac{7}{2}x+\frac{3}{2}\right)=0$

$$\Rightarrow x^2 - 2\times\frac{7}{2}\times\frac{1}{2}\times x + \frac{3}{2} = 0$$

$$\Rightarrow x^2 - 2\times\frac{7}{4}\times x + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$$

$$\Rightarrow x^2 - 2\times\frac{7}{4}\times x + \left(\frac{7}{4}\right)^2 - \frac{49}{16} + \frac{3}{2} = 0$$

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \frac{49}{16} + \frac{3}{2} = 0$$

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{49}{16} - \frac{3}{2}$$

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{49-86}{16}$$

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$$

$$\Rightarrow x - \frac{7}{4} = \pm\left(\frac{5}{4}\right)^2$$

$$\Rightarrow x - \frac{7}{4} = \pm\frac{5}{4}$$

$$\Rightarrow x - \frac{7}{4} = \frac{5}{4} \text{ or } x - \frac{7}{4} = -\frac{5}{4}$$

$$\Rightarrow x = \frac{5}{4} + \frac{7}{9} \text{ or } x = \frac{7}{4} - \frac{5}{4}$$

$$\Rightarrow x = \frac{12}{4} \text{ or } x = \frac{2}{4}$$

$$\Rightarrow x = 3 \text{ or } x = \frac{1}{2}$$

$\therefore x = 3$ and $x = \frac{1}{2}$ are the roots of the given quadratic equation.

3. $3x^2 + 11x + 10 = 0$

Sol:

We have,

$$3x^2 + 11x + 10 = 0$$

$$\Rightarrow x^2 + \frac{11}{3}x + \frac{10}{3} = 0$$

$$\Rightarrow x^2+2\times\frac{1}{2}\times\frac{11}{3}x+\frac{10}{3}=0$$

$$\Rightarrow x^2+2\times\frac{11}{6}\times 2+\left(\frac{11}{6}\right)^2-\left(\frac{11}{6}\right)^2+\frac{10}{3}=0$$

$$\Rightarrow\left(x+\frac{11}{6}\right)^2=\left(\frac{11}{6}\right)^2-\frac{10}{3}$$

$$\Rightarrow\left(x+\frac{11}{6}\right)^2=\frac{121}{36}-\frac{10}{3}$$

$$\Rightarrow\left(x+\frac{11}{6}\right)^2=\frac{121-120}{36}$$

$$\Rightarrow\left(x+\frac{11}{6}\right)^2=\frac{1}{36}$$

$$\Rightarrow\left(x+\frac{11}{6}\right)^2=\left(\frac{1}{6}\right)^2$$

$$\Rightarrow x+\frac{11}{6}=\pm\frac{1}{6}$$

$$\Rightarrow x+\frac{11}{6}=\frac{1}{6}\text{ or }x+\frac{11}{6}=-\frac{1}{6}$$

$$\Rightarrow x=\frac{1}{6}-\frac{11}{6}\text{ or }x=-\frac{1}{6}-\frac{11}{6}$$

$$\Rightarrow x=-\frac{10}{6}\text{ or }x=-\frac{12}{6}=-2$$

$$\Rightarrow x=-\frac{5}{3}\text{ or }x=-2$$

$\therefore x=-\frac{5}{3}$ or $x=-2$ are the two roots of the given equation.

4. $2x^2+x-4=0$

Sol:

We have,

$2x^2+x-4=0$

$$\Rightarrow 2\left(x^2+\frac{x}{2}-\frac{4}{2}\right)=0$$

$$\Rightarrow x^2+2\times\frac{1}{2}\times\frac{1}{2}\times x-2=0$$

$$\Rightarrow x^2+2x\times\frac{1}{4}\times x+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2-2=0$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2=\left(\frac{1}{4}\right)^2+2$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2=\frac{1}{16}+2$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2=\frac{1+2\times16}{16}$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2=\frac{1+32}{16}$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2=\frac{33}{16}$$

$$\Rightarrow \left(x+\frac{1}{4}\right)=\pm\sqrt{\frac{33}{16}}$$

$$\Rightarrow x+\frac{1}{4}=+\frac{\sqrt{33}}{4} \text{ or } x+\frac{1}{4}=-\frac{\sqrt{33}}{4}$$

$$\Rightarrow x=\frac{\sqrt{33}}{4}-\frac{1}{4} \text{ or } x=-\frac{\sqrt{33}}{4}-\frac{1}{4}$$

$$\Rightarrow x=\frac{\sqrt{33}-1}{4} \text{ or } x=\frac{\sqrt{33}-1}{4}$$

$\therefore x=\frac{\sqrt{33}-1}{4}$ or $x=-\frac{\sqrt{33}-1}{4}$ are the two roots of the given eqution

5. $2x^2+x+4=0$

Sol:

We have,

$$2x^2+x+4=0$$

$$\Rightarrow x^2+\frac{x}{2}+2=0$$

$$\Rightarrow x^2+2\times\frac{1}{2}\times\frac{1}{2}\times x+2=0$$

$$\Rightarrow x^2+2\times\frac{1}{4}\times x+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2-2$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2=\frac{1}{16}-2$$

$\Rightarrow \left(x+\frac{1}{4}\right)^2 = \frac{1-36}{16}$

$\Rightarrow \left(x+\frac{1}{4}\right)^2 = -\frac{31}{16}$

$\Rightarrow x+\frac{1}{4} = \pm\sqrt{\frac{-31}{16}}$

$\Rightarrow x+\frac{1}{4} = \frac{\sqrt{-31}}{4}$ or $x+\frac{1}{4} = -\frac{\sqrt{31}}{4}$

$\Rightarrow x = \frac{\sqrt{-31}-1}{4}$ or $x = \frac{-\sqrt{31}-1}{4}$

Since, $\sqrt{-31}$ is not a real number

$\therefore$ The roots are not real roots.

6. $4x^2 + 4\sqrt{3}x + 3 = 0$

Sol:

We have,

$4x^2 + 4\sqrt{3}x + 3 = 0$

$\Rightarrow x^2 + \frac{4\sqrt{3}}{4}x + \frac{3}{4} = 0$

$\Rightarrow x^2 + 2\times\frac{1}{2}\times\sqrt{3}\times x + \frac{3}{4} = 0$

$\Rightarrow x^2 + 2\times\frac{\sqrt{3}}{2}\times x + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + \frac{3}{4} = 0$

$\Rightarrow \left(x+\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} + \frac{3}{4} = 0$

$\Rightarrow \left(x+\frac{\sqrt{3}}{4}\right)^2 = 0$

$\Rightarrow x+\frac{\sqrt{3}}{2} = 0$ and $x+\frac{\sqrt{3}}{2} = 0$

$\Rightarrow x = \frac{-\sqrt{3}}{2}$ and $x - \frac{\sqrt{3}}{2}$

$\therefore x = \frac{-\sqrt{3}}{2}$ and $x = -\frac{\sqrt{3}}{2}$ are the two roots of the given equation as it is a perfect square.

7. $\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

Sol:

We have,

$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

$\Rightarrow x^2 - \frac{3x}{\sqrt{2}}x - \frac{2\sqrt{2}}{\sqrt{2}} = 0$

$\Rightarrow x^2 - \frac{3}{\sqrt{2}}x - 8 = 0$

$\Rightarrow x^2 - 2\times\frac{1}{2}\times\frac{3}{\sqrt{2}}x - 2 = 0$

$\Rightarrow x^2 - 2\times\frac{3}{2\sqrt{2}}\times x + \left(\frac{3}{2\sqrt{2}}\right)^2 - \left(\frac{3}{2\sqrt{2}}\right)^2 - 2 = 0$

$\Rightarrow \left(x - \frac{3}{2\sqrt{2}}\right)^2 = \frac{9}{8} + 2$

$\Rightarrow \left(x - \frac{3}{2\sqrt{2}}\right)^2 = \frac{9+16}{8}$

$\Rightarrow \left(x - \frac{3}{2\sqrt{2}}\right)^2 = \frac{25}{8}$

8. $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

Sol:

We have

$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

$\Rightarrow x^2 + \frac{10}{\sqrt{3}}x + \frac{7\sqrt{3}}{\sqrt{3}} = 0$

$\Rightarrow x^2 + 2\times\frac{1}{2}\times\frac{10}{\sqrt{3}}x + 7 = 0$

$\Rightarrow x^2 + 2\times\frac{5}{\sqrt{3}}\times x + \left(\frac{5}{\sqrt{3}}\right)^2 - \left(\frac{5}{\sqrt{3}}\right)^2 + 7 = 0$

$\Rightarrow \left(x + \frac{5}{\sqrt{3}}\right)^2 = \frac{25}{3} - 7$

$\Rightarrow \left(x + \frac{5}{\sqrt{3}}\right)^2 = \frac{25-21}{3}$

$\Rightarrow \left(x+\frac{5}{\sqrt{3}}\right)^2=\frac{4}{3}$

$\Rightarrow x+\frac{5}{\sqrt{3}}=\pm\sqrt{\frac{4}{3}}$

$\Rightarrow x+\frac{5}{\sqrt{3}}=+\frac{2}{\sqrt{3}}$ or $x+\frac{5}{\sqrt{3}}=\frac{-2}{\sqrt{3}}$

$\Rightarrow x=\frac{-3}{\sqrt{3}}$ or $x=-\frac{7}{\sqrt{3}}$

$\Rightarrow x=-\sqrt{3}$ or $x=-\frac{7}{\sqrt{3}}$

$\therefore x=-\sqrt{3}$ and $x=-\frac{7}{\sqrt{3}}$ are the roots of the given equation.

9. $x^2-\left(\sqrt{2}+1\right)x+\sqrt{2}=0$

Sol:

We have,

$x^2-\left(\sqrt{2}+1\right)x+\sqrt{2}=0$

$\Rightarrow x^2-2\times\frac{1}{2}\left(\sqrt{2}+1\right)x+\sqrt{2}=0$

$\Rightarrow x^2-2\times\frac{\sqrt{2}+1}{2}x+\left(\frac{\sqrt{2}-1}{2}\right)^2-\left(\frac{\sqrt{2}+1}{2}\right)^2+\sqrt{2}=0$

$\Rightarrow \left(x-\frac{\sqrt{2}+1}{2}\right)^2=\left(\frac{\sqrt{2}+1}{2}\right)^2-\sqrt{2}$

$\Rightarrow \left(x-\frac{\sqrt{2}+1}{2}\right)^2=\frac{3+2\sqrt{2}-4\sqrt{2}}{4}$

$\Rightarrow \left(x-\frac{\sqrt{2}+1}{2}\right)^2=\frac{3-2\sqrt{2}}{4}$

$\Rightarrow \left(x-\frac{\sqrt{2}+1}{2}\right)^2=\frac{2+1-2\sqrt{2}}{4}$

$\Rightarrow \left(x-\frac{\sqrt{2}+1}{2}\right)^2=\frac{\left(\sqrt{2}\right)^2-2\sqrt{2}+1}{2^2}$

$\Rightarrow\left(x-\frac{\sqrt{2}+1}{2}\right)^2=\frac{\left(\sqrt{2}-1^2\right)}{2^2}$

$\Rightarrow\left(x-\frac{\sqrt{2}+1}{2}\right)^2=\left(\frac{\sqrt{2}-1}{2}\right)^2$

$\Rightarrow x-\frac{\sqrt{2}+1}{2}=\pm\left(\frac{\sqrt{2}-1}{2}\right)$

$\Rightarrow x-\frac{\sqrt{2}+1}{2}=\frac{\sqrt{2}-1}{2}$ or $x-\frac{\sqrt{2}+1}{2}=-\frac{\sqrt{2}-1}{2}$

$\Rightarrow x=\frac{\sqrt{2}-1}{2}+\frac{\sqrt{2}+1}{2}$ or $x=\frac{\sqrt{2}-1}{2}+\frac{\sqrt{2}+1}{2}$

$\Rightarrow x=\frac{\sqrt{2}-1+\sqrt{2}+1}{2}$ or $x=\frac{-\sqrt{2}+1+\sqrt{2}+1}{2}$

$\Rightarrow x=\frac{\sqrt{2}}{2}$ or $x=\frac{1}{2}$

$\Rightarrow x=\sqrt{2}$ or $x=1$

$\therefore\ x=\sqrt{2}$ and $x=1$ are the roots of the given equation

10. $x^2-4ax+4a^2-b^2=0$

Sol:

We have,

$x^2-4ax+4a^2-b^2=0$

$\Rightarrow x^2-2\times(2a)\times x+(2a)^2-b^2=0$

$\Rightarrow (x-2a)^2=b^2$

$\Rightarrow x-2a=\pm b$

$\Rightarrow x-2a=b$ or $x-2a=-b$

$\Rightarrow x=2a+b$ or $x=2a-b$

$\therefore\ x=2a+b$ and $x=2a-b$ are the two roots of the given quadratic equation.

Exercise 8.5

1. Write the discriminant of the following quadratic equation:

(i) $2x^2-5x+3=0$ (ii) $x^2+2x+4=0$ (iii) $(x-1)(2x-1)=0$ (iv) $x^2-2x+k=0, K\in R$

(v) $\sqrt{3}x^2+2\sqrt{2}x-2\sqrt{3}=0$ (vi) $x^2-x+1=0$ (vii) $3x^2+2x+k=0$ (viii) $4x^2-3kx+1=0$

Sol: (i) $2x^2-5x+3=0$

The given equation is in the form of $ax^2+bx+c=0$

here $a=2, b=-5$ and $c=3$

The discriminant $\boxed{D=b^2-4ac}$

$\Rightarrow (-5)^2-4\times2\times3$

$\Rightarrow 25-24=1$

$\therefore$ The discriminant of the following quadratic equation is 1.

(ii) $x^2+2x+4=0$

The given equation is in the form of $ax^2+bx+c=0$

here $a=1, b=2$ and $c=4$

The discriminant is $\boxed{D=b^2-4ac}$

$\Rightarrow (2)^2-4\times1\times4$

$\Rightarrow 4-16=-12$

$\therefore$ The discriminant of the following quadratic equation is -12.

(iii) $(x-1)(2x-1)=0$

The given equation is $(x-1)(2x-1)=0$

By solving it, we get $2x^2-3x+1=0$

$\therefore$ This equation is in the form of $ax^2+bx+c=0$

here $a=2, b=-3, c=-1$

The discriminant is $\boxed{D=b^2-4ac}$

$\Rightarrow (-3)^2-4\times2\times1$

$\Rightarrow 9-8=1$

$\therefore$ The discriminant D, for the following quadratic equation is 1

(iv) $x^2-2x+k=0, K\in R$

The given equation is in the form of $ax^2+bx+c=0$

here $a=1, b=-2, c=k$ [given $k\in R$]

The discriminant is $\boxed{D = b^2 - 4ac}$

$\Rightarrow (-2)^2 - 4\times 1\times k$

$\Rightarrow 4-4k$

$\therefore$ The discriminant D, of the following quadratic equation is $4-4k$, where $K \in R$

(v) $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = \sqrt{3}, b = 2\sqrt{2}x$ and $c = -2\sqrt{3}$

The discriminant is $\boxed{D = b^2 - 4ac}$

$\Rightarrow \left(2\sqrt{2}\right)^2 - 4\times\sqrt{3}\times -2\sqrt{3}$

$\Rightarrow 8+24$

$\Rightarrow 32$

$\therefore$ The discriminant D, of the followig quadratic equation is 32.

(vi) $x^2 - x + 1 = 0$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = 1, b = -1$ and $c = 1$

The discriminant is $\boxed{D = b^2 - 4ac}$

$\Rightarrow (-1)^2 - 4\times 1\times 1$

$\Rightarrow 1-4=-3$

$\therefore$ The discriminant D, of the following quadratic equation is -3.

(vii) $3x^2 + 2x + k = 0$

The given equation has $3x^2 + 2x + k = 0$

here $a = 3, b = 2, c = k$

$\Rightarrow$ given that the quadratic equation has real roots.

i.e., $D = b^2 - 4ac \geq 0$

$\Rightarrow 4 - 4\times 3\times k \geq 0$

$\Rightarrow 4 - 12k \geq 0$

$\Rightarrow 4 \geq 12k$

$\Rightarrow 4 \leq \frac{4}{12}$

$\Rightarrow k \leq \frac{1}{3}$

The ‘k’ value should not exceed $\frac{1}{3}$ to have the real roots for the given equation.

(viii) $4x^2 - 3kx + 1 = 0$

The given equation has $4x^2 - 3k\,x + 1 = 0$

here $a = 3, b = 2, c = k$

given that quadratic equation has real roots

i.e., $D = b^2 - 4ac \geq 0$

$\Rightarrow (2)^2 - 4\times 3\times k \geq 0$

$\Rightarrow 4 - 4\times 3\times k \geq 0$

$\Rightarrow 4 - 12k \geq 0$

$\Rightarrow 4 \geq 12k$

$\Rightarrow k \leq \frac{12}{4} \Rightarrow k \leq 3$

2. In the following, determine whether the given quadratic equation have real roots and if so, find the roots:

(i) $16x^2 = 24x + 1$ (ii) $x^2 + x + 2 = 0$ (iii) $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$ (iv) $3x^2 - 2x + 2 = 0$

(v) $2x^2 - 2\sqrt{6}x + 3 = 0$ (vi) $3a^2x^2 + 8abx + 4b^2 = 0, a \neq 0$ (vii) $3x^2 + 2\sqrt{b}x - b = 0$

(viii) $x^2 - 2x + 1 = 0$ (ix) $2x^2 + 5\sqrt{3} + 6 = 0$ (x) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(xi) $2x^2 - 2\sqrt{2}x + 1 = 0$ (xii) $3x^2 - bx + 2 = 0$

Sol:

(i) $16x^2 = 24x + 1$

The given equation is in the form of $16x^2 - 24x - 1 = 0$

Hence, the equation is in the form of $ax^2 + bx + c = 0$

Here, $a = 16, b = -24, c = -1, D = b^2 - 4ac = (-24)^2 - 4\times 16\times -1 = 576 + 64 = 640 > 0$

As $D > 0,$ the given equation has real roots, given by

$$\boxed{\alpha = \frac{-b + \sqrt{D}}{2a}} \Rightarrow \frac{-(-24) + \sqrt{640}}{2\times 16} = \frac{3 + \sqrt{10}}{4}$$

$$\boxed{\beta = \frac{-b + \sqrt{D}}{2a}} \Rightarrow \frac{-(-24) + \sqrt{640}}{2\times 16} = \frac{3 - \sqrt{10}}{4}$$

$\therefore$ The roots of the equation are $\frac{3 \pm \sqrt{10}}{4}$

(ii) $x^2+x+2=0$

The given equation is in the form of $ax^2+bx+c=0$

$a=1,b=1,c=2.$

$D=b^2-4ac=(1)^2-4\times1\times2=1-8=-7<0$

As $Q<0,$ the equation has no real roots

(iii) $\sqrt{3}x^2+10x-8\sqrt{3}=0$

The given equation is in the form of $ax^2+bx+c=0$

here $a=\sqrt{3},b=10$ and $c=-8\sqrt{3}$

$\boxed{D=b^2-4ac}\Rightarrow(10)^2-4\times\sqrt{3}\times-8\sqrt{3}=100+96=19670$

As $Q>0,$ the given equation has real roots, given by

$\boxed{\alpha=\frac{-b+\sqrt{10}}{2a}}\Rightarrow\frac{10+\sqrt{196}}{2\times\sqrt{3}}=\frac{2\sqrt{3}}{3}=\frac{2}{\sqrt{3}}$ [$\because$ Multiplying and dividing by $\sqrt{3}$]

$\boxed{\beta=\frac{-b-\sqrt{10}}{2a}}\Rightarrow\frac{-10-\sqrt{196}}{2\times\sqrt{3}}=-4\sqrt{3}$

$\therefore$ The roots of the equation are $\frac{2}{\sqrt{3}}$ and $-4\sqrt{3}$

(iv) $3x^2-2x+2=0$

The given equation is in the form of $ax^2+bx+c=0$

here $a=3,b=-2,c=2$

The discriminant $\boxed{Q=b^2-4ac}$

$\Rightarrow(-2)^2-4\times3\times2=4-24$

$\Rightarrow-20<0$

Hence as $Q<0,$

The given equation has no reaal roots.

(v) $2x^2-2\sqrt{6}x+3=0$

The given equation is in the form of $ax^2+bx+c=0$

here $a=2,b=-2\sqrt{6},c=3$

The discriminant $\boxed{Q=b^2-4ac}$

$\Rightarrow\left(-2\sqrt{6}\right)^2-4\times2\times3=24-24$

$\Rightarrow0$

As $Q = 0$, the given equation has real and equal roots, They are

$$\boxed{\alpha = \frac{-b+\sqrt{D}}{2a}} \Rightarrow -\frac{\left(-2\sqrt{6}\right)+\sqrt{0}}{2\times 2} = \frac{\not{2}^1\sqrt{6}}{\not{4}_2} = \sqrt{\frac{3}{2}}$$

$$\boxed{\beta = \frac{-b-\sqrt{D}}{2a}} \Rightarrow -\frac{\left(-2\sqrt{6}\right)-\sqrt{0}}{2\times 2} = \frac{\not{2}^1\sqrt{6}}{\not{4}_2} = \sqrt{\frac{3}{2}}$$

$\therefore$ The roots of the given equation is $\sqrt{\frac{3}{2}}$

(vi) $3a^2x^2 + 8abx + 4b^2 = 0, a \neq 0$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = 3a^2, b = 8ab, c = 4b^2$ [given $a \neq 0$]

$$\boxed{D = b^2 - 4ac} = \left(8ab\right)^2 - 4\times 3a^2 \times 4b^2 = 64a^2b^2 - 48a^2b^2 = 16a^2b^2 > 0$$

As $Q = 0$, the given equation has real roots, given by

$$\boxed{\alpha = \frac{-b+\sqrt{D}}{2a}} \Rightarrow -\frac{\left(8ab\right)+\sqrt{16a^2b^2}}{2\times 3a^2} = \frac{-2b}{a}$$

$$\boxed{\beta = \frac{-b+\sqrt{D}}{2a}} \Rightarrow -\frac{\left(8ab\right)-\sqrt{16a^2b^2}}{2\times 3a^2} = \frac{-2b}{a}$$

$\therefore$ The roots of the given equation are $\frac{-2b}{a}, \frac{-2b}{3a}$

(vii) $3x^2 + 2\sqrt{b}x - b = 0$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = 3, b = 2\sqrt{5}, c = -5$

The discriminant $\boxed{Q = b^2 - 4ac}$

$$\Rightarrow \left(2\sqrt{5}\right)^2 - 4\times 3\times -5 = 20 + 4\times 3\times 5$$

$$\Rightarrow 20 + 60 - 80 > 0$$

As $Q = 0$, the given equation has real roots, given by

$$\boxed{\alpha = \frac{-b+\sqrt{D}}{2a}} \Rightarrow -\frac{-\left(2\sqrt{5}\right)+\sqrt{80}}{2\times 3} = \frac{\sqrt{5}}{3}$$

$$\boxed{\beta = \frac{-b+\sqrt{D}}{2a}} \Rightarrow -\frac{-\left(2\sqrt{5}\right)-\sqrt{80}}{2\times 3} = -\frac{\sqrt{5}}{3}$$

$\therefore$ The roots of the given equation is $\sqrt{\frac{3}{2}}$

(viii) $x^2 - 2x + 1 = 0$

The given equation is in the form of $ax^2 + bx + c = 0$

Here $a = 1, b = -2$ and $c = 1$

The discriminant $\boxed{D = b^2 - 4ac}$

$\Rightarrow (-2)^2 - 4\times1\times1 = 0$

As $Q = 0$, the given equation has real and equal roots

$\Rightarrow \alpha = -\frac{b+\sqrt{D}}{2a}, \beta = -\frac{b-\sqrt{D}}{2a}$ i.e., α and $\beta = -\frac{b}{2a} [\because 0 = 0]$

$\Rightarrow \alpha$ and $\beta = -\frac{b}{2a} = -\frac{(-2)}{2\times2} = \frac{2}{2} = 1$

$\therefore$ The roots of the given equation α and β is 1.

(ix) $2x^2 + 5\sqrt{3} + 6 = 0$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = 2, b = 5\sqrt{3}, c = 6$

The discriminant $\boxed{D = b^2 - 4ac}$

$\Rightarrow \left(5\sqrt{3}\right)^2 - 4\times2\times6 = 75 - 48$

$\Rightarrow 27 > 0$

As $Q = 0$, the given equation has real roots, given by

$$\boxed{\alpha = \frac{-b+\sqrt{D}}{2a}} \Rightarrow \frac{-\left(5\sqrt{3}\right)+\sqrt{27}}{2\times2} = \frac{\sqrt{3}(-5+3)}{4} = \frac{\sqrt{3}\times\cancel{-2}^{1}}{\cancel{4}_{2}} = \frac{\sqrt{3}}{2}$$

$$\boxed{\beta = \frac{-b+\sqrt{D}}{2a}} \Rightarrow \frac{-\left(5\sqrt{3}\right)-\sqrt{27}}{2\times2} = \frac{-\sqrt{3}[5+3]}{4} = \frac{\cancel{-8}^{2}}{\cancel{4}_{1}}\sqrt{3} = -2\sqrt{3}$$

$\therefore$ The roots of the given equation are $-\frac{\sqrt{3}}{2}, -2\sqrt{3}$

(x) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = \sqrt{2}, b = 7, c = 5\sqrt{2}$

The discriminant $\boxed{Q = b^2 - 4ac}$

$\Rightarrow (7)^2 - 4\times\sqrt{2}\times 5\sqrt{2} = 49 - 40$

$\Rightarrow 9 > 0$

As $0 = 0$, the given equation has real roots, given by

$\boxed{\alpha = \frac{-b+\sqrt{D}}{2a}} \Rightarrow \frac{-7+\sqrt{9}}{2\times\sqrt{2}} = \frac{-7+\sqrt{3}}{2\sqrt{2}} = \frac{\cancel{4}^{2}}{\cancel{2}\sqrt{2}} = -\sqrt{2}$

$\boxed{\beta = \frac{-b+\sqrt{D}}{2a}} \Rightarrow \frac{-7+\sqrt{9}}{2\times\sqrt{2}} = \frac{-7-\sqrt{3}}{2\sqrt{2}} = \frac{5}{2\sqrt{2}} = -\frac{5}{\sqrt{2}}$

$\therefore$ The roots of the given equation are $-\sqrt{2}, \frac{-5}{\sqrt{2}}$

(xi) $2x^2 - 2\sqrt{2}x + 1 = 0$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = 2, b = -2\sqrt{2}, c = 1$

$\boxed{D = b^2 - 4ac} \Rightarrow \frac{\left(-2\sqrt{2}\right)}{2\times 2} = \frac{\cancel{2}^{1}\sqrt{2}}{\cancel{4}_{2}} = \frac{1}{\sqrt{2}}$

as $D > 0$, the given equation has Real and equal roots

hence $\boxed{\alpha \text{ and } \beta = -\frac{b}{2a}} = -\frac{\left(-2\sqrt{2}\right)}{2\times 2} = \frac{\cancel{2}^{1}\sqrt{2}}{\cancel{4}_{2}} = \frac{1}{\sqrt{2}}$

$\therefore$ The roots $\frac{1}{\sqrt{2}}$ is obtained by multiplying and dividing $\frac{\sqrt{3}}{2}$ by $\sqrt{2}$

$\therefore$ The roots of the given equation is $\frac{1}{\sqrt{2}}$

(xii) $3x^2 - bx + 2 = 0$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a = 3, b = -5, c = 2$

$\boxed{D = b^2 - 4ac} \Rightarrow (-5)^2 - 4\times 3\times 2 = 25 - 24 = 1 > 0$

as $D > 0$, the given equation has Real roots, giving by

$\boxed{\alpha = -\frac{b+\sqrt{D}}{2a}} \Rightarrow \frac{-(-5)+\sqrt{1}}{2\times 3} = \frac{5+\sqrt{1}}{6} = \frac{6^{1}}{6} = 1$

$\boxed{\beta = -\frac{b-\sqrt{D}}{2a}} \Rightarrow \frac{-(-5)-\sqrt{1}}{2\times 3} = \frac{5-\sqrt{1}}{6} = \frac{\cancel{4}^{2}}{\cancel{6}_{3}} = \frac{2}{3}$

$\therefore$ The roots of the given equation are 1 and $\frac{2}{3}$

3. Solve for $x: \frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}; x\neq 2,4$

Sol: Given,

$$\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}$$

$$\Rightarrow \frac{(x-1)(x-4)+(x-3)(x-2)}{(x-2)(x-4)}=\frac{10}{3}$$ [Solving improper fraction]

$$\Rightarrow \frac{\left(x^2-5x+4\right)+\left(x^2-5x+6\right)}{(x-2)(x-4)}=\frac{10}{3}$$

This can also be written as. .

$$3\left[2x^2-10x+10\right]=10\left[(x-2)(x-4)\right]$$

$$\Rightarrow 6x^2-30x+30=10x^2-60x+80$$

By solving them, by taking all to one side, we get

$$\Rightarrow \left(10x^2-60x+80\right)-\left(6x^2-30x+30\right)=0$$

$$\Rightarrow 4x^2-30x+50=0,\text{ here } a=4, b=-30, c=50$$

Hence we get x by $\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$

$$\Rightarrow x=-\frac{(-30)+\sqrt{(-30)^2-4\times4\times50}}{2(4)}=5$$

$$x=-\frac{-(30)-\sqrt{(-30)^2-4\times4\times50}}{2\times4}=\frac{5}{2}$$

$\therefore$ The value of x are 5 and $\frac{5}{2}$

4. Solve for $x: \frac{1}{x}-\frac{1}{x-7}=3, x\neq 0,2$

Sol: Given $\frac{1}{x}-\frac{1}{x-2}=3$

$$\Rightarrow \frac{x-2}{x(x-2)}=3$$

$$\Rightarrow \frac{-2}{x(x-2)}=3$$

This can be written as $-2 = 3\left(x^2 - 2x\right)$

The equation hence is $3x^2 - 6x + 2 = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here $a = 3, b = -6$ and $c = 2$

$$\therefore \boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$\Rightarrow x = \frac{-(-6) + \sqrt{(-6)^2 - 4 \times 3 \times 2}}{2 \times 3} = \frac{6 + \sqrt{36 - 24}}{6} = \frac{3 + \sqrt{3}}{3}$$

$$\Rightarrow x = \frac{-(-6) - \sqrt{(-6)^2 - 4 \times 3 \times 2}}{2 \times 3} = \frac{6 - \sqrt{36 - 24}}{6} = \frac{3 - \sqrt{3}}{3}$$

$\therefore$ The value of x are $\frac{3 \pm \sqrt{3}}{3}$

5. $x + \frac{1}{x} = 3, x \neq 0$

Sol: Given $x + \frac{1}{x} = 3, x \neq 0$

Hence this equation can be written as $\frac{x^2 + 1}{x} = 3$

$\Rightarrow x^2 + 1 = 3x = x^2 - 3x + 1 = 0$

$\therefore$ The equation is in the form of $ax^2 + bx + c = 0$

Here $a = 1, b = -3, c = 1$.

The value of $'x'$ can be solved by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow x = \frac{-b + \sqrt{(-3)^2 - 4 \times 1 \times 1}}{2 \times 1} = \frac{3 + \sqrt{9 - 4}}{2} = \frac{3 + \sqrt{5}}{2}$$

$$\Rightarrow x = \frac{-b - \sqrt{(-3)^2 - 4 \times 1 \times 1}}{2 \times 1} = \frac{3 - \sqrt{9 - 4}}{2} = \frac{3 - \sqrt{5}}{2}$$

$\therefore$ The value of $'x'$ are $\frac{3 \pm \sqrt{5}}{2}$

Exercise 8.6

1. Determine the nature of the roots of the following quadratic equations:

(i) $2x^2-3x+5=0$ (ii) $2x^2-3x+5=0$ (iii) $\frac{3}{5}x^2-\frac{2}{3}x+1=0$ (iv) $3x^2-4\sqrt{3}x+4=0$

(v) $3x^2-2\sqrt{6}x+2=0$ (vi) $(x-2a)(x-2b)=4ab$ (vii) $2(a^2+b^2)x^2+2(a+b)x+1=0$

(viii) $2(a^2+b^2)x^2+2(a+b)x+1=0$ (ix) $(b+c)x^2-(a+b+c)x+a=0$

Sol:

(i) $2x^2-3x+5=0$

The given quadratic equation is $2x^2-3x+5=0$

here $a=2, b=-2, c=5$

$\boxed{D=b^2+4ac} \Rightarrow (-3)^2-4\times5\times1=9-20=-11<0$

As $D<0$, The discriminant of equation is negative, then the expression has no real roots

(ii) $2x^2-3x+5=0$

The given quadratic equation is $2x^2-3x+5=0$

here $a=2, b=-6$ and $c=3$

$\therefore \boxed{D=b^2-4ac} \Rightarrow (-6)^2-4\times2\times3=36-24=12>0$

As $D>0$, the discriminant of equation is positive, the equation has real and distinct roots

(iii) $\frac{3}{5}x^2-\frac{2}{3}x+1=0$

The given quadratic equation is $\frac{3}{5}x^2-\frac{2}{3}x+1=0$ can also be written as $9x^2-10x+15=0$

here $a=9, b=-10, c=15$

$\boxed{D=b^2-4ac} \Rightarrow (-10)^2-4\times5\times9 \Rightarrow 100-540=-440<0$

$\therefore$ as $D>0$, the equation has no real roots

(iv) $3x^2-4\sqrt{3}x+4=0$

The given quadratic equation is $3x^2-4\sqrt{3}x+4=0$

here $a=3, b=-4\sqrt{3}, c=4$

The discriminant $D=b^2-4ac$

$\Rightarrow \left(-4\sqrt{3}\right)^2-4\times3\times4=48-48=0$

as $D > 0,$ the equation has real and equal roots

(v) $3x^2 - 2\sqrt{6}x + 2 = 0$

The given quadratic equation is $3x^2 - 2\sqrt{6}x + 2 = 0$

Here. The equation is in the form of $ax^2 + b + c = 0$

Where $a = 3, b = -2\sqrt{6}$ and $c = 2$

$D = b^2 - 4ac \Rightarrow \left(-2\sqrt{6}\right)^2 - 4\times3\times2 = 24 - 24 = 0$

as $D = 0,$ the given quadratic equation has real and equal roots

(vi) $\left(x - 2a\right)\left(x - 2b\right) = 4ab$

The given equation $\left(x - 2a\right)\left(x - 2b\right) = 4ab$ can also be written as $x^2 - x\left(2a + 2b\right)$ and $c = 0\left[4ab - 4ab = 0\right]$

$D = b^2 - 4ac \Rightarrow \left[-\left(2a + 2a\right)\right]^2 - 4\times1\times0 = \left(2a + 2b\right)^2 > 0$

$\Rightarrow$ as equal root of any integers is always positive

$\Rightarrow D > 0,$ hence the discriminant of the equation is positive

(vii) $2\left(a^2 + b^2\right)x^2 + 2\left(a + b\right)x + 1 = 0$

The given equation is $2\left(a^2 + b^2\right)x^2 + 2\left(a + b\right)x + 1 = 0$

It is in the form of the equation $ax^2 + bx + c = 0$

Here $a = 2\left(a^2 + b^2\right), b = 2\left(a + b\right)$ and $c = 1$

$\therefore \boxed{D = b^2 - 4ac}$

$\Rightarrow \left(-2abcd\right)^2 - 9\times9a^2b^2\times16c^2d^2$

$\Rightarrow b + 6a^2b^2c^2d^2 - 576a^2b^2c^2d^2 = 0$

Hence as $D = 0,$ the equation has Real and equal roots

(viii) $2\left(a^2 + b^2\right)x^2 + 2\left(a + b\right)x + 1 = 0$

The given equation is $2\left(a^2 + b^2\right)x^2 + 2\left(a + b\right)x + 1 = 0$

It is in the farm of the equation $ax^2 + bx + c = 0$

Here $a = 2\left(a^2 + b^2\right), b = 2\left(a + b\right)$ and $c = 1$

$\therefore \boxed{D = b^2 - 4ac} \Rightarrow \left[2\left(a + b\right)\right]^2 - 4\times2\left(a^2 + b^2\right)\times1$

$\Rightarrow 4a^2 + ab^2 + 8ab - 8a^2 - 8b^2$

$\Rightarrow 8ab+4\left(a^2+b^2\right)<0$

as $D<0$, The discriminant is negative and the nature of the roots are not real

(ix) $\left(b+c\right)x^2-\left(a+b+c\right)x+a=0$

The given equation is $\left(b+c\right)x^2-\left(a+b+c\right)x+a=0$

Here $a=b+c, b=-\left(a+b+c\right)$ and $c=a$

$\therefore \boxed{D=b^2-4ac} \Rightarrow \left[-\left(a+b+c\right)\right]^2-4\times\left(b+c\right)\times a$

$\Rightarrow \left(a+b+c\right)^2-4abc>0$

$\therefore$ as $D>0$, the discriminant is positive and the nature of the roots are real and unequal

2. Find the values of k for which the roots are real and equal in each of the following equation:

(i) $kx^2+4x+1=0$ (ii) $kx^2-2\sqrt{5}x+4=0$ (iii) $3x^2-5x+2k=0$ (iv) $4x^2+kx+9=0$

(v) $2kx^2-40x+25=0$ (vi) $9x^2-24x+k=0$ (vii) $4x^2-3kx+1=0$

(viii) $x^2-2\left(5+2k\right)x+3\left(7+10k\right)=0$

Sol:

(i) $kx^2+4x+1=0$

Sol:

The given equation $kx^2+4x+1=0$ is in the form of $ax^2+bx+c=0$ where

$a=k, b=4, c=1$

$\Rightarrow$ given that, the equation has real and equal roots

i.e., $\boxed{D=b^2-4ac=0}$

$\Rightarrow \left(4\right)^2-4\times k\times 1=0$

$\Rightarrow 16=4k \Rightarrow \boxed{k=4}$

$\therefore$ The value of $k=4$

(ii) $kx^2-2\sqrt{5}x+4=0$

Sol:

The given equation $kx^2-2\sqrt{5}x+4=0$ is in the form of $ax^2+bx+c=0$ where

$a=k, b=-2\sqrt{3}$ and $c=4$

$\Rightarrow$ given that, the equation has real and equal roots

i.e., $\boxed{D=b^2-4ac=0}$

$\Rightarrow \left(-2\sqrt{5}\right)^2-4\times k\times 4=0$

$\Rightarrow 20 = 16k \Rightarrow k = \frac{20}{16} = \frac{5}{4} \quad \therefore k = \frac{5}{4}$

$\therefore$ The value of $k = \frac{5}{4}$

(iii) $3x^2 - 5x + 2k = 0$

Sol:

The given equation is $3x^2 - 5x + 2k = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here, $a = 3, b = -5$ and $c = 2k$

$\Rightarrow$ given that, the equation has real and equal roots

i.e., $\boxed{D = b^2 - 4ac = 0}$

$\Rightarrow (-5)^2 - 4 \times 3 \times (2k) = 0$

$\Rightarrow 25 = 24k$

$\Rightarrow \boxed{k = \frac{25}{24}}$

$\therefore$ The value of $k = \frac{25}{24}$

(iv) $4x^2 + kx + 9 = 0$

Sol:

The given equation is $4x^2 + kx + 9 = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here, $a = 4, b = k$ and $c = 9$

$\Rightarrow$ given that, the equation has real and equal roots

i.e., $\boxed{D = b^2 - 4ac = 0}$

$\Rightarrow k^2 - 4 \times 4 \times 9 = 0$

$\Rightarrow k^2 - 16 \times 9$

$\Rightarrow k = \sqrt{16 \times 9} = 4 \times 3 = 12$

$\therefore$ The value of $k = 12$

(v) $2kx^2 - 40x + 25 = 0$

Sol:

The given equation is $2kx^2 - 40x + 25 = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here, $a = 2k, b = -40$ and $c = 25$

$\Rightarrow$ given that, the equation has real and equal roots

i.e., $\boxed{D = b^2 - 4ac = 0}$

$\Rightarrow (-10)^2 - 4 \times 2k \times 25 = 0$

$\Rightarrow 1600 - 200k = 0$

$\Rightarrow k = \frac{1600}{200} = 8$ $\boxed{\therefore k = 8}$

$\therefore$ The value of $k = 8$

(vi) $9x^2 - 24x + k = 0$

Sol:

The given equation is $9x^2 - 24x + k = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here, $a = 9, b = -24,$ and $c = k$

$\Rightarrow$ given that, the nature of the roots of this equation is real and equal

i.e., $\boxed{D = b^2 - 4ac = 0}$

$\Rightarrow (-24)^2 - 4 \times 4 \times k = 0$

$\Rightarrow 576 - 36k = 0$

$\Rightarrow k = \frac{576}{36} = 16$ $\boxed{\therefore k = 16}$

$\therefore$ The value of $k = 16$

(vii) $4x^2 - 3kx + 1 = 0$

Sol:

The given equation is $4x^2 - 3kx + 1 = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here, $a = 4, b = -3k,$ and $c = 1$

$\Rightarrow$ given that, the nature of the roots of this equation is real and equal

i.e., $\boxed{D = b^2 - 4ac = 0}$

$\Rightarrow (-3k)^2 - 4 \times 4 \times 1 = 0$

$\Rightarrow k^2 = \frac{16}{9} \Rightarrow k = \sqrt{\frac{16}{9}} = \pm\frac{4}{3}$

$\Rightarrow \boxed{k = \pm\frac{4}{3}}$

$\therefore$ The value of k is $\pm\frac{4}{3}$

(viii) $x^2 - 2(5+2k)x + 3(7+10k) = 0$

Sol:

The given equation is $x^2 - 2(5+2k)x + 3(7+10k) = 0$

Here, $a = 1, b = -2, (5+2k)$ and $c = 3(7+10k)$

$\Rightarrow$ given that, the nature of the roots of this equation are real and equal

i.e., $D = b^2 - 4ac = 0$

$\Rightarrow \{[-2(5+2k)]\} - 4\times1\times3(7+10k) = 0$

$\Rightarrow 4(5+2k)^2 - 12(7+10k) = 0$

$\Rightarrow 25 + 4k^2 + 20k - 21 - 30k = 0$

$\Rightarrow 4k^2 - 10k + 4 = 0 = 2k^2 - 5k + 2 = 0$

$\Rightarrow k2(k-2) - 1(k-2) \Rightarrow k = 2$ or $k = \frac{1}{2}$

$\therefore$ The value of k is 2 or $\frac{1}{2}$

(ix) $(3k+1)x^2 + 2(k+1)x + k = 0$

Sol:

The given equation is $(3k+1)x^2 + 2(k+1)x + k = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here $a = 3k+1, b = 2(k+1)$ *and* $c = k$

$\Rightarrow$ Given that the nature of the roots of this equation are real and equal

i.e., $\boxed{D = b^2 - 4ac = 0}$

$\Rightarrow [2(k+1)]^2 - 4\times(3k+1)\times0$

$\Rightarrow 4[k+1]^2 = 4k[3k+1] = 0$

$\Rightarrow (k+1)^2 - k(3k+1) = 0$

$\Rightarrow k^2 + 1 + 2k - 3k^2 - k = 0$

$\Rightarrow -2k^2 + k + 1 = 0$

This equation can also be written as $2k^2 - k - 1 = 0$

The value of k can obtain by

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where $a = 2, b = -1, c = 1$ from equation 2

$$k=\frac{-(-1)+\sqrt{(-1)^2-4\times2\times(-1)}}{2\times2}=\frac{1+\sqrt{9}}{4}=\frac{4}{4}=1$$

$$k=\frac{-(-1)+\sqrt{(-1)^2-4\times2\times(-1)}}{2\times2}=\frac{1-\sqrt{9}}{4}=\frac{2^{1}}{4_{2}}=\frac{-1}{2}$$

$\therefore$ the value of k are 1 and $\frac{-1}{2}$

(x) $kx^2+kx+1=-4x^2-x$

Sol:

The given equation is $kx^2+kx+1=-4x^2-x$ bringing all the $'x'$ components to one side, we get the equation as $x^2(4+k)+x(k+1)+1=0$

This equation is in the form of the general quadratic equation i.e., $ax^2+bx+c=0$

Here $a=4+k, b=k+1$ and $c=1$

$\Rightarrow$ Given that the nature of the roots of the given equation are real and equal

i.e., $\boxed{D=b^2-4ac=0}$

$\Rightarrow(k+1)^2-4x(4+k)\times1=0$

$\Rightarrow k^2+1+2k-16-4k=0$

$\Rightarrow k^2-2k-10=0$ …………(2)

The equation (2) is as of the form ax^2+bx+c here $a=1, b=-2, c=-16+1=-15$

The value of k is obtained by $\boxed{k=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$

$$\Rightarrow k=\frac{-(-2)+\sqrt{(-2)^2-4\times1\times-15}}{2\times1}=5$$

$$\Rightarrow k=\frac{-(-2)+\sqrt{(-2)^2-4\times1\times-15}}{2\times1}==-3$$

$\therefore$ The value of k are 5 and -3 respectively for the given quadratic equation.

(xi) $(k+1)x^2+20=(k+3)x+(k+8)=0$

Sol:

The given equation is $(k+1)2^2+2(k+3)x+(k+8)=0$

Here $a=k+1, b=2(k+3)$ *and* $c=k+8$

$\Rightarrow$ given that the nature of the roots of this equation are real and equal i.e.,

$\boxed{D=b^2-4ac=0}$

$\Rightarrow \left[2(K+3)\right]^2 - 4\times(k+1)\times(k+8) = 0$

$\Rightarrow 4(k+3)^2 - 4(k+1)\times(k+8) = 0$

$\Rightarrow (k+3)^2 - (k+1)(k+8) = 0$

$\Rightarrow k^2 + 9 + 6k - \left[k^2 + 9k + 8\right] = 0$

$\Rightarrow k^2 + 9 + 6k - k^2 - 9k - 8 = 0$

$\Rightarrow -3k + 1 = 0 \Rightarrow k = \frac{1}{3}$

$\therefore$ The value of $'k'$ for the given equation is $\frac{1}{3}$

(xii) $x^2 - 2kx + 7k - 12 = 0$

Sol:

The given equation is $x^2 - 2kx + 7k - 12 = 0$

Here $a = 1, b = -2k$ and $c = 7k - 12$

$\Rightarrow$ given that the nature of the roots of this equation are real and equal i.e.,

$\boxed{D = b^2 - 4ac = 0}$

$\Rightarrow (-2k)^2 - 4\times1\times(7k - 12) = 0$

$\Rightarrow 4k^2 - 28k + 48 = 0$

$\Rightarrow k^2 - 7k + 12 = 0$

The value of k can be obtained by $\boxed{k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

$a = 1, b = -7, c = 12 \Rightarrow k = \frac{-(-7) \pm \sqrt{49 - 48}}{2} = \frac{7 \pm \sqrt{1}}{2} = 4, 3$

$\therefore$ The value of $'k'$ for the equation is 4 and 3

(xiii) $(k+1)x^2 - 2(3k+1)x + 8k + 1 = 0$

Sol:

The given equation is $(k+1)x^2 - 2(3k+1)x + 8k + 1 = 0$

It is in the form of the equation $ax^2 + bx + c = 0$

Here, $a = k + 1, b = -2(3k+1)$ and $c = 8k + 1$

$\Rightarrow$ given that the nature of the roots of the given equation are real and equal i.e.,

$D = b^2 - 4ac = 0$

$\Rightarrow \left[-2(3k+1)\right]^2 - 4\times(k+1)x(8k+1) = 0$

$\Rightarrow 4(3k+1)^2 - 4(k+1)(8k+1) = 0$

$\Rightarrow (3k+1)^2 - (k+1)(8k+1) = 0$

$\Rightarrow 9k^2 + 6k + 1 - [8k^2 + 9k + 1] = 0$

$\Rightarrow 9k^2 + 6k + 1 - 8k^2 - 9k - 1 = 0$

$\Rightarrow k^2 - 3k = 0$

$\Rightarrow k(k-3) = 0$

$\therefore k = 0 \text{ or } k = 3$

$\therefore$ The values of $'k'$ for the given quadratic equation are 0 and 3

(xiv) $5x^2 - 4x + 2 + k(4x^2 - 2x + 1) = 0$

Sol:

The given equation is $5x^2 - 4x + 2 + k(4x^2 - 2x + 1) = 0$

This can be written as $x^2[5+4k] - x[4+2k] + 2 - k = 0$

This equation is in the form of $ax^2 + bx + c = 0$ ………(1)

Here $a = 5+4k, b = -(4+2k)$ and $c = -2k$

$\Rightarrow$ given that the nature of the roots of this equation are real and equal i.e.,

$\boxed{D = b^2 - 4ac = 0}$

$\Rightarrow [-(4+2k)]^2 - 4(5+4k)(2-k) = 0$

$\Rightarrow (4+2k)^2 - 4(5+4k)(2-k) = 0$

$\Rightarrow 16 + 4k^2 + 16k - 4[10 - 5k + 8k - 4k^2] = 0$

$\Rightarrow 16 + 4k^2 + 16k - 40 + 20k - 32k + 16k^2 = 0$

$\Rightarrow 20k^2 - 4k - 24 = 0$

$5k^2 - k - 6 = 0$ ………..(2)

As equation (2) is of the form (1), k can be obtained

By $\boxed{k = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ where $a = 5, b = -1, c = -6$

$$\Rightarrow k = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) - \sqrt{1 - 4 \times 5 \times -6}}{2 \times 5} = +\frac{6}{5}$$

$$\Rightarrow k = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) - \sqrt{1 - 4 \times 5 \times -6}}{2 \times 5} = -1$$

The values of k for the given equation are $+\frac{6}{5}$ and -1

(xv) $(4-k)x^2+(2k+4)x+(8k+1)=0$

Sol:

The given equation is $(4-k)x^2+(2k+4)x+(8k+1)=0$

This equation is in the form of $ax^2+bx+c=0$

Here $a=4-k, b=2k+4$ and $c=8k+1$

$\Rightarrow$ given that the nature of the roots of this equation are real and equal i.e.,

$\boxed{D=b^2-4ac=0}$

$\Rightarrow (2k+4)^2-4(4-k)(8k+1)=0$

$\Rightarrow 4k^2+16+16k-4\left[-8k^2+32k+4-k\right]=0$

$\Rightarrow 4k^2+16+16k+(8k^2\times 4)-(31\times 4)k-16=0$

$\Rightarrow 4k^2+\cancel{16}+16k+32k^2-124k-\cancel{16}=0$

$\Rightarrow 36k^2-108k=0$

$\Rightarrow 9k^2-27k=0$

$\Rightarrow k^2-3k=0$

$\Rightarrow k(k-3)=0$

Hence $k=0$ or $k=3$

$\therefore$ The value of 'k' for the given quadratic equation is 0 and 3

(xvi) $(2k+1)x^2+2(k+3)x+(k+5)=0$

Sol:

The given equation is $(2k+1)x^2+2(k+3)x+(k+5)=0$

This equation is in the form of $ax^2+bx+c=0$

Here $a=2k+, b=2(k+3)$ and $c=k+5$

$\Rightarrow$ given that the nature of the roots for this equation are real and equal i.e.,

$\boxed{D=b^2-4ac=0}$

$\Rightarrow \left[2(k+3)\right]^2-4[2k+1][k+5]=0$

$\Rightarrow (k+3)^2-(2k+1)(k+5)=0$

$\Rightarrow k^2+9+6k-\left[2k^2+11k+5\right]=0$

$\Rightarrow -k^2-5k+4=0$

$\Rightarrow k^2+5k-4=0$ ……….(2)

$\Rightarrow k^2+4k+k-4=0$

The value of $'k'$ an be obtained by $\boxed{k=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$

Where from $(2), a=1, b=5, c=-4$

$$k=\frac{-5\pm\sqrt{25-4\times1\times-4}}{2\times1}=\frac{-5+\sqrt{25+16}}{2}=\frac{-5+\sqrt{41}}{2}$$

$$k=\frac{-5-\sqrt{25-4\times1\times-4}}{2\times1}=\frac{-5-\sqrt{25+16}}{2}=\frac{-5-\sqrt{41}}{2}$$

$\therefore$ The value of 'k' from the given equation are $\frac{-5\pm\sqrt{41}}{2}$

(vii) $4x^2-2(k+1)x+(k+4)=0$

Sol:

The given equation is $4x^2-2(k+1)x+(k+4)=0$

This equation is in the form of $ax^2+bx+c=0$

Here a = 4, b = -2(k + 1), c = k + 4

$\Rightarrow$Given that the nature of the roots of this equation is real and equal i.e. 0 = $b^2-4ac=0$

$\Rightarrow [-2(k+1)]^2-4x+x(k+4)=0$

$\Rightarrow 4(k+1)^2-16(k+4)=0$

$\Rightarrow (k+1)^2-4(k+4)=0$

$\Rightarrow\ k^2+1+2k-4k-16=0$

$\Rightarrow k^2-2k-15=0$

The value of 'k' can be obtained by the formula

$k=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ where a = 1, b = -2, c = -15

$\Rightarrow k=\frac{-b+\sqrt{b^2-4ac}}{2a}=\frac{-(-2)+\sqrt{4-4\times1\times-15}}{2a}=\frac{2+\sqrt{69}}{2\times1}=k=-3$

$\therefore$ The value of 'k' for the given equation are 5 and -3

(xviii) $x^2-2(k+1)x+(k+4)=0$

Sol:

The given equation is $x^2-2(k+1)x+(k+4)=0$

This equation is in the form of $ax^2+bx+c=0$

Here a = 1, b = $-2(k+1)$ and c = k + 4

$\Rightarrow$ The nature of the roots of this equation is given that it is real and equal

i.e., O = $b^2-4ac=0$

$\Rightarrow [-2(k+1)]^2-4\times1\times k+4=0$

$\Rightarrow 4(k+1)^2-4(k+4)-0$

$\Rightarrow 4(k^2+1+2k)-4k-16=0$

$\Rightarrow k^2 + k - 3 = 0$ …..(ii)

The value of 'k' can be obtained by formula $k = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ where a = 1, b = 1, c = -3

$k = \frac{-b+\sqrt{b^2-4ac}}{2a} = \frac{-1+\sqrt{1-4\times1-3}}{2\times1} = \frac{1}{2}$

$k = \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{-1-\sqrt{1-4\times1\times-3}}{2\times1} = \frac{1}{2}$

$The\ value\ of\ 'k'\ for\ the given\ equation\ are\ \frac{1}{2}$

(xix) $4x^2 - 2(k+1)x + 4 = 0$

Sol:

The given equation as $k^2x^2 - 2(2k-1)x + 4 = 0$

It is in the form of the equation $ax^2 + bx + c = 0$

Here $a = k^2, b = -2(2k-1)\ amd\ c = 4$

$\Rightarrow$ Given that the nature of the roots of the equation are real and equal

i.e., D = $b^2 - 4ac = 0$

$\Rightarrow [-2(2k-1)]^2 - 4 \times k^2 \times 4 = 0$

$\Rightarrow 4(2k-1)^2 - 16k^2 = 0$

$\Rightarrow (2k-1)^2 - 4k^2 = 0$

$\Rightarrow$ 4k – 1 = 0

k = $\frac{1}{4}$

∴ The value of 'k' for the given equation is $\frac{1}{4}$

(xx) $(k+1)x^2 - 2(k-1)x + 1 = 0$

Sol:

The given equation is $(k+1)x^2 - 2(k-1)x + 1 = 0$

It is in the form of the equation $ax^2 + bx + c = 0$

Here a = k + 1, b = $-2(k-1) and\ c = 1$

$\Rightarrow$ Given that the nature of the roots for the equation are real and equal

i.e., D = $b^2 - 4ac = 0$

$\Rightarrow [-2(k-1)]^2 - 4 \times [k+1] \times 1 = 0$

$\Rightarrow 4(k-1)^2 - 4(k+1) = 0$

$\Rightarrow (k-1)^2 - (k+1) = 0$

$\Rightarrow k^2 + 1 - 2k - k - 1 = 0$

$\Rightarrow k^2 - 3k = 0$

$\Rightarrow$ k(k – 3) = 0

∴ Here k = 0 or k = 3

∴ The value of 'k' for the given equation is k = 0 or k = 3

(xxi) $2x^2 + kx + 3 = 0$

Sol:

The given equation is $2x^2 + kx + 3 = 0$

It is in the form of the equation $ax^2 + bx + c = 0$

Here a = 2, b = k, and c = 3

⟹ Given that the roots of the equation are real and equal i.e., D = $b^2 - 4ac = 0$

⟹ $k^2 - 4 \times 2 \times 3 = 0$

⟹ $k^2 = 24$

⟹ k = $\sqrt{24} = \pm 2\sqrt{6}$

∴ The value of k for the given equation is $\pm 2\sqrt{6}$

(xxii) kx(x – 2) + 6 = 0

Sol:

The given equation is $kx^2 - 2kx + 6 = 0$

a = 6, b = −2k, c = 6

⟹ Given that the roots are real and equal

i.e., D = $b^2 - 4ac = 0 \Longrightarrow 4k^2 - 4k \times 6 = 0$

⟹ $k^2 - 6k = 0$

⟹ k(k – 6) = 0

⟹ k = 0 or 6

∴ The value of k for the given equation is 0 or 6

(xxiii) $x^2 - 4kx + k = 0$

Sol:

The given equation is $x^2 - 4kx + k = 0$

a = 1, b = −4k, c = k

⟹ Given that the roots are real and equal i.e., D = $b^2 - 4ac = 0$

⟹ $16k^2 - 4k = 0$

⟹ $4k^2 - k = 0$

⟹ $k(4k + 1) = 0$

k = 0, k = $\frac{1}{4}$

∴ The value of k for the given equation is 0 or $\frac{1}{4}$

3. In the following determine the set of values of k foe which the green quadratic equation has real roots:

(i) $2x^2 + 3x + k = 0$ (ii) $2x^2 + kx + 3 = 0$ (iii) $2x^2 - 5x - k = 0$ (iv) $kx^2 + 6x + 1 = 0$

(v) $x^2 - kx + 9 = 0$

Sol:

(i) $2x^2+3x+k=0$

Sol:

The given equation is $2x^2+3x+k=0$

$\Rightarrow$ given that the quadratic equation has real roots i.e., $D=b^2-4ac\geq 0$

Given here $a=2, b=3, c=k$

$\Rightarrow 9-4\times 2\times k\geq 0$

$\Rightarrow 9-8k\geq 0$

$\Rightarrow 9\geq 8k \Rightarrow k\leq \frac{9}{8}$

The value of k does not exceed $\frac{4}{8}$ to have roots

(ii) $2x^2+kx+3=0$

Sol:

The given equation is $2x^2+kx+3=0$

$\Rightarrow$ given that the quadratic equation has real roots i.e., $D=b^2-4ac\geq 0$

here $a=2, b=k, c=3$

$\Rightarrow k^2-4\times 2\times 3\geq 0$

$\Rightarrow k^2-24\geq 0$

$\Rightarrow k^2\geq 24$

$\Rightarrow k\geq \sqrt{24} \Rightarrow k\geq \pm 2\sqrt{6}$ or $k\leq -2\sqrt{6}$

$\therefore$ The value of k does not exceed $2\sqrt{6}$ and $-2\sqrt{6}$ to have real roots

(iii) $2x^2-5x-k=0$

Sol:

The given equation is $2x^2-5x-k=0$

$\Rightarrow$ given that the equation has real roots i.e., $D=b^2-4ac\geq 0$

$\Rightarrow 25-4\times 2\times -k\geq 0$

$\Rightarrow 25+8k\geq 0$

$\Rightarrow 8k\geq -25$

The value of k should not exceed $\frac{25}{8}$ to have real roots.

(iv) $kx^2+6x+1=0$

Sol:

The given equation is $kx^2+6x+1=0$

Here $a=k, b=6, c=1$

$\Rightarrow$ given that the equation has real roots

i.e., $D=b^2-4ac\geq 0$

$\Rightarrow 36-4\times k\times 1\geq 0$

$\Rightarrow 36\geq 4k$

$\Rightarrow k\geq \frac{36}{7}$

$\Rightarrow k\leq 9$

The value of k should not exceed the value $'9'$ to have real roots.

(v) $x^2-kx+9=0$

Sol:

The given equation is $x^2-kx+9=0$

Here $a=1, b=-k, c=9$

$\Rightarrow$ given that the equation has real roots

i.e., $D=b^2-4ac\geq 0$

$\Rightarrow (-k)^2-4\times 1\times 9\geq 0$

$\Rightarrow k^2-36\geq 0$

$\Rightarrow k^2\geq 36$

$\Rightarrow \boxed{k\geq 6}$ or $\boxed{k\leq -6}$

The 'k' value exits between -6 and 6 to have the real roots for the given equation.

(vi) $2x^2+kx+2=0$

Sol:

The given equation is $2x^2+kx+2=0$

Here a = 2, b = k, c = 2

$\Longrightarrow$ Given that the equation has real roots

i.e., D = $b^2-4ac\geq 0$

$\Longrightarrow k^2-4\times 2\times 2\geq 0$

$\Longrightarrow k^2-16\geq 0$

$\Longrightarrow k\geq 16$

$\Longrightarrow k\geq \sqrt{16}$

$\Longrightarrow k\geq 4$ or $k\leq -4$

$\therefore$ The k value lies between -4 and 4 to have the real roots for the given equation.

(vii) $3x^2+2x+k=0$

Sol:

The given equation has $3x^2+2x+k=0$

Here a = 3, b = 2, c= k

$\Longrightarrow$ Given that the quadratic equation has real roots i.e., D = $b^2-4ac\geq 0$

$\Longrightarrow 4-4\times 3\times k\geq 0$

$\Rightarrow 4 - 12k \geq 0$

$\Rightarrow k \leq \frac{4}{12}$

$\Rightarrow k \leq \frac{1}{3}$

The 'k' value should not exceed $\frac{1}{3}$ to have the real roots for the given equation

(viii) $4x^2 - 3k + 1 = 0$

Sol:

The given equation has $4x^2 - 3k + 1 = 0$

Here a = 4, b = -3k, c = 1

$\Rightarrow$ Given that that quadratic equation has real roots i.e., D = $b^2 - 4ac \geq 0$

= $9k^2 - 16 \geq 0$

$\Rightarrow 9k^2 \geq 16 = k^2 \geq \frac{16}{9}$

$\Rightarrow k \geq \sqrt{\frac{16}{9}} \Rightarrow k \geq \frac{4}{3} ork \leq -\frac{4}{3}$

∴ The value of k should be in between $-\frac{4}{3} and \frac{4}{3}$ to have real roots for the given equation.

(ix) $2x^2 + kx - 4 = 0$

Sol:

The given equation is $2x^2 + kx - 4 = 0$

Here a = 2, b = k, c = -4

$\Rightarrow$ Given that the quadratic equation has real roots i.e., D = $b^2 - 4ac \geq 0$

$\Rightarrow k^2 + 32 \geq 0$

$\Rightarrow k \leq \sqrt{32}$

$\Rightarrow k \in R$

∴ The k ∈ R for the equation to have the real roots

4. For what value of $k, (4-k)x^2 + (2k+4)x + (8k+1) = 0,$ is a perfect square

Sol:

The given equation is $(4-k)x^2 + (2k+4)x + (8k+1) = 0,$

Here $a = 4-k, b = 2k+4, c = 8\times+1$

The discriminant $D = b^2 - 4ac$

$= (2k+4)^2 - 4\times(4-k)(8k+1)$

$\Rightarrow 4k^2 + 16 + 4k - 4\left[32k + 4 - 8k^2 - k\right]$

$\Rightarrow \left[4k^2 + 8k^2 + 4k - 31k + 4 - 4\right]$

$\Rightarrow 4\left[9k^2 - 27k\right]$

$\Rightarrow D = 4\left[9k^2 - 27k\right]$

The given equation is a perfect square

$D = 0$

$\Rightarrow 4\left[9k^2 - 27k\right] = 0$

$\Rightarrow 9k^2 - 27k = 0$

$\Rightarrow 3k^2 - 9k = 0$

$\Rightarrow k^2 - 3k = 0$

$\Rightarrow k(k-3) = 0$

$\therefore k = 0$ or $k = 3$

$\therefore$ The value of k is '0' or '3' for the equation to be a perfect square

5. Find the least positive value of k for which the equation $x^2 + kx + 4 = 0$ has real roots

Sol:

The given equation is $x^2 + kx + 4 = 0$

$\Rightarrow$ given that the equation has real roots

i.e., $D = b^2 - 4ac \geq 0$

$\Rightarrow k^2 - 4 \times 1 \times 4 \geq 0$

$\Rightarrow k^2 - 16 \geq 0$

$\Rightarrow k \geq 4$ or $k \leq -4$

$\therefore$ The least positive value of $k = 4$, for the equation to have real roots

6. Find the value of k for which the gives quadratic equation has real and distinct roots

(i) $kx^2 + 2x + 1 = 0$ (ii) $kx^2 + 6x + 1 = 0$ (iii) $x^2 - kx + 9 = 0$

Sol:

(i) $kx^2 + 2x + 1 = 0$

The given equation is $kx^2 + 2x + 1 = 0$

Here $a = k, b = 2, c = 1$

$\Rightarrow$ given that the equation has real and distinct roots

i.e., $D = b^2 - 4ac \geq 0$

$\Rightarrow 4 - 4 \times 1 \times k \geq 0$

$\Rightarrow 4 - 4k \geq 0$ $\Rightarrow 4k \leq 4$

$\Rightarrow 4 \leq \frac{\not{4}}{\not{4}_1}$

$\therefore$ The value of k is 1 i.e., $k < 1$ for which the quadratic equation has real and distinct roots

(ii) $kx^2+6x+1=0$

The given equation is $kx^2+6x+1=0$

Here $a=k,b=6,c=1$

$\Rightarrow$ given that the equation has real and distinct roots

Hence $D=b^2-4ac\geq 0$

$\Rightarrow 36-4\times 1\times k\geq 0$

$\Rightarrow 36-4k\geq 0$

$\Rightarrow 4k\leq 36$

$\Rightarrow k\leq \frac{36}{4}\Rightarrow k>9$

$\therefore$ $k<9$ for the equation to have real and distinct roots

(iii) $x^2-kx+9=0$

The given equation is $x^2-kx+9=0$

Here $a=1,b=-k,c=9$

$\Rightarrow$ given that the equation is having real and distinct roots

Hence $D=b^2-4ac\geq 0$

$\Rightarrow k^2-4\times 1\times 9\geq 0$

$\Rightarrow k^2-36\geq 0$

$\Rightarrow k^2\geq 36$

$\Rightarrow k\geq 6$ or $k\leq -6$

$\therefore$ The value of "k" lies in between -6 and 6 to have the real roots for the given equation

7. If the roots of the equation $(b-c)x^2+(c-a)x+(a-b)=0$ are equal, then prove that $2b=a+c$

Sol:

The given equation is $(b-c)x^2+(c-a)x+(a-b)=0$

This equation has the general form i.e., $ax^2+bx+c=0$

Here $a=b-c,b=c-a$ and $c=a-b$

$\Rightarrow$ given that the equation has real and equal roots

Hence $b^2-4ac=D=0$

$\Rightarrow (c-a)^2-4\times(b-c)(a-b)=0$

$\Rightarrow c^2+a^2-2ac=4\left[ab-b^2-ac+cb\right]=0$

$\Rightarrow c^2+a^2-2ac-4ab+4b^2+4ac-4cb=0$

$\Rightarrow c^2+a^2+2ac-4ab+4b^2-4cb=0$

$\Rightarrow (a+c)^2 - 4ab + 4b^2 + 4cb = 0$

$\Rightarrow (c+a-2b)^2 = 0$

$\Rightarrow c+a-2b=0$

$\Rightarrow c+a=2b$

Hence, it is proved that $c+a=2b$

8. If the roots of the equation $\left(a^2+b^2\right)x^2 - 2\left(x+bd\right)x + \left(c^2+d^2\right) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$.

Sol:

The given equation is $\left(a^2+b^2\right)x^2 - 2\left(x+bd\right)x + \left(c^2+d^2\right) = 0$

This equation has the general form $ax^2 + bx + c = 0$

Here $a = a^2 + b^2, b = -2\left(ac+bd\right)c + \left(c^2+d^2\right) = 0$

$\Rightarrow$ given here that the nature of the real and equal

i.e., $D = b^2 - 4ac \geq 0$

$\Rightarrow \left[-2\left(ac+bd\right)\right]^2 - 4x\left(a^2+b^2\right)\left(c^2+d^2\right) = 0$

$\Rightarrow 4\left(ac+bd\right)^2 - 4\left(a^2+b^2\right)\left(c^2+d^2\right) = 0$

$\Rightarrow \left(ac+bd\right)^2 - \left(a^2+b^2\right)\left(c^2+d^2\right) = 0$

$\Rightarrow a^2c^2 + b^2d^2 + 2abcd - \left[a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2\right] = 0$

$\Rightarrow a^2c + 2^2d^2 + 2abcd - \left[a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2\right] = 0$

$\Rightarrow 2abcd - a^2d^2 - b^2c^2 = 0$

$\Rightarrow abcd + abcd - a^2d^2 - b^2c^2 = 0$

$\Rightarrow ad\left(bc-ad\right) + bc\left(ad-bc\right) = 0$

$\Rightarrow \left(ad-bc\right)\left(bc-ad\right) = 0$

Case i:

$\Rightarrow ab - bc = 0$

$\Rightarrow ad = bc$

$\Rightarrow \boxed{\frac{a}{b} = \frac{c}{d}}$

Case ii:

$\Rightarrow \left(bc-ad\right) = 0$

$\Rightarrow bc = ad$

$\Rightarrow \boxed{\frac{a}{b} = \frac{c}{d}}$

$\therefore$ Hence, it is proved that $\frac{a}{b} = \frac{c}{d}$

9. If the roots the equation $ax^2 + 2bx + c = 0$ and $bx^2 - 2\sqrt{ca}x + b = 0$ are simultaneously real, then prove that $b^2 - ac$

Sol:

Given equations are $ax^2 + 2bx + c = 0$ and $bx^2 - 2\sqrt{ca}x + b = 0$

Then two equations are of the form $ax^2 + bx + c = 0$

$\Rightarrow$ given that the roots of these two equations are real. Hence $D \geq 0$ i.e., $\boxed{b^2 - 4ac \geq 0}$

Let us assume that $ax^2 + 2bx + c = 0$ be equations …….(1)

and $bx^2 - 2\sqrt{ac}x + b = 0$ be equation …….(2)

from equation (1) $\Rightarrow b^2 - 4ac \geq 0$

$\Rightarrow 4b^2 - 4ac \geq 0$ …….(3)

From equation (2) $\Rightarrow b^2 - 4ac \geq 0$

$\Rightarrow \left(-2\sqrt{ac}\right)^2 - 4b^2 \geq 0$ …….(4)

Given that the roots of (1) and (2) are simultaneously real hence equation (3) equation (4)

$\Rightarrow 4b^2 - 4ac = 4ac - 4b^2$

$\Rightarrow 8ac = 8b^2$

$\Rightarrow \boxed{b^2 = ac}$

$\therefore$ Hence, it is proved that $b^2 - ac$

10. If p, q are real and $p \neq q$, then show that the roots of the equation $(p-q)x^2 + 5(p+q)x - 2(p-q) - 0$ are real and unequal

Sol:

The given equation is $(p-q)x^2 + 5(p+q) = 0$

$\Rightarrow$ given p, q are real and $p \neq q$

The discriminant $\boxed{D = b^{2-4ac}}$

$\Rightarrow \left[5(p+q)\right]^4 - 4 \times (p-q) \times (-2(p-q))$

$\Rightarrow 25(p+q)^2 + 8(p-q)^2$

We know that the square of nay integer is always positive i.e., greater than zero

Hence $D = b^2 - 4ac \geq 0$

As given that p, q are real and $p \neq q$

$\therefore 25(p+q)^2 + 8(p-q)^2 > 0$ i.e., $D > 0$

$\therefore$ The roots of this equation are real and unequal

11. If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ are equal, prove that either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$

Sol:

The given equation is $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here $a = c^2 - ab, b = -2(a^2 - bc)$ and $c = b^2 - ac$

$\Rightarrow$ given that the roots of this equation are equal

Hence $D = 0$ i.e., $b^2 - 4ac = 0$

$\Rightarrow \left[-2(a^2 - bc)\right]^2 - 4(c^2 - ab)(b^2 - ac) = 0$

$\Rightarrow 4(a^2 - bc)^2 - 4(c^2 - ab)(b^2 - ac) = 0$

$\Rightarrow 4a(a^3 + b^3 + c^3 - 3abc) = 0$

$\Rightarrow 4a = 0$ or $a^3 + b^3 + c^3 - 3abc = 0$

$\Rightarrow a = 0$ or $a^3 + b^3 + c^3 = 3abc$

$\therefore$ hence, it is proved

12. Show that the equation $2(a^2 + b^2)x^2 + 2(a+b)x + 1 = 0$ has no real roots, when $a \neq b$.

Sol:

The given equation is $2(a^2 + b^2)x^2 + 2(a+b)x + 1 = 0$

This equation is in the form of $ax^2 + bx + c = 0$

Here $a = 2(a^2 + b^2), b = 2(a+b)$ and $c = 1$

$\Rightarrow$ given that $a \neq b$

The discriminant $D = b^2 - 4ac$

$\Rightarrow \left[2(a+b)\right]^2 - 4 \times 2(a^2 + b^2) \times (1)$

$\Rightarrow 4(a+b)^2 - 8(a^2 + b^2)$

$\Rightarrow 4\left[a^2 + b^2 + 2ab\right] - 8a^2 - 8b^2$

$\Rightarrow -4a^2 - 4b^2 + 2ab$

As given that $a \neq b$, as the discriminant 0 has negative squares, 0 will be less than zero

Hence $0 < 0,$ when $a \neq b$

13. Prove that both the roots of the equation $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ are real but they are equal only when $a=b=c$

Sol:

The given equation is $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$

By solving the equation, we get it as

$3x^2-2x(a+b+c)+(ab+bc+ca)=0$

This equation is in the form of $ax^2+bx+c=0$

Here $a=3, b=-2(a+b+c)$ and $c=ab+bc+ca$

The discriminant $D=b^2-4ac$

$D=(-2\,a+b+c)^2-4(3)(ab+bc+ca)$

$\Rightarrow 4(\,a+b+c)^2-12(ab+bc+ca)$

$\Rightarrow 4\left[(\,a+b+c)^2-3(ab+bc+ca)\right]$

$\Rightarrow 4\left[a^2+b^2+c^2-ab-bc-ca\right]$

$\Rightarrow 2\left[2a^2+2b^2+2c^2-2ab-2bc-2ca\right]$

$\Rightarrow 2\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

Here clearly $0 \geq 0,$ if $0=0$ then

$(a-b)^2+(b-c)^2+(c-a)^2=0$

$\Rightarrow a-b=0, b-c=0, c-a=0$

Hence $a=b=c$

Hence, it is proved.

14. If a, b, c are real numbers such that $ac \neq 0,$ then show that at least one of the equations $ax^2+bx+c=0$ and $-ax^2+bx+c=0$ has real roots

Sol:

The given equations are $ax^2+bx+c=0$ ....(1)

And $-ax^2+bx+c=0$ ....(2)

Given equations are of the form $ax^2+bx+c=0$ also given that a, b, c are real numbers and $ac \neq 0$

The discriminant $D=b^2-4ac$

For equation (1) $\Rightarrow b^2-4ac$ ....(3)

For equation (2) $\Rightarrow b^2 - 4(-a)\times(c)$

$\Rightarrow b^2 + 4ac$ ....(4)

As a, b, c are real and given that $ac \neq 0$ hence $b^2 - 4ac > 0$ and $b^2 + 4ac > 0$

$\therefore 0 > 0$

Exercise 8.7

1. Find the consecutive numbers whose squares have the sum 85.

Sol:

Let the two consecutive natural numbers be 'x' and 'x + 1'

⇒ Given that the sum of their squares is 85.

Then by hypothesis, we get

$x^2 + (x+1)^2 = 85$

$\Rightarrow x^2 + x^2 + 2x + 1 = 85$

$\Rightarrow 2x^2 + 2x + 1 - 85 = 0$

$\Rightarrow 2x^2 + 2x + 84 = 0 \Rightarrow 2[x^2 + x - 42] = 0$

$\Rightarrow x^2 + 7x - 6x - 42 = 0$ [*by the method of factorisation*]

$\Rightarrow x(x+7) - 6(x+7) = 0$

$\Rightarrow (x-6)(x+7) = 0$

$\Rightarrow x = 6$ *or* $x = 7$

Case i: if x = 6 x + 1 = 6 + 1 = 7

Case ii: If x = 7 x + 1 = −7 + 1 = −6

∴ The consecutive numbers that the sum of this squares be 85 are 6,7 and $-6, -7$.

2. Divide 29 into two parts so that the sum of the squares of the parts is 425.

Sol:

Let the two parts be 'x' and 29 – x

⇒ Given that the sum of the squares of the parts is 425.

Then, by hypothesis, we have

$\Rightarrow x^2 + (29 - x)^2 = 425$

$\Rightarrow 2x^2 - 58x + 841 - 425 = 0$

$\Rightarrow 2x^2 - 58x + 416 = 0$

$\Rightarrow 2[x^2 - 29x + 208] = 0$

$\Rightarrow x^2 - 29x + 208 = 0$

$\Rightarrow x^2 - 13x - 16x + 208 = 0$ [*By the method of factorisation*]

$\Rightarrow x(x-13) - 16(x-13) = 0$

$\Rightarrow (x-13)(x-16) = 0$

⇒ x = 13 or x = 16

Case i: If x = 13; 29 – x = 29 – 13 = 16

Case ii: x = 16; 29 – x = 29 – 16 = 13

∴ The two parts that the sum of the squares of the parts is 425 are 13, 16.

3. Two squares have sides x cm and (x + 4)cm. The sum of this areas is 656 cm^2. Find the sides of the squares.

Sol:

The given sides of two squares = x cm and (x + 4) cm

The sum of their areas = 656 cm^2.

The area of the square = side × side.

∴ Area of the square = x (x + 4) cm^2.

⇒ Given that sum of the areas is 656 cm^2.

Hence by hypothesis, we have

⇒ x(x + 4) + x(x + 4) = 656

⇒ 2x (x + 4) = 656

⇒ $x^2 + 4x = 328$ [$dividing\ both\ sides\ by$ 2]

⇒ $x^2 + 4c - 328 = 0$

⇒ $x^2 + 20x - 16x - 328 = 0$ [∵ $By\ the\ method\ of\ factorisation$]

⇒ $x(x + 20) - 16(x + 20) = 0$

⇒ $(x + 20)(x - 16) = 0$ ⇒ $x = -20\ or\ x = 16$

Case i: If x = 16; x + 4 = 20

∴ The sides of the squares are 16 cm and 20 cm.

Note: No negative value is considered as the sides will never be measured negatively.

4. The sum of two numbers is 48 and their product is 432. Find the numbers

Sol:

Given the sum of two numbers is 48

Let the two numbers be x and 48 – x also given their product is 432.

Hence x(48 – x) = 432

⇒ $48x - x^2 = 432$

⇒ $48x - x^2 - 432 = 0$

⇒ $x^2 - 48x + 432 = 0$

⇒ $x^2 - 36x - 12x + 432 = 0$ [By method of factorisation]

⇒ $x(x - 36) - 12(x - 36) = 0$

⇒ $(x - 36)(x - 12) = 0$

⇒ x = 36 or x = 12

∴ The two numbers are 12, 36.

5. If an integer is added to its square, the sum is 90. Find the integer with the help of quadratic equation.

Sol:

Let the integer be 'x'

Given that if an integer is added to its square, the sum is 90.

$\Rightarrow x + x^2 = 90$

$\Rightarrow x + x^2 - 90 = 0$

$\Rightarrow x^2 + 10x - 9x - 90 = 0$

$\Rightarrow x(x + 10) - 9(x + 10) = 0$

⇒ x = −10 or x = 9

∴ The value of an integer are -10 or 9.

6. Find the whole numbers which when decreased by 20 is equal to 69 times the reciprocal of the members.

Sol:

Let the whole number be x as it is decreased by 20 ⇒ (x – 20) = 69. $\left(\frac{1}{2}\right)$

$\Rightarrow x.20 = 69.\left(\frac{1}{x}\right)$

$\Rightarrow x(x - 20) = 69$

$\Rightarrow x^2 - 20x - 69 = 0$

$\Rightarrow x^2 - 23 + 3x - 69 = 0$

$\Rightarrow x(x - 23) + 3(x - 623) = 0$

⇒ (x – 23) (x + 3) = 0

⇒ x = 23; x = −3

As the whole numbers are always positive, x = −3 is not considered.

∴ The whole number x = 23.

7. Find the two consecutive natural numbers whose product is 20.

Sol:

Let the two consecutive natural numbers be 'x' and 'x + 2'

⇒ Given that the product of the natural numbers is 20

Hence ⇒ x(x + 1) = 20

$\Rightarrow x^2 + x = 20$

$\Rightarrow x^2 + x - 20 = 0$

$\Rightarrow x^2 + 5x - 4x - 20 = 0$

$\Rightarrow x(x + 5) - 4(x + 5) = 0$

$\Rightarrow x = -5 \text{ or } x = 4$

Considering positive value of x as x ∈ N

For r = 4, x + 1 = 4 + 1 = 5

∴ The two consecutive natural numbers are 4 as 5.

8. The sum of the squares of the two consecutive odd positive integers as 394. Find them.

Sol:

Let the consecutive odd positive integers are 2x – 1 and 2x + 1

Given that the sum of the squares is 394.

$\Rightarrow (2x-1)^2 + (2x+1)^2 = 394$

$\Rightarrow 4x^2 + 1 - 4x + 4x^2 + 1 + 4x = 394$

$\Rightarrow 8x^2 + 2 = 394$

$\Rightarrow 4x^2 = 392$

$\Rightarrow x^2 = 36$

$\Rightarrow$ x = 6

As x = 6, 2x – 1 = 2 × 6 – 1 = 11

2x + 1 = 2 × 6 + 1 = 13

∴ The two consecutive odd positive numbers are 11 and 13.

9. The sum of two numbers is 8 and 15 times the sum of their reciprocals is also 8. Find the numbers.

Sol:

Let the numbers be x and 8 – x

Given that the sum of these numbers is 8

And 15 times the sum of their reciprocals as 8

$\Rightarrow 15\left(\frac{1}{x} + \frac{1}{8-x}\right) = 8$

$\Rightarrow 15\left(\frac{(8-x)+x}{x(8-x)}\right) = 8$

$\Rightarrow 15\left((8-x)+x\right) = 8\left(x(8-x)\right)$

$\Rightarrow 15\,[8 - x + x] = 8x(8-x)$

$\Rightarrow 120 = 64x - 8x^2$

$\Rightarrow 8x^2 - 64x + 120 = 0$

$\Rightarrow 8[x^2 - 8x + 15] = 0$

$\Rightarrow x^2 - 5x - 3x + 15 = 0$

$\Rightarrow (x-5)(x-3) = 0$

$\Rightarrow$ x = 5 or x = 3

∴ The two numbers are 5 and 3.

10. The sum of a numbers and its positive square root is $\frac{6}{25}$. Find the numbers.

Sol:

Let the number be x

By the hypothesis, we have

$\Rightarrow x + \sqrt{x} = \frac{6}{25}$

⇒ let us assume that $x = y^2$, we get

$\Rightarrow y^2 + y = \frac{6}{25}$

$\Rightarrow 25y^2 + 25y - 6 = 0$

The value of 'y' can be obtained by $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Where a = 25, b = 25, c = −6

$\Rightarrow y = \frac{-25 \pm \sqrt{625 - 600}}{50}$

$\Rightarrow y = \frac{-25 \pm 35}{50} \Rightarrow y = \frac{1}{5} or \frac{-11}{10}$

$x = y^2 = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$

$\therefore The\ number\ x = \frac{1}{25}$.

11. The sum of a number and its square is 63/4. Find the numbers.

Sol:

Let the number be x.

Given that the sum of x and its square = $\frac{63}{4}$

$\Rightarrow x + x^2 = \frac{63}{4}$

$\Rightarrow 4x + 4x^2 - 63 = 0$

$\Rightarrow 4x^2 + 4x - 63 = 0$

$\Rightarrow 4x^2 + 4x - 63 = 0$ ….(i)

The value of x can be found by the formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow here\ a = 4, b = 4\ and\ c = -63\ from$ (i)

$x = \frac{-4 \pm \sqrt{16 - 4 \times 4 \times -63}}{2 \times 4}$

$= \frac{-4 \pm \sqrt{16 + 16 \times 63}}{2 \times 4}$

$x = \frac{-4 \pm \sqrt{16 + 1008}}{8} = \frac{7}{2}; x = \frac{-4 - \sqrt{16 + 1008}}{8} = \frac{-9}{2}$

$\therefore$ The values of x i.e., the numbers is $\frac{7}{2}, \frac{-9}{2}$.

12. There are three consecutive integers such that the square of the first increased by the product of the first increased by the product of the others the two gives 154. What are the integers?

Sol:

Let the three consecutive numbers x, x+1 and x +8.

According to the hypothesis given

$x^2 + (x + 1)(x + 2) = 154.$

$\Rightarrow x^2 + [x^2 + 3x + 2] = 154$

$\Rightarrow 2x^2 + 3x = 152$
$\Rightarrow 2x^2 + 3x - 152 = 0$ …(i)
$The\ value\ of\ 'x'\ can\ be\ obtained\ by\ the\ formula\ x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$here\ a = 2, b = 3\ and\ c = 152\ from\ (i)$
$x = \frac{-3+\sqrt{9-4\times2\times-152}}{2\times3}$
$x = \frac{-3+\sqrt{9+8\times152}}{6} = 8, \frac{-19}{2}$
$considering\ the\ positive\ value\ of\ x$
If x = 8, x + 1 = 9, x + 2 = 10
∴ The three consecutive integers are 8, 9, and 10

13. The product of two successive integral multiples of 5 is 300. Determine the multiples.
Sol:
Given that the product of two successive integral multiples of 5 is 300.
Let the integers be 5x, and 5(x + 1)
Then, by the integers be 5x and 5(x + 1)
Then, by the hypothesis, we have
5x · 5(x + 1) = 300
⇒ 25x (x + 1) = 300
$\Rightarrow\ x^2 + x = 12$
$\Rightarrow x^2 + x - 12 = 0$
$\Rightarrow x^2 + 4x - 3x - 12 = 0$
⇒ x(x + 4) −3(x + 4) = 0
⇒ (x + 4) (x – 3) = 0
⇒ x = −4 or x = 3
If x = −4, 5x = −20, 5(x + 1) = −15
x = 3, 5x = 15, 5(x + 1) = 20
∴ The two successive integral multiples are 15, 20 or −15, −20.

14. The sum of the squares of two numbers as 233 and one of the numbers as 3 less than twice the other number find the numbers.
Sol:
Let the number be x
Then the other number = 2x – 3
According to the given hypothesis,
$\Rightarrow x^2 + (2x - 3)^2 = 233$
$\Rightarrow x^2 + 4x^2 + 9 - 12x = 233$
$\Rightarrow 5x^2 - 12x - 224 = 0$ …. (i)
The value of ‘x’ can be obtained by the formula $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Here a = 5, b = 12 and c = -224 from (i)

$x = \frac{-(-12)+\sqrt{144+20\times224}}{10} = 8$

$x = \frac{-(-12)-\sqrt{144+20\times224}}{10} = \frac{-28}{5}$

$considering\ the\ value\ of\ x = 8$

2x – 3 = 16 – 3 = 15

∴ The two numbers are 8 and 15.

15. Find the consecutive even integers whose squares have the sum 340.

Sol:

Let the consecutive even integers be 2x and 2x + 2.

Then according to the given hypothesis,

$(2x)^2 + (2x+2)^2 = 340$

$\Rightarrow 8x^2 + 8x - 336 = 0$

$\Rightarrow x^2 + x - 42 = 0$

$\Rightarrow x^2 + 7x - 6x - 42 = 0$

$\Rightarrow x(x+7) - 6(x+7) = 0$

$\Rightarrow (x+7)(x-6) = 0$

⇒ x = − 7 or x = 6

Considering, the positive integers of x.

⇒ x = 6; 2x = 12 and 2x + 2 = 14.

∴ The two consecutive even integers are 12 and 14.

16. The difference of two numbers is 4. If the difference of their reciprocals is $\frac{4}{21}$. Find the numbers.

Sol:

Let the two numbers be x and x – 4

Given that the difference of two numbers is 4.

By the given hypothesis, we have $\frac{1}{x-4} - \frac{1}{x} = \frac{4}{21}$

$\Rightarrow \frac{x-x+4}{x(x-4)} = \frac{4}{21}$

⇒ 84 = 4x(x – 4)

$\Rightarrow x^2 - 4x - 21 = 0$

$\Rightarrow x^2 - 7x + 3x - 21 = 0$

$\Rightarrow x(x-7) + 3(x-7) = 0$

$\Rightarrow (x-7)(x+3) = 0$

$\Rightarrow x = 7\ or\ x = -3$ and

If x = −3, x – 4 = −3 −4 = −7

Hence, required numbers are 3, 7 and −3, −7

17. Let us find two natural numbers which differ by 3 and whose squares have the sum 117.

Sol:

Let the numbers be x and x – 3

By the given hypothesis,

$x^2 + (x-3)^2 = 117$

$\Rightarrow x^2 + x^2 + 9 - 6x - 117 = 0$

$\Rightarrow 2x^2 - 6x - 108 = 0$

$\Rightarrow x^2 - 3x - 54 = 0$

$\Rightarrow x(x-9) + 6(x-9) = 0$

$\Rightarrow (x-9)(x+6) = 0$

$\Rightarrow x = 9$ or $x = -6$

Considering positive value of x

x = 9, x – 3 = 9 – 3 = 6

∴ The two numbers be 9 and 6.

18. The sum of the squares of three consecutive natural numbers as 149. Find the numbers

Sol:

Let the numbers be x, x + 1 and x + 2 according to the given hypothesis.

$x^2 + (x+1)^2 + (x+2)^2 = 149$

$\Rightarrow x^2 + x^2 + 1 + 2x + x^2 + 4 + 4x = 149$

$\Rightarrow 3x^2 + 6x + 5 - 149 = 0$

$\Rightarrow 3x^2 + x - 144 = 0$

$\Rightarrow x^2 + 2x - 48 = 0$

$\Rightarrow x(x+8) - 6(x+8) = 0$

$\Rightarrow (x+8)(x-6) = 0$

$\Rightarrow x = -8$ or $x = 6$

Considering the positive value of x

$x = 6,\ x + 1 = 7\ and\ x + 2 = 8$

∴ The three consecutive numbers are 6, 7, 8.

19. Sum of two numbers is 16. The sum of their reciprocals is $\frac{1}{3}$. Find the numbers.

Sol:

Given that the sum of two numbers is 16.

Let the two numbers be x and 16 – x

By the given hypothesis, we have

$\Rightarrow \frac{1}{x} + \frac{1}{16-x} = \frac{1}{3}$

$\Rightarrow \frac{16-x+x}{x(16-x)} = \frac{1}{3}$

$\Rightarrow 48 = 16x - x^2$

$\Rightarrow x^2 - 16x + 48 = 0$

$\Rightarrow x^2 - 12x - 4x + 48 = 0$
⇒ x(x – 12) – 4(x – 12) = 0
⇒ (x – 12) (x – 4) = 0
⇒ x = 12 or x = 4
∴ The two numbers are 4 and 12.

20. Determine two consecutive multiples of 3, whose product is 270.
Sol:
Let the two consecutive multiples of 3 are 3x and 3x + 3
Given that their product is 270
⇒ (3x) (3x + 3) = 270
⇒ x(3x + 3) = 90
$\Rightarrow x^2 + x - 30 = 0$
$\Rightarrow x^2 + 6x - 5x - 30 = 0$
⇒ x(x + 6) −5(x + 6) = 0
⇒ (x + 6) (x – 5) = 0
⇒ x = 5 or x = −6
Considering the positive value of x.
⇒ x = 5, 3x = 15 and 3x + 3 = 18
∴ The two consecutive multiples of 3 are 15 and 18.

21. The sum of a number and its reciprocal is $\frac{17}{4}$. Find the number.
Sol:
Let the number be 'x'
According to the given hypothesis
$x + \frac{1}{x} = \frac{17}{4}$
$\Rightarrow \frac{x^2+1}{x} = \frac{17}{4}$
$\Rightarrow 4(x^2 + 1) = 17x$
$\Rightarrow 4x^2 - 17x + 4 = 0$
$\Rightarrow 4x^2 - 16x - x + 4 = 0$
$\Rightarrow 4x(x - 4) - 1(x - 4) = 0$
$\Rightarrow x = \frac{1}{4}\ or\ x = 4$
$\therefore The\ value\ of\ x = 4$

22. A two-digit number is such that the products of its digits is 8. When 18 is subtracted from the number, the digits interchange their places. Find the number?
Sol:
Let the two digits be x and x – 2

Given that the product of their digits is 8.

$\Rightarrow x(x - 2) = 8$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow x^2 - 4x + 2x - 8 = 0$

$\Rightarrow x(x - 4) + 2(x - 4) = 0$

$\Rightarrow (x - 4)(x + 2) = 0$

$\Rightarrow x = 4 \text{ or } x = -2$

Considering the positive value x = 4, x – 2 = 2.

∴ The two digit number is 42.

23. A two digits number is such that the product of the digits is 12. When 36 is added to the number, the digits inter change their places determine the number

Sol:

Let the tens digit be x

Then, the units digit $= \frac{12}{x}$

$\therefore$ Number $= 10x + \frac{12}{x}$

And, number obtained by interchanging the

Digits $= 10 \times \frac{12}{x} + x = \frac{120}{x} + x.$

$$\Rightarrow 10x + \frac{12}{x} + 36 = \frac{120}{x} + x$$

$$\Rightarrow 9x + \frac{12 - 120}{x} + 36 = 0$$

$$\Rightarrow 9x^2 - 108 + 36x = 0$$

$$\Rightarrow 9\left(x^2 + 4x - 12\right) = 0$$

$$\Rightarrow x^2 + 6x - 2x - 12 = 0$$

$$\Rightarrow x\left(x + 6\right) - 2\left(x + 6\right) = 0$$

$$\Rightarrow \left(x - 2\right)\left(x + 6\right) = 0 \ \therefore x = 2 \text{ or } -6$$

But, a digit can never be negative, $80x = 2$

Hence, the digit $= 10 \times 2 + \frac{12}{2} = 20 + 6 = 26$

24. A two digit number is such that the product of the digits is 16. When 54 is subtracted from the number the digits are interchanged. Find the number

Sol:

Let the two digits be:

Tens digits be x and units $= \frac{16}{x}$

Number $= 10x + \frac{16}{x}$

Number obtained by interchanging $= 10 \times \frac{16}{x} + x$

$\Rightarrow \left(10x + \frac{16}{x}\right) - \left(10 \times \frac{16}{x} + x\right) = 5u$

$\Rightarrow 10x + \frac{16}{7} - \frac{160}{x} + x = 54$

$\Rightarrow 10x^2 + 16 - 160 + x^2 = 54x$

$\Rightarrow 9x^2 - 54x - 144 = 0$

$\Rightarrow x^2 - 6x - 16 = 0$

$\Rightarrow x^2 - 8x + 2x - 16 = 0$

$\Rightarrow x(x-8) + 2(x-8) = 0$

$\Rightarrow (x = 8)$ or $x = -2$

But, a digit can never be negative, hence $x = 8$

Hence the required number $= 10 \times 8 + \frac{16}{8} = 82$

25. Two numbers differ by 3 and their product is 504. Find the number

Sol:

Let the two numbers be x and $x-3$ given that $x(x-3) = 504$

$\Rightarrow x^2 - 3x - 504 = 0$

$\Rightarrow x^2 - 24x + 21x - 504 = 0$

$\Rightarrow x(x-24) + 21(x-24) = 0$

$\Rightarrow (x-24)(x+21) = 0$

$\Rightarrow x = 24$ or $x = 21$

Case 1: If $x = 24, x = 3 = 21$

Case 1: If $x = 21, x = 3 = 24$

$\therefore$ The two numbers are $21, 24$ or $-21, -24$

26. Two number differ by 4 and their product is $192.$ Find the numbers?

Sol: Let the two numbers be x and $x-4$

Given that their product is 192

$\Rightarrow x(x-4) = 192$

$\Rightarrow x^2 - 4x - 192 = 0$

$\Rightarrow x^2 - 16x + 12x - 192 = 0$

$\Rightarrow x(x-16) + 12(x-16) = 0$

$\Rightarrow (x-16)(x+12) = 0$

$\Rightarrow x = 16$ or $x = -12$

Considering the positive value of x

$x = 16, \Rightarrow x - 4 = 16 - 4 = 12$

$\therefore$ The two numbers are $12,16$

27. A two-digit number is 4 times the sum of its digits and twice the product of its digits. Find the numbers

Sol:

Let the digits at tens and units place of the number be x and y respectively then, it is given that $= 10x + y$

$\Rightarrow 10x + y = 4$ (sum of digits) and $2xy$

$\Rightarrow 10x + y = 4(x+y)$ and $10x + y = 3xy$

$\Rightarrow 10x + y = 4x + 4y$ and $10x + y = 3xy$

$\Rightarrow 6x - 3y = 0$ and $10x + y - 3xy = 0$

$\Rightarrow y = 2x$ and $10x + 2x = 2xy(2x)$

$\Rightarrow 12x = 4x^2$

$\Rightarrow 4x^2 - 12x = 0$

$\Rightarrow 4x(x-3) = 0$

$\Rightarrow 4x = 0$ or $x = 3$

$\Rightarrow$ here we have $y = 2x \Rightarrow 2\times 3 = 6$

$\therefore x = 3$ and $y = 6$

Hence $10x + y - 10\times 3 + 6 = 36$

$\therefore$ The required two digit number is 36

28. The sum of the squares of two positive integers is 208. If the square of the large numbr is 18 times the smaller. Find the numbers

Sol:

Let the smaller number be $x.$ Then square of a larger number $= 18x$

Also, square of the smaller number $= x^2$

It is given that the sum of the square of the integers is $208.$

$\therefore x^2 + 18x = 208$

$\Rightarrow x^2 + 18x - 208 = 0$

$\Rightarrow x^2 + 26x - 8x - 208 = 0$

$\Rightarrow (x+26)(x-8) = 0 \Rightarrow x = 8$ or $x = -26$

But, the numbers are positive. Therefore $x = 8$

$\therefore$ square of the larger number $= 18x = 18\times 8 = 144$

$\Rightarrow$ larger number are 8 and 18.

29. The sum of two numbers is $18.$ The sum of their reciprocals is $\frac{1}{4}.$ Find the numbers

Sol: Let The numbers be x and $18 - x$

$\Rightarrow$ according to the given hypothesis

$$\frac{1}{x} + \frac{1}{18-x} = \frac{1}{4}$$

$$\Rightarrow \frac{18 - x + x}{x(18-x)} = \frac{1}{4}$$

$\Rightarrow 7_2 = 18x - x^2$

$\Rightarrow x^2 - 18x - 72 = 0$

$\Rightarrow x^2 - 6x - 12x - 72 = 0$

$\Rightarrow x(x-6) - 12(x-12) = 0$

$\Rightarrow x = 6$ or $x = 12$

$\therefore$ The two number are $6,12$

30. The sum of two numbers a and $b.$ is $15.$ and the sum of their reciprocals $\frac{1}{a}$ and $\frac{1}{b}$ is $\frac{3}{10}$.
Find the numbers a and $b.$

Sol:

Let us assume a number $'x'$

Such that $\frac{1}{x} + \frac{1}{15-x} = \frac{3}{10}$

Hence $\Rightarrow \frac{15-x+x}{x(15-x)} = \frac{3}{10}$

$\Rightarrow 150 = 45x + 3x^2$

$\Rightarrow 3x^2 - 45x + 150 = 0$

$\Rightarrow x^2 - 15x + 50 = 0$

$\Rightarrow 2^2 - 10x - 5x + 50 = 0$

$\Rightarrow x(x-10) - 5(x-10) = 0$

$\Rightarrow (x-10)(x-5) = 0$

$\Rightarrow x = 5$ or $x = 10$

Case i: If $x = a,\ a = 5$ and

$b = 15 - x, b = 10.$

Case ii: if $x = 15 + a = 15 + 10 = 5x\ x = a = 10$

$b = 15 - 10 = 5$

$\therefore a=5, b=10$ or $a=10$ and $b=5$

31. The sum of two numbers is 9. The sum of their reciprocals is $\frac{1}{2}$. Find the numbers.

Sol:

Given that the sum of two numbers is 9 Let the two numbers be x and $9-x$

By the given hypothesis, we have

$$\frac{1}{x}+\frac{1}{9-x}=\frac{1}{2}$$

$$\Rightarrow \frac{9-x+x}{x(9-x)}=\frac{1}{2}$$

$$\Rightarrow 18=9x-x^2$$

$$\Rightarrow x^2-9x+18=0$$

$$\Rightarrow x^2-6x-3x+18=0$$

$$\Rightarrow x(x-6)-3(x-6)=0$$

$$\Rightarrow (x-6)(x-3)=0$$

$$\Rightarrow x=6 \text{ or } x=3$$

$\therefore$ The two numbers are 3 and 6

32. Three consecutive positive integers are such that the sum of the square of the first and the product of other two is 46. Find the integers.

Sol:

Let the three consecutive positive integers be x, $x+1$ and $x+2$

According to the hypothesis, we have

$$\Rightarrow x^2+(x+1)(x+2)=46$$

$$\Rightarrow x^2+x^2+3x+2=46$$

$$\Rightarrow 2x^2+3x-44=0$$

$$\Rightarrow 2x^2-8x+11x-44=0$$

$$\Rightarrow 2x(x-4)+11(x-4)=0$$

$$\Rightarrow (2x+11)(x-4)=0$$

$\Rightarrow x = 4$ or $x = -\frac{11}{2}$

Considering the positive value of x

$\Rightarrow x = 4, x+1 = 4$ and $x+2 = 6$

$\therefore$ The three consecutive numbers are $4, 5$ and 6.

33. The difference of squares of two numbers is 88. If the larger number is 5 less than twice the smaller number, then find the two numbers

Sol:

Let the smaller number be x. Then, larger number $= 2x - 5$

It is given that the difference of the square of the number is 88.

$\Rightarrow (2x-5)^2 - x^2 = 88$

$\Rightarrow 4x^2 + 25 - 20x - x^2 = 88$

$\Rightarrow 3x^2 - 20x - 63 = 0$

$\Rightarrow 3x^2 - 27x + 7x - 63 = 0$

$\Rightarrow 3x(x-9) + 7(x-9) = 0$

$\Rightarrow (x-9)(3x+7) = 0$

$\therefore x = 9$ or $-\frac{7}{3}$

As a digit can never be negative, $x = 9$

$\Rightarrow \therefore$ The numbers $= 2x - 5$

$= 2 \times 9 - 5 = 13$

$\therefore$ Hence, required numbers are 9 and 13

34. The difference of square of two numbers is 180 . the square of the smaller number is 8 times the large numbers find two numbers

Sol:

Let the number be x

By the given hypothesis, we have

$x^2 - 8x = 180$

$\Rightarrow x^2 - 8x - 180 = 0$

$\Rightarrow x^2 + 10x + -18x - 180 = 0$

$\Rightarrow x(x+10) - 18(x+10) = 0$

$\Rightarrow (x+10)(x-18) = 0$

$\Rightarrow x = -10$ or $x = 18$

Case (i): $x = 18$

$8x = 8 \times 18 = 144$

$\therefore$ Larger number $= \sqrt{144} = \pm 12$

Case (ii): $x = -10$

Square of larger number $8x = -80$ here no perfect square exist, hence the numbers are 18,12

Exercise 8.8

1. The speed of a boat in still water is 8 km/hr It can go 15 km upstream and 22 km downstream in 5 hours. Find the speed of the stream.

Sol:
Let the speed of the stream be x km/hr
Given that,
Speed of the boat in still water = 8 km/hr
Now,
Speed of the boat in upstream = (8 – x) km/hr
And speed of the boat in downstream = (8 + x) km/hr
Time taken for going 15 km upstream = $\frac{15\ km}{(8-x) km/hr} = \frac{15}{8-x} hours$
Time taken for going 22 km downstream = $\frac{22\ km}{(8+x)\ km/hr} = \frac{22}{8+x} hours$
Given that,
Time taken for upstream + downstream = 5 hours
$\Rightarrow \frac{15}{8-x} hours + \frac{22}{8+x} hours = 5 hours$
$\Rightarrow \frac{15}{8-x} + \frac{22}{8+x} = 5$
$\Rightarrow \frac{15(8+x)+22(8-x)}{(8-x)(8+x)} = 5$
$\Rightarrow \frac{120+15x+176-22x}{8^2-x^2} = 5$
$\Rightarrow 296 - 7x = 5\ (64 - x^2)$
$\Rightarrow 296 - 7x = 320 - 5x^2$

$\Rightarrow 5x^2 - 7x + 296 - 320 = 0$
$\Rightarrow 5x^2 - 7x - 24 = 0$ $[5 \times -24 = -120 \Rightarrow -180 = 8 \times -15 - 7 = -15 + 8]$
$\Rightarrow 5x^2 - (15 - 8)x - 24 = 0$
$\Rightarrow 5x^2 - 15x + 8x - 24 = 0$
⇒ 5x (x – 3) +8 (x – 3) = 0
⇒ (x – 3) (5x + 8) = 0
⇒ x – 3 = 0 or 5x + 8 = 0
⇒ x = 3 or x = $\frac{-8}{5}$
Since, x cannot be a negative value So, x = 3
∴ Speed of the stream is 3 km/hr

2. A passenger train takes 3 hours less for a journey of 360 km, if its speed is increased by 10 km/hr from its usual speed. What is the usual speed?
Sol:
Let the usual speed be x km/hr.
Distance covered in the journey = 360 km
Now,
Time taken by the train with the usual speed = $\frac{360\ km}{x\ km/hr} = \frac{360}{x} hr$
Given that if speed is increased by 10 km/hr, the same train takes 3 hours less.
⇒ Speed of the train = (x + 10) km/hr and time taken by the train after increasing the speed = $\frac{360\ km}{(x+10) km/hr} = \frac{360}{x+10} hr$

3. A fast train takes one hour less than a slow train for a journey of 200 km. If the speed of the slow train is 10 km/hr less than that of the fast train, find the speed of the two trains.
Sol:
Let the speed of the slow train be x km/hr.
Given that speed of the slow train is 10 km/hr less than that of fast train
⇒ Speed of the fast train = (x + 10) km/hr
Total distance covered in the journey = 200 km
Time taken by fast train = $\frac{200\ km}{(x+10)km/hr} = \frac{200}{x+10} hr$ and
Time taken by slow train = $\frac{200\ km}{x\ km/hr} = \frac{200}{x} hr$
Given that faster train takes 1 hour less than that of slow train
i.e., $\frac{800}{x} - \frac{800}{x+10} = 1$
⇒ $800\left(\frac{1}{x} - \frac{1}{x+10}\right) = 1$
⇒ $800\left(\frac{x+10-x}{x((x+10)}\right) = 1$
⇒ 800 (10) = x (x + 10) = 1

⇒ 8000 = x^2 + 10x
⇒ $x^2 + 10x - 2000 = 0$
⇒ $x^2 + (50 - 40)x + (50x - 40) = 0$
⇒ $\left(x^2 + 50x - 40x + (50x - 40)\right) = 0$
⇒ x (x + 50) −40(x + 50) = 0
⇒ (x + 50) (x – 40) = 0
⇒ x + 50 = 0 or x – 40 = 0
⇒ x = −50 or x = 40
Clearly x cannot be a negative volume since it is speed. So, x = 40
∴ Speed of slow train is 40 km/hr
Now,
Speed of fast train = (x + 10) km/hr = (40 + 10) km/hr = 50 km/hr

4. A passenger train takes one hour less for a journey of 150 km if its speed is increased by 5 km/hr from its usual speed. Find the usual speed of the train.
Sol:
Let the usual speed of the train be x km/hr
Distance covered in the journey = 150 km
⇒ Time taken by the train with usual speed = $\frac{150\ km}{x\ km/hr} = \frac{150}{x}\ hr$
Given that, if the speed is increased by 5 km/hr from i to usual speed, the train takes one hour less for the same journey.
⇒ Speed of the train = (x + 5) km/hr
Now, time taken by the train after increasing the speed = $\frac{150\ km}{(x+5) km/hr} = \frac{150}{x+5} hr$
We have, $\frac{150}{x} - \frac{150}{x+5} = 1$
⇒ $150\left(\frac{1}{x} - \frac{1}{x+5}\right) = 1$
⇒ $150\left(\frac{x+5-x}{x((x+5)}\right) = 1$
⇒ 150 (5) = x (x + 5)
⇒ 750 = x^2 + 5x
⇒ $x^2 + 5x - 750 = 0$
⇒ $x^2 + 30x - 25x + (30 \times -25) = 0$
⇒ $x(x + 30) - 25(x + 30) = 0$
⇒ $(x + 30)(x - 25) = 0$
⇒ $x = -30\ or\ (x - 25) = 0$
⇒ $Since, speed\ cannot\ be\ negative\ values, so\ x = 25.$
∴ usual speed of the train = 25 km/hr

5. The time taken by a person to cover 150 km was 2.5 hrs more than the time taken in the return journey. If he returned at a speed of 10 km/hr more than the speed of going, what was the speed per hour in each direction?

Sol:

Let the going speed of the person be x km/hr

Given that, the return speed is 10 km/hr more than the going speed

⇒ Return speed of the person = (x + 10) km/hr

Total distance covered = 150 km.

Time taken for going = $\frac{150\ km}{x\ km/hr} = \frac{150}{x} hr$

Time taken for returning = $\frac{150\ km}{(x+10) km/hr} = \frac{150}{(x+10)} hr$

Given that, time taken for going is 2.5 hours more than the time for returning

i.e. $\frac{150}{x} hr - \frac{150}{x+10} hr = 2.5\ hr$

⇒ $150\left(\frac{1}{x} - \frac{1}{x+10}\right) = \frac{25}{10}$

⇒ $150\left(\frac{x+10-x}{x(x+10)}\right) = \frac{25}{10}$

⇒ 6 (10) = $\frac{x(x+10)}{10}$

⇒ 60 × 10 = $x^2 + 10x$

⇒ $x^2 + 10x - 600 = 0$

⇒ $x^2 + (30 - 20)x + (30 \times -20) = 0$

⇒ $x^2 = 30x - 20x + (30 \times -20) = 0$

⇒ $x(x + 30) - 20(x + 30) = 0$

⇒ (x + 30) (x – 20) = 0

⇒ x + 30 = 0 or x – 20 = 0

⇒ x = −30 or x = 20

Since, speed cannot be negative. So x = 20

∴ speed of the person when going = 20 km/hr

Now, speed of the person when returning = (x + 10) km/hr

= (20 + 10) km/hr

= 30 km/hr

6. A plane left 40 minutes late due to bad weather and in order to reach its destination, 1600 km away in time, it had to increase its speed by 400 km/hr from its usual speed. Find the usual speed of the plane.

Sol:

Let the usual speed of the plane be x km/hr

Total distance travelled = 1600 km

⇒ Time taken by the plane with usual speed = $\frac{1600\ km}{x\ km/hr} = \frac{1600}{x}\ hr$

Given that, if speed is increased by 400 km/hr, the plane takes 40 minutes less than that of the usual time. Speed of the plane after increasing = (x + 400) km/hr

⇒ Time taken by the plane with increasing speed = $\frac{1600\ km}{(x+400)\frac{km}{hr}} = \frac{1600}{x+400} hr$

Now,

$\frac{1600}{x} hr - \frac{1600}{x+400} hr = \frac{40}{60} hr$ $\left[\because 40\ minutes = \frac{40}{60} hr\ as\ 1\ hr = 60\ min\right]$

$\Rightarrow 1600\left[\frac{1}{x} - \frac{1}{x+400}\right] = \frac{40}{60}$

$\Rightarrow 1600\left[\frac{x+400-x}{x(x+400)}\right] = \frac{40}{60}$

$\Rightarrow 40(400 \times 60) = x(x+400)$

$\Rightarrow x^2 + 400x - 960000 = 0$

$\Rightarrow x^2 + (1800 - 800)x + \left(1800 \times (-800)\right) = 0$

$\Rightarrow (x^2) + 1800x - 800x + (1800 \times -800) = 0$

$\Rightarrow x(x + 1800) - 800(x + 1800) = 0$

⇒ (x + 1800) (x – 800) = 0

⇒ x = −1800 or x – 800 = 0

⇒ x = −1800 or x = 800

Since, speed cannot be negative. So, x = 800

∴ Usual speed of the plans is 800 km/hr.

7. An aeroplane takes 1 hour less for a journey of 1200 km if its speed is increased by 100 km/hr from its usual speed. Find its usual speed.

Sol:

Let the usual speed of the plane be x km/hr.

Distance covered in the journey = 1800 km

⇒ Time taken by the plane with usual speed = $\frac{1200\ km}{x\frac{km}{hr}} = \frac{1200}{x} hr$

Now, speed is increased by 100 km/hr and the time taken is 1 hour less for the same journey.

⇒ Speed of the plane after increased = (x + 100) km/hr and Time taken by plane with increased speed = $\frac{1800\ km}{\frac{(x+100)km}{hr}} = \frac{1800}{x+100} hr$

Now, we have

$\frac{1800}{x} - \frac{1800}{x+100} = 1$

⇒ 1800 $\left(\frac{1}{x} - \frac{1}{x+100}\right) = 1$

⇒ 1800 $\left(\frac{x+100-x}{x(x+100)}\right) = 1$

⇒ 1200(100) = x(x + 100)

⇒ 120000 = $x^2 + 100x$

$\Rightarrow x^2 + 100x - 120000 = 0$

$\Rightarrow x^2 + (400 - 300)x + (400 \times -300) = 0$
$\Rightarrow x^2 + 400x - 300x + (400 \times \ -300) = 0$
$\Rightarrow x(x + 400) - 300(x + 400) = 0$
⇒ (x + 400) (x – 300) = 0
⇒ x + 400 = 0 or x – 300 = 0
⇒ x = −400 or x = 300
Since, speed cannot be negative so, x = 300 ∴ usual speed of the plane = 300 km/hr

8. A passenger train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. Find the usual speed of the train.

Sol:
Let the usual speed of the train be x km/hr
Distance covered in the journey = 300 km
Time taken by the train with usual speed = $\frac{300\ km}{x\ km/hr} = \frac{300}{x}\ hr$
Now,
If the speed is increased by 5 km/hr, the train takes 2 hours less for the same journey.
⇒ speed of the train after increasing = (x + 5) km/hr
And time taken by the train after increasing the speed = $\frac{300\ km}{(x+5)km/hr} = \frac{300}{x+5} hr$
We have,
$\frac{300}{x} hr - \frac{300}{x+5} hr = 2hrs$
$\Rightarrow 300\ \left(\frac{1}{x} - \frac{1}{x+5}\right) = 2$
$\Rightarrow 300\ \left(\frac{x+5-x}{x(x+5)}\right) = 2$
$\Rightarrow 300(5) = 2\big(x(x + 5)\big)$
$\Rightarrow 750 = x^2 + 5x$
$\Rightarrow x^2 + 5x - 750 = 0$
$\Rightarrow x^2 + 30x - 25x + (30 \times -25) = 0$
⇒ (x + 30) (x – 25) = 0
⇒ x + 30 = 0 or x – 25 = 0
⇒ x = −30 or x = 25
Since, speed cannot be negative. So x = 25
∴ The usual speed of the train = 25 km/hr.

9. A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more, it would have taken 30 minutes less for the journey. Find the original speed of the train.

Sol:
Let the original speed of the train be x km/hr

Distance covered = 90 km.

⇒ Time taken by the train with original speed = $\frac{90\ km}{x\ km/hr} = \frac{90}{x} hr$

Now, if the speed of the train is increased by 15 km/hr, the train takes 30 minutes less for the same journey

⇒ Speed of the train after increasing = (x + 15) km/hr and the time taken by the train after increasing the speed = $\frac{90\ km}{(x+15)\ km/hr} = \frac{90}{x+15} hr$

Now,

$\frac{90}{x} hr - \frac{90}{x+15}\ hr = 30\ min$

$\Rightarrow 90\ hr \left(\frac{1}{x} - \frac{1}{x+15} hr\right) = 30 min$

$\Rightarrow 90 \left(\frac{1}{x} - \frac{1}{x+15}\right) = \frac{30}{60} hr \quad [\because 1hr = 30\ min]$

$\Rightarrow 90\ \left(\frac{x+15-x}{x(x+15)}\right) = \frac{1}{2}$

$\Rightarrow 90 \times 15 \times 2 = x(x + 15)$

$\Rightarrow 2700 = x^2 + 15x$

$\Rightarrow x^2 + 15x - 2700 = 0$

$\Rightarrow x^2 + (60 - 45)x + \left(60 \times (-45)\right) = 0$

$\Rightarrow x^2 + 60x - 45x + (60 \times\ -45) = 0$

$\Rightarrow x(x + 60) - 45(x + 60) = 0$

$\Rightarrow (x + 60)(x - 45) = 0$

$\Rightarrow x + 60 = 0\ or\ x - 45 = 0$

$\Rightarrow x = -60\ or\ x = 45$

Since, speed cannot be negative. So, $x = 45$

∴ Original speed of the train = 45 km/hr

10. A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Sol:

Let the speed of the train be x km/hr

Distance covered by the train = 360 km

⇒ Time taken by the train with initial speed = $\frac{360\ km}{x\ km/hr} = \frac{360}{x} hr$

Now, if the speed is 5 km/hr more, the train takes 1 hour less for the same journey.

⇒ Speed of the train after increasing the speed = (x + 5) km/hr

And time taken by the train with increased speed $= \frac{360\ km}{\frac{(x+5)km}{hr}} = \frac{360}{x+5} hr$

Now,

$\frac{360}{x} hr - \frac{360}{x+5} hr = 1hr$

$\Rightarrow 360 \left(\frac{1}{x} - \frac{1}{x+5}\right) = 1$

$\Rightarrow 360\left(\frac{x+5-x}{x(x+5)}\right)=1$
$\Rightarrow 360 \times 5 = 1 \times x(x+5)$
$\Rightarrow x^2 + 5c - 1800 = 0$
$\Rightarrow x^2 + 45x - 40x + (x+45) = 0$
$\Rightarrow x(x+45) - 40(x+45) = 0$
$\Rightarrow (x+45)(x-40) = 0$
$\Rightarrow x + 45 = 0 \ or \ x - 40 = 0$
$\Rightarrow x = -45 \ or \ x = 40$
Since, speed is always a positive value i.e. x = 0 ⇒ x = 40
∴ The speed of the train = 40 km/hr

11. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 kin/hr more than that of the passenger train, find the average speeds of the two trains.
Sol:
Let the speed of the passenger train be x km/hr
Given that the average speed of the express train is 11 km/hr more than that of passenger train.
⇒ Average speed of express train = (x + 11) km/hr
Now,
Time taken by the passenger train = $\frac{138\ km}{x\ km/hr} = \frac{13}{x} hr$
And time taken by the express train = $\frac{132\ km}{(x+11)km/hr} = \frac{138}{x+11} hr$
Given that, express train takes 1 hour less than that of passenger train to reach the destiny.
$\Rightarrow \frac{132}{x} hr - \frac{132}{x+11} hr = 1hr$
$\Rightarrow 132\left(\frac{1}{x} - \frac{1}{x+11}\right) = 11$
$\Rightarrow 132\left(\frac{x+11-x}{x(x+11)}\right) = 1$
$\Rightarrow 132 \times 11 = x(x+11) \times 1$
$\Rightarrow x^2 - 11x - 1452 = 0$
$\Rightarrow x^2 + (44x - 33x) + (44 \times -33) = 0$
$\Rightarrow x^2 + 44x - 33x + (44 \times -33) = 0$
$\Rightarrow x(x+4) - 33(x+44) = 0$
$\Rightarrow (x+44)(x-33) = 0$
⇒ x + 44 = 0 or x – 33 = 0
⇒ x = −44 or x = 33
Since, speed cannot be in negative values. So, x = 33
∴ Average speed of the slower train i.e. passenger train = 33 km/hr

And average speed of express train = (x + 11) km/hr = (33 + 11) km/hr = 44 km/hr.

12. An aeroplane left 50 minutes later than its scheduled time, and in order to reach the destination, 1250 km away, in time, it had to increase its speed by 250 km/hr from its usual speed. Find its usual speed.

Sol:

Let the usual speed of the plane be x km/hr

Distance covered by the plane = 1250 km

⇒ Time taken by the plane with usual speed = $\frac{1250\ km}{x\ km/hr} = 1250\ hr$

To cover the delay of 50 minutes, the speed of the plane is increased by 250 km/hr

Now,

Speed of the plane after increasing = (x + 250) km/hr and

Time taken by the plane with increased speed = $\frac{1250\ km}{(x+250)\ km/hr} = \frac{1250}{x+250} hr$

From the data we have,

$\frac{1250}{x} hr - \frac{1250}{x+250} hr = 50\ min$

$\Rightarrow 1250\ hr\ \left(\frac{1}{x} - \frac{1}{x+250}\right) = \frac{50}{60} hr\ [\because 1hr = 50\ min]$

$\Rightarrow 250\left(\frac{x+250-x}{x(x+250)}\right) = \frac{1}{6}$

$\Rightarrow 250 \times 250 \times 6 = x(x+250) \times 1$

$\Rightarrow 375000 = x^2 + 250x$

$\Rightarrow x^2 + 250x - 375000 = 0$

$\Rightarrow x^2 + (750 - 500)x + (750 \times -500) = 0$

$\Rightarrow x^2 + 750x - 500x + (750 \times -500) = 0$

$\Rightarrow (x + 750)(x - 500) = 0$

$\Rightarrow (x + 750) = 0\ or\ x = 500 = 0$

$\Rightarrow x = -750\ or\ x = 500$

Since, speed cannot be a negative value. So, x = 500

∴ the usual speed of the plane = 500 km/hr.

Exercise 8.9

1. Ashu is x years old while his mother Mrs Veena is x years old. Five years hence Mrs Veena will be three times old as Ashu. Find their present ages.

Sol:

Given that, Ashu is x years old while his mother Mrs. Veena is x^2 years old.

⇒ Ashu’s present age = x years and Mrs. Veena’s present age = x^2 years

And also given that, after 5 years Mrs. Veena will be three times old as Ashu.

⇒ Ashu's age after 5 years = (x + 5)years
And Mrs. Veena's age after 5 years = (x^2 + 5)years
But given that,
⇒ $(x^2 + 5) = 3(x + 5)$
⇒ $x^2 + 5 = 3x + 15$
⇒ $x^2 - 3x - 10 = 0$
⇒ $x^2 - 5x + 2x - 10 = 0$
⇒ x(x – 5) +2(x – 5) = 0 ⇒ (x – 5) (x + 2) = 0
⇒ x – 5 = 0 or x + 2 = 0
⇒ x = 5 or x = −2
Since, age cannot be in negative values. So, x = 5 years.
∴ Present age of Ashu is x = 5 years and
Present age of Mrs. Veena is $x^2 \Rightarrow 5^2 years \ \Rightarrow 25 years$.

2. The sum of the ages of a man and his son is 45 years. Five years ago, the product of their ages was four times the man's age at the time. Find their present ages.
Sol:
Let the present age of the son be x years
Given that,
sum of present ages of man and his son is 45 years.
⇒ Man's present age = (45 – x)years
And also given that,
five years ago, the product of their ages was four times the man's age at the time.
⇒ Man's age before 5 years = (45 – x – 5) years = (40 – x) years
And son's age before 5 years = (x – 5) years
But, given that (40 – x) (x – 5) = 4(40 – x)
⇒ x – 5 = 4
⇒ x = 9 years
⇒ Son's present age ⇒ x = 9 years
Now, Man's present age ⇒ (45 – x) years = (45 – 9) years = 36 years
∴ The present ages of man and son are 36 years and 9 years respectively.

3. The product of Shikha's age five years ago and her age 8 years later is 30, her age at both times being given in years. Find her present age.
Sol:
Let the present age of shika be x years.
Given that,
The product of her age five years ago and her age 8 years later is 30
Now,

Shika's age five years ago = (x – 5) years
And Shika's age 8 years later = (x + 8) years
Given that.
$(x-5)(x+8)=30$
$\Rightarrow x^2+8x-5x-40=30$
$\Rightarrow x^2+3x-70=0$
$\Rightarrow x^2+10x-7x-70=0$
$\Rightarrow x(x+10)-7(x+10)=0$
$\Rightarrow (x+10)(x-7)=0$
$\Rightarrow x+10=0 \; or \; x-7=0$
$\Rightarrow x=-10 \; or \; x=7$
Since, age cannot be in negative values, So x = 7 years
∴ The present age of shika is 7 years.

4. The product of Ramu's age (in years) five years ago and his age (in years) nine years later is 15. Determine Ramu's present age.

Sol:
Let the present age of Ramu be a x years
Given that,
The product of his age five years ago and his age y nine years later is 15.
Now, Ramu's age five years ago = (x – 5) years
And Ramu's age nine years later = (x + 9) years
Given that,
$(x-5)(x+9)=15$
$\Rightarrow x^2+9x-5x-45=15$
$\Rightarrow x^2+4x-60=0$
$\Rightarrow x^2+10x-6x-60=0$
$\Rightarrow x(x+10)-6(x+10)=0$
$\Rightarrow (x+10)(x-6)=0$
$\Rightarrow x+10=0 \; or \; x-6=0$
$\Rightarrow x=-10 \; or \; x=6$
Since, age cannot be in negative values, So x = 6 years
∴ The present age of shika is 6 years.

5. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Sol:
Let the present age of friend 1 be a x years
Given that,
Sum of the ages of two friends = 20 years

⇒ Present age of friend 2 = (20 – x) years
And also given that, four years ago, the product of their age was 48.
⇒ Age of friend 1 before 4 years = (x – 4) years
And age of friend 2 before 4 years = (20 – x – 4) years = (16 – x) years
Given that,
$(x-4)(16-x)=48$
$\Rightarrow 16x-x^2-64+4x=48$
$\Rightarrow x^2-20x+112=0$
Let D be the discriminant of this quadratic equation.
Then, D = $(-20)^2-4\times112\times1=400-448=-48\angle0$
We know that, to have real roots for a quadratic equation that discriminant D must be greater than or equal to 0 i.e. D ≥ 0
But D ∠ 0 in the above. So, above equation does not have real roots
Hence, the given situation is not possible.

6. A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages.

Sol:
Let the age of girls sister be a x years
Given that,
Girl is twice as old as her sister
⇒ Girls age = $2\times x$ years = 2x years
Given that, after 4 years, the product of their ages will be 160.
⇒ Girls age after 4 years = (2x + 4) years
And sisters age after 4 years = (x + 4) years
Given that,
$(2x+4)(x+4)=160$
$\Rightarrow 2x^2+8x+4x+16=160$
$\Rightarrow 2x^2+12x-144=0$
$\Rightarrow 2(x^2+6x-72)=0$
$\Rightarrow x^2+6x-72=0$
$\Rightarrow x^2+12x-6x-72=0$
$\Rightarrow x(x+12)-6(x+12)=0$
$\Rightarrow (x+12)(x-6)=0$
$\Rightarrow x+12=0 \; or \; x-6=0$
$\Rightarrow x=-12 \; or \; x=6$
Since, age cannot be in a negative value.
So, x = 6.
∴ Age of girls sister is x = 6 years.
And age of girl is 2x = 2 × 6 years = 12 years

Hence, the present ages of girl and her sister are 12 years and 6 years respectively.

7. The sum of the reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Sol:

Let the present age of Rehman be x years.

Now,

Rehman's age 3 years ago = (x – 3) years

And Rehman's age 5 years later = (x + 5) years

Given that,

The sum of reciprocals of Rehman's ages 3 years ago and 5 years later is $\frac{1}{3}$

$\Rightarrow \frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$

$\Rightarrow (2x+2) \times 3 = 1(x-3)(x+5)$

$\Rightarrow 6x + 6 = x^2 + 5x - 3x - 15$

$\Rightarrow x^2 + 2x - 6x - 15 - 6 = 0$

$\Rightarrow x^2 - 4x - 21 = 0$

$\Rightarrow x^2 - 7x + 3x - 21 = 0$

$\Rightarrow x(x-7)^2 - 3(x-7) = 0$

$\Rightarrow (x-7)(x+3) = 0 \;\Rightarrow x - 7 = 0 \; or \; x + 3 = 0$

$\Rightarrow$ x = 7 or x = −3

Since, age cannot be in negative values. So, x = 7 years Hence, the present age of Rehman is 7 years.

Exercise 8.10

1. The hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 5 cm. Find the lengths of these sides.

Sol:

Let the length of the shortest side be x cm

Given that the length of the largest side is 5cm more than that of smaller side

$\Rightarrow$ longest side = (x + 5)cm

And also, given that

Hypotenuse = 25cm

So, let us consider a right angled triangle ABC right angled at B

We have, hypotenuse (AC) = 25 cm

BC = x cm and AB = (x + 5)cm

Since, ABC is a right angled triangle

We have, $(BC)^2 + (AB)^2 = (AC)^2$
$\Rightarrow x^2 cm^2 + (x+5)^2 cm^2 = (25)^2 cm^2$
$\Rightarrow x^2 + x^2 + 10x + 25 = 625$
$\Rightarrow 2x^2 + 10x - 600 = 0$
$\Rightarrow 2(x^2 + 5x - 300) = 0$
$\Rightarrow x^2 + 5x - 300 = 0$
$\Rightarrow x^2 + 20x - 15x + (20x - 15) = 0$
$\Rightarrow x(x+20) - 15(x+20) = 0$
$\Rightarrow (x+20)(x-15) = 0$
$\Rightarrow (x+20) = 0 \; or \; (x-15) = 20$

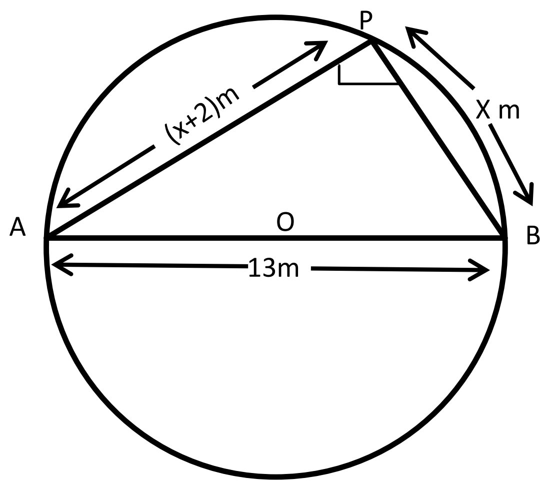
2. The hypotenuse of a right triangle is 3.Ji cm. If the smaller leg is tripled and the longer leg doubled, new hypotenuse will be 9Iš cm. How long are the legs of the triangle?

Sol:

Using Pythagoras theorem,
$(AB)^2 + (BC)^2 = (AC)^2$
$\Rightarrow (2y)^2 cm^2 + (3x)^2 cm^2 = \left(9\sqrt{5}\right)^2 cm^2$
$\Rightarrow 4y^2 + 9x^2 = 81 \times 5$
$\Rightarrow 4y^2 + 9x^2 = 405$
$\Rightarrow 4(90 - x^2) + 9x^2 = 405 \quad [\because x^2 + y^2 = 90]$
$\Rightarrow 4 \times 90 - 4x^2 + 9x^2 = 405$
$\Rightarrow 5x^2 = 405 - 360$
$\Rightarrow 5x^2 = 405 - 360$
$\Rightarrow 5x^2 = 45$
$\Rightarrow x^2 = 9$
$\Rightarrow x = \sqrt{3^2} \Rightarrow x = \pm 3$
$Since, x \; cannot \; be \; a \; negative \; value. So \; x = 3cm$
We have,
$x^2 + y^2 = 90$
$\Rightarrow y^2 = 90 - (3)^2$
$\Rightarrow y^2 = 90 - 9$
$\Rightarrow y^2 = 81 \Rightarrow y = \sqrt{81} \Rightarrow y = \pm 9$
Since, y cannot be a negative value. So, y = 9cm
∴ hence, the length of the smaller side is 3 cm and the length of the longer side is 9cm.

3. A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the difference of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it the possible to do so? If yes, at what distances from the two gates should the pole be erected?

Sol:
Yes, it is possible to do so as in the given condition
This can be proved as below,



Let P be the required location of the pole such that its distance from gate B is x meter i.e.
BP = x meters and also AP – BP = 7m
⇒ AP = BP + 7m = (x + 7)m
Since, AB is a diameter and P is a point on the boundary of the semi-circle, ΔAPB is right angled triangle, right angled at P.
Using Pythagoras theorem,
$(AB)^2 = (AP)^2 + (BP)^2$
⇒ $(13)^2m^2 = (x + 7)^2m^2 + (x)^2m^2$
⇒ 169 = $x^2 + 14x + 49 + x^2$
⇒ $2x^2 + 14x + 49 - 169 = 0$
⇒ $2x^2 + 14x - 120 = 0$
⇒ $2(x^2 + 7x - 60) = 0$
⇒ $x^2 + 7x - 60 = 0$
⇒ $x^2 + 12x - 5x - (12 \times -5) = 0$
⇒ $x(x + 12) - 5(x + 12) = 0$
⇒ $(x + 12)(x - 5) = 0$
⇒ x + 12 = 0 or x – 5 = 0
⇒ x = −12 or x = 5
Since, x cannot be a negative value, So x = 5
⇒ BP = 5m
Now, AP = (BP + 7)m = (5 + 7)m = 12 m
∴ The pole has to be erected at a distance 5 mtrs from the gate B and 12 m from the gate A.

4. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Sol:

120 m, 90 m

Exercise 8.11

1. The perimeter of a rectangular field is 82 m and its area is 400 m2. Find the breadth of the rectangle.

Sol:

Let the breadth of the rectangle be x meters

Given that,

Perimeter = 82 m and Area = $400m^2$

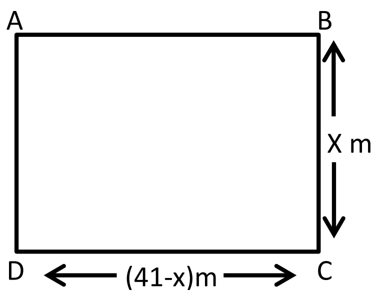
We know that

Perimeter of a rectangle = 2(length + breadth)

$\Rightarrow 82 = 2\ (\text{length} + x)$

$\Rightarrow 41 = \text{length} + x$

$\Rightarrow \text{length} = (41 - x)m$



We have

Area of rectangle = length × breadth

$\Rightarrow 400\ m^2 = (41 - x)\ m \times xm$

$\Rightarrow 400 = 41x - x^2$

$\Rightarrow 400\ x^2 - 41x + 400 = 0$

$\Rightarrow x^2 - 25x - 16x + (-25 \times -16) = 0$

$\Rightarrow x(x - 25) - 16(x - 25) = 0$

$\Rightarrow (x - 25)(x - 16) = 0$

$\Rightarrow (x - 25)\ (x - 16) = 0$

$\Rightarrow x - 25 = 0$ or $x - 16 = 0$

$\Rightarrow x = 25$ or $x = 16$

Hence, breadth of the rectangle is 25m or 16m

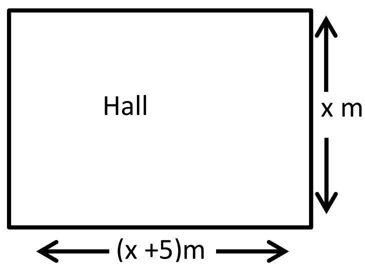
2. The length of a hail is 5 m more than its breadth. If the area of the floor of the hail is 84 m^2, what are the length and breadth of the hail?

Sol:

Let the breadth of the rectangle (hall) be x meter.

Given that,

Length of the hall is 5m more than its breadth i.e. length = (x + 5)m



And also given that,

Area of the hall = $84m^2$

Since, hall is in the shape of a rectangle,

Area of the rectangular hall = length × breadth

$\Rightarrow 84m^2 = xm \times (x+5)m$

$\Rightarrow 84 = x(x + 5)$

$\Rightarrow 84 = x^2 + 5x$

$\Rightarrow x^2 + 5x - 84 = 0$

$\Rightarrow x^2 + 12x - 7x - 84 = 0$

$\Rightarrow x(x+12) - 7(x+12) = 0$

$\Rightarrow (x - 7)(x + 12) = 0$

$\Rightarrow$ x = 7m or x = −12m

Since, x cannot be negative. So, breadth of the hall = 7m

Hence, length of the hall = (x + 5)m = (7 + 5)m = 12m.

3. Two squares have sides x cm and (x + 4) cm. The sum of their areas is 656 cm^2. Find the sides of the squares.

Sol:

Let S_1 and S_2 be two squares.

Let x cm be the side of square S_1 and (x + 4)cm be the side of square S_2.

We know that,

Area of a square = $(Side)^2$

$\Rightarrow$ Area of square S_1 = $(x)^2 = x^2 cm^2$

$\Rightarrow$ Area of square S_2 = $(x+4)^2 cm^2$

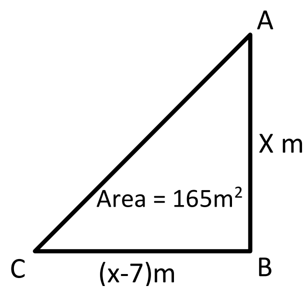
Given that,

Area of square S_1 + Area of square S_2 = 656 cm^2

$\Rightarrow x^2 cm^2 + (x+4) cm^2 = 656\ cm^2$

$\Rightarrow x^2 + x^2 + 8x + 16 = 656$
$\Rightarrow 2x^2 + 8x + 16 - 656 = 0$
$\Rightarrow 2x^2 + 8x - 640 = 0$
$\Rightarrow 2(x^2 + 4x - 320) = 0$
$\Rightarrow x^2 + 4x - 320 = 0$
$\Rightarrow x^2 + 20x - 16x + (20x - 16) = 0$
$\Rightarrow x + 20 = 0 \; or \; x - 16 = 0$
$\Rightarrow x = -20 \; cm \; or \; x = 16 \; cm$
Since, x cannot be negative. So, x = 16cm
∴ Side of square S_1 ⇒ x = 16cm and
Side of square S_2 ⇒ (x + 4) = (16 + 4)cm = 20 cm

4. The area of a right angled triangle is 165 m2. Determine its base and altitude if the latter exceeds the former by 7m.
Sol:
Let the altitude of the right angled triangle be denoted by x meter
Given that altitude exceeds the base of the triangle by 7m.
⇒ Base = (x – 7)m



We know that,
Area of a triangle = $\frac{1}{2} \times base \times height$
⇒ 165m² = $\frac{1}{2} \times (x - 7)m \times x\; m$ $[\because Area = 165m^2 \; given]$
$\Rightarrow 2 \times 165 = x(x - 7)$
$\Rightarrow x^2 - 7x = 330$
$\Rightarrow x^2 - 7x - 330 = 0$
$\Rightarrow (x - 22) + 15(x - 22) = 0$
$\Rightarrow (x - 22)(x + 15) = 0$
⇒ x = 22 or x = − 15
Since, x cannot be negative. So, x = 22 m
∴ Altitude of the triangle ⇒ x = 22m
And base of the triangle ⇒ (x – 7)m = (22 – 7)m = 15m

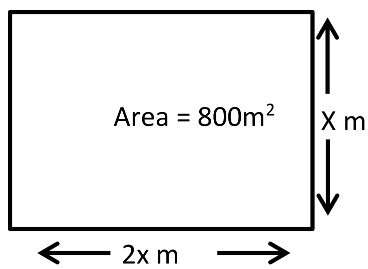
5. Is it possible to design a rectangular mango grove whose length is twice its breadth and the area is 800 m^2? If so, find its length and breadth.

Sol:

Let the breadth of the rectangular mango grove be x meter.

Given that length is twice that of breadth ⇒ length = 2 × x m = 2x m

Given that area of the grove is $800m^2$.



But we know that

Area of a rectangle = length × breadth

$\Rightarrow 800m^2 = 2x\ m \times x\ m$

$\Rightarrow 2x^2 = 800$

$\Rightarrow x^2 = 400$

$\Rightarrow x = \sqrt{400} = \sqrt{(20)^2} = \pm 20$

⇒ x = 20 or x = −20

Since, x cannot be a negative value.

So, x = 20 m

∴ Breadth of the mango grove = 20m and length of the mango grove

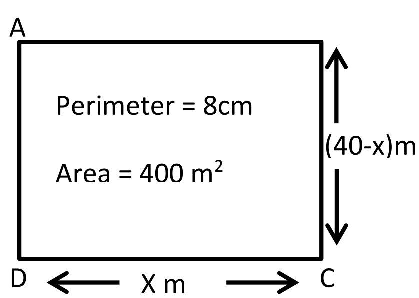
= 2x m = 2 × 20 m = 40m

Yes. It is possible to design a rectangular mango grove whose length is twice its breadth and the area is $800m^2$.

6. Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2? If so, find its length and breadth.

Sol:

To prove the given condition, let us assume that the length of the rectangular park be denoted by x m.



Given that
Perimeter = 80m and Area = 400m^2
We know that,
Perimeter of a rectangle = 2(length + breadth)
⇒ 80m = 2 (x + breadth)
⇒ breadth = $\left(\frac{80}{2} - x\right)m$
⇒ breadth = (40 – x)m
And also,
Area of a rectangle = length × breadth
⇒ 400m^2 = x m × $(40 - x)m$
⇒ 400 = x(40 – x)
⇒ 400 = 40x – x^2
⇒ $x^2 - 40x + 400 = 0$
⇒ $x^2 - 2 \times 20 \times x + (20)^2 = 0$
⇒ $(x - 20)^2 = 0$
⇒ $(x - 20 = 0)$ ⇒ $x = 20$
∴ length of the rectangular park ⇒ x = 20 m and breadth of the rectangular park ⇒ (40 – x)m = (40 – 20)m = 20m
Yes. It is possible to design a rectangular park of perimeter 80m and area 400m^2.

7. Sum of the areas of two squares is 640 m^2. If the difference of their perimeters is 64 m, find the sides of the two squares.

Sol:
Let the two squares be denoted as S_1 and S_2 and let side of squares S_1 be denoted as x meter and that of square S_2 be y m.
Given that,
Difference of their perimeter is 64m.
We know that
Perimeter of a square = 4 × side
⇒ Perimeter of square S_1 = $4 \times x\ m = 4x\ m$
⇒ Perimeter of square $S_2 = 4 \times y\ m = 4y\ m$
Now, difference of perimeter = perimeter of square S_1 − Perimeter of square S_2
⇒ 64 m = (4x – 4y)m
⇒ 64 = 4(x – y)
⇒ x – y = 16
⇒ x = y + 16
And also,
Given that sum of areas of two squares = 640 m^2.
We know that,
Area of a square = (Side)^2

⇒ Area of square $S_1 = x^2m^2$
⇒ Area of square $S_2 = y^2m^2$
Now,
Sum of areas of two squares = Area of square S_1+Area of square S_2
⇒ $640m^2 = x^2m^2 + y^2m^2$
⇒ 640 = $(y + 16)^2 + y^2$ [$\because x = y + 16$]
⇒ $y^2 + 32y + 256 + y^2 = 640$
⇒ $2y^2 + 32y + 256 - 640 = 0$
⇒ $2y^2 + 32y - 384 = 0$
⇒ $2(y^2 + 16y - 192) = 0$
⇒ $y^2 + 16y - 192 = 0$
⇒ $y^2 + 24y - 8y + (24 \times -8) = 0$
⇒ $y(y + 24) - 8(y + 24) = 0$
⇒ (y + 24) (y – 8) = 0
⇒ y + 24 = 0 or y – 8 = 0
⇒ y = − 24 or y = 8
Since, y cannot be a negative value. So, y = 8m
∴ Side of the square S_2 is y = 8m
And side of the square S_1 is x = (y + 16)m = (8 + 16)m = 24 m
Hence, sides of the two squares is 24m and 8m.

Exercise 8.12

1. A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days, find the time taken by B to finish the work.

Sol:
Let B takes x days to complete the piece of work.
⇒ B’s one days work = $\frac{1}{x}$
Now, A takes 10 days less than that of B to finish the same piece of work i.e. (x – 10)days
⇒ A’s one days work = $\frac{1}{x-10}$
Given that, both A and B together can finish the same work in 12 days.
⇒ (A and B)’s one days work = $\frac{1}{12}$
Now,
(A’s one days work) + (B’s one days work) = $\frac{1}{x} + \frac{1}{x-10}$ and $(A + B)$’s one days work = $\frac{1}{12}$
⇒ $\frac{1}{x} + \frac{1}{x-10} = \frac{1}{12}$
⇒ $\frac{x-10+x}{x(x-10)} = \frac{1}{12}$
⇒ $(2x - 10) \times 12 = x(x - 10)$

$\Rightarrow 24x - 120 = x^2 - 10x$
$\Rightarrow x^2 - 10x - 24x + 120 = 0$
$\Rightarrow x^2 - 34x + 120 = 0$
$= x^2 - 30x - 4x + (-30 \times -4) = 0$
$\Rightarrow x(x - 30) - 4(x - 30) = 0$
$\Rightarrow (x - 30)(x - 4) = 0$
$\Rightarrow (x - 30) = 0 \; or \; (x - 4) = 0$
$\Rightarrow x = 30 \; or \; x = 4$
We can observe that, the value of x cannot be less than 10.
∴ The time taken by B to finish the work is 30 days.

2. If two pipes function simultaneously, a reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours will the second pipe take to fill the reservoir?
Sol:
Let x be no. of students planned for a picnic
Given that budget for food was Rs 480
⇒ Share of each student = $\frac{Total\ budget}{No.of\ students} =$ Rs $\frac{480}{x}$
Given that 8 students foiled to go
⇒ No. of students will be (x – 8)
Now,
Share of each student will be equal to
$= \frac{total\ budget}{No.of\ students} = Rs.\frac{480}{x-8}$
Given that if 8 students failed to go, then cost of food for each member increased by Rs. 10.

3. Two water taps together can fill a tank in 9 hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.
Sol:
Let the time taken by the top of smaller diameter to fill the tank be x hours
⇒ Portion of tank filled by smaller pipe in one hour = $\frac{1}{x}$
Now, larger diameter pipe takes 10 hours less than that of smaller diameter pipe i.e. (x – 10) hours
⇒ Portion of tank filled by larger diameter pipe in one hour = $\frac{1}{x-10}$
Given that,
Two tops together can fill the tank in $9\frac{3}{8}$ hours ⇒ $\frac{75}{8}$ hours
Now,

Portion of tank filled by both the tops together in one hour = $\frac{1}{75/8} = \frac{8}{75}$

We have,

Portion of tank filled by smaller pipe in 1 hr + Portion of tank filled by larger pipe in 1 hr.

$= \frac{1}{x} + \frac{1}{x-10} \Rightarrow \frac{8}{75} = \frac{1}{x} + \frac{1}{x-10}$

4. Two pipes running together can fill a tank in 11 minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately.

Sol:

Let us take the time taken by the faster pipe to fill the tank as x minutes.

⇒ Portion of tank filled by faster pipe in one minute = $\frac{1}{x}$

Now, time taken by the slower pipe to fill the same tank is 5 minutes more than that of faster pipe i.e. (x + 5) minutes.

⇒ Portion of tank filled by slower pipe in one minute = $\frac{1}{x+5}$

Given that,

The two pipes together can fill the tank in $11\frac{1}{9}$ minutes ⇒ $\frac{100}{9}$ minutes

⇒ portion of tank filled by faster pipe in 1min + Portions of tank filled by slower pipe in 1min i.e. $\frac{9}{100} = \frac{1}{x} + \frac{1}{x+5}$

$\Rightarrow \frac{9}{100} = \frac{x+5+x}{x(x+5)}$

Exercise 8.13

1. A piece of cloth costs Rs. 35. If the piece were 4 m longer and each metre costs Rs. one less, the cost would remain unchanged. How long is the piece?

Sol:

Let initial length of the cloth be x m, and cost per each meter of cloth be Rs y

⇒ Total cost of piece of cloth will be length of cloth x cost per each meter

⇒ xy

But given that xy = Rs. 35 ⇒ y = Rs. $\frac{35}{x}$

And also,

Given that if the piece were 4m longer and each meter costs Rs. 1 less the cost would remain unchanged.

⇒ Length of the cloth will be (x + 4)m and cost per each meter of cloth will be Rs (y..)

⇒ Total cost of piece of cloth will be Rs. (x + 4) (y – 1)

But,

Rs (x + 4) (y – 1) = Rs 35

⇒ xy + 4y – x – 4 = 35

$\Rightarrow 35 + 4\left(\frac{35}{2}\right) - x - 4 = 35$ $\left[\because xy = 35 \; \& y = \frac{35}{2}\right]$

$\Rightarrow \frac{140 - x^2 - 4x}{x} = 0$

$\Rightarrow x^2 + 4x - 140 = 0$

$\Rightarrow x^2 + 14x - 10x + (14x - 10) = 0$ $[\because 140 = 14x - 10 = 4x = 14x - 10x]$

$\Rightarrow x(x + 14) - 10(x + 14) = 0$

$\Rightarrow (x + 14)(x - 10) = 0$

$\Rightarrow (x + 14) = 0 \; or \; (x - 10) = 0$

$\Rightarrow x = -14 \; or \; x = 10$

Since length of the cloth cannot be in negative integers, the required length of cloth is 10m.

2. Some students planned a picnic. The budget for food was Rs. 480. But eight of these failed to go and thus the cost of food for each member increased by Rs. 10. How many students attended the picnic?

Sol:

Let x be no. of students planned for a picnic

Given that budget for food was Rs 480

$\Rightarrow$ Share of each student = $\frac{Total\ budget}{No.of\ students}$ = Rs $\frac{480}{x}$

Given that 8 students foiled to go

$\Rightarrow$ No. of students will be (x – 8)

Now,

Share of each student will be equal to

$= \frac{total\ budget}{No.of\ students} = Rs.\frac{480}{x-8}$

Given that if 8 students failed to go, then cost of food for each member increased by Rs. 10.

3. A dealer sells an article for Rs. 24 and gains as much percent as the cost price of the article. Find the cost price of the article.

Sol:

Let the cost price of the article be Rs x

Given that gain percentage of the article is as much as cost price i.e. x

$\Rightarrow$ Selling price = cost price + gain

= Rs x + cost price × gain percentage

= Rs x + Rs x × $\frac{x}{100}$

= Rs $\left(x + \frac{x^2}{100}\right)$

Given that selling price = Rs 24

$\Rightarrow$ Rs 24 = Rs $\left(x + \frac{x^2}{100}\right)$

$\Rightarrow$ 24 = $x + \frac{x^2}{100}$

$\Rightarrow \frac{x^2}{100} + x - 24 = 0$
$\Rightarrow x^2 + 100x - 2400 = 0$
$\Rightarrow x^2 + 120x - 20x + (120 \times -80) = 0$
$\Rightarrow x^2(x + 180) - 80(x + 180) = 0$
$\Rightarrow (x + 180)(x - 20) = 0$
⇒ x + 120 = 0 or x – 20 = 0
⇒ x = -120 or x = 20
Since, cost price of the article cannot be negative, the required cost price of the article is Rs 20
$\Rightarrow Rs\,\frac{480}{x-8} - Rs\,\frac{480}{x} = Rs\ 10$
$\Rightarrow \frac{480}{x-8} - \frac{480}{x} = 10$
$\Rightarrow 480\left(\frac{1}{x-8} - \frac{1}{x}\right) = 10$
$\Rightarrow 48\left(\frac{x-(x-8)}{x(x-8)}\right) = 1$
$\Rightarrow 48\left(\frac{x-x+8}{x^2-8x}\right) = 1$
$\Rightarrow 48\ (8) = x^2 - 8x$
$\Rightarrow x^2 - 8x - 384 = 0$
$\Rightarrow\ x^2 - 24x + 16x + (-24 \times 16) = 0$
$\Rightarrow\ x(x - 24) + 16(x - 24) = 0$
$\Rightarrow (x - 24)(x + 16) = 0$
$\Rightarrow (x - 24) = 0$ or $(x + 16) = 0$
$\Rightarrow x - 24$ or $x = -16$
Since the value of number of students cannot be negative, the required number of students attended the picnic is 24.

4. Out of a group of swans, 7/2 times the square root of the total number are playing on the share of a pond. The two remaining ones are swinging in water. Find the total number of swans.

Sol:

Let total number of swans be x
Given that 7/2 times the square root of the total number of swans are playing on the share of a pond i.e. $\frac{7}{2}\sqrt{x}$ and the two remaining ones are swinging in water
⇒ Total number of swans x = $\frac{7}{2}\sqrt{x} + 2$
$\Rightarrow x = \frac{7}{2}\sqrt{x} + 2 \qquad \left[Let\ \sqrt{x} = y\ \Rightarrow x = y^2\right]$
$\Rightarrow y^2 = \frac{7}{2}y + 2$
$\Rightarrow y^2 - \frac{7}{2}y - 2 = 0$

$\Rightarrow 2y^2 - 7y - 4 = 0$
$\Rightarrow 2y^2 - 8y + y - 4 = 0$
$\Rightarrow 2y(y - 4) + 1(y - 4) = 0$
$\Rightarrow (y - 4)(2y + 1) = 0$
$\Rightarrow (y - 4) = 0\ or\ (2y + 1) = 0$
$\Rightarrow$ y = 4 or y = $\frac{-1}{2}$
$\Rightarrow y^2 = 4^2 = 16\ or\ y^2 = \left(\frac{-1}{2}\right)^2 = \frac{1}{4}$
Since, the value of number of swans cannot be a fraction, the required number of swans
x = 16

5. If the list price of a toy is reduced by Rs. 2, a person can buy 2 toys more for Rs. 360. Find the original price of the toy.
Sol:
Let initial list price of the toy be Rs x
Given that total cost of toys = Rs 360
$\Rightarrow$ Initially number of toys a person can buy = $\frac{Total\ cost}{list\ price\ of\ each\ toy} = \frac{Rs\ 360}{Rs\ x} \Rightarrow \frac{360}{x}$
Now, if the list price is reduced by Rs 2 i.e. Rs. (x – 2)
Number of toys a person can buy is 2 more for Rs 360
$\Rightarrow$ Number of toys a person can buy when price is reduced = $\frac{Total\ cost}{list\ price} = \frac{Rs\ 360}{Rs\ x-2} = \frac{360}{x-2}$
Now,
$\frac{360}{x-2} - \frac{360}{x} = 2$
$\Rightarrow 360\left(\frac{1}{x-2} - \frac{1}{x}\right) = 2$
$\Rightarrow 360\left(\frac{x-(x-2)}{x(x-2)}\right) = 2$
$\Rightarrow 360\left(\frac{x-x+2}{x^2-2x}\right) = 2$
$\Rightarrow 360\left(\frac{2}{x^2-2x}\right) = 2$
$\Rightarrow 360 = x^2 - 2x$

6. Rs. 9000 were divided equally among a certain number of persons. Had there been 20 more persons, each would have got Rs. 160 less. Find the original number of persons.
Sol:
Let the original number of persons be x,
Total amount to be divided equally is Rs. 9000
$\Rightarrow$ Share of each person will be equal to = $\frac{Total\ amount}{No.of\ persons} = Rs\ \frac{900}{x}$
Given that if there had been 20 more persons
$\Rightarrow$ Final number of persons will be x + 20, then each would have got Rs 160 less

Now,

Final share of each person will be equal to $= \frac{Total\ amount}{No.of\ persons} = Rs\, \frac{9000}{x+20}$

We have,

Rs $\frac{9000}{x} - Rs\, \frac{9000}{x+20} = Rs\ 160$

$\Rightarrow 9000\left(\frac{1}{x} - \frac{1}{x+20}\right) = 160$

$\Rightarrow 9000\left(\frac{7+80-x}{x(x+20)}\right) = 160$

$\Rightarrow 9000\left(\frac{20}{x^2+20x}\right) = 160$

$\Rightarrow 1125 = x^2 + 20x$

$\Rightarrow x^2 + 20x - 1125 = 0$

$\Rightarrow x^2 + 45x - 25x + (45 \times -25) = 0$

$\Rightarrow x(x+45) - 25(x+45) = 0$

$\Rightarrow (x+45)(x-25) = 0$

$\Rightarrow x + 45 = 0$ or $x – 25 = 0$

$\Rightarrow x = -45$ or $x = 25$

Since, share of each person cannot be negative value, the required share of each person is Rs 25.

7. Some students planned a picnic. The budget for food was Rs. 500. But, 5 of them failed to go and thus the cost of food for each member increased by Rs. 5. How many students attended the picnic?

Sol:

Let the number of students planned for the picnic be x

Given budget for food = Rs. 500

$\Rightarrow$ Initially share of food for each student = $\frac{total\ budget}{no.of\ students} = Rs\, \frac{500}{x}$

Given that 5 students failed to go for the picnic

$\Rightarrow$ No. of students attended the picnic will be (x – 5)

Now, share of food for each student will be equal to = $\frac{total\ budget}{no.of\ students\ attended} = Rs\, \frac{500}{x-5}$

Given that, share of food for each student is increased

$\Rightarrow$ Rs $\frac{500}{x-5} - Rs\, \frac{500}{x} = Rs\ 5$

$\Rightarrow \frac{500}{x-5} - \frac{500}{x} = 5$

$\Rightarrow 500\left(\frac{1}{x-5} - \frac{1}{x}\right) = 5$

$\Rightarrow 500\left(\frac{s-(x-5)}{x(x-5)}\right) = 5$

$\Rightarrow 500\left(\frac{x-x+5}{x^2-5x}\right) = 5$

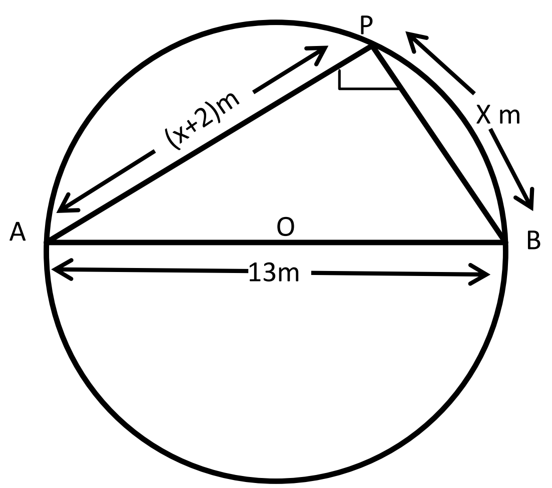
$\Rightarrow 500\left(\frac{5}{x^2-5x}\right) = 5$

⇒ $500 = x^2 - 5x$
⇒ $x^2 - 5x - 500 = 0$
⇒ $x^2 - 25x + 20x - 500 = 0$
⇒ $x(x - 25) + 20(x - 25) = 0$
⇒ $(x - 25)(x + 20) = 0$
⇒ $(x - 25) = 0$ *or* $(x + 20) = 0$
⇒ x = 25 or x = −20
Since, the value of x cannot be negative,
⇒ x = 25
Here, x is the no. of students planned,
Given that 5 students failed to go
⇒ No. of students attended the picnic = x – 5 = 25 – 5 = 20
∴ No. of students attended the picnic = 20

8. A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the difference of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it the possible to do so? If yes, at what distances from the two gates should the pole be erected?

Sol:

Let P be the required location of the pole such that its distance from gets B is x meters i.e. BP = x meters and also AP – BP = 7m ⇒ AP = (x + 7)m



Since, AB is a diameter and P is a point in the semi-circle ΔAPB is right angled at P.
Now, $(x + 7)^2 + (x)^2 = (13)^2$ $[\because AP^2 + BP^2 = AB^2 \ and \ AB = 13m]$
⇒ $x^2 + 14x + 49 + x^2 = 169$
⇒ $2x^2 + 14x + (49 - 169) = 0$
⇒ $2x^2 + 14x - 180 = 0$
⇒ $2(x^2 + 7x - 60) = 0$
⇒ $x^2 + 7x - 60 = 0$

$\Rightarrow x^2 + 12x - 5x + (12x - 5) = 0$
$\Rightarrow x(x + 12) - 5(x + 12) = 0$
$\Rightarrow (x + 12)(x - 5) = 0$
⇒ x + 12 = 0 or x – 5 = 0
⇒ x = - 12m or x = 5m
Since, BP cannot be in negative value (or) distances cannot be negative values,
The required values of BP and AP are 5m and 12m respectively.
∴ The pole has to be erected at a distance 5meters from the gate B.

9. In a class test, the sum of the marks obtained by P in Mathematics and science is 28. Had he got 3 marks more in Mathematics and 4 marks less in Science. The product of his marks, would have been 180. Find his marks in the two subjects.

Sol:
Let number of marks obtained by P in mathematics and science be x and y respectively.
Given that sum of these two is 28
⇒ x +y = 28 ⇒ x = 28 – y
Given that if x becomes (x + 3) i.e. marks in mathematics is increased by 3 and y becomes (y – 4) i.e. marks in science is decreased by 4, The product of these two becomes by 4,
$\Rightarrow (x + 3)(y - 4) = 180$
$\Rightarrow (28 - y + 3)(y - 4) = 180 \qquad [\because x = 28 - y]$
$\Rightarrow (31 - y)(y - 4) = 180$
$\Rightarrow 31y - 31 \times 4 - y^2 + 4y = 180$
$\Rightarrow 35y - y^2 - 124 = 180$
$\Rightarrow y^2 - 35y + 180 + 124 = 0$
$\Rightarrow y^2 - 35y + 304 = 0$
$\Rightarrow y^2 - 19y - 16y + (-19 \times -16) = 0$
$\Rightarrow y(y - 19) - 16(y - 19) = 0$
$\Rightarrow (y - 19)(y - 16) = 0$
⇒ y – 19 = 0 or y – 16 = 0
⇒ y = 19 or y = 16
We have,
x + y = 28
if y = 19 ⇒ x = 28 – y = 28 – 19 = 9 and
if y = 16 ⇒ x = 28 – y = 28 – 16 = 12
∴ Marks in mathematics = 9 and Marks in Science = 19 or
Marks in mathematics = 12 and Marks in Science = 16

10. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210. Find her marks in two subjects.

Sol:

Let marks of shefali in Mathematics and English be x and y respectively.

Given that sum of these two is 30 ⇒ x + y = 30 ⇒ x = 30 – y

Given that if x becomes (x + 2) i.e. marks in mathematics is increased by 2

and y becomes (y – 3) i.e. marks in English is decreased by 3,

the product at these two becomes 210

i.e. (x + 2) (y – 3) = 210

⇒ (30 – y + 2) (y – 3) = 210 [∵ x = 30 – y]

⇒ (32 – y) (y – 3) = 210

⇒ (32 – y) (y – 3) = 210

⇒ 32y – 32 × 3 – y × 3y = 210

⇒ 35y – 96 – y^2 = 210

$\Rightarrow y^2 - 35y + 210 + 96 = 0$

$\Rightarrow y^2 - 35 + 306 = 0$

$\Rightarrow y^2 - 17y - 18y + (-17 \times -18) = 0$ $[\because 306 = 17 \times 18 = -17 \times -18]$

$\Rightarrow y(y-17) - 18(y-17) = 0$

$\Rightarrow (y-17)(y-18) = 0$

⇒ y – 17 = 0 or y – 18 = 0

⇒ y = 17 or y = 18

We have,

x + y = 30

if y = 17 ⇒ x = 30 – y = 30 – 17 = 13 and

if y = 18 ⇒ x = 30 – y = 30 – 18 = 18

∴ Marks in Mathematics = 13 and marks in English = 17 or

Marks in Mathematics = 12 and marks in English = 18.

11. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs. 90, find the number of articles produced and the cost of each article.

Sol:

Let the number of articles produced on a particular day be x.

Total cost of production on that particular day = Rs 90

Given ⇒ Cost of production of each article = $\frac{Total\ cost\ of\ production}{no.of\ articles\ produced} = Rs\,\frac{90}{x}$

But given that, the cost of production of each article was 3 more than twice the no. of articles produced on that day i.e. Rs (2x +3)

⇒ Rs (2x + 3) = Rs $\frac{90}{x}$

⇒ 2x + 3 = $\frac{90}{x}$

⇒ x(2x + 3) = 90 ⇒ $2x^2 + 3x - 90 = 0$

⇒ $2x^2 + 15x - 12x - 90 = 0$

⇒ $x(2x + 15) - 6(2x + 15) = 0$

⇒ (2x + 15) (x – 16) = 0

⇒ 2x + 15 = 0 or x – 6 = 0

⇒ x = $\frac{-15}{2}$ $or\ x = 6$

Since, number of articles x cannot be a negative value, the required value of number of articles produced on a particular day x = 6.

Exercise – 9.1

1. Write the first terms of each of the following sequences whose n[th] term are
   (i) $a_n = 3n + 2$
   (ii) $a_n = \frac{n-2}{3}$
   (iii) $a_n = 3^n$
   (iv) $a_n = \frac{3n-2}{5}$
   (v) $a_n = (-1)^n 2^n$
   (vi) $a_n = \frac{n(n-2)}{2}$
   (vii) $a_n = n^2 - n + 1$
   (viii) $a_n = n^2 - n + 1$
   (ix) $a_n = \frac{2n-3}{6}$

Sol:

We have to write first five terms of given sequences

(i) $a_n = 3n + 2$

Given sequence $a_n = 3n + 2$

To write first five terms of given sequence put n = 1, 2, 3, 4, 5, we get

$a_1 = (3 \times 1) + 2 = 3 + 2 = 5$

$a_2 = (3 \times 2) + 2 = 6 + 2 = 8$

$a_3 = (3 \times 3) + 2 = 9 + 2 = 11$

$a_4 = (3 \times 4) + 2 = 12 + 2 = 14$

$a_5 = (3 \times 5) + 2 = 15 + 2 = 17$

∴ The required first five terms of given sequence $a_n = 3n + 2$ are 5, 8, 11, 14, 17.

(ii) $a_n = \frac{n-2}{3}$

Given sequence $a_n = \frac{n-2}{3}$

To write first five terms of given sequence $a_n = \frac{n-2}{3}$

put n = 1, 2, 3, 4, 5 then we get

$a_1 = \frac{1-2}{3} = \frac{-1}{3}; a_2 = \frac{2-2}{3} = 0$

$a_3 = \frac{3-2}{3} = \frac{1}{3}; a_4 = \frac{4-2}{3} = \frac{2}{3}$

$a_5 = \frac{5-2}{3} = 1$

∴ The required first five terms of given sequence $a_n = \frac{n-2}{3}$ are $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, 1$.

(iii) $a_n = 3^n$

Given sequence $a_n = 3^n$

To write first five terms of given sequence, put n = 1, 2, 3, 4, 5 in given sequence.

Then,

$a_1 = 3^1 = 3; a_2 = 3^2 = 9; a_3 = 27; a_4 = 3^4 = 81; a_5 = 3^5 = 243.$

(iv) $a_n = \frac{3n-2}{5}$

Given sequence, $a_n = \frac{3n-2}{5}$

To write first five terms, put n = 1, 2, 3, 4, 5 in given sequence $a_n = \frac{3n-2}{5}$

Then, we ger

$a_1 = \frac{3\times1-2}{5} = \frac{3-2}{5} = \frac{1}{5}$

$a_2 = \frac{3\times2-2}{5} = \frac{6-2}{5} = \frac{4}{5}$

$a_3 = \frac{3\times3-2}{5} = \frac{9-2}{5} = \frac{7}{5}$

$a_4 = \frac{3\times4-2}{5} = \frac{12-2}{5} = \frac{10}{5}$

$a_5 = \frac{3\times5-2}{5} = \frac{15-2}{5} = \frac{13}{5}$

∴ The required first five terms are $\frac{1}{5}, \frac{4}{5}, \frac{7}{5}, \frac{10}{5}, \frac{13}{5}$

(v) $a_n = (-1)^n 2^n$

Given sequence is $a_n = (-1)^n 2^n$

To get first five terms of given sequence an, put n = 1, 2, 3, 4, 5.

$a_1 = (-1)^1 . 2^1 = (-1). 2 = -2$

$a_2 = (-1)^2 . 2^2 = (-1). 4 = 4$

$a_3 = (-1)^3 . 2^3 = (-1). 8 = -8$

$a_4 = (-1)^4 . 2^4 = (-1). 16 = 16$

$a_5 = (-1)^5 . 2^5 = (-1). 32 = -32$

∴ The first five terms are -2, 4, -8, 16, -32.

(vi) $a_n = \frac{n(n-2)}{2}$

The given sequence is, $a_n = \frac{n(n-2)}{2}$

To write first five terms of given sequence $a_n = \frac{n(n-2)}{2}$

Put n = 1, 2, 3, 4, 5. Then, we get

$a_1 = \frac{1(1-2)}{2} = \frac{1-1}{2} = \frac{-1}{2}$

$a_2 = \frac{2(2-2)}{2} = \frac{2.0}{2} = 0$

$a_3 = \frac{3(3-2)}{2} = \frac{3.1}{2} = \frac{3}{2}$

$a_4 = \frac{4(4-2)}{2} = \frac{4.2}{2} = 4$
$a_5 = \frac{5(5-2)}{2} = \frac{5.3}{2} = \frac{15}{2}$
∴ The required first five terms are $\frac{-1}{2}, 0, \frac{3}{2}, 4, \frac{15}{2}$.

(vii) $a_n = n^2 - n + 1$
The given sequence is, $a_n = n^2 - n + 1$
To write first five terms of given sequence a_{n1} we get put n = 1, 2, 3, 4, 5. Then we get $a_1 = 1^2 - 1 + 1 = 1$
$a_2 = 2^2 - 2 + 1 = 3$
$a_3 = 3^2 - 3 + 1 = 7$
$a_4 = 4^2 - 4 + 1 = 13$
$a_5 = 5^2 - 5 + 1 = 21$
∴The required first five terms of given sequence $a_n = n^2 - n + 1$ are 1, 3, 7, 13, 21

(viii) $a_n = 2n^2 - 3n + 1$
The given sequence is $a_n = 2n^2 - 3n + 1$
To write first five terms of given sequence a_n, we put n = 1, 2, 3, 4, 5. Then we get
$a_1 = 2.1^2 - 3.1 + 1 = 2 - 3 + 1 = 0$
$a_2 = 2.2^2 - 3.2 + 1 = 8 - 6 + 1 = 3$
$a_3 = 2.3^2 - 3.3 + 1 = 18 - 9 + 1 = 10$
$a_4 = 2.4^2 - 3.4 + 1 = 32 - 12 + 1 = 21$
$a_5 = 2.5^2 - 3.5 + 1 = 50 - 15 + 1 = 36$
∴ The required first five terms of given sequence $a_n - 2n^2 - 3n + 1$ are 0, 3, 10, 21, 36

(ix) $a_n = \frac{2n-3}{6}$
Given sequence is, $a_n = \frac{2n-3}{6}$
To write first five terms of given sequence we put n = 1, 2, 3, 4, 5. Then, we get,
$a_1 = \frac{2.1-3}{6} = \frac{2-3}{6} = \frac{-1}{6}$
$a_2 = \frac{2.2-3}{6} = \frac{4-3}{6} = \frac{1}{6}$
$a_3 = \frac{2.4-3}{6} = \frac{8-3}{6} = \frac{5}{6}$
$a_4 = \frac{2.4-3}{6} = \frac{8-3}{6} = \frac{5}{6}$
$a_5 = \frac{2.5-3}{6} = \frac{10-3}{6} = \frac{7}{6}$
∴ The required first five terms of given sequence $a_n = \frac{2n-3}{6}$ are $\frac{-1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{7}{6}$.

2. Find the indicated terms in each of the following sequences whose nth terms are:

(i) $a_n = 5n - 4; a_{12}\ and\ a_{15}$

(ii) $a_n = \frac{3n-2}{4n+5}; a_7\ and\ a_8$

(iii) $a_n = n(n-1)(n-2);\ a_5\ and\ a_8$

(iv) $a_n = (n-1)(2-n)(3+n); a_{11}\ a_{21}\ a_3$

(v) $a_n = (-1)^n n; a_3, a_5, a_8$

Sol:

We have to find the required term of a sequence when n^th term of that sequence is given.

(i) $a_n = 5n - 4; a_{12}\ and\ a_{15}$

Given n^th term of a sequence $a_n = 5n - 4$

To find 12^th term, 15^th terms of that sequence, we put n = 12, 15 in its n^th term.

Then, we get

$a_{12} = 5.12 - 4 = 60 - 4 = 56$

$a_{15} = 5.15 - 4 = 15 - 4 = 71$

∴ The required terms $a_{12} = 56, a_{15} = 71$

(ii) $a_n = \frac{3n-2}{4n+5}; a_7\ and\ a_8$

Given n^th term is (a_n) = $\frac{3n-2}{4n+5}$

To find 7^th, 8^th terms of given sequence, we put n = 7, 8.

$a_7 = \frac{(3.7)-2}{(4.7)+5} = \frac{19}{33}$

$a_8 = \frac{(3.8)-2}{(4.8)+5} = \frac{22}{37}$

∴ The required terms $a_7 = \frac{19}{33}$ and $a_8 = \frac{22}{37}$.

(iii) $a_n = n(n-1)(n-2);\ a_5\ and\ a_8$

Given n^th term is $a_n = n(n-1)(n-2)$

To find 5^th, 8^th terms of given sequence, put n= 5, 8 in an then, we get

$a_5 = 5(5-1).(5-2) = 5.4.3 = 60$

$a_8 = 8.(8-1).(8-2) = 8.7.6 = 336$

∴ The required terms are $a_5 = 60\ and\ a_8 = 336$

(iv) $a_n = (n-1)(2-n)(3+n); a_{11}\ a_{21}\ a_3$

The given n^th term is $a_n = (n+1)(2-n)(3+n)$

To find a_1, a_2, a_3 of given sequence put n = 1, 2, 3 is an

$a_1 = (1-1)(2-1)(3+1) = 0.1.4 = 0$

$a_2 = (2-1)(2-2)(3+2) = 1.0.5 = 0$

$a_3 = (3-1)(2-3)(3+3) = 2.-1.6 = -12$

∴ The required terms a_1 = 0, $a_2 = 0$, $a_3 = -12$

(v) $a_n = (-1)^n n; a_3, a_5, a_8$

The given n^th term is, $a_n = (-1)^n.n$

To find $a_3.a_5, a_8$ of given sequence put n = 3, 5, 8, in a_n.

$a_3 = (-1)^3.3 = -1.3 = -3$
$a_5 = (-1)^5.5 = -1.5 = -5$
$a_8 = (-1)^8 = 1.8 = 8$
∴ The required terms $a_3 = -3, a_5 = -5, a_8 = 8$

3. Find the next five terms of each of the following sequences given by:
(i) $a_1 = 1, a_n = a_{n-1} + 2,\ n \geq 2$
(ii) $a_1 = a_2 = 2, a_n = a_{n-1} - 3, n > 2$
(iii) $a_1 = -1, a_n = \frac{a_n - 1}{n}, n \geq 2$
(iv) $a_1 = 4, a_n = 4\ a_{n-1} + 3, n > 1$

Sol:

We have to find next five terms of following sequences.

(i) $a_1 = 1, a_n = a_{n-1} + 2,\ n \geq 2$
Given, first term (a_1) = 1,
n[th] term $a_n = a_{n-1} + 2, n \geq 2$
To find 2[nd], 3[rd], 4[th] , 5[th] , 6[th] terms, we use given condition n ≥ 2 for n[th] term $a_n = a_{n-1} + 2$
$a_2 = a_{2-1} + 2 = a_1 + 2 = 1 + 2 = 3$ (∴ $a_1 = 1$)
$a_3 = a_{3-1} + 2 = a_2 + 2 = 3 + 2 = 5$
$a_4 = a_{4-1} + 2 = a_3 + 2 = 5 + 2 = 7$
$a_5 = a_{5-1} + 2 = a_4 + 2 = 7 + 2 = 9$
$a_6 = a_{6-1} + 2 = a_5 + 2 = a + 2 = 11$
∴ The next five terms are,
$a_2 = 3, a_3 = 5, a_4 = 7, a_5 = a, a_6 = 11$

(ii) $a_1 = a_2 = 2, a_n = a_{n-1} - 3, n > 2$
Given,
First term $(a_1) = 2$
Second term $(a_2) = 2$
n[th] term $(a_n) = a_{n-1} - 3$
To find next five terms i.e., a_3, a_4, a_5, a_6, a_7 we put n = 3, 4, 5, 6, 7 is a_n
$a_3 = a_{3-1} - 3 = 2 - 3 = -1$
$a_4 = a_{4-1} - 3 = a_3 - 3 = -1 - 3 = -4$
$a_5 = a_{5-1} - 3 = a_4 - 3 = -4 - 3 = -7$
$a_6 = a_{6-1} - 3 = a_5 - 3 = -7 - 3 = -10$
$a_7 = a_{7-1} - 3 = a_6 - 3 = -10 - 3 = -13$
∴ The next five terms are, $a_3 = -1, a_4 = -4, a_5 = -7, a_6 = -10, a_7 = -13$

(iii) $a_1 = -1, a_n = \frac{a_n - 1}{n}, n \geq 2$
Given, first term $(a_1) = -1$

n^{th} term $(a_n) = \frac{a_n - 1}{n}, n \geq 2$

To find next five terms i.e., a_2, a_3, a_4, a_5, a_6 we put n = 2, 3, 4, 5, 6 is an

$a_2 = \frac{a_{2-1}}{2} = \frac{a_1}{2} = \frac{-1}{2}$

$a_3 = \frac{a_{3-1}}{3} = \frac{a_2}{3} = \frac{-1/2}{3} = \frac{-1}{6}$

$a_4 = \frac{a_{4-1}}{4} = \frac{a_3}{4} = \frac{-1/6}{4} = \frac{-1}{24}$

$a_5 = \frac{a_{5-1}}{5} = \frac{a_4}{5} = \frac{-1/24}{5} = \frac{-1}{120}$

∴ The next five terms are,

$a_2 = \frac{-1}{2}, a_3 = \frac{-1}{6}, a_4 = \frac{-1}{24}, a_5 = \frac{-1}{120}, a_6 = \frac{-1}{720}$

(iv) $a_1 = 4, a_n = 4\, a_{n-1} + 3, n > 1$

Given,

First term $(a_1) = 4$

n^{th} term $(a_n) = 4\, a_{n-1} + 3$, n > 1

To find next five terms i.e., a_2, a_3, a_4, a_5, a_6 we put n= 2, 3, 4, 5, 6 is a_n

Then, we get

$a_2 = 4a_{2-1} + 3 = 4.a_1 + 3 = 4.4 + 3 = 19\ (\because a_1 = 4)$

$a_3 = 4a_{3-1} + 3 = 4.a_2 + 3 = 4(19) + 3 = 79$

$a_4 = 4\, a_{4-1} + 3 = 4.a_3 + 3 = 4(79) + 3 = 319$

$a_5 = 4\, a_{5-1} + 3 = 4.a_4 + 3 = 4(319) + 3 = 1279$

$a_6 = 4.a_{6-1} + 3 = 4.a_5 + 3 = 4(1279) + 3 = 5119$

∴ The required next five terms are,

$a_2 = 19, a_3 = 79, a_4 = 319, a_5 = 1279, a_6 = 5119$

Exercise – 9.2

1. For the following arithmetic progressions write the first term a and the common difference d:

(i) $-5, -1, 3, 7, \ldots\ldots\ldots\ldots$

(ii) $\frac{1}{5}, \frac{3}{5}, \frac{5}{5}, \frac{7}{5}, \ldots\ldots\ldots\ldots$

(iii) 0.3, 0.55, 0.80, 1.05,.........

(iv) $-1.1, -3.1, -5.1, -7.1, \ldots..$

Sol:

We know that if a is the first term and d is the common difference, the arithmetic progression is $a, a+d, a+2d+a+3d, \ldots\ldots\ldots\ldots$

(i) $-5, -1, 3, 7, \ldots\ldots\ldots\ldots$

Given arithmetic series is

$-5, -1, 3, 7 \ldots\ldots\ldots\ldots$

This is in the form of $a, a+d, a+2d+a+3d,$............. by comparing these two

$a=-5, a+d=1, a+2d=3, a+3d=7,$........

First term $(a)=-5$

By subtracting second and first term, we get

$(a+d)-(a)=d$

$-1-(-5)=d$

$4=d$

Common difference $(d)=4.$

(ii) $\frac{1}{5},\frac{3}{5},\frac{5}{5},\frac{7}{5},$.............

Given arithmetic series is,

$\frac{1}{5},\frac{3}{5},\frac{5}{5},\frac{7}{5},$.............

This is in the form of $\frac{1}{5},\frac{2}{5},\frac{5}{5},\frac{7}{5},$.............

$a, a+d, a+2d, a+3d,$......

By comparing this two, we get

$a=\frac{1}{5}, a+d=\frac{3}{5}, a+2d=\frac{5}{5}, a+3d=\frac{7}{5}$

$\boxed{First\ term\ \cos=\frac{1}{5}}$

By subtracting first term from second term, we get

$d=(a+d)-(a)$

$d=\frac{3}{5}-\frac{1}{5}$

$d=\frac{2}{5}$

$\boxed{common\ difference(d)=\frac{2}{5}}$

(iii) 0.3, 0.55, 0.80, 1.05,.........

Given arithmetic series,

0.3, 0.55, 0.80, 1.05,.........

General arithmetic series

$a, a+d, a+2d, a+3d,$........

By comparing,

$a=0.3, a+d=0.55, a+2d=0.80, a+3d=1.05$

First term $(a) = 0.3.$

By subtracting first term from second term. We get

$d = (a+d) - (a)$

$d = 0.55 - 0.3$

$d = 0.25$

Common difference (d) $= 0.25$

(iv) $-1.1, -3.1, -5.1, -7.1, ...$

General series is

$a, a+d, a+2d, a+3d, ...$

By comparing this two, we get

$a = -1.1, a+d = -3.1, a+2d = -5.1, a+3d = -71$

First term $(a) = -1.1$

Common difference $(d) = (a+d) - (a)$

$= -3.1 - (-1.1)$

Common difference (d) $= -2$

2. Write the arithmetic progressions write first term a and common difference d are as follows:

(i) $a = 4, d = -3$

(ii) $a = -1, d = \frac{1}{2}$

(iii) $a = -1.5, d = -0.5$

Sol:

We know that, if first term $(a) = a$ and common difference $= d$, then the arithmetic series is, $a, a+d, a+2d, a+3d, ...$

(i) $a = 4, d = -3$

Given first term $(a) = 4$

Common difference $(d) = -3$

Then arithmetic progression is,

$a, a+d, a+2d, a+3d, ...$

$\Rightarrow 4, 4-3, a+2(-3), 4+3(-3), ...$

$\Rightarrow 4, 1, -2, -5, -8, ...$

(ii) $a = -1, d = \frac{1}{2}$

Given,

First term $(a) = -1$

Common difference $(d) = \frac{1}{2}$

Then arithmetic progression is,

$\Rightarrow a, a+d, a+2d, a+3d, ...$

$\Rightarrow -1, -1+\frac{1}{2}, -1+2\frac{1}{2}, -1+3\frac{1}{2}, ...$

$\Rightarrow -1, \frac{-1}{2}, 0, \frac{1}{2}, ...$

(iii) $a = -1.5, d = -0.5$

Given

First term $(a) = -1.5$

Common difference $(d) = -0.5$

Then arithmetic progression is

$\Rightarrow a, a+d, a+2d, a+3d, ...$

$\Rightarrow -1.5, -1.5-0.5, -1.5+2(-0.5), -1.5+3(-0.5)$

$\Rightarrow -1.5, -2, -2.5, -3, ...$

Then required progression is

$-1.5, -2, -2.5, -3, ...$

3. In which of the following situations, the sequence of numbers formed will form an A.P.?

(i) The cost of digging a well for the first metre is Rs 150 and rises by Rs 20 for each succeeding metre.

(ii) The amount of air present in the cylinder when a vacuum pump removes each time $\frac{1}{4}$ of their remaining in the cylinder.

Sol:

(i) Given,

Cost of digging a well for the first meter $(c_1) = Rs.150.$

Cost rises by $Rs.20$ for each succeeding meter

Then,

Cost of digging for the second meter $(c_2) = Rs.150 + Rs\ 20$

$= Rs\ 170$

Cost of digging for the third meter $(c_3) = Rs.170 + Rs\ 20$

$= Rs\ 210$

Thus, costs of digging a well for different lengths are

$150, 170, 190, 210, ...$

Clearly, this series is in $A\cdot p.$

With first term $(a)=150,$ common difference $(d)=20$

(ii) Given

Let the initial volume of air in a cylinder be V liters each time $\frac{3}{4}^{th}$ of air in a remaining i.e.,

$1-\frac{1}{4}$

First time, the air in cylinder is $\frac{3}{4}V.$

Second time, the air in cylinder is $\frac{3}{4}V.$

Third time, the air in cylinder is $\left(\frac{3}{4}\right)^2 V.$

Therefore, series is $V,\ \frac{3}{4}V, \left(\frac{3}{4}\right)^2 V, \left(\frac{3}{4}\right)^3 V, ...$

4. Show that the sequence defined by $a_n = 5n - 7$ is an A.P., find its common difference.

Sol:

Given sequence is

$a_n = 5n-7$

n^{th} term of given sequence $(a_n)=5n-7$

$(n+1)^{th}$ term of given sequence $(a_n+1)-a_n$

$=(5n-2)-(5n-7)$

$=5$

$\therefore d=5$

5. Show that the sequence defined by $a_n = 3n^2 - 5$ is not an A.P.

Sol:

Given sequence is,

$a_n = 3n^2-5.$

n^{th} term of given sequence $(a_n)=3n^2-5.$

$(n+1)^{th}$ term of given sequence $(a_n+1)=3(n+1)^2-5$

$=3\left(n^2+1^2+2n.1\right)-5$

$=3n^2+6n-2$

$\therefore$ The common difference $(d) = a_n + 1 - an$

$d = (3n^2 + 6n - 2) - (3n^2 - 5)$

$= 3a^2 + 6n - 2 - 3n^2 + 5$

$= 6n + 3$

Common difference (d) depends on $'n'$ value

$\therefore$ given sequence is not in $A.p$

6. The general term of a sequence is given by $a_n = -4n + 15$. Is the sequence an A.P.? If so, find its 15th term and the common difference.

Sol:

Given sequence is,

$a_n = -4n + 15.$

n^{th} term is $(a_n) = -4n + 15$

$(n+1)^{th}$ term is $(a_{n+1}) = -4(n+1) + 15$

$= -4n - 4 + 15$

$= -4n + 11$

Common difference $(d) = a_{n+1} - an$

$= (-4n + 11) - (-4n + 15)$

$= -4n + 11 + 4n - 15$

$d = -4$

Common difference $(d) = a_{n+1} - an$

$= (-4n + 11) + (-4n + 15)$

$= -4n + 11 + 4n - 15$

$d = -4.$

Common difference (d) does not depend on $'n'$ value

$\therefore$ given sequence is in $A.P$

$\Rightarrow 15^{th}$ term $a_{15} = -4(15) + 15$

$= -60 + 15$

$= -45$

$a_{15} = -45$

7. Find the common difference and write the next four terms of each of the following arithmetic progressions:

(i) $1, -2, -5, -8, \ldots\ldots.$

(ii) $0, -3, -6, -9, \ldots\ldots\ldots..$

(iii) $-1, \frac{1}{4}, \frac{3}{2}, ...$

(iv) $-1, \frac{-5}{6}, \frac{-2}{3}, ...$

Sol:

(i) $1, -2, -5, -8, ...$

Given arithmetic progression is,

$a_1 = 1, a_2 = -2, a_3 = -5, a_4 = -8.........$

Common difference $(d) = a_2 - a_1$

$= -2 - 1$

$d = -3$

To find next four terms

$a_5 = a_4 + d = -8 - 3 = -11$

$a_6 = a_5 + d = -11 - 3 = -14$

$a_7 = a_6 + d = -14 - 3 = -17$

$a_8 = a_7 + d = -17 - 3 = -20$

$\therefore d = -3, a_5 = -11, a_6 = -16, a_7 = -17, a_8 = -20$

(ii) $0, -3, -6, -9, ...$

Given arithmetic progression is.

$0, -3, -6, a_4 = -9...........$

Common difference $(d) = a_2 - a_1$

$= -3 - 0$

$d = -3$

To find next four terms

$a_5 = a_4 + d = -9 - 3 = -12$

$a_6 = a_5 + d = -12 - 3 = -15$

$a_7 = a_6 + d = -15 - 3 = -18$

$a_8 = a_7 + d = -18 - 3 = -21$

$\therefore d = -3, a_5 = -12, a_6 = -15, a_7 = -17, a_8 = -21$

(iii) $-1, \frac{1}{4}, \frac{3}{2}, ...$

Given arithmetic progression is,

$-1, \frac{1}{4}, \frac{3}{2}, ...$

$a_1 = -1, a_2 = \frac{1}{4}, a_3 = \frac{3}{2}, ...$

Common difference $(d) = a_2 - a_1$

$= \frac{1}{4} - (-1)$

$= \frac{1+4}{4}$

$d = \frac{5}{4}$

To find next four terms,

$a_4 = a_3 + d = \frac{3}{2} + \frac{5}{4} = \frac{6+4}{4} = \frac{11}{4}$

$a_5 = a_4 + d = \frac{11}{4} + \frac{5}{4} = \frac{16}{4}$

$a_6 = a_5 + d = \frac{16}{4} + \frac{5}{4} = \frac{21}{4}$

$a_7 = a_6 + d = \frac{21}{4} + \frac{5}{4} = \frac{26}{4}$

$\therefore d = \frac{5}{4}, a_4 = \frac{11}{4}, a_5 = \frac{16}{4}, a_6 = \frac{21}{4}, a_7 = \frac{26}{4}.$

(iv) Given arithmetic progression is,

$-1, \frac{-5}{6}, \frac{-2}{3}, ...$

$a_1 = -1, a_2 = \frac{-5}{6}, a_3 = \frac{-2}{3}, ...$

Common difference $(d) = a_2 - a_1$

$= \frac{-5}{6} - (-1)$

$= \frac{-5+6}{6}$

$= \frac{1}{6}$

To find next four terms,

$a_4 = a_3 + d = \frac{-2}{3} + \frac{1}{6} = \frac{-4+1}{6} = \frac{-3}{6} = -\frac{1}{2}.$

$a_5 = a_4 + d = \frac{-1}{2} + \frac{1}{6} = \frac{-3+1}{6} = \frac{-\cancel{2}^{1}}{\cancel{6}_{3}} = -\frac{1}{2}.$

$a_6 = a_5 + d = \frac{-1}{3} + \frac{1}{6} = \frac{-2+1}{6} = -\frac{1}{16}.$

$a_7 = a_6 + d = \frac{-1}{6} + \frac{1}{6} = 0.$

$\therefore d = \frac{1}{6}, a_4 = -\frac{1}{2}, a_5 = -\frac{1}{3}, a_6 = -\frac{1}{6}, a_7 = 0$

8. Prove that no matter what the real numbers a and b are, the sequence with nth term a + nb is always an A.P. What is the common difference?

Sol:

Given sequence $(a_n) = a_n + 6n$

n^{th} term $(a_n) = a + nb$

$(n+1)^{th}$ term $(a_{n+1}) = a + (n+1)b.$

Common difference (d) $= a_{n+1} - an$

$d = (a + (n+1)b) - (a + nb)$

$= \cancel{a} + \cancel{n}b + b - \cancel{a} - \cancel{nb}$

$= b$

$\therefore$ common difference (d) does not depend on n^{th} value so, given sequence I sin Ap with $(d) = b$

9. Write the sequence with nth term :

(i) $a_n = 3 + 4n$

Sol:

(i) $a_n = 3 + 4n$

Given, n^{th} term $a_n = 3 + 4n$

$(n+1)^{th}$ term $a_{n+1} = 3 + 4(n+1)$

Common difference $(d) = a_{n+1} - a_n$

$= (3 + 4(n+1)) - 3 + 4n$

$= 4.$

$d = 4$ does not depend on n value so, the given series is in $A.p$ and the sequence is

$a_1 = 3 + 4(1) = 3 + 4 = 7$

$a_2 = a_1 + d = 7 + 4 = 11; a_3 = a_2 + d = 11 + 4 = 15$

$\Rightarrow 7, 11, 15, 19, ...$

(ii) $a_n = 5 + 2n$

Given, n^{th} term $(a_n) = 5 + 2n$

$(n+1)^{th}$ term $(a_{n+1}) = 5 + 2(n+1)$

$= 7 + 2n$

Common difference $(d) = 7 + 2n - 5 - 20$

$= 2.$

$\because d = 2$ does not depend on n value given sequence is in $A.p$ and the sequence is B_1

$a_1 = 5 + 2.1 = 7$

$a_2 = 7 + 2 = 9, a_3 = 9 + 2 = 11, a_4 = 11 + 2 = 13$

$\Rightarrow 7, 9, 11, 13, ...$

(iii) $a_n = 6 - n$

Given, n^{th} term $a_n = 6 - n$

$(n+1)^{th}$ term $a_{n+1} = 6 - (n+1)$

$= 5 - n$

Common difference $(d) = a_{n+1} - a_n$

$= (5 - n) - (6 - n)$

$= -1$

$\therefore d = -1$ does not depend on n value given sequence is in $A.p$ the sequence is

$a_1 = 6 - 1 = 5, a_2 = 5 - 1 = 4, a_3 = 4 - 1 = 3, a_4 = 3 - 1 = 2$

$\Rightarrow 5, 4, 3, 2, 1, ...$

(iv) $a_n = 9 - 5n$

Given, n^{th} term $a_n = 9 - 5n$

$(n+1)^{th}$ term $a_{n+1} = 9 - 5(n+1)$

$= 4 - 5n$

Common difference $(d) = a_{n+1} - a_n$

$= (4 - 5n) - (4 - 5n)$

$= -5$

$\because d = -1$ does not depend on n value given sequence is in $A.p$

The sequence is,

$a_1 = 9 - 1.1 = 4$
$a_2 = 9 - 5.2 = -1$
$a_3 = 9 - 5.3 = -6$
$\Rightarrow 4, -1, -6, -11, ...$

10. Find out which of the following sequences are arithmetic progressions. For those which are arithmetic progressions, find out the common differences.

Sol:

(i) $3, 6, 12, 24, ...$

General arithmetic progression is $a, a+d, a+2d, a+3d, ...$

Common difference $(d) =$ Second term – first term

$= (d+d) - a = d$ (or)

$=$ Third term – second term

$= (a+2d) - (a+d) = d$

To check given sequence is in $A.p$ or not we use this condition.

Second term – First term = Third term – Second term

$a_1 = 3, a_2 = 6, a_3 = 12, a_4 = 24$

Second term – First term = $6 - 3 = 3$

Third term – Second term $= 12 - 6 = 6$

This two are not equal so given sequence is not in $A.p$

(ii) $0, -4, -8, -12, ...$

In the given sequence

$a_1 = 0, a_2 = -4, a_3 = -8, a_4 = -12$

Check the condition

Second term – first term = third term – second term

$a_2 - a_1 = a_8 - a_2$

$-4 - 0 = -8 - (-4)$

$-4 = +8 + 4$

$-4 = -4$

Condition is satisfied $\therefore$ given sequence is in $A.p$ with common difference

$(d) = a_2 - a_1 = -4$

(iii) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, ...$

In the given sequence

$a_1 = \frac{1}{2}, a_2 = \frac{1}{4}, a_3 = \frac{1}{6}, a_4 = \frac{1}{8}.$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$\frac{1}{4} - \frac{1}{2} = \frac{1}{6} - \frac{1}{4}$

$\frac{1-2}{4} = \frac{4-6}{24}$

$\frac{-1}{4} = -\frac{2}{24}$

$\frac{-1}{4} \neq \frac{-1}{12}$

Condition is not satisfied

$\therefore$ given sequence not in $A.p$

(iv) $12, 2, -8, -18, ...$

In the given sequence

$a_1 = 12, a_2 = 2, a_3 = -8, a_4 = -18$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$2 - 12 = -8 - 2$

$-10 = -10$

$\therefore$ given sequence is in $A.p$ with common difference $d = -10$

(v) $3, 3, 3, 3, ...$

In the given sequence

$a_1 = 3, a_2 = 3, a_3 = 3, a_4 = 3$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$3 - 3 = 3 - 3$

$0 = 0$

$\therefore$ given sequence is in $A.p$ with common difference $d = 0$

(vi) $p, p+90, p+80, p+270, ...$ where $p = (999)$

In the given sequence

$a_1 = p, a_2 = p+90, a_3 = p+180, a_4 = p+270$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$\cancel{p} + 90 \cancel{- p} = \cancel{p} + 180 - \cancel{p} - 90$

$90 = 180 - 90$

$90 = 90$

(vii) 1.0, 1.7, 2.4, 3.1….,

In the given sequence

$a_1 = 1.0, a_2 = 1.7, a_3 = 2.4, a_4 = 31$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$1.7 - 1.0 = 2.4 - 1.7$

$0.7 = 0.7$

$\therefore$ The given sequence is in $A.p$ with $d = 0.7$

(viii) $-225, -425, -625, -825, ...$

In the given sequence

$a_1 = 225, a_2 = -425, a_3 = -625, a_4 = -825$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$-425 + 225 = -625 + 425$

$-200 = -200$

$\therefore$ The given sequence is in $A.p$ with $d = -200$

(ix) $10, 10 + 2^5, 10 + 2^6, 10 + 2^7, ...$

In the given sequence

$a_1 = 10, a_2 = 10 + 2^5, a_3 = 10 + 2^6, a_4 = 10 + 2^7$

Check the condition

$a_2 - a_1 = a_3 - a_2$

$10 + 2^5 - 10 = 10 + 2^6 - 10 - 2^5$

$2^5 \neq 2^6 - 2^5.$

$\therefore$ The given sequence is not in $A.p$

Exercise – 9.3

1. Find:

(i) 10th term of the AP 1,4 , 7, 10….

(ii) 18th term of the AP $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, ...$

(iii) nth term of the AP $13, 8, 3, -2, ...$

(iv) 10th term of the AP $-40, -15, 10, 35, ...$

(v) 8th term of the AP 11, 104, 91, 78………

(vi) 11th term of the AP 10.0, 10.5, 11.0, 11.2…….

(vii) 9th term of the AP $\frac{3}{4}, \frac{5}{4}, \frac{7}{4} + \frac{9}{4}, ...$

Sol:

(i) Given $A.p$ is

$1,4,7,10,..........$

First term $(a)=1$

Common difference (d) = second term first term

$=4-1$

$=3.$

n^{th} term in an $A.p=a+(n-1)d$

10^{th} term in an $1+(10-1)3$

$=1+9.3$

$=1+27$

$=28$

(ii) Given $A.p$ is

$\sqrt{2},3\sqrt{2},5\sqrt{2},............$

First term $(a)=\sqrt{2}$

Common difference $=$ Second term – First term

$=3\sqrt{2}-\sqrt{2}$

$d=2\sqrt{2}$

n^{th} term in an $A.p=a+(n-1)d$

18^{th} term of $A.p=\sqrt{2}+(18-1)2\sqrt{2}$

$=\sqrt{2}+17.2\sqrt{2}$

$=\sqrt{2}(1+34)$

$=35\sqrt{2}$

$\therefore 18^{th}$ term of $A.p$ is $35\sqrt{2}$

(iii) Given $A.p$ is

$13,8,3,-2,...........$

First term $(a)=13$

Common difference $(d)=$ Second term first term

$=8-13$

$=-5$

n^{th} term of an $A.p$ $a_n=a+(n-1)d$

$=13+(n-1)-5$

$=13-5n+5$

$a_n = 18 - 5n$

(iv) Given $A.p$ is

$-40, -15, 10, 35, ...$

First term $(a) = -40$

Common difference $(d) =$ Second term – first term

$= -15 - (-40)$

$= 40 - 15$

$= 25$

n^{th} term of an $A.p$ $a_n = a + (n-1)d$

10^{th} term of $A.p$ $a_{10} = -40 + (10-1)25$

$= -40 + 9.25$

$= -40 + 225$

$= 185$

(v) Given sequence is

$117, 104, 91, 78, ...$

First learn can $= 117$

Common difference $(d) =$ Second term – first term

$= 104 - 117$

$= -13$

n^{th} term $a_n = a + (n-1)d$

8^{th} term $a_8 = a + (8-1)d$

$= 117 + 7(-13)$

$= 117 - 91$

$= 26$

(vi) Given $A.p$ is

$10.0, 10.5, 11.0, 11.5, ...$

First term $(a) = 10.0$

Common difference (d) $=$ Second term – first term

$= 10.5 - 10.0$

$= 0.5$

n^{th} term $a_n = a + (n-1)d$

11^{th} term $a_{11} = 10.0 + (11-1)0.5$

$= 10.0 + 10 \times 0.5$

$= 10.0 + 5$

$=15.0$

(vii) Given $A.p$ is

$\frac{3}{4},\frac{5}{4},\frac{7}{4}+\frac{9}{4},.............$

First term $(a)=\frac{3}{4}$

Common difference $(d)=$ Second term – first term

$=\frac{5}{4}-\frac{3}{4}$

$=\frac{2}{4}$

n^{th} term $a_n=a+(n-1)d$

9^{th} term $a_9=a+(9-1)d$

$=\frac{3}{4}+8.\frac{2}{4}$

$=\frac{3}{4}+\frac{16}{4}$

$=\frac{19}{4}$

2. (i) Which term of the AP 3, 8, 13, …. is 248?
(ii) Which term of the AP 84, 80, 76, … is 0?
(iii) Which term of the AP 4, 9, 14, …. is 254?
(iv) Which term of the AP 21, 42, 63, 84, … is 420?
(v) Which term of the AP 121, 117, 113, … is its first negative term?

Sol:

(i) Given $A.p$ is $3,8,13,.........$

First term $(a)=3$

Common difference $(d)=$ Second term – first term

$=8-3$

$=5$

n^{th} term $(a_n)=a+(n-1)d$

Given n^{th} term $a_n=248$

$248=3+(n-1).5$

$248=-2+5n$

$5n=250$

$n=\frac{250}{5}=50$

50^{th} term is $248.$

(ii) Given $A.p$ is $84,80,76,.........$

First term $(a)=84$

Common difference $(d)=a_2-a$

$=80-84$

$=-4$

n^{th} term $(a_n)=a+(n-1)d$

Given n^{th} term is 0

$0=84+(n-1)-4$

$+84=+4(n-1)$

$n-1=\frac{84}{4}=21$

$n=21+1=22$

22^{nd} term is 0.

(iii) Given $A.p$ $4,9,14,.......$

First term $(a)=4$

Common difference $(d)=a^2-a$

$=9-4$

$=5$

n^{th} term $(a_n)=a+(n-1)d$

Given n^{th} term is 254

$4+(n-1)5=254$

$(n-1)\cdot 5=250$

$n-1=\frac{250}{5}=50$

$n=51$

$\therefore 51^{st}$ term is $254.$

(iv) Given $A.p$

$21,42,63,84,..........$

$a=21, d=a_2-a$

$=42-21$

$=21$

n^{th} term $(a_n) = a + (n-1)d$

Given n^{th} term $= 420$

$21 + (n-1)21 = 420$

$(n-1)21 = 399$

$n - 1 = \frac{399}{21} = 19$

$n = 20$

$\therefore 20^{th}$ term is $420.$

(v) Given $A.p$ is $121, 117, 113, ...$

First term $(a) = 121$

Common difference $(d) = 117 - 121$

$= -4$

n^{th} term $(a) = a + (n-1)d$

Given n^{th} term is negative i.e., $a_n < 0$

$121 + (n-1) - 4 < 0$

$121 + 4 - 4n < 0$

$125 - 4n < 0$

$4n > 125$

$n > \frac{125}{4}$

$n > 31.25$

The integer which comes after 31.25 is 32.

$\therefore 32^{nd}$ term is first negative term

3. (i) Is 68 a term of the AP 7, 10, 13,….?

(ii) Is 302 a term of the AP 3, 8, 13, …. ?

(iii) Is −150 a term of the AP 11, 8, 5, 2, …?

Sol:

In the given problem, we are given an $A.p$ and the Value of one of its term

We need to find whether it is a term of the $A.p$ or not so here we will use the formula

$a_n = a + (n-1)d$

(i) Here, $A.p$ is $7, 10, 13, ...$

$a_n = 68, a = 7$ and $d = 10 - 7 = 3$

Using the above mentioned formula, we get

$68 = 7 + (n-1)3$

$\Rightarrow 68-7=3n-3$

$\Rightarrow 31+3=3n$

$\Rightarrow 64=3n$

$\Rightarrow n=\frac{64}{3}$

Since, the value of n is a fraction.

Thus, 68 is not the team of the given $A.p$

(ii) Here, $A.p$ is $3,8,13,..........$

$a_n=302, a=3$

Common difference $(d)=8-3=5$ using the above mentioned formula, we get

$302=3+(n-1)5$

$\Rightarrow 302-3=5n-5$

$\Rightarrow 299=5n-5$

$\Rightarrow 5n=304$

$\Rightarrow n=\frac{305}{5}$

Since, the value of $'n'$ is a fraction. Thus, 302 is not the term of the given $A.p$

(iii) Here, $A.p$ is $11,8,5,2,..........$

$a_n=-150, a=1$ and $d=8-11=-3$

Thus, using the above mentioned formula, we get

$-150=11+(x-1)(-3)$

$\Rightarrow -150-11=-34+3$

$\Rightarrow -161-3=-34$

$\Rightarrow -34=-164$

$\Rightarrow n=\frac{164}{3}$

Since, the value of n is a fraction. Thus, -150 is not the term of the given $A.p$

4. How many terms are there in the AP?

(i) $7,10,13,..............43$

(ii) $-1,\frac{-5}{6},\frac{-2}{3},\frac{-1}{2},........,\frac{10}{3}.$

(iii) $7,13,19,...........05$

(iv) $18,15\frac{1}{2},13,..........-47$

Sol:

(i) $7,10,13,..............43$

From given $A.p$

$a = 7, d = 10 - 7 = 3, a_n = a + (n-1)d.$

Let, $a_n = 43$ (last term)

$7 + (n-1)3 = 43$

$(n-1) = \frac{\not{2}6^{12}}{\not{3}} = 12$

$n = 13$

$\therefore 13$ terms are there in given $A.p$

(ii) $-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, ..., \frac{10}{3}.$

From given $A.p$

$a = -1, d = -\frac{5}{6} + 1, \quad a_n = a + (n-1)d$

$= \frac{1}{6}$

Let, $a_n = \frac{10}{3}$ (last term)

$-1 + (n-1)\frac{1}{6} = \frac{10}{3}$

$(n-1) \times \frac{1}{6} = \frac{10}{3}$

$(n+1) = \frac{13 \times \not{6}^{2}}{\not{3}} = 26$

$n = 27$

$\therefore 27$ terms are there in given $A.p$

(iii) $7, 13, 19, ...05$

From the given $A.p$

$a = 7, d = 13 - 7 = 6, a_n = a + (n-1)d$

Let, $a_n = 205$ (last term)

$7 + (n-1)6 = 205$

$(n-1).6 = 198$

$n - 1 = 33$

$n = 34$

$\therefore 34$ terms are there in given $A.p$

(iv) $18, 15\frac{1}{2}, 13, ... -47$

From the given $A.p.$,

$a = 18, d = 15\frac{1}{2} - 18 = \frac{31}{2} - 18 = 15\cdot 5 - 18 = -2\cdot 5$

$a_n = a + (n-1).d$

Let $a_n = -47$ (last term)

$18 + (n-1), 2.5 = -47$

$12.5(n-1) = +65$

$n - 1 = \frac{65}{2\times 5} = \frac{65\times 10}{25} = 26$

$n = 27$

$\therefore 27$ terms are there in given $A.p$

5. The first term of an AP is 5, the common difference is 3 and the last term is 80, find the number of terms.

Sol:

Given

First term $(a) = 5$

Common difference $(d) = 3$

Last term $(1) = 80$

To calculate no of terms in given $A.p$

$a_n = a + (n-1)d$

Let $a_n = 80$,

$80 = 5 + (n-1)\cdot 3$

$75 = (n-1)\cdot 3$

$n - 1 = \frac{75}{3} = 25$

$n = 26$

$\therefore$ There are 26 terms.

6. The 6th and 17 terms of an A.P. are 19 and 41 respectively, find the 40th term.

Sol:

Given, $a_6 = 19, a_{17} = 41$

$\Rightarrow a_6 = a + (6-1)d$

$19 = a + 5d$ ……….(1)

$\Rightarrow a_{17} = a + (17-1)\cdot d$

$41 = a + 16d$ ………..(2)

Subtract (1) from (2)

$$\begin{array}{l} a + 16d = 41 \\ \underline{a + 5d = 19} \\ 0 + 11d = 22 \end{array}$$

$d = \frac{22}{11} = 2$

Substitute $d = 2$ in (1)

$19 = a + 5(2)$

$9 = a$

$\therefore 40^{th}$ term $a_{40} = a + (40-1)\cdot d$

$= 9 + 39 \cdot 2$

$= 9 + 78$

$= 87$

$\therefore a_{40} = 87$

7. If 9th term of an A.P. is zero, prove that its 29th term is double the 19th term.

Sol:

Given

9^{th} term of an $A.p$ $a_9 = 0, a_n = a + (n-1)d$

$a + (a-1)\cdot d = 0$

$a + 8d = 0$

$a = -8d$

We have to prove

24^{th} term is double the 19^{th} term $a_{29} = 2 \cdot a_{19}$

$a + (29-1)d = 2\left[a + (1a-1).d\right]$

$a + 28d = 2[a + 18a]$

Put $a = -8d$

$-8d + 28d = 2[-8d + 18d]$

$20d = 2 \times 10d$

$20d = 20d$

Hence proved

8. If 10 times the 10th term of an A.P. is equal to 15 times the 15th term, show that 25th term of the A.P. is zero.

Sol:

Given,

10 times of 10^{th} term is equal to 15 times of 15^{th} term.

$10a_{10} = 15.a_{15}$

$10[a+(10-1)d] = 15[a+(15-1).d] (\therefore a_n = a+(n-1)d)$

$10(a+9d) = 15(a+14\cdot d)$

$a+9d = \frac{15}{10}(a+14d)$

$a - \frac{3}{2}a = \frac{42d}{2} - 9d$

$-\frac{1}{2}a = \frac{24}{2}\cdot d$ (12)

$-a = +24\cdot d$

$a = -24\cdot d$

We have to prove 25^{th} term of $A.p$ is 0

$a_{25} = 0$

$a+(25-1)d = 0$

$a+24d = 0$

Put $a = -24d$

$-24\times d + 24d = 0$

$0 = 0$

Hence proved.

9. The 10^{th} and 18^{th} terms of an A.P. are 41 and 73 respectively. Find 26^{th} term.

Sol:

Given,

$a_{10} = 41, a_{18} = 73, a_n = a+(n-1)\cdot d$

$\Rightarrow a_{10} = a+(10-1)\cdot d$

$41 = a+9d$ ………..(1)

$\Rightarrow a_{18} = a+(18-1)d$

$73 = a+17d$ ………..(2)

Subtract (1) from (2)

(2) (1)

$a+17d=73$

$a+9d=41$

$0+8d=32$

$d=\frac{32}{8}=4$

Substitute $d=4$ in (1)

$a+9\cdot 4=41$

$a=41-36$

$a=5$

26^{th} term $a_{26}=a+(26-1)d$

$=5+25.4$

$=5+100$

$=105$

$\therefore 26^{th}$ term $a_{26}-105.$

10. In a certain A.P. the 24^{th} term is twice the 10^{th} term. Prove that the 72^{nd} term is twice the 34^{th} term.

Sol:

Given

24^{th} term is twice the 10^{th} term

$a_{24}=2\ a_{10}$

Let, first term of a square $=a$

Common difference $=d$

n^{th} term $a_n=a+(n-1)d$

$a+(24-1)d=(a+(10-1).d)2$

$a+23d=2(a+9d)$

$(23-18)d=a$

$a=5d$

We have to prove

72^{nd} term is twice the 34^{th} term

$a_{12}=2a_{34}$

$a+(12-1)d=2[a+(34-1)d]$

$a+71d=2a+66d$

Substitute $a=5d$

$5d+71d=2(5d)+66d$

$76d=10d+66d$

$76d = 76d$

Hence proved.

11. If (m + 1)th term of an A.P. is twice the (n + 1)th term, prove that (3m + 1)th term is twice the (m+n+l)th term.

Sol:

Given

$(m+1)^{th}$ term is twice the $(m+1)^{th}$ term.

First term $= a$

Common difference $= d$

n^{th} term $a_n = a + (n-1).d$

$a_{m+1} = 2\ a_n + 1$

$a + (m+1-1)\cdot d = 2(a + (n+1-1)\cdot d)$

$a + md = 2(a + nd)$

$a = (m-2n)d$

We have to prove

$(3m+1)^{th}$ term is twice the $(m+n+1)^{th}$ term

$a_{3m+1} = 2\cdot a_{m+n+1}$

$a + (3m+1-1)\cdot d = (a + (m+n+1-1)\cdot d)$

$a + 3m\cdot d = 2a + 2(m+n)d$

Substitute $a = (m-2n)\cdot d$

$(m-2n)\not{d} + 3m\not{d} = 2(m-2n)\not{d} + 2(m+n)\cdot\not{d}$

$4m - 2n = 4m - 4n + 2n$

$4m - 2n = 4m - 2n$

Hence proved.

12. If the n term of the A.P. 9, 7, 5, ... is same as the th term of the A.P. 15, 12, 9, ... find n.

Sol:

Given,

First sequence is $9, 7, 5, ...$

$a = 9, d =\neq -9 = -2, a_n = a + (n+1)d$

$a_n = 9 + (n-1). - 2$

Second sequence is $15, 12, 9, ...$

$a = 15, d = 12 - 15 = -3, a_n = a + (n-1)d$

$a_n = 15 + (n-1) - 3$

Given an. a_n are equal

$9 - 2(n-1) = 15 - 3(n-1)$

$3(n-1) - 2(n-1) - 15 - 9$

$n - 1 = 6$

$n = 7$

$\therefore 7^{th}$ term of two sequence are equal

13. Find the 12th term from the end of the following arithmetic progressions:
(i) 3 51 7, 9, ... 201 (ii) 3, 8, 13, ... , 253 (iii) 1, 4, 7, 10, ..., 88

Sol:

(i) $3, 5, 7, 9, ...2d$

First term $(a) = 3$

Common difference $(d) = 5 - 3 = 2$

12^{th} term from the end is can be considered as (1) last term = first term and common differnce $= d^1 = -d$ n^{th} term from the end = last term $+(n-1) - d$

12^{th} term from end $= 201 + (12-1)(-2)$

$= 201 - 22$

$= 179$

(ii) $3, 8, 13, ...253$

First term $= a = 3$

Common difference $d = 8 - 3 = 5$

Last term $(1) = 253$

n^{th} term of a sequence on $= a + (n-1) \cdot d$

To find n^{th} term from the end, we put last term (1) as $'a'$ and common difference as $-d$

n^{th} term from the end = last term $+(n-1) \cdot -d$

12^{th} term from the end $= 253 + (12-1) - 5$

$= 253 - 55$

$= 198$

$\therefore 12^{th}$ term from the end $= 198$

(iii) $1, 4, 7, 10, ...88$

First term $a = 1$

Common differnce $d = 4 - 1 = 3$

Last term $(1) = 88$

n^{th} term $a_n = a + (n+1) \cdot d$

n^{th} term from the end = last term $+ (n-1) \cdot -d$

12^{th} term from the end $= 88 + (12-1) \cdot -3$

$= 88 - 33$

$= 55$

$\therefore 12^{th}$ term from the end $= 55.$

14. The 4th term of an A.P. is three times the first and the 7 term exceeds twice the third term by 1. Find the first term and the common difference.

Sol:

Given,

4^{th} term of an $A.p$ is three times the times the first term

$a_4 = 3.\, a$

n^{th} term of a sequence $a_n = a + (n-1) \cdot d$

$a + (4-1) \cdot d = 3a$

$a + 3d = 3a$

$3d = 2a$

$a = \frac{3}{2} d.$ …………(1)

Seventh term exceeds twice the third term by 1.

$a_7 + 1 = 2.a_3$

$a + (7-1) \cdot d + 1 = 2(\alpha + \beta - 1 \cdot d)$

$a + 6d + 1 = 2a + 4d$

$a = 2d + 1$ …………(2)

By equating (1), (2)

$\frac{3}{2} d = 2d + 1$

$\frac{3}{2} d - 2d = 1$

$\frac{3d - 4d}{2} = 1$

$-d = 2$

$d = -2$

Put $d = -2$ in $a = \frac{3}{2} d$

$= \frac{3}{2}.\ x$

$= -3$

$\therefore$ First term $a = -3,$ common difference $d = -2.$

15. Find the second term and nth term of an A.P. whose 6th term is 12 and the 8th term is 22.

Sol:

Given

$a_6 = 12, a_8 = 22$

n^{th} term of an $A.p$ $a_n = a + (n-1)d$

$a_6 = a + (n-1)\cdot d = a + (6-1)d = a + 5d = 12$ ....(1)

Subtracting (1) from (2)

$$a + 7d = 22$$

(2) (1) $\Rightarrow$
$$\frac{a + 5d = 12}{0 + 2d = 10}$$

$$d = 5$$

$$a + 5d = 12$$

Put $d = 5$ in
$$a + 55 = 12$$
$$a = 12 - 25$$
$$a = -13$$

Second term $a_2 = a + (2-1)\cdot d$

$= a + d$

$= -13 + 5$

$a_1 = -8$

n^{th} term $a_n = a + (n-1)d$

$= -13 + (n-1) - 5$

$a_n = -18 + 5n$

n^{th} term $a_n = a + (n-1)d$

$= -13 + (n-1) - 5$

$a_n = -18 + 5n$

$\therefore a_2 = -8, a_n = -18 + 5n$

16. How many numbers of two digits are divisible by 3?

Sol:

We observe that 12 is the first two-digit number divisible by 3 and 99 is the last two digit number divisible by 3. Thus, the sequence is

$12, 15, 18, \ldots\ldots\ldots\ldots\ldots 99$

This sequence is in $A.p$ with

First term $(9) = 12$

Common difference $(d) = 15 - 12 = 3$

n^{th} term $a_n = 99$

n^{th} term of an $A.p(a_n) = a + (n-1) \cdot d$

$99 = 12 + (n-1) \cdot 3$

$99 - 12 = n - 1 \cdot 3$

$\frac{87}{3} = n = 1$

$n = 30$

$\therefore 30$ term are there in the sequence

17. An A.P. consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32nd term.

Sol:

Given

No. of terms $= n = 60$

First term $(a) = 7$

Last term $a_{10} = 125$

$a_{60} = a + (60-1) \cdot d \qquad (\therefore a_n = a + (n-1)d)$

$125 = 7 + 59 \cdot d$

$118 = 59d$

$d = \frac{118}{59} = 2$

52^{nd} term $a_{32} = a + (32-1)d$

$= 7 + 31.2$

$= 7 + 62$

$= 69$

18. The sum of 4 and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 34. Find the first term and the common difference of the A.P.

Sol:

Given

$a_4 + a_8 = 24$

$a_6 + a_{10} = 34$

$\Rightarrow a + (4-1)d + a + (18-1)d = 24$

$2a + 10d = 24$

$a + 5d = 12$ …………(1)

$\Rightarrow a_6 + a_{10} = 34$

$a + (6-1)d + a + (10-1)d = 34$

$2a + 14d = 34$

$a + 7d = 17$ …………(2)

Subtract (1) from (2)

$a + 7d = 17$

$\underline{a + 5d = 12}$

$2d = 5$

$d = \frac{5}{2}$

Put $d = \frac{5}{2}$ in $a + 5d = 12$

$a = 12 - 5 \cdot \frac{5}{2}$

$a = \frac{24 - 25}{2} = \frac{-1}{2}$

$\therefore a = -\frac{1}{2}, d = \frac{b}{2}$

19. The first term of an A.P. is 5 and its 100th term is — 292. Find the 50th term of this A.P.

Sol:

Given,

$a_{30} - a_{20} = a + (30-1)d - (a + (20-1)d)(\therefore a_n = a + (n-1)d)$

$= a + 29d - a - 19d$

$= 10d$

(i) $-9, -14, -19, -24, ...$

Common difference (d) = second tern – first term

$= -14 - (-9)$

$= -14 + 9$

$d = 5$

Then $a_{30}-a_{20}=10d$

$=10.5$

$a_{30}-a_{20}=50$

(ii) $a,a+d,a+2d,a+3d$

First term $(a)=a$

Common difference $(d)=d$

$a_{30}-a_{20}=a+(30-1)d-(a+(20-1)d)$

$=a+29d-0-19d$

$a_{30}-a_{20}=10d$

20. Find a_{30} — a_{20} for the A.P.

(i) $-9,-14,-19,-24,........$

(ii) a, a + d, a + 2d, a + 3d,...

Sol:

Given,

$a_{30}-a_{20}=a+(30-1)d-(a+(20-1)d)(\therefore a_n=a+(n-1)d)$

$=a+29d-a-19d$

$=10d$

(i) $-9,-14,-19,-24,........$

Common difference (d) = second term – first term

$=-14-(-9)$

$=-14+9$

$d=5$

Then $a_{30}-a_{20}=10d$

$=10.5$

$a_{30}-a_{20}=50$

(ii) $a,a+d,a+2d,a+3d,............$

First term $(a)=a$

$a_{30}-a_{20}=a+(30-1)d-(a+(20-1)d)$

$=a+29d-a-19d$

$a_{30}-a_{20}=10d$

21. Write the expression a_n-a_k for the A.P. a, a + d, a + 2d, ...

Hence, find the common difference of the AP for which

(i) 11^{th} term $a_n=5$ and 13^{th} term $a_{13}=79$

(ii) $a_{10} - a_5 = 200$

(iii) 20th term is 10 more than the 18th term.

Sol:

General arithmetic progression

$a, a+d, a+2d, ...$

$a_b - a_k = a + (n-1)d - (a + (k-1)d)(\therefore a_n = a + (n-1)d)$

$= a + (n-1)d - 2\not{a} + (k-1)d$

$a_n - a_k = (n-k)d$ ....(1)

(i) Given

11^{th} term $a_n = 5$

13^{th} term $a_{13} = 79$

By using (1) put $n = 13, k = 11$

$a_n - a_k = (n-k)\cdot d$

$79 - 5 = (13-11)\cdot d$

$74 = 2 \times d$

$d = \frac{74}{2} = 37$

(ii) Given

$a_{10} - a_5 = 200$

From (1) $a_{10} - a_5 = (10-5)d$

$200 = 5 \cdot d$

$d = \frac{200}{5} = 40 \Rightarrow d = 40$

(iii) Given

$a_{20} \neq 10 = a_{18}$

$a_{20} - a_{18} = 10$

By (1) $a_n - a_k = (n-k)\cdot d$

$a_{20} - a_{18} = (20-18)\cdot d$

$10 = 2 \cdot d$

$d = \frac{10}{2} = 5$

$\therefore d = 5$

22. Find n if the given value of x is the n term of the given A.P.

(i) $1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, \ldots\ldots\ldots, x = \frac{141}{11}$

(ii) $5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \ldots\ldots\ldots\ldots : x = 550$

(iii) $-1, -3, -5, -7, \ldots\ldots : x = -151$

(iv) $25, 50, 75, 100 \ldots\ldots\ldots : c = 1000$

Sol:

(i) $25, 50, 75, 100 \ldots\ldots\ldots : c = 1000$

First term $(a) = 25$

Common difference $(d) = 50 - 25 = 25$

n^{th} term $a_n = a + (n-1) \times d$

Given, $a_n = 1000$

$1000 = 25 + (n-1) \cdot 25$

$975 = (n-1) \times 25$

$n - 1 = \frac{975}{25} = 39$

$n = 40$

(ii) Given sequence $-1, -3, -5, -7, \ldots\ldots : x = -151$

First term $(a) = -1$

Common difference $(d) = -3 - (-1) = -3 + 1 = -2$

n^{th} term $a_n = a + (n-1)d$

Given $a_n = -151,$

$-151 = -1 + (n-1) - 2$

$-150 = 1(n+1)2$

$n + 1 = \frac{150}{2} = 75$

$n = 74$

(iii) Given sequence is

$5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \ldots\ldots\ldots\ldots : x = 550$

First term (a) $= 5\frac{1}{2} = \frac{11}{2}$

$= \frac{22-11}{2}$

$= \frac{11}{2}$

n^{th} term $a_n = a + (n-1)d$

$550 = \frac{11}{2} + (n-1) \cdot \frac{11}{2}$

$550 = \frac{11}{2}[1 + n - 1]$

$n = 550 \times \frac{2}{11}$

$n = 100$

(iv) Given sequence is

$1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, ..., x = \frac{141}{11}$

First term $(a) = 1$

Common difference $(d) = \frac{21}{11} - 1$

$= \frac{21-11}{11}$

$= \frac{10}{11}$

n^{th} term $a_n = a + (n-1) \times d$

$\frac{171}{11} = 1 + (n-1) . \frac{10}{11}$

$\frac{171}{11} - 1 = (n-1)\frac{10}{11}$

$\frac{171-11}{11} = (n-1)\frac{10}{11}$

$\frac{160}{11} = (n-1) . \frac{10}{11}$

$n - 1 = \frac{160}{11} \times \frac{11}{10}$

$n = 17$

23. If an A.P. consists of n terms with first term a and n^{th} term 1 show that the sum of the m^{th} term from the beginning and the m^{th} term from the end is (a + l).

Sol:

First term of a sequence is a

Last term $= 1$

Total no. of terms $= n$
Common difference $= d$
m^{th} term from the beginning $a_m = a + (n-1) \cdot d$
m^{th} term from the end = last term $+(n-1) - d$
$a_n - m + 1 = 1 - (n-1) \times d$
$\Rightarrow a_m + a_n - m + 1 = a + (n-1)d + (l - (n-1)d)$
$= a + (n-1)d + l - (n-1)d$
$a_m + a_n - m + 1 = a + l$
Hence proved

24. Find the arithmetic progression whose third term is 16 and seventh term exceeds its fifth term by 12.

Sol:
Given, $a_3 = 16$
$a + (3-1)d = 16$
$a + 2d = 16.$ ……..(1)
And $a_7 - 12 = a_5$
$a + (7-1)d - 12 = a + (5-1)d$ $\left(\because a_n = a + (n-1)d\right)$
$\not{a} + 6d - 12 = \not{a} + 4d$
$2d = +12$
$d = +\frac{12}{2} = +6$
Put $d = -6$ in (1)
$a + 2(+6) = 16$
$a + 12 = 6$
$a = 284$
Then the sequence is $a, a+d, a+2d, a+3d, ...$
$\Rightarrow 28, 4, 10, 16, 22, ...$

25. The 7th term of an A.P. is 32 and its 13th term is 62. Find the A.P.

Sol:
Given,
$a + = 32$
$a + (7-1)d = 32$
$a + 6d = 32$ …….(1)

And $a_{13} = 62$

$a + (13-1)d = 62$

$a + 12d = 62$ ……..(2)

Subtract (1) from (2)

$$a + 12d = 62$$

$$(2)-(1) \Rightarrow \frac{a+6d = 32}{0+6d = 32}$$

$d = \frac{30}{6} = 5$

Put $d = 5$ in $a + 6d = 32$

$a + 6 \cdot 5 = 32$

$a = 2$

Then the sequence is $a, a+d, a+2d, a+3d, \ldots\ldots\ldots.$

$\Rightarrow 2, 7, 12, 17, \ldots\ldots$

26. Which term of the A.P. 3, 10, 17, ... will be 84 more than its 13th term?

Sol:

Given $A.p$ is $3, 10, 17, \ldots\ldots.$

First term $(a) = 3,$ Common difference $(d) = 10 - 3$

$= 7$

Let, n^{th} term of $A.p$ will be 84 more than 13^{th} term

$a_n = 84 + a_{13}$

$\not{a} + (n-1)d = \not{a} + (13-1)d + 84$

$(n-1)7 = 12.7 + 84$

$(n-1) \cdot 7 = 168$

$n - 1 = \frac{168}{7} = 24$

$n = 25$

Hence 25^{th} term of given $A.p$ is 84 more than 13^{th} term

27. Two arithmetic progressions have the same common difference. The difference between their 100th terms is 100, what is the difference between their l000th terms?

Sol:

Let the two $A.p$ is be $a_1, a_2, a_3, \ldots\ldots\ldots.$ and $b_1, b_2, b_3, \ldots\ldots\ldots..$

$a_n = a_1 + (n-1)d$ and $b_n = b_1 + (n-1) \cdot d$

Since common difference of two equations is same given difference between 100^{th} terms is 100

$a_{100} - b_{100} = 100$

$a_1 + (99)d - b_1 - 99d = 100$

$a_1 - b_1 = 100$ …………..(1)

Difference between. 1000^{th} terms is

$a_{1000} - b_{1000} = a_1 + (1000-1)d - (b_1 + (1000-1)d)$

$= a_1 + 999d - b_1 - 999d$

$= a_1 - b_1$

$= 100$ (from (1))

$\therefore$ Hence difference between 1000^{th} terms of two $A.p$ is 100.

28. For what value of n, the nth terms of the arithmetic progressions 63, 65, 67, . . . and 3, 10, 17, . . . are equal?

Sol:

Given two $A.p$ is are

$63, 65, 67..........$ and $3, 10, ...$

First term of sequence 1 is $a_1 = 63$

Common difference $d_1 = 65 - 63$

$= 2.$

n^{th} term $(a_n) = a_1 + (n-1)d$

$= 63 + (n-1)d$

First term of sequence 2 is $b_1 = 3.$

Common difference $d_2 = 10 - 3$

$= 7$

n^{th} term $(b_n) = b_1 + (n-1)d_2$

$= 3 + (n-1)\cdot 7$

Let n^{th} terms of two sequence is equal

$63 + (n-1)2 = 3 + (n-1)\times 7$

$60 = 5(n-1)$

$n - 1 = \frac{60}{5} = 12$

$n = 13$

$\therefore$ 13^{th} term of both the sequence are equal.

29. How many multiples of 4 lie between 10 and 250?

Sol:

Multiple of 4 after 10 is 12 and multiple of 4 before 250 is $\frac{250}{4}$ remainder is 2, so,

$250 - 2 = 248$

248 is the last multiple of 4 before 250.

The sequence is

$12, ..., 248$

With first term $(a) = 12$

Last term $(1) = 248$

Common difference $(d) = 4$

n^{th} term $a_n = a + (n-1) \cdot d$

Here, n^{th} term $a_n = 248$

$248 = 12 + (n-1) \times 4$

$236 = (n-1) \times 4$

$n - 1 = \frac{236}{4} = 59$

$n = 60$

$\therefore$ There are 60 terms between 10 and 250 which are multiples of 4

30. How many three digit numbers are divisible by 7?

Sol:

The three digit numbers are $100, ...999$ 105 is the first 3 digit number which is divisible by 7 when we divide 999 with 7 remainder is 5. So, 999 – 5 = 994 is the last three digits divisible by 7 so, the sequence is

$105, ..., 994$

First term $(a) = 105$

Last term $(1) = 994$

Common difference $(d) = -7$

Let there are n numbers in the sequence

$a_n = 994$

$a + (n-1)d = 994$

$a + (n-1)d = 994$

$105+(n-1)7=994$

$(n-1)\cdot 7=889$

$n-1=\frac{889}{7}=127$

$n=128$

$\therefore$ there are 128 numbers between 105, 994 which are divisible by 7

31. Which term of the arithmetic progression 8, 14, 20, 26, . . . will be 72 more than its 41st term?

Sol:

Given sequence

$8,14,20,26,.........$

Let n^{th} term is 72 more than its 41^{st} term

$a_n=a_{41}+72$

For the given sequence

$a=8, d=14-8=6$

$a+(n-1)d=8+(a+1)6+72$

$8+(n-1)6=8+(90)\cdot 6+72$

$(n-1)6=312$

$n-1=\frac{312}{6}=52$

$n=53$

$\therefore 53^{rd}$ term is 72 more than 41^{st} term

32. Find the term of the arithmetic progression 9, 12, 15, 18,. . . which is 39 more than its 36th term.

Sol:

Given $A.p$ is $9,12,15,........$

For this $a=9, d=12-9=3$

Let n^{th} term is 39 more than its 36^{th} term

$a_n=39+a_{36}$

$\not{a}+(n-1)3=39+\not{9}+(36-1)\cdot 3 \qquad (\therefore a_n=a+(n-1)d)$

$(n-1)3=39+35\cdot 3$

$(n-1)\times 3=144$

$n-1=\frac{144}{3}=48$

$n=49$

$\therefore 49^{th}$ term is 39 more than its 36^{th} term

33. Find the 8th term from the end of the A.P. 7,1õ13, . . ., 184

Sol:

Given $A.p$ is $7,10,13,.........184$

$a=7, d=10-7=3, l=184$

n^{th} term from the end $=l+(n-1)-d$

8^{th} term from the end $=184+(8-1)-3$

$=184-21$

$=163$

$\therefore 8^{th}$ term from the end -163

34. Find the 10th term from the end of the A.P. 8, 10, 12,. . ., 126.

Sol:

Given $A.p$ is $8,10,12,........126$

$a=8, d=10-8=2, l=126$

n^{th} term from the end $=l+(n-1)-d$

10^{th} term from the end $=126+(10-1)-2$

$=126-18$

$=108$

$\therefore 10^{th}$ term from the end $=108$

35. The sum of 4th and 8th terms of an A.P. is 24 and the sum of 6th and 10th terms is 44. Find the A.P.

Sol:

Given, $a_4+a_8=24$

$(a+(4-1)d)+(a+(8-1)d)=24(\therefore a_n=a+(n-1)d)$

$2a+10d=24$

$a+5d=12$ ....(1)

And $a_6+a_{10}=44$

$a+(6-1)+a+(10-1)d=44$ $(\therefore a_n=a+(n-1)d)$

$2a+14d=44$

$a+7d=22$ ....(2)

Subtract (1) from (2)

$$\begin{aligned} a+7d &= 22 \\ (2)-(1) \Rightarrow \quad a+5d &= 12 \\ \hline 0+2d &= 10 \end{aligned}$$

$d=5$

Put $d=5$ in (1) $a+5\cdot 5=12$

$a=-13$

36. Which term of the A.P. 3, 15, 27, 39, . . . will be 120 more than its 21st term?

Sol:

Given $A.p$ is

$3,15,27,39,........$

Let n^{th} term is 120 more than 21^{st} term

Then $a_n = 120 + a_{21}$

For the given sequence

$a=3, d=15-3=12$

$\cancel{a}+(n-1)d = 120+\cancel{a}+(21-1)d$

$(n-1)12 = 120+20(2)$

$(n-1)12 = 360$

$(n-1) = \frac{360}{12} = 30$

$n=31$

$\therefore$ 31^{st} term is 120 more than 21^{st} term

37. The 17th term of an A.P. is 5 more than twice its 8th term. If the 11th term of the A.P. is 43, find the nth term.

Sol:

Given

17^{th} term of an $A.p$ is 5 more than twice its 8^{th} term

$a_{17} = 5+2a_8$

$a+(17-1)d = 5+2\left(a+(8-1)\cdot d\right)$

$a+16d = 5+2a+14d$

$a+5=2d$ ....(1)

And 11^{th} term of the $A.p$ is 43

$a_{11}=43$

$a+(11-1)d=43$

$a_{11} = 43$

$a + (11-1)d = 43$

$a + 10d = 43$ ....(2)

$$(2)-(1) \Rightarrow \begin{array}{l} a+10d = 43 \\ a-2d = +5 \\ \hline a+12d = 48 \end{array}$$

$d = \frac{48}{12} = 4$

Put $d = 4$ in (1)

$a + 5 = 2(4)$

$a = 3$

$\therefore n^{th}$ term of given sequence is an $= a = 1 + (n-1)d$

$= 3 + (n-1)4$

$= 3 + 4n - 4$

$= 4n - 1$

$\therefore n^{th}$ term of given sequence $a_n = 4n - 1$

Exercise – 9.4

1. The sum of three terms of an A.P. is 21 and the product of the first and the third terms exceeds the second term by 6, find three terms.

Sol:

Given,

Sum of three terms of on A.P is 21.

Product of first and the third term exceeds the second term by 6.

Let, the three numbers be a-d, a, a+d, with common difference d: then,

(a – d) + a + (a + d) = 21

3a = 21

a = $\frac{21}{3} = 7$

and (a – d) (a + d) = a + 6

$a^2 - d^2 = a + 6$

Put a = 7 $\Longrightarrow 7^2 - d^2 = 7 + 6$

49 – 13 = d^2

d = ± 6

∴ The three terms are a – d, a, a + d, i.e., 1, 7, 13.

2. Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers.

Sol:

Let, the three numbers are a – d, a, a + d.

Given,

(a – d) + a + (a + d) = 27

3a = 27

a = $\frac{27}{3} = 9$

and, $(a - d)(a)(a + d) = 648$

$(a^2 - d^2)(a) = 648$

Put a = 9, then

$(9^2 - d^2)\ 9 = 648$

81 – d^2 = $\frac{648}{9} = 72$

$d^2 = 81 - 72$

$d^2 = 9$

d = 3

∴ The three terms are a – d, a, a + d i.e. 6, 9, 12.

3. Find the four numbers in A.P., whose sum is 50 and in which the greatest number is 4 times the least.

Sol:

Let, the four numbers be a – 3d, a – d, a + d, a + 3d, with common difference 2d.

Given, sum is 50.

(a – 3d) + (a – d) + (a + d) + (a + 3d) = 50

4a = 50

a = 12.5

greater number is 4 time the least

(a + 3d) = 4(a – 3d)

a + 3d = 4a – 12d

15d = 3a

Put a = 12.5

d = $\frac{3}{15} \times 12.5$

d = 2.5

∴ The four numbers are a – 3d, a – d, a + 3d i.e., 12.5 – 3(2.5), 12.5 – 2.5, 12.5 + 2.5, 12.5 + 3(2.5)

⟹ 5, 10, 15, 20

4. The angles of a quadrilateral are in A.P. whose common difference is 10°. Find the angles.
Sol:
A quadrilateral has four angles. Given, four angles are in A.P with common difference 10.
Let, the four angles be, a – 3d, a – d, a + d, a + 3d with common difference = 2d.
2d = 10
d = $\frac{10}{2} = 5$
In a quadrilateral, sum of all angles = 360°
(a – 3d) + (a – d) + (a + d) + (a + 3d) = 360
4a = 360
a = 360/4 = 90°
∴ The angles are a – 3d, a – d, a + d, a + 3d with a = 90, d = 5
i.e. 90 – 3(5), 90 – 5, 90 + 3(5)
⟹ 75°, 85°, 95°, 105°.

5. The sum of three numbers in A.P. is 12 and the sum of their cubes is 288. Find the numbers.
Sol:
2, 4, 6, or 6, 4, 2.

6. Find the value of x for which (8x + 4), (6x — 2) and (2x + 7) are in A.P.
Sol:
Given,
8x + 4, 6x – 2, 2x + 7 are are A.P.
If the numbers a, b, c are in A.P. then condition is 2b = a + c.
Then, 2(6x – 2) = 8x + 4 + 2x + 7
12x – 4 = 10 + 11
2x = 15
x = $\frac{15}{2}$

7. If x + 1 , 3x and 4x + 2 are in A.P., find the value of x.
Sol:
Given numbers
x + 1, 3x, 4x + 2 are in AP
If a, b, c are in AP then 2b = a + c
Then 2(3x) = x + 1 + 4x + 2
6x = 5x + 3
x = 3

8. Show that $(a - b)^2, (a^2 + b^2)$ and $(a + b)^2$ are in A.P.

Sol:

We have to show, (a – b)2, (a2 + b2) and (a + b)2 are in AP.

If they are in AP. Then they have to satisfy the condition

2b = a + c

$2(a^2 + b^2) = (a - b)^2 + (a + b)^2$

$2a^2 + 2b^2 = a^2 + 2ab + b^2 + a^2 + 2ab + b^2$

$2a^2 + 2b^2 = 2a^2 + 2b^2$.

They satisfy the condition means they are in AP.

Exercise – 9.5

1. Find the sum of the following arithmetic progressions:
   (i) 50, 46, 42, ... to 10 terms
   (ii) 1, 3, 5, 7, ... to 12 terms
   (iii) 3, 9/2, 6, 15/2, ... to 25 terms
   (iv) 41, 36, 31, ... to 12 terms
   (v) a + b, a – b, a − 3b, ... to 22 terms
   (vi) $(x - y)^2, (x^2 + y^2), (x + y)^2, ...,$ to n terms
   (vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, ... to\ n\ terms$
   (viii) $-26, -24, -22, ... to\ 36\ terms$

Sol:

In an A.P let first term = a, common difference = d, and there are n terms. Then, sum of n terms is,

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

(i) Given progression is,

50, 46, 42, ……to 10 term.

First term (a) = 50

Common difference (d) = 46 – 50 = -4

n^{th} term = 10

Then $S_{10} = \frac{10}{2}\{2.50 + (10-1) - 4\}$

$= 5\{100 - 9.4\}$

$= 5\{100 - 36\}$

$= 5 \times 64$

$\therefore S_{10} = 320$

(ii) Given progression is,

1, 3, 5, 7, …..to 12 terms

First term difference (d) = 3 – 1 = 2

n^{th} term = 12

Sum of n[th] terms $S_{12} = \frac{12}{2} \times \{2.1 + (12-1).2\}$

$= 6 \times \{2 + 22\} = 6.24$

$\therefore S_{12} = 144.$

(iii) Given expression is

$3, \frac{9}{2}, 6, \frac{15}{2}, \dots \dots \text{ to } 25 \text{ terms}$

First term (a) = 3

Common difference (d) = $\frac{9}{2} - 3 = \frac{3}{2}$

Sum of n[th] terms $S_n, given\ n = 25$

$S_{25} = \frac{n}{2}(2a + (n-1).d)$

$S_{25} = \frac{25}{2}\left(2.3 + (25-1).\frac{3}{2}\right)$

$= \frac{25}{2}\left(6 + 24.\frac{3}{2}\right)$

$= \frac{25}{2}(6 + 36)$

$= \frac{25}{2}(42)$

$\therefore S_{25} = 525$

(iv) Given expression is,

41, 36, 31, …… to 12 terms.

First term (a) = 41

Common difference (d) = 36 – 41 = − 5

Sum of n[th] terms $S_n, given\ n = 12$

$S_{12} = \frac{n}{2}(2a(n-1).d)$

$= \frac{12}{6}(2.41 + (12-1).-5)$

$= 6(82 + 11.(-5))$

= 6(27)

= 162

$\therefore S_{12} = 162.$

(v) a + b, a – b, a – 3b, …… to 22 terms

First term (a) = a + b

Common difference (d) = a – b – a – b = −2b

Sum of n[th] terms $S_n = \frac{n}{2}\{2a(n-1).d\}$

Here n = 22

$S_{22} = \frac{22}{2}\{2.(a+b) + (22-1).-2b\}$

$= 11\{2(a+b) - 22b\}$

= 11 {2a – 20b}

= 22a – 440b

$\therefore S_{22} = 22a - 440b$

(vi) $(x-y)^2, (x^2+y^2), (x+y)^2, \ldots\ldots.. to\ n\ terms$
First term (a) = $(x-y)^2$
Common difference (d) = $x^2+y^2-(x-y)^2$
= $x^2+y^2-(x^2+y^2-2xy)$
= $x^2+y^2-x^2+y^2+2xy$
= 2xy
Sum of n^{th} terms $S_n = \frac{n}{2}\{2a(n-1).d\}$
= $\frac{n}{2}\{2(x-y)^2+(n-1).2xy\}$
= $n\{(x-y)^2+(n-1)xy\}$
$\therefore S_n = n\{(x-y)^2+(n-1).xy\}$

(vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \ldots.. to\ n\ terms$
$First\ term\ (a) = \frac{x-y}{x+y}$
Common difference (d) = $\frac{3x-2y}{x+y} - \frac{x-y}{x+y}$
= $\frac{3x-2y-x+y}{x+y}$
= $\frac{2x-y}{x+y}$
Sum of n terms S_n = $\frac{n}{2}\{2a+(n-1).d\}$
= $\frac{n}{2}\left\{2.\frac{x-y}{x+y}+(n-1).\frac{2x-y}{x+y}\right\}$
= $\frac{n}{2(x+y)}\{2(x-y)+(n-1)(2x-y)\}$
= $\frac{n}{2(x+y)}\{2x-2y+2nx-ny-2x+y\}$
= $\frac{n}{2(x+y)}\{n(2x-y)-y\}$
$\therefore S_n = \frac{n}{2(x+y)}\ \{n(2x-y)-y\}$

(viii) Given expression $-26, -24, -22, \ldots\ldots.$ To 36 terms
First term (a) = -26
Common difference (d) = $-24-(-26) = -24+26 = 2$
Sum of n terms $S_n = \frac{n}{2}\{2a+(n-1)d\}$
Sum of n terms $S_n = \frac{36}{2}\{2.-26+(36-1)2\}$
= $18[-52+70]$
= 18.18
= 324
$\therefore S_n = 324$

2. Find the sum to n term of the A.P. 5, 2, — 1, —4, —7, ...
Sol:
Given AP is 5, 2, -1, -4, -7, …..

a = 5, d = 2 – 5 = −3

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$= \frac{n}{2}\{2.5 + (n-1) - 3\}$

$= \frac{n}{2}\{10 - 3(n-1)\}$

$= \frac{n}{2}\{13 - 3n\}$

$\therefore S_n = \frac{n}{2}(13 - 3n)$

3. Find the sum of n terms of an A.P. whose th terms is given by a_n = 5 - 6n.

Sol:

Given nth term $a_n = 5 - 6n$

Put n = 1, $a_1 = 5 - 6.1 = -1$

We know, first term (a_1) = −1

Last term (a_n) = 5 – 6n = 1

Then $S_n = \frac{n}{2}(-1 + 5 - 6n)$

$= \frac{n}{2}(4 - 6n) = \frac{n}{2}(2 - 3n)$

4. If the sum of a certain number of terms starting from first term of an A.P. is 25, 22, 19, ... is 116. Find the last term.

Sol:

Given AP is 25, 22, 19, ……

First term (a) = 25, d = 22 – 25 = −3.

Given, $S_n = \frac{n}{2}(2a + (n-1)d)$

$116 = \frac{n}{2}(2 \times 25 + (n-1) - 3)$

$232 = n(50 - 3(n-1))$

232 = n(53 – 3n)

$232 = 53n - 3n^2$

$3n^2 - 53n + 232 = 0$

(3n – 29) (n – 8) = 0

∴ n = 8

$\Longrightarrow a_8 = 25 + (8 - 1) - 3$

∴ n = 8, a_8 = 4

= 25 – 21 = 4

5. (i) How many terms of the sequence 18, 16, 14, ... should be taken so that their

(ii) How many terms are there in the A.P. whose first and fifth terms are −14 and 2 respectively and the sum of the terms is 40?

(iii) How many terms of the A.P. 9, 17, 25, . . . must be taken so that their sum is 636?

(iv) How many terms of the A.P. 63, 60, 57, . . . must be taken so that their sum is 693?

Sol:

(i) Given sequence, 18, 16, 14, …

a = 18, d = 16 – 18 = −2.

Let, sum of n terms in the sequence is zero

$S_n = 0$

$\frac{n}{2}(2a + (n-1)d) = 0$

$\frac{n}{2}(2.18 + (n-1) - 2) = 0$

$n\big(18 - (n-1)\big) = 0$

$n(19 - n) = 0$

n = 0 or n = 19

(ii) $\because n = 0$ is not possible. Therefore, sum of 19 numbers in the sequence is zero.

Given, a = -14, a 5 = 2

a + (5 – 1)d = 2

−14 + 4d = 2

$4d = 16 \implies d = 4$

Sequence is −14, −10, -6, −2, 2, ……

Given $S_n = 40$

40 = $\frac{n}{2}\{2(-14) + (n-1)4\}$

80 = $n(-28 + 4n - 4)$

80 = $n(-32 + 4n)$

4(20) = 4n(−8 + n)

$n^2 - 8n - 20 = 0$

$(n-10)(n+2) = 0$

$n = 10 \ or \ n = -2$

∴ Sum of 10 numbers is 40 (Since -2 is not a natural number)

(iii) Given AP 9, 17, 25, …….

a = 9, d = 17 – 9 = 8, and $S_n = 636$

636 = $\frac{n}{2}(2.9 + (n-1)8)$

1272 = $n(18 - 8 + 8n)$

1272 = n(10 + 8n)

2 × 636 = 2n(5 + 4n)

636 = 5n + $4n^2$

$4n^2$ + 5n – 636 = 0

(4n + 53) (n – 12) = 0

∴ n = 12 (Since n $\frac{-53}{4}$ is not a natural number)

Therefore, value of n is 12.

(iv) Given AP, 63, 60, 57, ……

a =63, d = 60 – 63 = -3 $S_n = 693$

$S_n = \frac{n}{2}(2a + (n-1)d)$

693 = $\frac{n}{2}(2.63 + (n-1) - 3)$

1386 = n(126 – 3n + 3)

1386 = (129 – 3n)n

$3n^2 - 129n + 1386 = 0$

$n^2 - 43n + 462 = 0$

n = 21, 22

∴ Sum of 21 or 22 term is 693

6. The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Sol:

Given, a = 17, l = 350, d = 9

l = $a_n = a + (n-1)d$

350 = 17 + (n – 1)9

333 = (n – 1)9

n – 1 = $\frac{333}{9} = 37$

n = 38

∴ 38 terms are there

$S_n = \frac{n}{2}\{a + l\}$

= $\frac{38}{2}\{17 + 350\}$

= 19.367

∴ $S_n = 6973$

7. The third term of an A.P. is 7 and the seventh term exceeds three times the third term by 2. Find the first term, the common difference and the sum of first 20 terms.

Sol:

Given, $a_3 = 7$ and $3a_3 + 2 = a_7$

$a_7 = 3.7 + 2$

$a_7 = 21 + 2 = 23$

∴ $a_n = a + (n-1)d$

$a_3 = a(3-1)d \ and \ a_7 = a + (7-1)d$

$7 = a + 2d \ldots..(i)$ $23 = a + 6d \ \ldots.(ii)$

$Subtract\ (i) from\ (ii)$

$(ii) - (i) \Longrightarrow$ a + 6d = 23

a + 2d = 7 _

$4d = 16$

$d = 4$

Put d = 4 in (i) $\Longrightarrow 7 = a + 2.4$

$a = 7 - 8 = -1$

Given to find sum of first 20 terms.

$S_{20} = \frac{20}{2}\{-2 + (10-1)4\}$

$= 10(-2 + 76)$

$\therefore S_{20} = 740$

8. The first term of an A.P. is 2 and the last term is 50. The sum of all these terms is 442. Find the common difference.

Sol:

Given a = 2, l = 50, $S_n = 442$

$S_n = \frac{n}{2}(a+1)$

$442 = \frac{n}{2}(2+50)$

$442 = \frac{n}{2}.52$

$\therefore n = \frac{442}{26} = 17$

Given, $a_n = 1 = 50$

$50 = 2 + (17 - 1)\,d$

$48 = 16 \times d$

$d = \frac{48}{16} = 3$

$\therefore d = 3$

9. If 12[th] term of an A.P. is—13 and the sum of the first four terms is 24, what is the sum of first 10 terms?

Sol:

Given, $a_{12} = -13, a + a_2 + a_3 + a_4 = 24$

$S_4 = \frac{4}{2}(2a + 3d) = 24$

$2a + 3d = \frac{24}{2} = 12 \quad \dots.(i)$

$\Longrightarrow a + (12-1)d = -13$

$a + 11d = -13 \dots\dots(ii)$

Subtract (i) from (ii) $\times$ 2

$2 \times$ (ii) – (i) $\Longrightarrow$

$$\begin{aligned} 2a + 22d &= -28 \\ \underline{2a + 3d} &= \underline{\ \ 12} \\ 198d &= -38 \end{aligned}$$

$d = \frac{-38}{19} = -2$

put $d = -2$ in (ii)

a + 11 $(-2) = -13$
a = $-13 + 22$
a = 9
Given to find sum of first 10 terms.
$S_{10} = \frac{10}{2}\{2(a) + (10-1) - 2\}$
= 5(18 – 18)
= 0
$\therefore S_{10} = 0$

10. Find the sum of first 22 terms of an A.P. in which d = 22 and a = 149.
Sol:
Given, d = 22, $a_{22} = 149$
$a + (22-1)\ d$ = 149
$a = -313$
Given, to find $S_{22} = \frac{22}{2}[2a + (22-1)d]$
= $11[2(-313) + 21.22]$
= $11[-626 + 462]$
= 11 – 164
= -1804
$\therefore S_{22} = -1804$

11. Find the sum of all natural numbers between 1 and 100, which are divisible by 3.
Sol:
The numbers between 1 and 100 which are divisible by 3 are 3, 6, 9, ….99.
In this sequence, a = 3, d = 3, $a_n = 99$
99 = $a + (n-1)d$
99 = 3 + (n – 1)3
99 = 3[1 + n – 1]
n = $\frac{99}{3} = 33$
∴ There are 33 numbers in the given sequence
$S_{33} = \frac{33}{2}(2.3 + (33-1)3)\ \left(\therefore S_n = \frac{n}{2}(2a + (n-1)d)\right)$
= $\frac{33}{2}(6 + 96)$
$= \frac{33}{2} \times 102$
= 1683
∴ Sum of all natural numbers between 1 and 100, which are divisible by 3 is 1683.

12. Find the sum of first n odd natural numbers.

Sol:

The sequence is, 1, 3, 5, …..n.

In this first term (a) = 1, common difference (d) = 2

$S_n = \frac{n}{2}(2a + (n-1)d)$

$= \frac{n}{2}(2.1 + (n-1)2)$

$= \frac{n}{2} \times 2(1 + n - 1)$

$= n^2.$

∴ Sum of first n odd natural numbers is n^2.

13. Find the sum of all odd numbers between (i) 0 and 50 (ii) 100 and 200.

Sol:

(i) Odd numbers between 0 and 50 are 1, 3, 5, ….., 49

In this a = 1, d = 2, l = 49 = a_n

49 = 1 + (n – 1)2 (∴ a_n = a + (n – 1)d)

48 = (n – 1)2

n – 1 = $\frac{48}{2} = 24$

n = 25.

∴ There are 25 terms

$S_{25} = \frac{25}{2}(1 + 49)$ $\left(\therefore S_n = \frac{n}{2}(a + 1)\right)$

$= \frac{25}{2} \times 50 = 625$

∴ Sum of all odd numbers between 0 and 50 is 625.

(ii) Odd numbers between 100 and 200 are 101, 103, …. 199

In this a = 101, d = 2, l = a_n = 199

199 = 101 + (n – 1)2

n – 1 = $\frac{98}{2} = 49$

n = 50

∴ There are 50 terms.

$S_{50} = \frac{50}{2}(101 + 199)$ $\left(\therefore S_n = \frac{n}{2}(a + 1)\right)$

= $\frac{50}{2} \times 300$

$= 7500$

∴ Sum of all odd numbers between 100 and 200 is 7500.

14. Show that the sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667.

Sol:

Odd integers between 1 and 1000 which are divisible by 3 are 3, 6, 9, 15 …… 999.

In this a = 3, d = 5, l = $a_n = 999$

999 = 3 + (n – 1)5 $(\therefore a_n = a + (n-1)d)$

999 = 3[1 + (n – 1)2]

$\therefore$ 2n – 1 = $\frac{999}{3} = 333 \Rightarrow n = \frac{334}{2} = 167$

$\therefore$ There are 333 numbers 167.

$S_{167} = \frac{333}{2}[3 + 999]$

= $\frac{333}{2} \times 100$ = 83667

$\therefore S_{167} = 83667$

$\therefore$ Sum of all odd integers between 1 and 4000 which are divisible by 3 is 83667.

15. Find the sum of all integers between 84 and 719, which are multiples of 5.

Sol:

The numbers between 84 and 719, which are multiples of 5 are 85, 90, 95,…..715.

In this, a = 85, d = 5, $a_n = l = 715$

715 = 85 + (n – 1)5 $(\therefore a_n = a + (n-1)d)$

630 = (n – 1)5

$n - 1 = 126$

n = 127

$\therefore S_n = \frac{127}{2}(85 + 115)$ $\left(\therefore S_n = \frac{n}{2}(a + 1)\right)$

= $\frac{127}{2} \times 800 = 50800$

$\therefore$ Sum of all integers between 84 and 719, which are multiples of 5 is 50800.

16. Find the sum of all integers between 50 and 500, which are divisible by 7.

Sol:

Numbers between 50 and 500, which are divisible by 7 are 56, 63, ….., 497.

In this a = 56, d = 7, l = $a_n = 497$

497 = 56 + (n – 1)7

441 = (n – 1)7

n – 1 = $\frac{441}{7}$ = 63

n = 64

$\therefore$ There are 64 terms.

$S_{64} = \frac{64}{2}(56 + 497)$

= $32 \times 553 = 17696$

∴ Sum of all integers between 50 and 500, which are divisible by 7 is 17696.

17. Find the sum of all even integers between 101 and 999.

Sol:

Even integers between 101 and 999 are 102, 104, …..998

a = 102, d = 2, $a_n = l = 998$

998 = 102 + (n – 1)× 2 ($\therefore a_n = a + (n-1)d$)

896 = (n – 1)(2)

n – 1 = 448

n = 449.

∴ 449 terms are there

$S_{449} = \frac{449}{2}[102 + 998]$

$= \frac{449}{2} \times 1100 = 246950$

∴ Sum of all even integers between 101 and 999 is 24690

18. Find the sum of all integers between 100 and 550, which are divisible by 9.

Sol:

Integers between 100 and 550 which are divisible by 9 are 108, 117, …., 549.

In this a = 108, d = 9, $a_n = l = 549$

549 = 108 + (n – 1)× 9 ($\therefore a_n = a + (n+1)d$)

441 = (n – 1) × 9

n – 1 = $\frac{449}{9} = 49$

n = 50.

$\therefore S_{50} = \frac{50}{2}\{108 + 549\}$ $\left(\therefore S_n = \frac{n}{2}(10 + 1)\right)$

= 25 × 657

= 16425

∴ Sum of all integers between 100 and 550, which are divisible by 9 is 16425.

19. In an A.P., if the first term is 22, the common difference is — 4 and the sum to n terms is 64, find n.

Sol:

Given, a = 22, d = -4, $S_n = 64$

$S_n = \frac{n}{2}(2a + (n-1)d)$

64 = $\frac{n}{2} \times (2.22 + (n-1) - 4)$

64 = n(24 – 2n)

64 = 2n (12 – n)

12n – n^2 = $\frac{64}{2} = 32$

$n^2 - 12n + 32 = 0$

$(n-4)(n-8) = 0$

∴ n = 4 or 8

20. In an A.P., if the 5th and 12th terms are 30 and 65 respectively, what is the sum of first 20 terms?

Sol:

Given, $a_5 = 30, a_{12} = 65$

⟹ 30 = a + (5 – 1)d

30 = a + 4d .....(i)

⟹ 65 = a + (12 – 1)d

65 = a + 11d …..(ii)

(ii) – (i) ⟹ a + 11d = 65

a + 4d = 30

0 + 7d = 35

d = $\frac{35}{7} = 5$

put d = 5 in …(i) ⟹ 80 = a + 4 (5)

a = 30 – 20 = 10

$S_{20} = \frac{20}{2}(2(10) + (20-1)5)$ $\left(\therefore S_n = \frac{n}{2}(20 + (n-1)d)\right)$

$= 10[20 + 95]$

$= 10 \times 115$

= 1150

∴ Sum of first 20 terms $S_{20} = 1150$

21. Find the sum of the first

(i) 11 terms of the A.P : 2, 6, 10, 14, ...

(ii) 13 terms of the A.P : — 6, 0, 6, 12,....

(iii) 51 terms of the A.P : whose second term is 2 and fourth term is 8.

Sol:

(i)

Given AP, 2; 6, 10, 14, ……

a = 2, d = 4, $S_n = S_{11} = \frac{11}{2}(2.2 + (11-1).4)$ $\left(\therefore S_n = \frac{n}{2}(2a + (n-1)d)\right)$

$= \frac{11}{2}(4 + 40)$

$= \frac{11}{2} \times 49$

$\therefore S_{11} = 242$

(ii)

Given AP $-6, 0, 6, 12, \dots \dots$

a = -6, d = 6, $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_n = S_{13} = \frac{13}{2}(2 \times -6 + (13-1) \times 6)$

= $\frac{13}{2}(-12 + 72)$

= $\frac{13}{2} \times 60$

= 390

$\therefore S_{13} = 890$

(iii)

Given, $a_2 = 2$ and $a_4 = 8$

a + d = 2 ….(i)   a + 3d = 8 ….(ii)

(ii) – (i) ⟹ a + 3d = 8

a + d = 2

2d = 6

d = 3

put d = 3 in ….(i) ⟹ a + d = 2

a + 3 = 2

a = -1

$S_{51} = \frac{51}{2}(2 \times -1 + (51-1) \times 3)$ $\left(\therefore S_n = \frac{n}{2}(2a + (n-1)d)\right)$

= $\frac{51}{2}(-2 + 50 \times 3)$

= $\frac{51}{2} \times 148$

= 3774.

$\therefore S_n = 3774$

22. Find the sum of

(i) the first 15 multiples of 8

(ii) the first 40 positive integers divisible by (a) 3 (b) 5 (c) 6.

(iii) all 3 — digit natural numbers which are divisible by 13.

(iv) all 3-digit natural numbers, which are multiples of 11.

Sol:

The first 15 multiples of 8 are 8, 16, 24, ……

a = 8, d = 8, n = 15

$S_{15} = \frac{15}{2}(28 + (15-1) \times 8)$ $\left(\therefore S_n = \frac{n}{2}(2a + (n-1)d)\right)$

= $\frac{15}{2}(16 + 112)$

$= \frac{15}{2} \times 128$

$= 960$

$\therefore Sum\ of\ first\ 15\ multiples\ of\ 8\ is\ 960.$

Given, $a_2 = 2 \; and \; a_4 = 8$

a + d = 2 ….(i) a + 3d = 8 …..(ii)

(ii) – (i) ⟹ a + 3d = 8

a + d = 2

2d = 6

d = 3

Put d = 3 in ….(i) ⟹ a + d = 2

a + 3 = 2

a = −1

$S_{51} = \frac{51}{2}\left(2 \times -1 + (51 - 1 \times 3)\right) \quad \left(\therefore S_n = \frac{n}{2}(20 + (n-1)d)\right)$

$= \frac{51}{2}(-2 + 50 \times 3)$

$= \frac{51}{2} \times 148$

$= 3774$

$= 44550$

∴ Sum of all 3 – digit natural numbers which are multiples of 11 is 44550.

23. Find the sum:

(i) 2 + 4 + 6 + …… + 200

(ii) 3 + 11 + 19 + …..+ 803

(iii) 34 + 32 + 30 + ….. + 10

(iv) 25 + 28 + 31 + ….. + 100

Sol:

(i) 2 + 4 + 6 + …… + 200

a = 2, d = 4 – 2 = 2, l = 200 = a_n

$\therefore S_n = \frac{n}{2}(a + l) \; and \; a_n = a + (n-1)d$

200 = 2 + (n – 1)2

198 = (n – 1)2

n – 1 = $\frac{198}{2} = 99$

n = 100

$S_n = \frac{100}{2}(2 + 200)$

= 50×202

= 10100

(ii) 3 + 11 + 19 + …..+ 803

a = 3, d = 11 – 3 = 8, l = a_n = 803

803 = 3+ (n – 1)8

$\frac{800}{8} = n - 1$

$n = 101$

$S_n = \frac{101}{2}(3 + 803)$
$= \frac{101}{2} \times 806$
= 504
$S_n = 504$

(iii) 34 + 32 + 30 + ….. + 10
a = 34, d = −2, l = a_n = 10
10 = 34 + (n – 1) × 2
+24 = 2 (n – 1)
n – 1 = 12
n = 13
$\therefore S_{13} = \frac{13}{2}(34 + 10)$
$= \frac{13}{2} \times 44$
$= 286$

(iv) 25 + 28 + 31 + ….. + 100
a = 25, d = 8, l = a_n = 100
100 = 25 + (n – 1) × 3
75 = (n – 1) × 3
n – 1 = 25
n = 26

24. Find the sum of the first 15 terms of each of the following sequences having n^{th} term as
(i) $a_n = 3 + 4n$
(ii) $b_n = 5 + 2n$
(iii) $Y_n = 9 - 5n$

Sol:

(i) Given $a_n = 3 + 4n$
Put n = 1, $a_1 = 3 + 4(1) = 7$
Put n = 15, $a_{15} = 3 + 4(15) = 63 = 8$
Sum of 15 terms, $S_{15} = \frac{15}{2}(7 + 63)$ $\left(\therefore S_n = \frac{n}{2}(a + l)\right)$
$= \frac{15}{2} \times 70$
$\therefore S_{15} = 525$

(ii) Given $b_n = 5 + 2n$
Put n = 1, $b_1 = 5 + 2(1) = 7$
Put n = 15, $b_{15} = 5 + 2(15) = 35 = l$
Sum of 15 terms , $S_{15} = \frac{15}{2}(7 + 35)$ $\left(\therefore S_n = \frac{n}{2}\ (a + l)\right)$
$= \frac{15}{2} \times 42$

$= 315$

$\therefore S_{15} = 315$

(iii) Given, $Y_n = 9 - 5n$

Put n = 1, $y_1 = 9 - 5.1 = -4$

Put n = 15, $y_{15} 9 - 5.15 = 9 - 75 = -66 = (l)$

$\therefore S_{15} = \frac{15}{2}(-4 - 66) \quad \left(\therefore S_n = \frac{n}{2}(a + l)\right)$

$= \frac{15}{2} \times -70$

$= -465$

$\therefore S_{15} = -465$

25. Find the sum of first 20 terms of the sequence whose n^th term is a = An + B.

Sol:

Given, n^th term $a_n = A_n + B$

Put n = 1, $a_1 = A + B$

Put n = 20, $a_{20} = 20\,A + B = (l)$

$\therefore S_{20} = \frac{20}{2}(A + B + 20A + B) \quad \left(\therefore S_n = \frac{n}{2}(a + l)\right)$

= 10 (21A + 2B)

= 210A + 20B

$\therefore S_n = 210A + 2B$

26. Find the sum of the first 25 terms of an A.P. whose n^th term is given by $a_n = 2 - 3n$.

Sol:

Given, n^th term $a_n = 2 - 3n$

Put n = 1, $a_1 = 2 - 3.1 = -1$

Put n = 25, $a_{15} = l = 2 - 3.15 = -43$

$\therefore 25 = \frac{25}{2}\,(-1 - 43) = \frac{25}{2}(-44) = -925$

$\therefore S_{25} = -925$

27. Find the sum of the first 25 terms of an A.P. whose n^th term is given by $a_n = 7 - 3n$.

Sol:

Given, $a_n = 7 - 3n$

Put n = 1, $a_1 = 7 - 3.1 = 4$

Put n = 25, $a_{25} = l = 7 - 3.25 = -68$

$\therefore S_{25} = \frac{25}{2}(4 - 68) \quad \left(\therefore S_n = \frac{n}{2}(a + l)\right)$

$= \frac{25}{2} \times -64$

$= -800$

$\therefore S_{25} = -800$

28. Find the sum of first 51 terms of an A.P. whose second and third terms are 14 and 18 respectively.

Sol:

Given, $a_2 = 14 \Longrightarrow a + d = 14 \ldots\ldots(i)$

$a_3 = 18 \Longrightarrow a + 2d = 18 \ldots\ldots(ii)$

(ii) – (i) ⟹ a + 2d = 18

a + d = 14

0 + d = 4

Put d = 4 is (i) a + 4 = 4

a = 10

$\therefore S_{50} = \frac{51}{2}\{2.10 + (51-1) \times 4\} \quad \left(S_n = \frac{n}{2}\{2a + (n-1)d\}\right)$

$= \frac{51}{2}\{20 + 200\}$

$= \frac{51}{2} \times 220$

$= 5610$

$\therefore S_{51} = 5610$

29. If the sum of 7 terms of an A.P. is 49 and that of 17 terms is 289, find the sum of n terms.

Sol:

Given, $S_7 = 49$

$\frac{7}{2}(2a + (7-1)d) = 49 \quad \left(\therefore S_n = \frac{n}{2}\{2a + (n-1)d\}\right)$

$\frac{7}{2}(2a + 6d) = 49$

$\frac{7}{2} \times 2(a + 3d) = 49$

$a + 3d = \frac{49}{7} = 7 \ldots.(i)$ and

$S_{17} = 289$

$\frac{17}{2}(2a + (17-1)d) = 289$

$\frac{17}{2} \times 2(a + 8d) = 289$

$a + 8d = \frac{289}{17} = 17$ …..(ii)

$Subtract\ (i) from\ (ii)$

a + 8d = 17

a + 3d = 7

5d = 10

d = 2

put d = 2, in (i) ⟹ a + 3 × 2 = 7

a = 1

$\therefore S_n = \frac{n}{2}\{2.1 + (n-1).2\} \quad \left(\therefore S_n = \frac{n}{2}(2a + (n-1)d)\right)$

$= n\{1 + n - 1\}$
$\therefore S_n = n^2$.

30. The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Sol:

Given, a = 5, l = 45, Sum of terms = 400

$\therefore S_n = 400$

$\frac{n}{2}\{5 + 45\} = 400$

$\frac{n}{2} = 50 = 400$

$n = 40 \times \frac{2}{5}$

$\therefore n = 16$

16th term is 45

$a_{16} = 45 \Longrightarrow 5 + (16 - 1) \times d = 45 = 15 \times d = 40$

d = $\frac{408}{15} = \frac{8}{3}$

$\therefore$ n = 16, d = $\frac{8}{3}$

31. In an A.P., the sum of first n terms is $\frac{3n^2}{2} + \frac{13}{2}$n. Find its 25th term.

Sol:

Given, sum of n terms $S_n = \frac{3n^2}{2} + \frac{13}{2}n$

Let, $a_n = S_n - S_{n-1}$ ($\therefore$ Replace n by (n – 1) is S_n to get $S_{n-1} = \frac{3(n-1)^2}{2} + \frac{13}{2}\ (n-1)$)

$a_n = \frac{3n^2}{2} + \frac{13}{2}n - \frac{3(n-1)^2}{2} - \frac{13}{2}(n-1)$

$= \frac{3}{2}\{n^2 - (n-1)^2\} + \frac{13}{2}\{n - (n-1)\}$

$= \frac{3}{2}\{n^2 - n^2 + 2n - 1\} + \frac{13}{2}\{1\}$

$= 3n + \frac{10}{2} = 3n + 5$

Put n = 25, $a_{25} = 3(25) + 5 = 75 + 5 = 80$

$\therefore$ 25th term $a_{25} = 80$

32. Let there be an A.P. with first term 'a', common difference 'd'. If a denotes its nth term and S_n the sum of first n terms, find.

(i) n and S_n and if a = 5, d = 3 and a = 50

(ii) n and a, if a_n = 4, d = 2 and $S_n = -14$

(iii) d, if a = 3, n = 8 and S_n = 192

(iv) a, if a_n = 28, S_n = 144 and n = 9

(v) n and d, if a = 8, a = 62 and S_n = 210

(vi) n and a_n, if a = 2, d = 8 and S_n = 90

Sol:

(i) Given a = 5, d = 3, $a_n = 50$

$a_n = 50$

$a + (n-1)d = 50$

5 + (n – 1)3 = 50

(n – 1)3 = 45

n – 1 = $\frac{45}{3} = 15$

n = 16

Some of n terms $S_n = \frac{n}{2}[a + l]$

= $\frac{16}{2}[5 + 50]$

$= 8 \times 55$

= 440

(ii) Given, $a_n = 4, d = 2, S_n = -14$

a + (n – 1).2 = 4 and $\frac{n}{2}[2a + (n-1).2] = -14$

a + 2n = 6 $\quad n[2a + 2n - 2] = -14$

(or)

$\frac{n}{2}[a + a_n] = -14$

$\frac{n}{2}[a + 4] = -14$

$n[6 - 2n + 4] = -28$

$n[10 - 2n] = -28$

$2n^2 - 10n - 28 = 0$

$2(n^2 - 5n - 14) = 0$

(n + 2) (n – 7) = 0

N = −2, n = 7

∵ n = −2 is not a natural number. So, n = 7.

(iii) Given, a = 3, n = 8, S_n = 192.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$192 \times 2 = 8[6 + (8-1)d]$

$\frac{192\times2}{8} = 6 + 7d$

$48 = 6 + 7d$

7d = 42

d = 6

(iv) Given, $a_n = 28, S_n = 144, n = 9$

$S_n = \frac{n}{2}[a + l]$

144 = $\frac{9}{2}[a + 28]$

144 = $\frac{2}{9} = a + 28$

a + 28 = 32

a = 4

(v) Given, a = 8, 62 and S_n = 210

$S_n = \frac{n}{2}[a + l]$

210 = $\frac{n}{2}[8 + 62]$

210 × 2 = n[70]

n = $\frac{210\times2}{70} = 6$

a + (n – 1) d = 62

8 + (6 – 1) d = 62

5d = 54

d = 10.8

(vi) Given

a = 2, d = 8 and S_n = 90

90 = $\frac{n}{2}[4 + (n-1)8]$ $\left(\therefore S_n = \frac{n}{2}[2a + (n-1)d]\right)$

180 = n [4 + 8n – 8]

$8n^2 - 4n - 180 = 0$

$4(2n^2 - n - 45) = 0$

$2n^2 - n - 45 = 0$

(2n + 1) (n – 5) = 0

∵ n = $-\frac{1}{2}$ is not a natural no. n = 5

$a_n = 2 + 4(8)$ $(\therefore a_n = a + (n-1)d)$

$a_n = 32$

33. A man saved Rs 16500 in ten years. In each year after the first he saved Rs 100 more than he did in the preceding year. How much did he save in the first year?

Sol:

Let ‘a’ be the money he saved in first year

⟹ First year he saved the money = Rs a

He saved Rs 100 more than, he did in preceding year.

⟹ Second year he saved the money = Rs (a + 100)

⟹ Third year he saved the money = Rs. (a + 2 (100))

So, the sequence is a, a + 100, a + 2(100), ……, This is in AP with common difference (d) = 100.

⟹ Sum of money he saved in 10 years $S_{10} = 16{,}500\ rupees$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{10} = \frac{10}{2}(2a + (10-1).100)$

16,500 = 5 (2a + 9 × 100)

$2a + 900 = \frac{16500}{5} = 3300$

$2a = 2400$

$a = \frac{2400}{2} = 1200$

∴ He saved the money in first year (a) = Rs. 1200

34. A man saved Rs 32 during the first year, Rs 36 in the second year and in this way he increases his savings by Rs 4 every year. Find in what time his saving will be Rs 200.

Sol:

Given

Saving in 1st yr (a_1) = Rs 32

Saving in 2nd yr (a_2) = Rs 36

Increase in salary every year (d) = Rs 4

Let in n years his saving will be Rs 200

$\Rightarrow S_n = 200$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 200$

$\Rightarrow \frac{n}{2}[64 + 4n - 4] = 200$

$\Rightarrow \frac{n}{2}[4n + 60] = 200$

$\Rightarrow 2n^2 + 30n = 200$

$\Rightarrow n^2 + 15 - 100 = 0$ [Divide by 2]

$\Rightarrow n^2 + 20n - 5n - 100 = 0$

$\Rightarrow n(n + 20) - 5(n + 20) = 0$

$\Rightarrow (n + 20)(n - 5) = 0$

If n + 20 = 0 or n – 5 = 0

n = −20 or n = 5 (Rejected as n cannot be negative)

∴ In 5 years his saving will be Rs 200

35. A man arranges to pay off a debt of Rs 3600 by 40 annual installments which form an arithmetic series. When 30 of the installments are paid, he dies leaving one-third of the debt unpaid, find the value of the first installment.

Sol:

Given

A man arranges to pay off a debt of Rs 3600 by 40 annual installments which form an A.P

i.e., sum of all 40 installments = Rs 3600

$S_{40} = 3600$

Let, the money he paid in first installment is a, and every year he paid with common difference = d

Then,

S40 = 3600 $(\therefore S_n = \frac{n}{2}[2a + (n-1)d])$

$\frac{40}{x}[2a+(40-1)d]=3600$

$2a+39d=\frac{3600}{20}=180\ \ldots\ldots(i)$

$but,$

He died by leaving one third of the debt unpaid that means he paid remaining money in 30 installments.

∴ The money he paid in 30 installments = 3600 $-\frac{3600}{3}=3600-1200$

$\therefore S_{30}=2400$

$S_{30}=2400$

$=\frac{30}{2}[2a+(30-1)d]=2400 \quad \left(\therefore S_n=\frac{n}{2}(2a+(n-1)d)\right)$

2a + 2ad = $\frac{2400}{15}=160\ \ldots.(ii)$

(i) – (ii) ⟹ 2a + 39d = 180

2a + 29d = 160

0 + 10 d = 20

d = $\frac{20}{10}=2$

put d = 2 in (ii) 2a + 29 (2) = 160

2a = 102

a = $\frac{102}{2}=51$

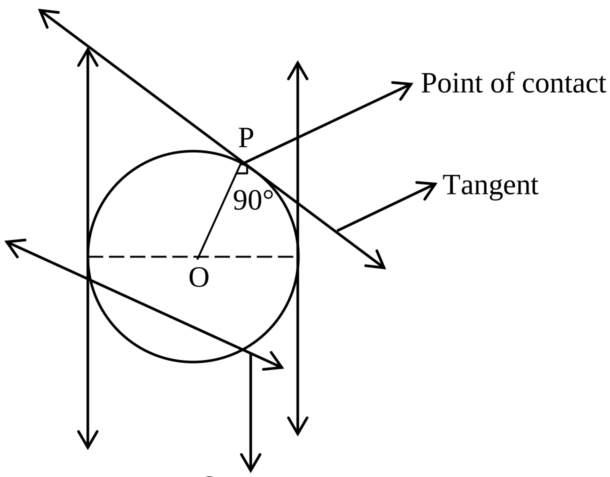
∴ The value of his first installment = 51.

Exercise – 9.1

Exercise – 10.1

1. Fill in the blanks
   (i) The common point of tangent and the circle is called point of contact.
   (ii) A circle may have two parallel tangents.
   (iii) A tangent to a circle intersects it in one point.
   (iv) A line intersecting a circle in two points A called a secant.
   (v) The angle between tangent at a point P on circle and radius through the point is 90°.

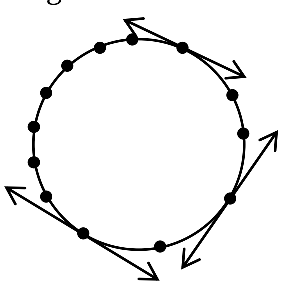
   Sol:



2. How many tangents can a circle have?

   Sol:

   Tangent: A line intersecting circle in one point is called a tangent.

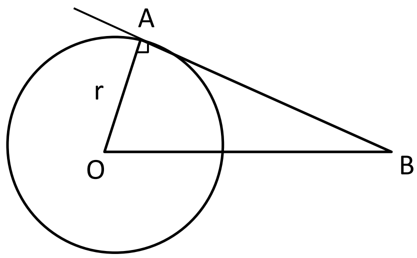


   As there are infinite number of points on the circle a circle has many (infinite) tangents.

3. O is the center of a circle of radius 8cm. The tangent at a point A on the circle cuts a line through O at B such that AB = 15 cm. Find OB

   Sol:

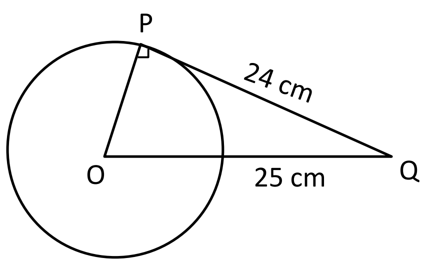
   Consider a circle with center O and radius OA = 8cm = r, AB = 15 cm.



(AB) tangent is drawn at A (point of contact)
At point of contact, we know that radius and tangent are perpendicular.
In ΔOAB, ∠OAB = 90°, By Pythagoras theorem
$OB^2 = OA^2 + AB^2$
$OB = \sqrt{8^2 + 15^2}$
$= \sqrt{64 + 225} = \sqrt{229} = 17\ cm$
$\therefore OB = 17\ cm$

4. If the tangent at point P to the circle with center O cuts a line through O at Q such that PQ = 24cm and OQ = 25 cm. Find the radius of circle

Sol:
Given,
PQ = 24 cm
OQ = 25 cm
OP = radius = ?



P is point of contact, At point of contact, tangent and radius are perpendicular to each other
∴ ΔPOQ is right angled triangle ∠OPQ = 90°
By Pythagoras theorem,
$PQ^2 + OP^2 = OQ^2$
$\Rightarrow\ 24^2 + OP^2 = 25^2$
$\Rightarrow OP = \sqrt{25^2 - 24^2} = \sqrt{625 - 576}$
$= \sqrt{49} = 7cm$
$\therefore OP = radius = 7cm$

Exercise – 10.2

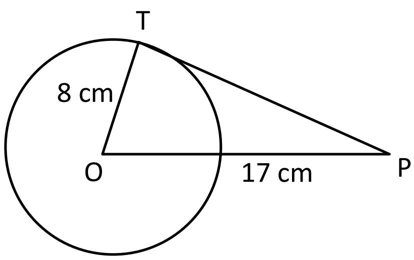
1. If PT is a tangent at T to a circle whose center is O and OP = 17 cm, OT = 8 cm. Find the length of tangent segment PT.

Sol:

OT = radius = 8cm

OP = 17cm

PT = length of tangent = ?



T is point of contact. We know that at point of contact tangent and radius are perpendicular.

∴ OTP is right angled triangle ∠OTP = 90°, from Pythagoras theorem $OT^2 + PT^2 = OP^2$

$8^2 + PT^2 = 17^2$

PT $\sqrt{17^2 - 8^2} = \sqrt{289 - 64}$

= $\sqrt{225} = 15cm$

∴ PT = length of tangent = 15 cm.

2. Find the length of a tangent drawn to a circle with radius 5cm, from a point 13 cm from the center of the circle.

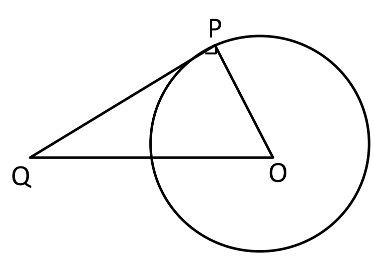
Sol:

Consider a circle with center O.

OP = radius = 5 cm.

A tangent is drawn at point P, such that line through O intersects it at Q, OB = 13cm.

Length of tangent PQ = ?



A + P, we know that tangent and radius are perpendicular.

ΔOPQ is right angled triangle, ∠OPQ = 90°

$By\ pythagoras\ theorem, OQ^2 = OP^2 + PQ^2$

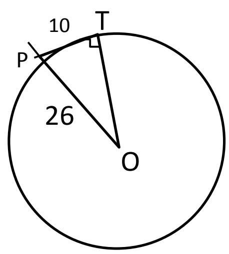
$\Rightarrow 13^2 = 5^2 + PQ^2$

$\Rightarrow PQ^2 = 169 - 25 = 144$

⇒ PQ = $\sqrt{144} = 12cm$
Length of tangent = 12 cm

3. A point P is 26 cm away from O of circle and the length PT of the tangent drawn from P to the circle is 10 cm. Find the radius of the circle.

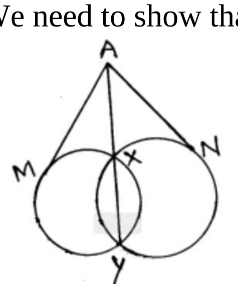
Sol:
Given OP = 26 cm
PT = length of tangent = 10cm
radius = OT = ?



At point of contact, radius and tangent are perpendicular ∠OTP = 90°, ΔOTP is right angled triangle.
By Pythagoras theorem, $OP^2 = OT^2 + PT^2$
$26^2 = OT^2 + 10^2$
$OT^k = \left(\sqrt{676 - 100}\right)^k$
$OT = \sqrt{576}$
= 24 cm
OT = length of tangent = 24 cm

4. If from any point on the common chord of two intersecting circles, tangents be drawn to circles, prove that they are equal.

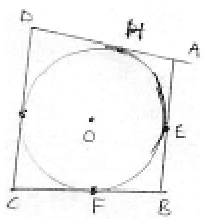
Sol:
Let the two circles intersect at points X and Y.
XY is the common chord.
Suppose 'A' is a point on the common chord and AM and AN be the tangents drawn A to the circle
We need to show that AM = AN.



In order to prove the above relation, following property will be used.

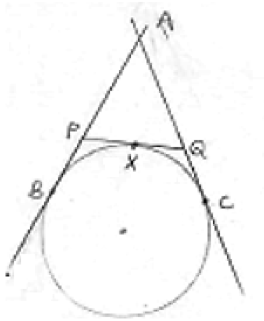
"Let PT be a tangent to the circle from an external point P and a secant to the circle through P intersects the circle at points A and B, then $PT^2 = PA \times PB$"
Now AM is the tangent and AXY is a secant $\therefore AM^2 = AX \times AY \ldots\ldots (i)$
AN is a tangent and AXY is a secant $\therefore AN^2 = AX \times AY \ldots\ldots (ii)$
From (i) & (ii), we have $AM^2 = AN^2$
∴ AM = AN

5. If the quadrilateral sides touch the circle prove that sum of pair of opposite sides is equal to the sum of other pair.
Sol:
Consider a quadrilateral ABCD touching circle with center O at points E, F, G and H as in figure.



We know that
The tangents drawn from same external points to the circle are equal in length.
1. Consider tangents from point A [AM ⊥ AE]
   AH = AE …. (i)
2. From point B [EB & BF]
   BF = EB …. (ii)
3. From point C [CF & GC]
   FC = CG …. (iii)
4. From point D [DG & DH]
   DH = DG …. (iv)
   Adding (i), (ii), (iii), & (iv)
   (AH + BF + FC + DH) = [(AC + CB) + (CG + DG)]
   ⇒ (AH + DH) + (BF + FC) = (AE + EB) + (CG + DG)
   ⇒ AD + BC = AB + DC [from fig.]
   Sum of one pair of opposite sides is equal to other.

6. If AB, AC, PQ are tangents in Fig. and AB = 5cm find the perimeter of ΔAPQ.
Sol:
Perimeter of ΔAPQ, (P) = AP + AQ + PQ
= AP + AQ + (PX + QX)



We know that

The two tangents drawn from external point to the circle are equal in length from point A,

AB = AC = 5 cm

From point P, PX = PB

From point Q, QX = QC

Perimeter (P) = AP + AQ + (PB + QC)

= (AP + PB) + (AQ + QC)

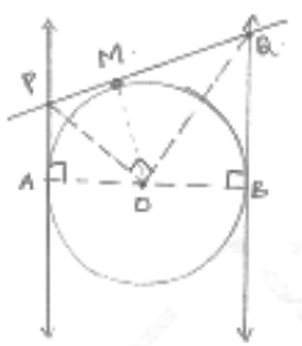
= AB + AC = 5 + 5

= 10 cms.

7. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at center.

Sol:

Consider circle with center 'O' and has two parallel tangents through A & B at ends of diameter.



Let tangents through M intersects the tangents parallel at P and Q required to prove is that ∠POQ = 90°.

From fig. it is clear that ABQP is a quadrilateral

∠A + ∠B = 90° + 90° = 180° [At point of contact tangent & radius are perpendicular]

∠A + ∠B + ∠P + ∠Q = 360° [Angle sum property]

∠P + ∠Q = 360°−180° = 180° …..(i)

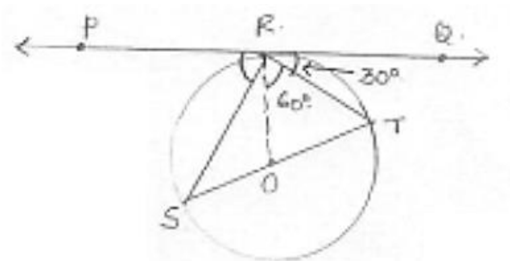
At P & Q ∠APO = ∠OPQ = $\frac{1}{2}\angle P$

∠BQO = ∠PQO = $\frac{1}{2}\angle Q$ in (i)

2∠OPQ + 2 ∠PQO = 180°

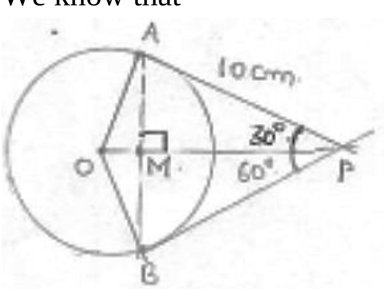
∠OPQ + ∠PQO = 90° …. (ii)
In ΔOPQ, ∠OPQ + ∠PQO + ∠POQ = 180° [Angle sum property]
90° + ∠POQ = 180° [from (ii)]
∠POQ = 180° − 90° = 90°
∴ ∠POQ = 90°

8. In Fig below, PQ is tangent at point R of the circle with center O. If ∠TRQ = 30°. Find ∠PRS.



Sol:
Given ∠TRQ = 30°.
At point R, OR ⊥ RQ.
∠ORQ = 90°
⇒ ∠TRQ + ∠ORT = 90°
⇒ ∠ORT = 90° − 30° = 60°
ST is diameter, ∠SRT = 90° [∵ Angle in semicircle = 90°]
∠ORT + ∠ SRO = 90°
∠SRO + ∠PRS = 90°
∠PRS = 90° − 30° = 60°

9. If PA and PB are tangents from an outside point P. such that PA = 10 cm and ∠APB = 60°. Find the length of chord AB.
Sol:
AP = 10 cm ∠APB = 60°
Represented in the figure
We know that



A line drawn from center to point from where external tangents are drawn divides or bisects the angle made by tangents at that point ∠APO = ∠OPB = $\frac{1}{2} \times 60° = 30°$

The chord AB will be bisected perpendicularly

∴ AB = 2AM

In ΔAMP,

$$\sin 30° = \frac{opp.side}{hypotenuse} = \frac{AM}{AP}$$

AM = AP sin 30°

$= \frac{AP}{2} = \frac{10}{2} = 5cm$

AP = 2 AM = 10 cm ---- Method (i)

In ΔAMP, ∠AMP = 90°, ∠APM = 30°

∠AMP + ∠APM + ∠MAP = 180°

$90° + 30° + \angle MAP = 180°$

$\angle MAP = 180°$

In ΔPAB, ∠MAP = ∠BAP = 60°, ∠APB = 60°

We also get, ∠PBA = 60°

∴ΔPAB is equilateral triangle

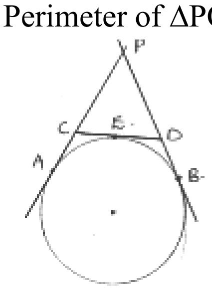
AB = AP = 10 cm. -----Method (ii)

10. From an external point P, tangents PA and PB are drawn to the circle with centre O. If CD is the tangent to the circle at point E and PA = 14 cm. Find the perimeter of ABCD.

Sol:

PA = 14 cm

Perimeter of ΔPCD = PC + PD + CD = PC + PD + CE + ED



We know that

The two tangents drawn from external point to the circle are equal in length.

From point P, PA = PB = 14cm

From point C, CE = CA

From point D, DB = ED

Perimeter = PC + PD + CA +DB

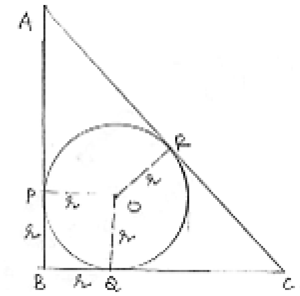
= (PC + CA) + (PD + DB)

= PA + PB = 14 + 14 = 28 cm.

11. In the fig. ABC is right triangle right angled at B such that BC = 6cm and AB = 8cm. Find the radius of its in circle.

Sol:

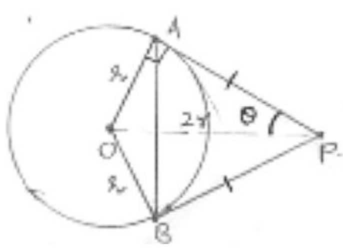
BC = 6cm AB = 8cm
As ABC is right angled triangle



By Pythagoras theorem
$AC^2 = AB^2 + BC^2 = 6^2 + 8^2 = 100$
$AC = 10\ cm$
Consider BQOP ∠B = 90°,
∠BPO = ∠OQB = 90° [At point of contact, radius is perpendicular to tangent]
All the angles = 90° & adjacent sides are equal
∴ BQOP is square BP = BQ = r
We know that
The tangents drawn from any external point are equal in length.
AP = AR = AB – PB = 8 – r
QC = RC = BC – BQ = 6 – r
AC = AR + RC ⇒ 10 = 8 – r + 6 – r
⇒ 10 = 14 – 2r
⇒ 2r = 4
⇒ Radius = 2cm

12. From a point P, two tangents PA and PB are drawn to a circle with center O. If OP = diameter of the circle shows that ΔAPB is equilateral.

Sol:
OP = 2r
Tangents drawn from external point to the circle are equal in length
PA = PB

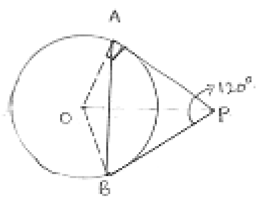


At point of contact, tangent is perpendicular to radius.
In ΔAOP, sin $\theta = \frac{opp.side}{hypotenuse} = \frac{r}{2r} = \frac{1}{2}$
θ = 30°

∠APB = 20 = 60°, as PA = PB ∠BAP = ∠ABP = x.
In ΔPAB, by angle sum property
∠APB + ∠BAP + ∠ABP = 180°
2x = 120° ⇒ x = 60°
In this triangle all angles are equal to 60°
∴ ΔAPB is equilateral.

13. Two tangent segments PA and PB are drawn to a circle with center O such that ∠APB = 120°. Prove that OP = 2AP

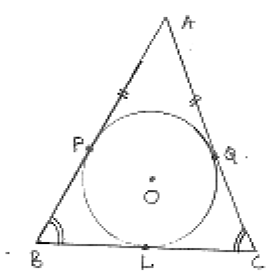
Sol:
A + P
OP bisects ∠APB
∠APO = ∠OPB = $\frac{1}{2}\angle APB = \frac{1}{2} \times 120° = 60°$



At point A
OA ⊥ AP, ∠OAP = 90°
In ΔPDA, cos 60° = $\frac{AP}{DP}$
$\frac{1}{2} = \frac{AP}{DP} \Rightarrow DP = 2AP$

14. If ΔABC is isosceles with AB = AC and C (0, 2) is the in circle of the ΔABC touching BC at L, prove that L, bisects BC.

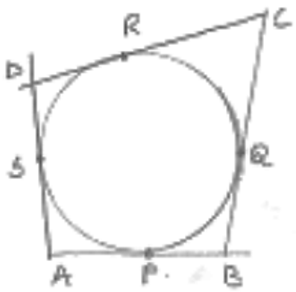
Sol:
Given ΔABC is isosceles AB = AC



We know that
The tangents from external point to circle are equal in length
From point A, AP = AQ
But AB = AC ⇒ AP + PB = AQ + QC
⇒ PB = PC …. (i)

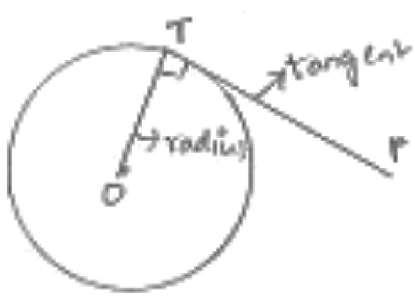
From B, PB = BL; ….(ii) from C, CL = CQ …..(iii)
From (i), (ii) & (iii)
BL = CL
∴ L bisects BC.

15. In fig. a circle touches all the four sides of quadrilateral ABCD with AB = 6cm, BC = 7cm, CD = 4cm. Find AD.
Sol:
We know that the tangents drawn from any external point to circle are equal in length.



From A ⟶ AS = AP ….(i)
From B ⟶ QB = BP …. (ii)
From C ⟶ QC = RC …..(iii)
From D ⟶ DS = DR …. (iv)
Adding (i), (ii), (iii) & (iv)
(AS + QB + QC + DS) = (AB + BP + RC + OR)
(AS + DS) + (QB + QC) = (AP + BP) + (RC + DR)
AD + BC = AB + CD
⇒ AD + 7 = 6 + 4 AD = 3cm
⇒ AD = 10 – 7 = 3cm

16. Prove that the perpendicular at the point of contact to a circle passes through the centre of the circle.
Sol:
We know that



The at point of contact, the tangent is perpendicular to the radius. Radius is line from center to point on circle. Therefore, perpendicular to tangent will pass through center of circle.

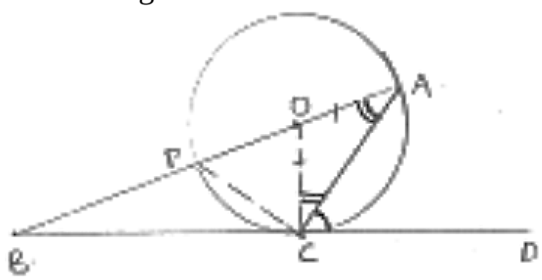
17. In fig.. O is the center of the circle and BCD is tangent to it at C. Prove that ∠BAC + ∠ACD = 90°

Sol:

Given

O is center of circle

BCD is tangent.



Required to prove: ∠BAC + ∠ACD = 90°

Proof: OA = OC [radius]

In ΔOAC, angles opposite to equal sides are equal.

∠OAC = ∠OCA …. (i)

∠OCD = 90° [tangent is radius are perpendicular at point of contact]

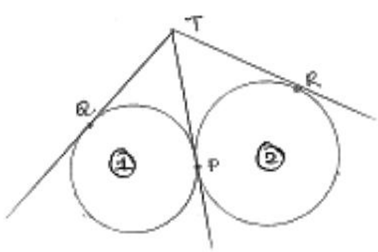
∠ACD + ∠OCA = 90°

∠ACD + ∠OAC = 90° [∵ ∠OAC = ∠BAC]

∠ACD + ∠BAC = 90° ⟶ Hence proved

18. Two circles touch externally at a point P. from a point T on the tangent at P, tangents TQ and TR are drawn to the circles with points of contact Q and E respectively. Prove that TQ = TR.

Sol:



Let the circles be represented by (i) & (ii) respectively

TQ, TP are tangents to (i)

TP, TR are tangents to (ii)

We know that

The tangents drawn from external point to the circle will be equal in length.

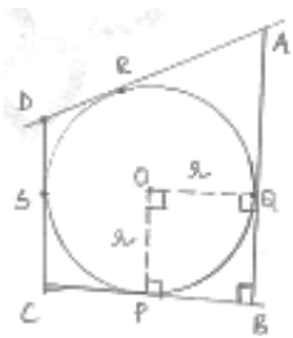
For circle (i), TQ = TP …. (i)

For circle (ii), TP = TR …. (ii)
From (i) & (ii) TQ = TR

19. In the fig. a circle is inscribed in a quadrilateral ABCD in which ∠B = 90° if AD = 23cm, AB = 29cm and DS = 5cm, find the radius of the circle.

Sol:

Given AD = 23 cm
AB = 29 cm
∠B = 90°
DS = 5cm

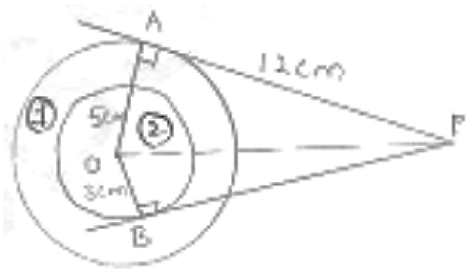


From fig in quadrilateral POQB
∠OPB = ∠OQB = 90° = ∠B = ∠POQ
and PO = OQ. ∴ POQB is a square PB = BQ = r
We know that
Tangents drawn from external point to circle are equal in length.
We know that
Tangents drawn from external point to circle are equal in length.
From A, AR = AQ …. (i)
From B, PB = QB …. (ii)
From C, PC = CS …. (iii)
From D, DR = DS …. (iv)
(i) + (ii) + (iv) ⇒ AR + DB + DR = AQ + QB + DS
⇒ (AR + DR) + r = (AQ + QB) + DS
AD + r = AB + DS
⇒ 23 + r = 29 + 5
⇒ r = 34 – 23 = 11 cm
∴ radius = 11 cm

20. In fig. there are two concentric circles with Centre O of radii 5cm and 3cm. From an external point P, tangents PA and PB are drawn to these circles if AP = 12cm, find the tangent length of BP.

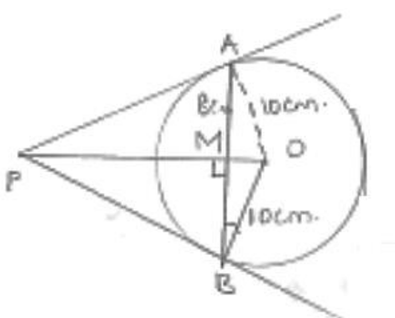
Sol:

Given



OA = 5 cm
OB = 3 cm
AP = 12 cm
BP = ?
We know that
At the point of contact, radius is perpendicular to tangent.
For circle 1, ΔOAP is right triangle
By Pythagoras theorem, $OP^2 = OA^2 + AP^2$
$\Rightarrow OP^2 = 5^2 + 12^2 = 25 + 144$
= 169
$\Rightarrow$ OP = $\sqrt{169} = 13\ cm$
For circle 2, ΔOBP is right triangle by Pythagoras theorem,
$OP^2 = OB^2 + BP^2$
$13^2 = 3^2 + BP^2$
$BP^2 = 169 - 9 = 160$
$BP = \sqrt{160} = 4\sqrt{10}\ cm$

21. In fig. AB is chord of length 16cm of a circle of radius 10cm. The tangents at A and B intersect at a point P. Find the length of PA.
Sol:
Given length of chord AB = 16cm.
Radius OB = OA = 10 cm.



Let line through Centre to point from where tangents are drawn be intersecting chord AB at M. we know that the line joining Centre to point from where tangents are drawn be intersecting chord AB at M. we know that

The line joining Centre to point from where tangents are drawn bisects the chord joining the points on the circle where tangents intersects the circle.

AM = MB = $\frac{1}{2}(AB) = \frac{1}{2} \times 16 = 8cm$

Consider ΔOAM from fig. ∠AMO = 90°

By Pythagoras theorem, $OA^2 = AM^2 + OM^2$

$10^2 = 8^2 + OM^2$

$OM = \sqrt{100 - 64} = \sqrt{36} = 6cm$

In ΔAMP, ∠AMP = 90° by Pythagoras theorem $AP^2 = AM^2 + PM^2$

$AP^2 = 8^2 + (OP - OM)^2$

$PA^2 = 64 + (OP - 6)^2$

$(OP - 6)^2 = -64 + PA^2$ ….(i)

$In\ \Delta APO, \angle PAO = 90°$ [At point of contact, radius is perpendicular to tangent]

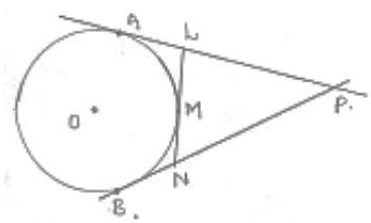
$OP^2 = OA^2 + PA^2$ [Pythagoras theorem]

$PA^2 = OP^2 - 10^2$

$= OP^2 - 100$ ….. (ii)

22. In figure PA and PB are tangents from an external point P to the circle with centre O. LN touches the circle at M. Prove that PL + LM = PN + MN

Sol:



Given

O is Centre of circle

PA and PB are tangents

We know that

The tangents drawn from external point to the circle are equal in length.

From point P, PA = PB

⇒ PL + AL = PN + NB …. (i)

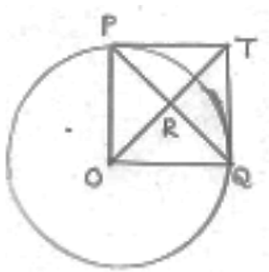
From point L & N, AL = LM and MN = NB } …. Substitute in (i)

PL + Lm = PN + MN

⇒ Hence proved.

23. In the fig. PO ⊥ QO. The tangents to the circle at P and Q intersect at a point T. Prove that PQ and OT are right bisectors of each other.

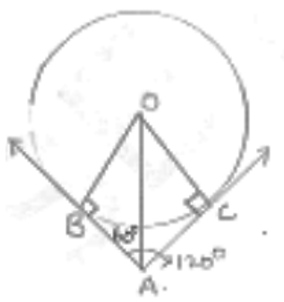
Sol:



Given
PO ⊥ OQ
Consider quadrilateral OQTP.
∠POQ = 90°
∠OPT = ∠OQT = 90° [At point of contact, tangent and radius are perpendicular]
∴ ∠PTO = 90°
OP = OQ = radius
In this quadrilateral, all the angles are equal and pair of adjacent sides are equal.
∴ OQTP is a square.

24. In the fig two tangents AB and AC are drawn to a circle O such that ∠BAC = 120°. Prove that OA = 2AB.

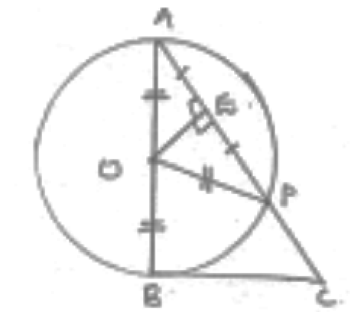
Sol:



Consider Centre O for given circle
∠BAC = 120°
AB and AC are tangents
From the fig.
In ΔOBA, ∠OBA = 90° [radius perpendicular to tangent at point of contact]
∠OAB = ∠OAC = $\frac{1}{2}\angle BAC = \frac{1}{2} \times 120° = 60°$
[Line joining Centre to external point from where tangents are drawn bisects angle formed by tangents at that external point1]
In ΔOBA, cos 60° = $\frac{AB}{OA}$
$\frac{1}{2} = \frac{AB}{OA} \Rightarrow OA = 2AB$

25. In the fig. BC is a tangent to the circle with Centre O. OE bisects AP. Prove that ΔAEO ~ ΔABC.

Sol:



Given

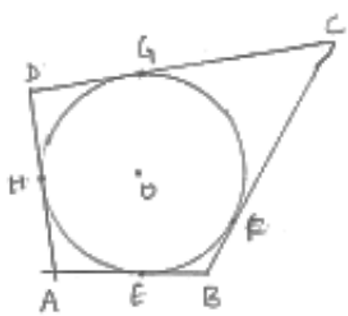
BC is tangent to circle

OE bisects AP, AE = EP

Consider ΔAOP

26. The lengths of three consecutive sides of a quadrilateral circumscribing a circle are 4cm, 5cm and 7cm respectively. Determine the length of fourth side.

Sol:



Let us consider a quadrilateral ABCD, AB = 4cm, BC = 5 cm, CD = 7cm, CD as sides circumscribing circle with centre O. and intersecting at points E, F, G, H. as in fig.

We know that the tangents drawn from external point to the circle are equal in length.

From point A, AE = AH …. (i)

From point B, BE = BF ….. (ii)

From point C, GC = CE ….(iii)

From point D, GD = DH ….(iv)

(i) + (ii) + (iii) + (iv) ⇒ (AE + BE + GC + GD) = (AH + BF + CF + DH)

⇒ (AE + BE) + (GC + GD) = (AH + DH) + (BF + CF)

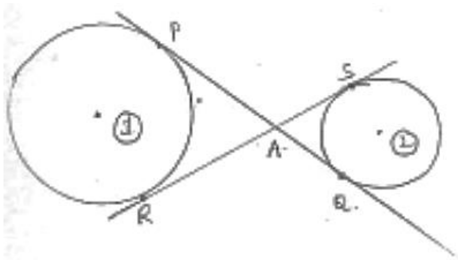
⇒ AB + CD = AD + BC

⇒ 4 + 7 = 5 + AD

⇒ AD = 11 – 5 = 6 cm

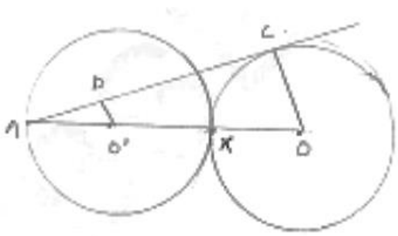
Fourth side = 6 cm

27. In fig common tangents PQ and RS to two circles intersect at A. Prove that PQ = RS.
Sol:
Consider



Two circles namely (i) & (ii) as shown with common tangents as PQ and RS.
We know that
The tangents from external point to the circle are equal in length.
From A to circle (i) AP = AR … (i)
From A to circle (ii), AQ = AS …. (ii)
(i) + (ii) ⇒ AP + AQ = AR + RS
⇒ PQ = RS

28. Equal circles with centers O and O' touch each other at X. OO' produced to meet a circle with Centre O' at A. AC is tangent to the circle whose Centre is a O'D is perpendicular to AC. Find the value of DO'/CO
Sol:



Given circles with centers O and O'
O'D ⊥ AC. Let radius = r
$O'A = O'X = OX = r$
$In\ triangles, \Delta AO'D$ and ΔAOC
∠A = ∠A [Common angle]
$\angle ADO' = \angle ACO = 90°$ [O'D ⊥ AC and at point of contact C, radius ⊥ tangent]
$By\ A \cdot A\ similarity\ \Delta AO'D \sim \Delta AOC.$
$when\ two\ triangles\ are\ similar\ then\ their\ corresponding\ sides\ will\ be\ in\ proportion$
By A.A similarity $\Delta AO'D \sim \Delta AOC$
When two triangles are similar then their corresponding sides will be in proportion
$\frac{AO'}{AO} = \frac{DO'}{CO}$
$\Rightarrow \frac{DO'}{CO} = \frac{r}{r+r+r} = \frac{r}{3r} = \frac{1}{3}$

$\Rightarrow \frac{DO'}{CO} = \frac{1}{3}$

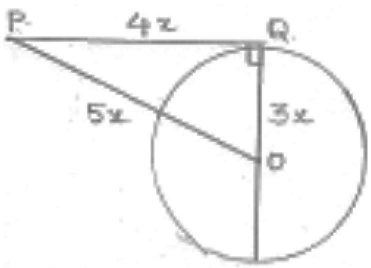
29. In figure OQ : PQ = 3 : 4 and perimeter of ΔPDQ = 60cm. determine PQ, QR and OP.

Sol:

Given OQ: PQ = 3 : 4

Let OQ = 3x PQ = 4x

OP = y



∠OQP = 90° [since at point of contact, tangent is perpendicular to radius]

In ΔOQP, by Pythagoras theorem

$OP^2 = OQ^2 + QP^2$

$\Rightarrow y^2 = (3x)^2 + (4x)^2$

$\Rightarrow y^2 = 9x^2 + 16x^2 = 25x^2$

$\Rightarrow y^2 = \sqrt{25x^2} = 5x$

Perimeter = OQ + PQ + OP = 3x + 4x + 5x = 12x

According to problem perimeter = 60

∴ 12x = 60

x = $\frac{60}{12} = 5cm$

OQ = $3 \times 5 = 15cm$

PQ = $4 \times 5 = 20\ cm$

OP = $5\ \times 5 = 25cm$

Exercise – 12.1

1. A tower stands vertically on the ground. From a point on the ground, 20 m away from the foot of the tower, the angle of elevation of the top of the tower is 600. What is the height of the tower?

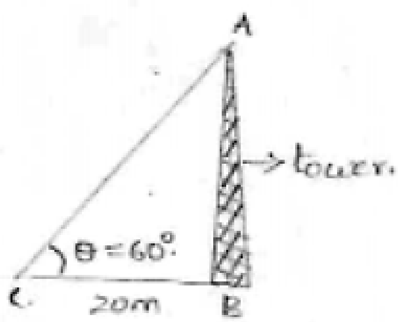
Sol:

Given

Distance between point of observation and foot of tower $=20m=BC$

Angle of elevation of top of tower $=60^\circ=0$

Height of tower $H=?=AB$



Now from fig ABC

ΔABC is a right angle

$$\frac{1}{\tan}=\frac{\text{Adjacent side}}{\text{Opposite side}}$$

$$\Rightarrow \boxed{\tan\theta=\frac{\text{Opposite side}(AB)}{\text{Adjacent side}(BC)}}$$

$$i.e., \tan 60^\circ=\frac{AB}{20}$$

$\Rightarrow AB=20\tan 60^\circ$

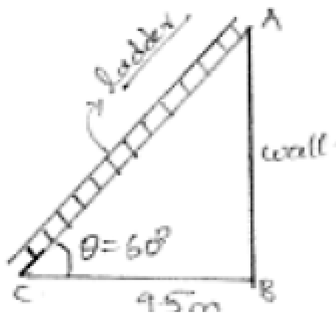
$\Rightarrow H=20\sqrt{3}$

$=20\sqrt{3}$

$\therefore$ Height of tower $H=20\sqrt{3}m$

2. The angle of elevation of a ladder leaning against a wall is 600 and the foot of the ladder is 9.5 m away from the wall. Find the length of the ladder.

Sol:



Distance between foot ladder and wall $=9.5m=BC$

Angle of elevation $0=60°$

Length of ladder $=l=?=AC.$

Now fig. forms a right angle triangle ABC

We know

$$\boxed{\cos\theta=\frac{\text{Adjacent side}}{\text{hypotenuse}}}$$

$$\Rightarrow \cos 60°=\frac{BC}{AC}$$

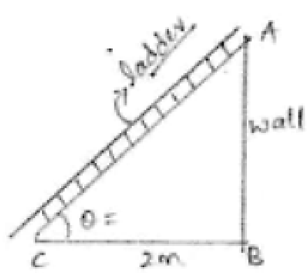
$$\Rightarrow \frac{1}{2}=\frac{9\cdot 5}{AC}$$

$$\Rightarrow AC=2\times 9.5=19m$$

$\therefore$ length of ladder $l=19m$

3. A ladder is placed along a wall of a house such that its upper end is touching the top of the wall. The foot of the ladder is 2 m away from the wall and the ladder is making an angle of 600 with the level of the ground. Determine the height of the wall.

Sol:



Distance between foot and ladder and wall $=2m=BC$

Angle made by ladder with ground

$\theta=60°$

Height of wall $H=?=AB$

Now fig ABC forms a right angled triangle

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

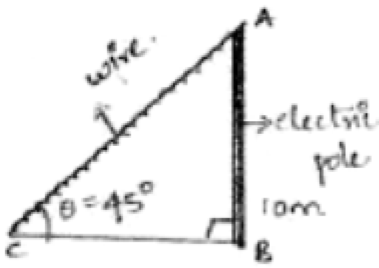
$\therefore$ height of wall $H = 2\sqrt{3}m.$

$$\Rightarrow \tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{AB}{2} \Rightarrow AB = 2\sqrt{3}m.$$

4. An electric pole is 10 m high. A steel wire tied to top of the pole is affixed at a point on the ground to keep the pole up right. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire.

Sol:



Height of the electric pole $H = 10m = AB$ angle made by steel wire with ground (horizontal) $\theta = 45°$

Let length of rope wire $= l = AC$

If we represent above data is

Form of figure thin it forms a right triangle ABC

Here $\boxed{\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}}$

$$\Rightarrow \sin 45° = \frac{AB}{AC}$$

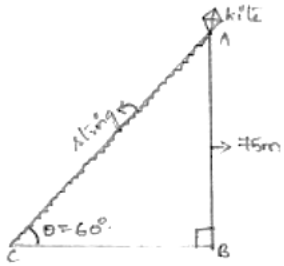
$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{10m}{l}$$

$$\Rightarrow l = 10\sqrt{2}m$$

$\therefore$ length of wire $l = 10\sqrt{2}m$

5. A kite is flying at a height of 75 meters from the ground level, attached to a string inclined at 600 to the horizontal. Find the length of the string to the nearest meter.

Sol:



Given

Height o kite from ground $= 75m = AB$

Inclination of string with ground

$\theta = 60°$

Length of string $l = ? = AC$

If we represent the above data is form of figure as shown then its form a right angled triangle ABC here

$$\boxed{\sin\theta = \frac{\text{Opposite side}}{\text{hypotenuse}}}$$

$$\sin 60° = \frac{AB}{AC}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{75}{l}$$

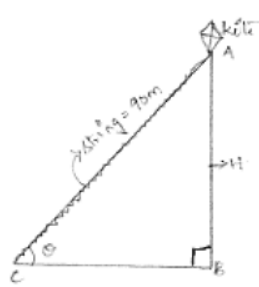
$$\Rightarrow l = \frac{75\times 2}{\sqrt{3}} = \frac{3\times 50}{\sqrt{3}}$$

$$\Rightarrow l = 50\sqrt{3}m$$

Length of string $l = 50\sqrt{3}m.$

6. The length of a string between a kite and a point on the ground is 90 meters. If the string makes an angle O with the ground level such that tan O = 15/8, how high is the kite? Assume that there is no slack in the string.

Sol:



Length of string between point on ground and kite $=90.$

Angle made by string with ground is θ and $\tan\theta=\frac{15}{8}$

$\Rightarrow \theta=\tan^{-1}\left(\frac{15}{8}\right)$

Height of the kite be Hm

If we represent the above data in figure as shown then it forms right angled triangle ABC.

We have,

$$\boxed{\tan\theta=\frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$\tan\theta=\frac{AB}{BC}$

$\Rightarrow \frac{15}{8}=\frac{\text{H}}{BC}$

$\Rightarrow BC=\frac{84}{15}$ …………(1)

in $\Delta ABC,$ by Pythagoras theorem we have

$$\boxed{AC^2=BC^2+AB^2}$$

$\Rightarrow 90^2=\left(\frac{8H}{15}\right)^2+H^2$

$\Rightarrow 90^2=\frac{(8H)^2+(15H)^2}{15^2}$

$\Rightarrow H^2\left(8^2+15^2\right)=90^2\times 15^2$

$\Rightarrow H^2(64+225)=(90\times 15)^2$

$\Rightarrow H^2=\frac{(90\times 15)^2}{289}$

$\Rightarrow H^2=\left(\frac{90\times 15}{17}\right)^2$

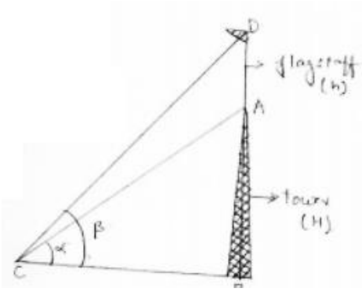
$\Rightarrow H=\frac{90\times 15}{17}=79\cdot 41$

$\therefore$ height of kite from ground $H=79\cdot 41m.$

7. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff. At a point on the plane 70 metres away from the tower, an observer notices that the angles of

elevation of the top and the bottom of the flag-staff are respectively 600 and 45°. Find the height of the flag-staff and that of the tower.

Sol:



Given

Vertical tower is surmounted by flag staff distance between tower and observer $= 70m = BC$. Angle of elevation of top of tower $\alpha = 45°$

Angle of elevation of top of flag staff $\beta = 60°$

Height of flagstaff $= h = AD$

Height of tower $= H = AB$

If we represent the above data in the figure then it forms right angled triangles ΔABC and ΔCBD

When θ is angle in right angle triangle we know that

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha = \frac{AB}{BC}$$

$$\Rightarrow \tan 45° = \frac{H}{70}$$

$$\Rightarrow H = 70\times 1$$

$$= 70m.$$

$$\tan\beta = \frac{DB}{BC}$$

$$\Rightarrow \tan 60° = \frac{AD + AB}{70} = \frac{h+H}{70}$$

$$\Rightarrow h + 70 = 70\left(\sqrt{3}\right)$$

$$\Rightarrow h = 70\left(\sqrt{3} - 1\right)$$

$$= 70\left(1\times 32 - 1\right) = 70\times 0\cdot 732$$

$$= 51\cdot 24m. \qquad \therefore h = 51\cdot 24m$$

Height of tower $= 70m$ height of flagstaff $= 51\cdot 24m$

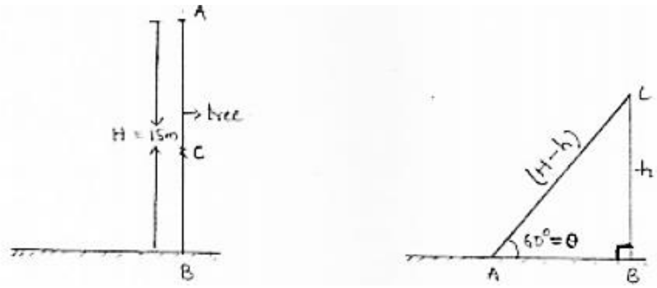
8. A vertically straight tree, 15 m high, is broken by the wind in such a way that its top just touches the ground and makes an angle of 60° with the ground. At what height from the ground did the tree break?

Sol:

Initial height of tree $H = 15m$

$= AB$

Let us assume that it is broken at pointe.



Then given that angle made by broken part with ground $\theta = 60°$

Height from ground to broken pointe $= h = BC$

$AB = AC + BC$

$\Rightarrow H = AC + h \Rightarrow AC = (H - h)m$

If we represent the above data in the figure as shown then it forms right angled triangle ABC from fig

$$\boxed{\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}}$$

$$\Rightarrow \sin 60° = \frac{BC}{CA}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{H - h}$$

$$\Rightarrow \sqrt{3}(15 - h) = 2h$$

$$\Rightarrow 15\sqrt{3} - h\sqrt{3} = 2h$$

$$\Rightarrow (2 + \sqrt{3})h = 15\sqrt{3}$$

$$\Rightarrow h = \frac{15\sqrt{3}}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

Rationalizing denominator rationalizing factor of $a + \sqrt{b}$ is $a - \sqrt{b}$

$$\Rightarrow h = \frac{(15\sqrt{3})(2 - \sqrt{3})}{2^2 - (3)^2}$$

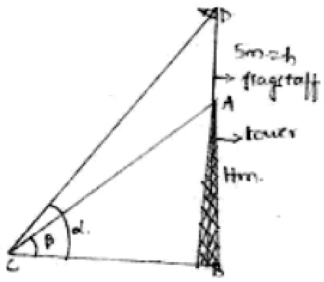
$$= 15(2\sqrt{3} - 3)$$

$\therefore h = 15\left(2\sqrt{3} - 3\right)$

$\therefore$ height of broken point from ground $= 15\left(2\sqrt{3} - 3\right)m$

9. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 5 meters. At a point on the plane, the angles of elevation of the bottom and the top of the flag-staff are respectively 30^0 and 60^0. Find the height of the tower.

Sol:



Height of the flagstaff $h = 5m = AP$

Angle of elevation of the top of flagstaff $= 60° = \alpha$

Angle of elevation of the bottom of flagstaff $= 30° = \beta$

Let height of tower be $Hm = AB.$

If we represent the above data in forms of figure then it from triangle CBD in which ABC is included with $\angle B = 90°$

In right angle triangle, if

Angle is θ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha = \frac{BD}{BC}$$

$$\Rightarrow \tan 60° = \frac{AB + AD}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{H + 5}{BC} \qquad \ldots\ldots\ldots\ldots..(1)$$

$$\tan\beta = \frac{AB}{BC}$$

$$\Rightarrow \tan 30° = \frac{H}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{H}{BC} \qquad \ldots\ldots\ldots\ldots..(2)$$

(1) and (2) $\Rightarrow \dfrac{\sqrt{3}}{\dfrac{1}{\sqrt{3}}} = \dfrac{H15/BC}{H/BC}$

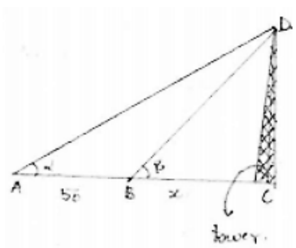
$\Rightarrow 3 = \dfrac{H+5}{H} \Rightarrow 3H = H+5$

$\Rightarrow 2H = 5 \Rightarrow H = \dfrac{5}{2} = 2 \times 5m.$

Height of tower $H = 2 \cdot 5m.$

10. A person observed the angle of elevation of the top of a tower as 30°. He walked 50 m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60^0. Find the height of the tower.

Sol:



Given,

Angle of elevation of top of tour, from first point of elevation $(A)\alpha = 30°$

Let the walked $50m$ from first point (A) to B then $AB = 50m$

Angle of elevation from second point $B \Rightarrow Gb = 60°$

Now let is represent the given data in form of then it forms triangle ACD with triangle BCD in it $\angle c = 90°$

Let height of tower, be

$Hm = CD$

$BC = xm.$

If in a right angle triangle θ is the angle then $\boxed{\tan\theta = \dfrac{\text{Opposite side}}{\text{Adjacent side}}}$

$\tan\alpha = \dfrac{CD}{AC}$

$\Rightarrow \tan 30° = \dfrac{H}{AB+BC}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{H}{50+x}$

$\Rightarrow 50 + x = H\sqrt{3}$ …………..(1)

$$\tan\beta = \frac{CD}{BC}$$

$$\Rightarrow \tan 60° = \frac{H}{x}$$

$$\Rightarrow \sqrt{3} = \frac{H}{x}$$

$$\Rightarrow x = \frac{11}{\sqrt{3}} \quad \text{…………..(2)}$$

(2) in (1)

$$\Rightarrow 50 + \frac{H}{\sqrt{3}} = H\sqrt{3}$$

$$\Rightarrow H\sqrt{3} - \frac{H}{\sqrt{3}} = 50$$

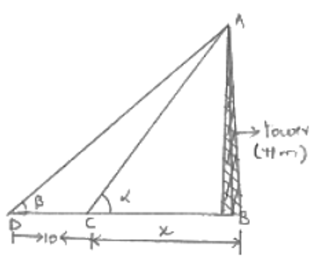
$$\Rightarrow H\left(\frac{3-1}{\sqrt{3}}\right) = 50$$

$$\Rightarrow H = \frac{50\sqrt{3}}{2} = 25\sqrt{3}$$

$\therefore$ Height of tower $H = 25\sqrt{3}m$

11. The shadow of a tower, when the angle of elevation of the sun is 45°, is found to be 10 m. longer than when it was 60^0. Find the height of the tower.

Sol:



Let the length of shadow of tower when angle of elevation is $(\alpha = 60°)$ be $x\,m = BC$ then according to problem

Length of the shadow with angle of elevation $(\beta = 45°)$ is $(10 + x)m = BD$.

If we represent the, above data in form of figure then it forms a triangle ABD is which triangle ABC is included with $\angle B = 90°$

Let height of tower be $Hm = AB$

If in right angle triangle one of the angle is θ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$\tan\alpha = \frac{AB}{BC}$

$\Rightarrow \tan 60° = \frac{H}{x}$

$\Rightarrow x = \frac{H}{\sqrt{3}}$ ……………(1)

$\tan\beta = \frac{AB}{BD}$

$\Rightarrow \tan 45° = \frac{H}{x+10}$

$\Rightarrow x+10 = H$

$\Rightarrow x = H-10$ ……………(2)

Substitute $x = H-10$ in (1)

$H-10 = \frac{H}{\sqrt{3}}$

$\Rightarrow \sqrt{3}H - 10\sqrt{3} = H$

$\Rightarrow \left(\sqrt{3}-1\right)H = 10\sqrt{3}$

$\Rightarrow H = \frac{10\sqrt{3}}{\sqrt{3}-1}$

$\Rightarrow H = \frac{10\sqrt{3}\times\sqrt{3}+1}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}$

$= \frac{10\sqrt{3}\left(\sqrt{3}+1\right)}{2}$

$= 5\left(3+\sqrt{3}\right)$

$= 23\cdot 66m$

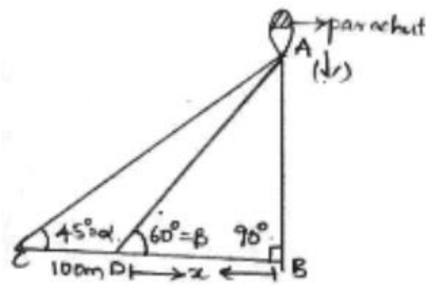
Rationalize denominator rationalizing factor of $a+\sqrt{b}$ is $a-\sqrt{b}$

$\therefore$ Height of tower

$= 23\cdot 66m$

12. A parachutist is descending vertically and makes angles of elevation of 45° and 60° at two observing points 100 m apart from each other on the left side of himself. Find the maximum height from which he falls and the distance of the point where he falls on the ground from the just observation point.

Sol:



Let is the parachutist at highest point A. Let C and D be points which are $100m$ a part on ground where from then $CD = 100m$

Angle of elevation from point $C = 45°[\alpha]$

Angle of elevation from point $B = 60°[\beta]$

Let B be the point just vertically down the parachute

Let us draw figure according to above data then it forms the figure as shown in which ABC is triangle and ABD included in it with

ABD triangle included

Maximum height of parachute

From ground $= AB = Hm$

Distance of point where parachute falls to just nearest observation point $= xm$

If in right angle triangle one of the included angle θ. Then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha = \frac{AB}{CB}$$

$$\tan 45° = \frac{H}{100+x}$$

$$100 + x = H \quad \ldots\ldots\ldots(1)$$

$$\tan\beta = \frac{AB}{DB}$$

$$\tan 60° = \frac{H}{x}$$

$$H = \sqrt{3}x \quad \ldots\ldots\ldots(2)$$

From (1) and (2)

$$\sqrt{3}x = 100 + x$$

$\left(\sqrt{3}-1\right)x = 100$

$x = \frac{100}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{100\left(\sqrt{3}+1\right)}{2}$

$\Rightarrow x = 50\left(\sqrt{3}+1\right)m.$

$\Rightarrow x = 50\left(1\times 732+1\right)$

$\Rightarrow x = 50\left(2\times 732\right)$

$\Rightarrow x = 136.6m$ in (2)

$H = \sqrt{3}\times 136\times 6 = 1\cdot 732\times 136\cdot 6 = 236\cdot 6m$

Maximum height of parachute from ground

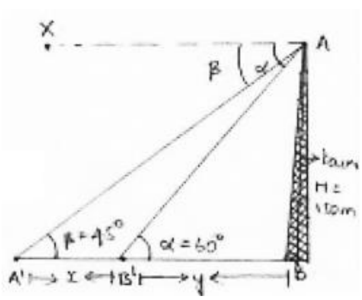
$H = 236\cdot 6m$

Distance between point where parachute falls on ground and just observation is

$x = 136\cdot 6m$

13. On the same side of a tower, two objects are located. When observed from the top of the tower, their angles of depression are 45° and 60°. If the height of the tower is 150 m, find the distance between the objects.

Sol:



Height of tower, $H = AB = 150m.$

Let A and B be two objects m the ground

Angle of depression of objects $A^{'}\left[\angle A' Ax\right] = \beta = 45° = \angle AA^{'}B\left[Ax\right]\left[A'B\right]$

Angle of depression of objects B ’

$\angle xAB' = \alpha = 60° = \angle AB'B\left[Ax\right]\left[A'B\right]$

Let $A'B' = x \quad B'B = y$

In we figure the above data in figure, then it is as shown with $\angle B = 90°$

In any right angled triangle if one of the included angle is θ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha = \frac{AB}{BB'}$$

$$\Rightarrow \tan 60^\circ = \frac{150}{y}$$

$$\Rightarrow y = \frac{150}{\sqrt{3}} \qquad \ldots\ldots\ldots(1)$$

$$\tan\beta = \frac{AB}{A'B}$$

$$\Rightarrow \tan 45^\circ = \frac{150}{x+y}$$

$$\Rightarrow x + y = 150 \qquad \ldots\ldots\ldots(2)$$

(1) and (2) $\Rightarrow x + \frac{150}{\sqrt{3}} = 150$

$$\Rightarrow x + \frac{50\times 3}{\sqrt{3}} = 150$$

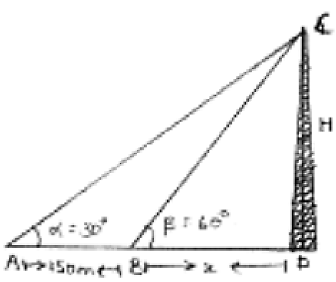
$$\Rightarrow x = 150 - 50\sqrt{3} = 150 - 50(1732)$$

$$= 150 - 86\cdot 6 = 63\cdot 4m$$

Distance between objects $A'B' = 63\cdot 4m$

14. The angle of elevation of a tower from a point on the same level as the foot of the tower is 30°. On advancing 150 meters towards the foot of the tower, the angle of elevation of the tower becomes 60°. Show that the height of the tower is 129.9 meters (Use $\sqrt{3}$ = 1.732).

Sol:



Angle of elevation of top of tower from first point $A, \alpha = 30^\circ$

Let we advanced through A to b by 150m then $AB = 150m$

Angle of elevation of top of lower from second point $B, \beta = 60^\circ$

Let height of tower $CD = H\ m$

If we represent the above data in from of figure then it forms figure as shown with $\angle D = 90^\circ$

If in right angled triangle, one of included angle is θ then $\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$

$\tan\alpha = \frac{CD}{AD}$

$\Rightarrow \tan 30° = \frac{H}{150+x}$

$150 + x = H\sqrt{3}$ ……….(1)

$\tan\beta = \frac{CD}{BD}$

$\Rightarrow \tan 60° = \frac{H}{x}$

$\Rightarrow H = x\sqrt{3} \Rightarrow x = \frac{H}{\sqrt{3}}$ ……….(2)

(2) in (1)

$150 + \frac{H}{\sqrt{3}} = H\sqrt{3} \Rightarrow H\left(\sqrt{3} - \frac{1}{3}\right) = 150$

$\Rightarrow H\left(\frac{3-1}{\sqrt{3}}\right) = 150 \Rightarrow H = \frac{150\times\sqrt{3}}{2} = 75\sqrt{3} = 75\times 1\cdot 732$

$= 129\cdot 9m$

$\therefore$ height of tower $= 129\cdot 9m$

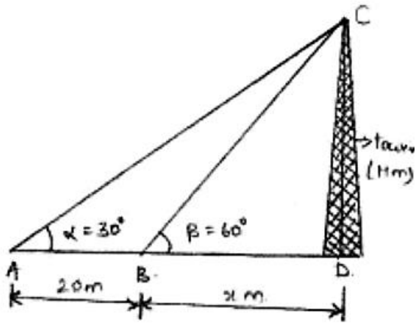
15. The angle of elevation of the top of a tower as observed from a point in a horizontal plane through the foot of the tower is 32°. When the observer moves towards the tower a distance of 100 m, he finds the angle of elevation of the top to be 63°. Find the height of the tower and the distance of the first position from the tower. [Take tan 32° = 0.6248 and tan 63° = 1.9626]

Sol:

91.65m, 146.7m

16. The angle of elevation of the top of a tower from a point A on the ground is 30°. Moving a distance of 20metres towards the foot of the tower to a point B the angle of elevation increases to 60°. Find the height of the tower & the distance of the tower from the point A.

Sol:



Angle of elevation of top of tower from points A $\alpha = 30°$

Angle of elevation of top of tower from points B $\beta = 60°$

Distance between A and B, $AB = 20m$

Let height of tower $CD = 'h'm$

Distance between second point B from foot of tower $bc\ 'x'm$

If we represent the above data in the figure, then it forms figure as shown with $\angle D = 90°$

In right angled triangle if one of the included angle is θ then $\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$

$$\tan\alpha = \frac{CD}{AD}$$

$$\tan 30° = \frac{h}{20+x}$$

$$20 + x = h\sqrt{3} \qquad \ldots\ldots\ldots\ldots(1)$$

$$\tan\beta = \frac{CD}{BD}$$

$$\tan 60° = \frac{h}{x}$$

$$x = \frac{h}{\sqrt{3}} \qquad \ldots\ldots\ldots\ldots(2)$$

(2) in (1) $\Rightarrow 20 + \frac{h}{\sqrt{3}} = h\sqrt{3} \Rightarrow h\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 20$

$$\Rightarrow h\left(\frac{3-1}{\sqrt{3}}\right) = 20 \Rightarrow h = \frac{20\sqrt{3}}{2} = 10\times\sqrt{3} = 17\cdot 32m$$

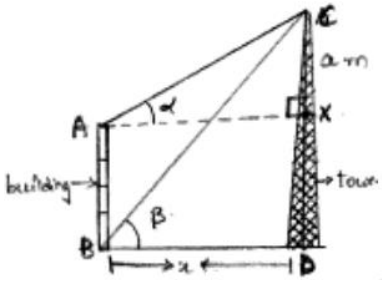
$$x = \frac{10\sqrt{3}}{\sqrt{3}} = 10m.$$

Height of tower $h = 17\times 32m$

Distance of tower from point $A = (20+10) = 30m$

17. From the top of a building 15 m high the angle of elevation of the top of a tower is found to be 30°. From the bottom of the same building, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower and the distance between the tower and building.

Sol:



Let AB be the building and CD be the tower height of the building is $15m = h = AB.$

Angle of elevation of top of tower from top of building $\alpha = 30°$

Angle of elevation of top of tower from bottom of building $\beta = 60°$

Distance between tower and building $BD = x$

Let height of tower above building be $'a'$ m

If we represent the above data is from of figure then it forms figure as shown with $\angle D = 90°$ also draw $AX \parallel BD, \angle AXC = 90°$

Here $ABDX$ is a rectangle

$\therefore BD = DX = 'x'm \qquad AB = XD = h = 15m$

In right triangle if one of the included angle is θ then $\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$

$$\tan\alpha = \frac{CX}{AX}$$

$$\Rightarrow \tan 30° = \frac{a}{x}$$

$$\Rightarrow x = a\sqrt{3} \qquad \ldots\ldots\ldots\ldots..(1)$$

$$\tan\beta = \frac{CD}{BD}$$

$$\Rightarrow \tan 60° = \frac{a+15}{x}$$

$$\Rightarrow x\sqrt{3} = a + 15 \qquad \ldots\ldots\ldots\ldots..(2)$$

(1) and (2)

$$\Rightarrow \frac{x}{x\sqrt{3}} = \frac{a\sqrt{3}}{a+15} \Rightarrow a + 15 = a\sqrt{3}\left(\sqrt{3}\right)$$

$$a + 15 = 3a$$

$$\Rightarrow 2a = 15$$

$\Rightarrow a = \frac{15}{2} = 7 \cdot 5m$

$x = a\sqrt{3}$

$= 7 \cdot 5 \times 1 \cdot 732$

$= 12 \cdot 99m$

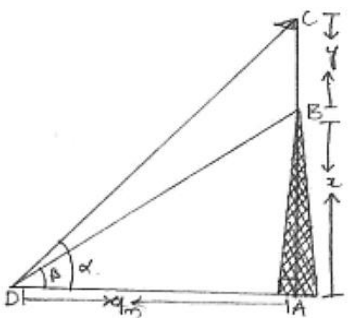
Height of tower above ground $= h + a$

$= 15 + 7 \cdot 5 = 22 \cdot 5m$

Distance between tower and building $= 12 \cdot 99m$

18. On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. At a point 9 meters away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are 60° and 30° respectively. Find the height of the tower and the flag pole mounted on it.

Sol:



Let AB be the tower and BC be flagstaff on the tower

Distance of point of observation from foot of tower $BD = 9m$

Angle of elevation of top of flagstaff $[c]\alpha = 60°$

Angle of elevation of bottom of flag pole $[B]\beta = 30°$

Let height of tower $= 'x' = AB$

Height of pole $= 'y' = BC$

The above data is represented in form of figure a shown with $\angle A = 90°$

If in right triangle one of the included is $\theta,$ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$\tan\alpha = \frac{AC}{AD}$

$\tan 60° = \frac{x+y}{9}$

$x + y = 9\sqrt{3}$

$y = 9\sqrt{3} - 3\sqrt{3}$

$\tan\beta = \dfrac{AB}{AD}$

$\tan 30° = \dfrac{x}{9}$

$x = \dfrac{9}{\sqrt{3}} = 3\sqrt{3} = 5\cdot 196m$

$= 6\sqrt{3} = 6\times 1\cdot 732$

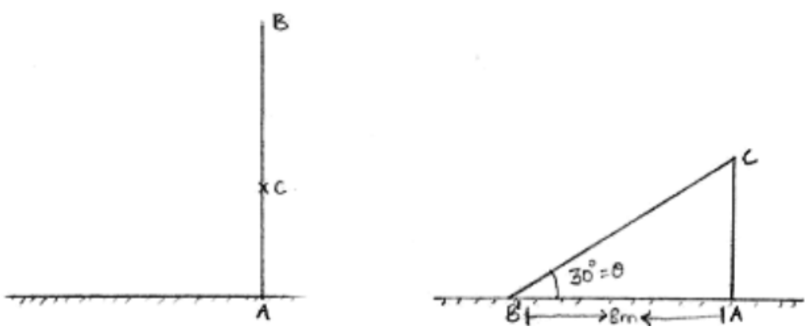
$= 10\cdot 392m$

Height of tower $x = 5\cdot 196m$

Height of pole $y = 10\cdot 392m$

19. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle of 30° with the ground. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Sol:



Let initially tree height be AB

Let us assumed that the tree is broken at point C

Angle made by broken part CB' with ground is $30° = \theta$

Distance between foot of tree of point where it touches ground $B'A = 8m$

Height of tree $= h = AC + CB' = AC + CB$

The above information is represent in the form of figure as shown

$\cos\theta = \dfrac{\text{Adjacent side}}{\text{Hypotenuse}}$	$\tan\theta = \dfrac{\text{Opposite side}}{\text{Adjacent side}}$

$\cos 30° = \dfrac{AB'}{CB'}$

$\dfrac{\sqrt{3}}{2} = \dfrac{B}{CB'}$

$CB' = \dfrac{16}{\sqrt{3}}$

$\tan 30° = \frac{CA}{AB'}$

$\frac{1}{\sqrt{3}} = \frac{CA}{8}$

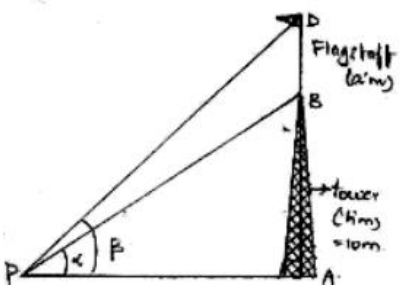
$CA = \frac{8}{\sqrt{3}}$

Height of tree $= CB' + CA = \frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}} = \frac{24}{\sqrt{3}} = \frac{8\times 3}{\sqrt{3}}$

$= 8\sqrt{3}m$

20. From a point P on the ground the angle of elevation of a 10 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flag-staff from P is 45°. Find the length of the flag-staff and the distance of the building from the point P.(Take $\sqrt{3}$= 1.732).

Sol:



Let AB be the tower and 80 be the flagstaff Angle of elevation of top of building from P $\alpha = 30°$

$AB =$ height of tower $= 10m$

Angle of elevation of top of flagstaff from P $\beta = 45°$

Let height of flagstaff $BD = 'a'm$

The above information is represented in form of figure as shown with $\angle A = 90°$

In a right angled triangle if one of the included

Angle is θ

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$\tan\alpha = \frac{AB}{AP'}$

$\tan 30° = \frac{10}{AP}$

$AP = 10\sqrt{3}$

$= 10 \times 1\cdot 732$

$= 17 \cdot 32$

$\tan \beta = \frac{AD}{AP}$

$\tan 45° = \frac{10 + a}{AP}$

$10 + a = AP$

$a = 17 \cdot 32 - 10$

$= 7 \cdot 32m$

Height of flagstaff $'\theta' = 7 \cdot 32m$

Distance between P and foot of tower $= 17 \cdot 32m.$

21. A 1 .6 m tall girl stands at a distance of 3.2 m from a lamp-post and casts a shadow of 4.8 m on the ground. Find the height of the lamp-post by using (i) trigonometric ratios (ii) property of similar triangles.

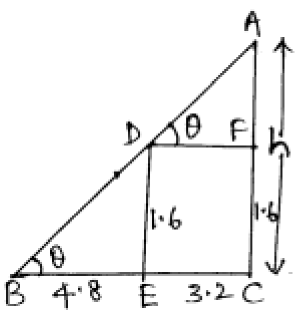
Sol:

Let AC be the lamp past of height $'h'$

We assume that $ED = 1 \cdot 6m, BE = 4 \cdot 8m$ and $EC = 3 \cdot 2m$

We have to find the height of the lamp post

Now we have to find height of lamp post using similar triangles



Since triangle BDE and triangle ABC are similar,

$\frac{AC}{BC} = \frac{ED}{BE}$

$\Rightarrow \frac{h}{4 \cdot 8 + 3 \cdot 2} = \frac{1 \cdot 6}{4 \cdot 8}$

$\Rightarrow h = \frac{8}{3}$

Again we have to find height of lamp post using trigonometry ratios

In ΔADE, $\tan \theta = \frac{1 \cdot 6}{4 \cdot 8}$

$\Rightarrow \tan \theta = \frac{1}{3}$

Again in ΔABC,

$$\tan\theta = \frac{h}{4\cdot 8 + 3\cdot 2}$$

$$\Rightarrow \frac{1}{3} = \frac{h}{8}$$

$$\Rightarrow h = \frac{8}{3}$$

Hence the height of lamp post is $\frac{8}{3}$.

22. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Sol:

$\mathbf{19\sqrt{3}}$

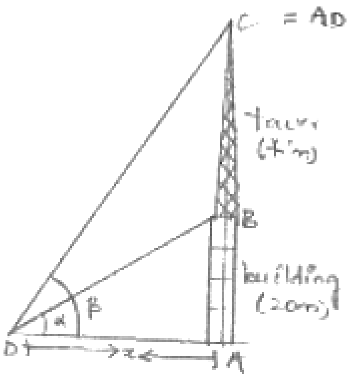
23. The shadow of a tower standing on a level ground is found to be 40 m longer when Sun's altitude is 30° than when it was 60°. Find the height of the tower

Sol:

$\mathbf{20\sqrt{3}}$

24. From a point on the ground the angles of elevation of the bottom and top of a transmission tower fixed at the top of 20 m high building are 45° and 60° respectively. Find the height of the transmission tower.

Sol:



Given height of building $= 20m = AB$

Let height of tower above building $= 'h' = BC$

Height of tower + building $= (h+20)m$ [from ground] $= CA$

Angle of elevation of bottom of tour, $\alpha = 45°$

Angle of elevation of top of tour, $\beta = 60°$

Let distance between tower and observation point $='x'm$
The above data is represented in $= AD$
The form of figure as shown is one of the included angle is right angle triangle is a then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha = \frac{AB}{AD}$$

$$\Rightarrow \tan 45° = \frac{20}{x}$$

$$\Rightarrow x = 20m$$

$$\tan\beta = \frac{CA}{DA}$$

$$\Rightarrow \tan 60° = \frac{h+20}{x}$$

$$\Rightarrow h+20 = 20\sqrt{3}$$

$$\Rightarrow h = 20\left(\sqrt{3}-1\right)$$

Height of tower $h = 20\left(\sqrt{3}-1\right)$

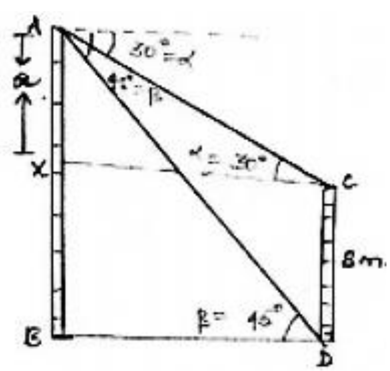
$$= 20(1\cdot 732 - 1)$$

$$= 20\times 0\cdot 732$$

$$= 14\cdot 64m$$

25. The angles of depression of the top and bottom of 8 m tall building from the top of a multistoried building are 30° and 45° respectively. Find the height of the multistoried building and the distance between the two buildings.

Sol:



Let height of multistoried building $'h'm = AB$
Height of tall building $= 8m = CD$
Angle of depression of top of tall building $\alpha = 30°$

Angle of depression of bottom of tall building $\beta = 45°$

Distance between two building $= 'x'm = BD$

Let $Ax = x$

$AB = AX + XB$ but $XB = CD$ [$\because AXCD$ is rectangle]

$AB = 'a'm + 8m$

$AB = (a+8)m$

The above information is represented in the form of figure e as shown

If in right triangle are of included angle is θ

Then $\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$

In ΔAXB

$\tan 30° = \frac{AX}{CX}$

$\frac{1}{\sqrt{3}} = \frac{a}{BD} = \frac{a}{x}.$

$\Rightarrow x = a\sqrt{3}$ ....(1)

In ΔABD

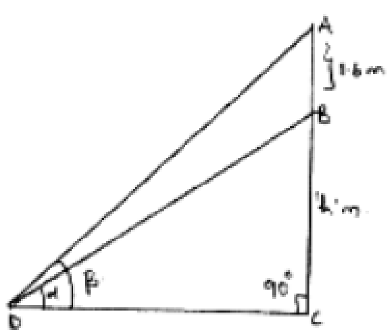
$\tan 45° = \frac{AB}{BD} = \frac{a+8}{x}$

$1 = \frac{a+8}{x}$

$\Rightarrow a + 8 = x$ ....(2)

26. A statue I .6 m tall stands on the top of pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Sol:



Let height of pedestal be $'h'm$

Height of status $= 1\cdot 6m$

Angle of elevation of top of status $\alpha = 60°$

Angle of elevation of pedestal of status $\alpha = 60°$

The above data is represented in the form of figure as shown.

If in right angle triangle one of the included angle is θ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha = \frac{BC}{BD}$$

$$\tan 45° = \frac{h}{DC}$$

$$DC = h\sqrt{8}.1$$

$$DC = 'h'm \qquad ....(1)$$

$$\tan\beta = \frac{AC}{DC}$$

$$\tan 60° = \frac{h + 1\cdot 6}{DC}$$

$$DC = \frac{h + 1\cdot 6}{BC} \qquad ....(2)$$

From (1) and (2) $h = \dfrac{h + 1\cdot 6}{\sqrt{3}}$

$$\Rightarrow h\sqrt{3} = h + 1\cdot 6$$

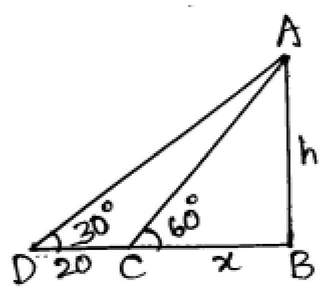
$$\Rightarrow h\left(\sqrt{3} - 1\right) = 1\cdot 6$$

$$\Rightarrow h = \frac{1\cdot 6}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = 0.5\left(\sqrt{3} + 1\right)$$

Height of pedestal $= 0\cdot 6\left(\sqrt{3} + 1\right)m.$

27. A T.V. Tower stands vertically on a bank of a river. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From a point 20 m away this point on the same bank, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the river.

Sol:



Let AB be the T.V tower of height $'h'm$ on a bank of river and $'D'$ be the point on the opposite of the river. An angle of elevation at top of tower is $60°$ and form the point $20m$ away them angle of elevation of tower at the same point is $30°$

Let $AB = h$ and $BC = x$

Here we have to find height and width of river the corresponding figure is here

In ΔCAB,

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \sqrt{3}x = h$$

$$\Rightarrow \boxed{x = \frac{h}{\sqrt{3}}}$$

Again in ΔDBA,

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20+x}$$

$$\Rightarrow \sqrt{3}h = 20 + x$$

$$\Rightarrow \sqrt{3}h = 20 + \frac{h}{\sqrt{3}} \left[\because x = \frac{h}{\sqrt{3}}\right]$$

$$\Rightarrow \sqrt{3}h - \frac{h}{\sqrt{3}} = 20$$

$$\Rightarrow \frac{2h}{\sqrt{3}} = 20$$

$$\Rightarrow h = 10\sqrt{3}$$

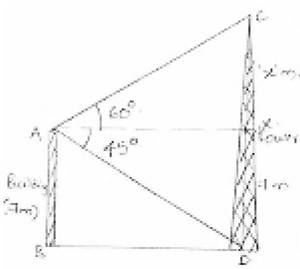
$$\Rightarrow x = \frac{h}{\sqrt{3}} = \frac{10\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow \boxed{x = 10}$$

Hence the height of the tower is $10\sqrt{3}m$ and width of the river is $10\,m.$

28. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Sol:



Given

Height of building $=7m=AB$

Height of cable tower $='H'm=CD$

Angle of elevation of top of tower, from top of building $\alpha=60°$

Angle of depression of bottom of tower, from top of building $\beta=45°$

The above data is represented in form of figure as shown

Let $CX='x'm$

$CD=DX+XC=7m+'x'm$

$=x+7m.$

In ΔADX

$$\tan 45° = \frac{\text{Opposite side}(\text{XD})}{\text{Adjacent side}(\text{AX})}$$

$$1=\frac{7}{AX}$$

$\Rightarrow AX=7m$

In ΔAXD

$$\tan 60° = \frac{XC}{AX}$$

$$\sqrt{3}=\frac{x}{H}$$

$\Rightarrow x=7\sqrt{3}$

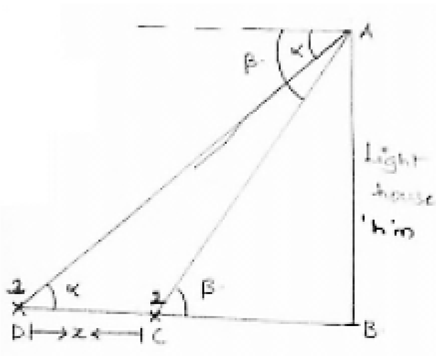
But $CD=x+7$

$=7\sqrt{3}+7=7\left(\sqrt{3}+1\right)m.$

Height of cable tower $=7\left(\sqrt{3}+1\right)m$

29. As observed from the top of a 75 m tall lighthouse, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Sol:



Given

Height of light house $=75m='h'm=AB$

Angle of depression of ship 1 $\alpha=30°$

Angle of depression of ship 2 $\beta=45°$

The above data is represented in form of figure as shown.

Let distance between ships be $'x'm=CD$

In right triangle if one of included angle is θ then

$$\boxed{\tan\theta=\frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha=\frac{AB}{DB}$$

$$\tan 30°=\frac{75}{x+BC}$$

$$x+BC=75\sqrt{3} \qquad \ldots\ldots\ldots\ldots(1)$$

$$\tan\beta=\frac{AB}{CB}$$

$$\tan 45°=\frac{75}{BC}$$

$$BC=75 \qquad \ldots\ldots\ldots\ldots(2)$$

(2) in (1) $\Rightarrow x+75=75\sqrt{3}$

$\Rightarrow x=75\left(\sqrt{3}-1\right)$

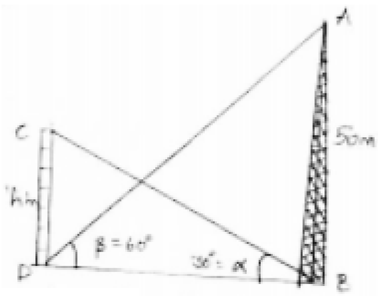
$\therefore$ Distance between ships

$='x'm$

$=75\left(\sqrt{3}-1\right)m.$

30. The angle of elevation of the top of the building from the foot of the tower is 30° and the angle of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Sol:



Angle of elevation of top of building from foot of tower $= 30° = \alpha$

Angle of elevation of top of tower, from foot of building $= 60° = \beta$

Height of tower $= 50m = AB$

Height of building $= 'h'm$

$= CD$

The above information is represented in form of figure as shown

In right triangle if one of the included angle is θ then $\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$

In ΔABC

$$\tan\beta = \frac{AB}{BD}$$

$$\tan 60° = \frac{50}{BD}$$

$$BD = \frac{50}{\sqrt{3}}$$

In ΔCBD

$$\tan\alpha = \frac{CD}{BD}$$

$$\tan 30° = \frac{h}{\frac{50}{\sqrt{3}}}$$

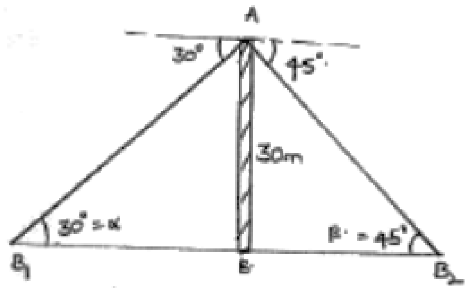
$$h = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$$

$$= \frac{50}{3}$$

$\therefore$ height of building $= \frac{50}{3} m$

31. From a point on a bridge across a river the angles of depression of the banks on opposite side of the river are 30° and 45° respectively. If bridge is at the height of 30 m from the banks, find the width of the river.

Sol:



Height of the bridge $= 30m[AB]$

Angle of depression of bank 1 i.e., $\alpha = 30°.[B_1]$

Angle of depression of bank 2 i.e., $\beta = 30°.[B_2]$

Given banks are on opposite sides

Distance between banks $B_1B_2 = B_1B + BB_2$

The above information is represented is the form of figure as shown in right angle triangle if one of the included angle is O then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

In ΔABB_1

$$\tan\alpha = \frac{AB}{B_1B}$$

$$\tan 30 = \frac{30}{B_1B}$$

$$B_1B = 30\sqrt{3}m$$

In ΔABB_2

$$\tan\beta = \frac{AB}{BB_2}$$

$$\tan 45° = \frac{30}{BB_2}$$

$$BB_2 = 30m$$

$$B_1B_2 = B_1B + BB_2 = 30\sqrt{3} + 30$$

$$= 30\left(\sqrt{3}+1\right)$$

Distance between banks $= 30\left(\sqrt{3}+1\right)m$

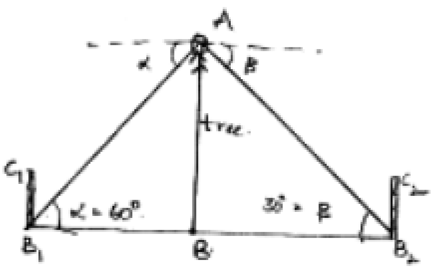
32. Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point between them on the road the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

Sol:

$\mathbf{20\sqrt{3}m}$

33. A man sitting at a height of 20 m on a tall tree on a small island in the middle of a river observes two poles directly opposite to each other on the two banks of the river and in line with the foot of tree. If the angles of depression of the feet of the poles from a point at which the man is sitting on the tree on either side of the river are 60° and 30° respectively. Find the width of the river.

Sol:



Height of tree $AB = 20m$

Angle of depression of pole 1 feet $\alpha = 60°$

Angle of depression of pole 2 feet $\beta = 30°$

B_1C_1 be one pole and B_1C_2 be other sides width of river $= B_1B_2$

$= B_1B + BB_2$

The above information is G represent in from of figure as shown

In right triangle, if one of included angle is 0

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha = \frac{AB}{B_1B}$$

$$\tan 60° = \frac{20}{B_1B}$$

$$B_1B = \frac{20}{\sqrt{3}}$$

$$\tan\beta = \frac{AB}{BB_2}$$

$\tan 30° = \frac{20}{BB_2}$

$BB_2 = 20\sqrt{3}$

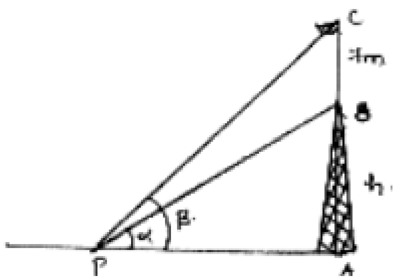
$B_1B_2 = B_2B + BB_2 = \frac{20}{\sqrt{3}} + 20\sqrt{3} = 20\left[\frac{1+3}{\sqrt{3}}\right] = \frac{20}{\sqrt{3}}$

Width of river $= \frac{80}{\sqrt{3}} m.$

$= \frac{80\sqrt{3}}{3} m.$

34. A vertical tower stands on a horizontal plane and is surmounted by a flag-staff of height 7 m. From a point on the plane, the angle of elevation of the bottom of the flag-staff is 30° and that of the top of the flag-staff is 45°. Find the height of the tower.

Sol:



Given

Height of flagstaff $= 7m = BC$

Let height of tower $= 'h' m = AB$

Angle of elevation of bottom of flagstaff $\alpha = 30°$

Angle of elevation of top of flagstaff $\beta = 45°$

Points of desecration be $'p'$

The above data is represented in form of figure as shown

In right angle triangle if one of the induced angle is θ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$\tan\alpha = \frac{AB}{AP}$

$\tan 30° = \frac{h}{AP}$

$AP = h\sqrt{3}$ …….(1)

$\tan\beta = \frac{AC}{AP}$

$\tan 45° = \frac{h+7}{AP}$

$AP = h+7$ …….(2)

From (1) and (2)

$h\sqrt{3} = h+7$

$h\sqrt{3} - h = 7$

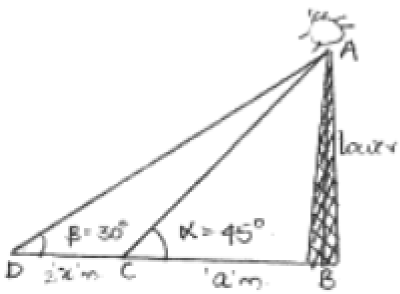
$h\left(\sqrt{3}-1\right) = 7 \Rightarrow h = \frac{7}{3-1} + \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{7\times\left(\sqrt{3}+1\right)}{2} = 3\cdot 5\left(\sqrt{3}+1\right)$

Height of tower $= 3\cdot 5\left(\sqrt{2}+1\right)m.$

35. The length of the shadow of a tower standing on level plane is found to be 2x metres longer when the sun's altitude is 30° than when it was 45°. Prove that the height of tower is x (√3 + 1) metres.

Sol:



Let

Length of shadow be $'a' m\left[BC\right]$ when sun attitude be $= 45°$

Length of shadow will be $\left(2x+a\right)m = 80$ when sun attitude is $\beta = 30°$

Let height of tower be $'h' m = AB$ the above information is represented in form of figure as shown

In right triangle one of the included angle is θ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

In ABC

$\tan\alpha = \frac{AB}{BC}$

$\tan 45° = \frac{h}{a}$

$h = a$ ……….(1)

In ADB

$$\tan\beta = \frac{AB}{(2x+a)BC}$$

$$\tan 30^\circ = \frac{h}{2x+a}$$

$2x + a = h\sqrt{3}$ ……….(2)

(1) in (2) $\Rightarrow 2x + h = h\sqrt{3}$

$$\Rightarrow h\left(\sqrt{3}-1\right) = 2x$$

$$\Rightarrow h = \frac{2x}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

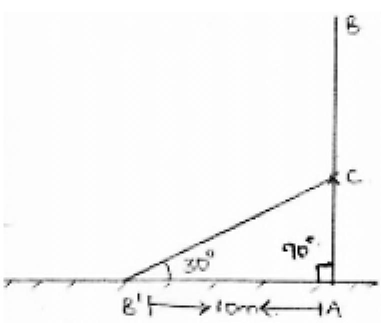
$$\Rightarrow \frac{2x\left(\sqrt{3}+1\right)}{2}$$

$$\Rightarrow x\left(\sqrt{3}+1\right)$$

Height of tower $= x\left(\sqrt{3}+1\right)m$

36. A tree breaks due to the storm and the broken part bends so that the top of the tree touches the ground making an angle of 30° with the ground. The distance from the foot of the tree to the point where the top touches the ground is 10 meters. Find the height of the tree.

Sol:



Let AB be height of tree it is broken at pointe and top touches ground at B'

Angle made by top $\alpha = 30^\circ$

Distance from foot of tree from point where A touches ground = O meter

The above information is represented in form of figure as shown

Height of tree $= AB = AC + CB$

$= AC + CB'$

In right triangle If one of angle is θ then

$$\boxed{\tan\theta = \frac{\text{Adjacent side}}{\text{Opposite side}}}\ \boxed{\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}}$$

$$\tan 30° = \frac{AC}{B'A}$$

$$AC = \frac{10}{\sqrt{3}} m$$

$$\cos 30 = \frac{AB'}{B'C}$$

$$\frac{\sqrt{3}}{2} = \frac{10}{B'C}$$

$$B'C = \frac{20}{\sqrt{3}} m.$$

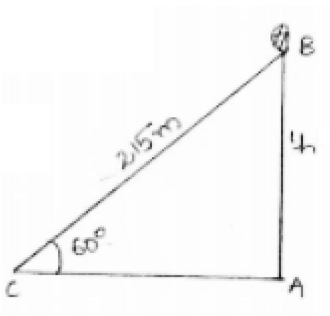
$$AB = CA + CB' = \frac{10}{\sqrt{3}} + \frac{20}{\sqrt{3}}$$

$$= \frac{30}{\sqrt{3}} = 10\sqrt{3}$$

Height of tree $= 10\sqrt{3}m$

37. A balloon is connected to a meteorological ground station by a cable of length 215 m inclined at 600 to the horizontal. Determine the height of the balloon from the ground. Assume that there is no slack in the cable.

Sol:



Length of cable connected to balloon $= 215m[CB]$

Angle of inclination of cable with ground $\alpha = 60°$

Height of balloon from ground $= 'h'm = AB$

The above data is represented in form of figure as shown

In right triangle one of the included angle is θ then

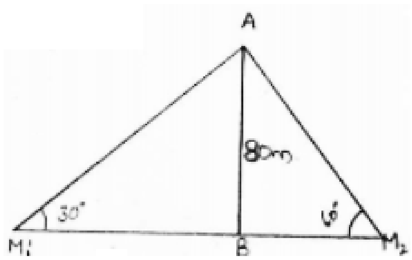
$$\boxed{\sin\theta = \frac{\text{Opposite side}}{\text{hypotenuse}}}$$

$\sin 60° = \frac{AB}{BC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{215} \Rightarrow h = \frac{215\sqrt{3}}{2} = 107 \cdot 5\sqrt{3}m$

$\therefore$ Height of balloon from ground $= 107 \cdot 5\sqrt{3}m.$

38. Two men on either side of the cliff 80 m high observes the angles of elevation of the top of the cliff to be 300 and 600 respectively. Find the distance between the two men.

Sol:



Height of cliff $= 80m = AB.$

Angle of elevation from Man 1, $\alpha = 30°[M_1]$

Angle of elevation from Man 2, $\beta = 60°[M_2]$

Distance between two men $= M_1M_2 \ = BM_1 + BM_2.$

The above information is represented in form of figure as shown

In right angle triangle one of the included angle is θ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha = \frac{AB}{M_1B}$$

$$\tan 30° = \frac{80}{M_1B}$$

$$M_1B = 80\sqrt{3}$$

$$\tan\beta = \frac{AB}{BM_2}$$

$$\tan 60° = \frac{80}{BM_2}$$

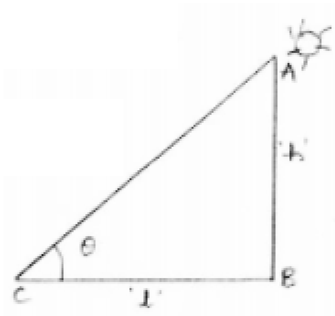
$$BM_2 = \frac{80}{\sqrt{3}}$$

$$M_1M_2 = M_1B + BM_1 = 80\sqrt{3} + \frac{80}{\sqrt{3}} = \frac{80 \times 4}{\sqrt{3}} = \frac{320}{\sqrt{3}}$$

Distance between men $= \frac{320\sqrt{3}}{3}$ meters

39. Find the angle of elevation of the sun (sun’s altitude) when the length of the shadow of a vertical pole is equal to its height.

Sol:



Let

Height of pole $='h'm=$ sun’s altitude from ground length of shadow be $'l'$

Given that $l=h.$

Angle of elevation of sun’s altitude be θ the above data is represented in form of figure as shown

In right triangle if one of the included angle is 0 then.

$$\boxed{\tan\theta=\frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\theta=\frac{AB}{BC}\Rightarrow\tan\theta=\frac{h}{l}$$

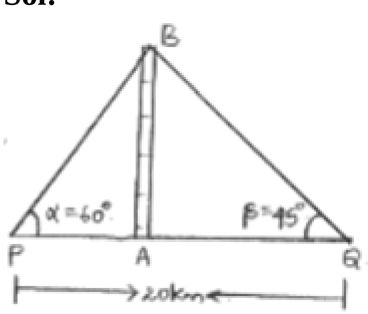
$$\Rightarrow\tan\theta=\frac{l}{l}\left[\because h=1\right]$$

$$\Rightarrow\theta=\tan^{-1}\left(1\right)=45°$$

Angle of sun’s altitude is $45°$

40. A fire in a building B is reported on telephone to two fire stations P and 20 km apart from each other on a straight road. P observes that the fire is at an angle of 60° to the road and Q observes that it is at an angle of 45° to the road. Which station should send its team and how much will this team have to travel?

Sol:



Let AB be the building

Angle of elevation from point P [Fire station 1] $\alpha = 60°$
Angle of elevation from point Q [Fire station 1] $\beta = 45°$
Distance between fire stations $PQ = 20km$
The above information is represented in form of figure as shown
In right triangle if one of the angle is θ then.

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha = \frac{AB}{AP}$$

$$\tan 60° - = \frac{AB}{AP}$$

$$AP = \frac{AB}{\sqrt{3}} \qquad \ldots\ldots\ldots..(1)$$

$$\tan\beta = \frac{AB}{AQ}$$

$$\tan 45° - = \frac{AB}{AQ}$$

$$AQ = AB \qquad \ldots\ldots\ldots..(2)$$

$$(1) + (2) \Rightarrow AP + AQ = \frac{AB}{\sqrt{3}} + AB = AB\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right)$$

$$\Rightarrow 20 = AB\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right) \Rightarrow AB = \frac{20\sqrt{3}}{\sqrt{3}+1}$$

$$AB = \frac{20\sqrt{3}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = 10\sqrt{3}\left(\sqrt{3}-1\right) = 10\left(3-\sqrt{3}\right)$$

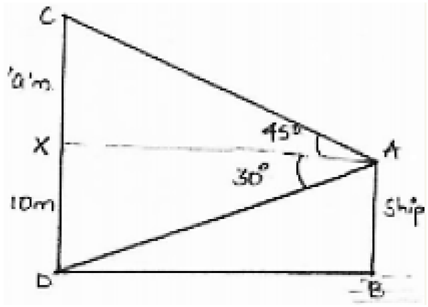
$$AQ = AB = 10\left(3-\sqrt{3}\right) = 10\left(3-1\cdot 732\right) = 12\cdot 64km$$

$$Ap = \frac{AB}{\sqrt{3}} = 10\left(\sqrt{3}-1\right) = 10 \times 0\cdot 732 = 7\cdot 32km$$

Station 1 should send its team and they have to travel $7\cdot 32km$

41. A man on the deck of a ship is 10 m above the water level. He observes that the angle of elevation of the top of a cliff is 45° and the angle of depression of the base is 300. Calculate the distance of the cliff from the ship and the height of the cliff.

Sol:

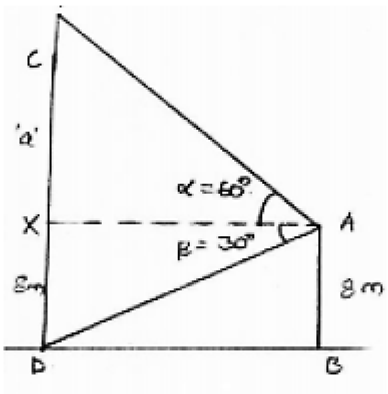


Height of ship from water level $=10cm=AB$
Angle of elevation of top of cliff $\alpha=45°$
Angle of depression of bottom of cliff $\alpha=30°$
Height of cliff $CD='h'm.$
Distance of ship from foot of tower cliff
Height of cliff above ship be $'a'm$
Then height of cliff $=DX+XC$
$=(10+0)m$
The above data is represented in form of figure as shown
In right triangle, if one of the included angle is $\theta,$ then $\boxed{\tan\theta=\frac{\text{Opposite side}}{\text{Adjacent side}}}$

$$\tan 45°=\frac{CX}{AX}$$
$$1=\frac{a}{AX}$$
$$AX='a'm$$
$$\tan 30°=\frac{XD}{AX}$$
$$\frac{1}{\sqrt{3}}=\frac{10}{AX}$$
$$AX=10\sqrt{3}$$
$$\therefore a=10\sqrt{3}m.$$
Height of cliff $=10+10\sqrt{3}=10+\left(\sqrt{3}+1\right)m.$

Distance between ship and cliff $=10\sqrt{3}m.$

42. A man standing on the deck of a ship, which is 8 m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.
Sol:



Height of ship above water level $= 8m = AB$

Angle of elevation of top of cliff (hill) $\alpha = 60°$

Angle of depression of bottom of hill $\beta = 30°$

Height of hill $= CD$

Distance between ship and hill $= AX$.

Height of hill above ship $= CX = 'a' m$

Height of hill $= (a+8)m.$

The above data is represented in form of figure as shown

In right triangle if one of included angle is θ then $\boxed{\tan\theta = \dfrac{\text{Opposite side}}{\text{Adjacent side}}}$

$$\tan\alpha = \frac{CX}{AX}$$

$$\tan 60° = \frac{a}{AX}$$

$$AX = \frac{a}{\sqrt{3}}$$

$$\tan\beta = \frac{XD}{AX}$$

$$\tan 30° = \frac{8}{AX}$$

$$AX = 8\sqrt{3}$$

$$\therefore \frac{a}{\sqrt{3}} = 8\sqrt{3} \Rightarrow a = 24m.$$

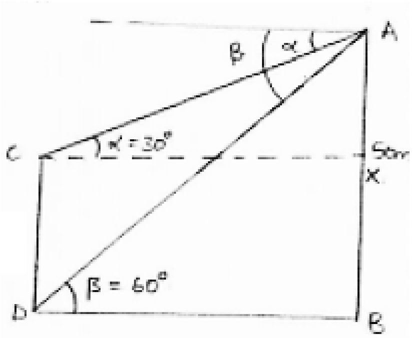
$$AX = 8\sqrt{3}m$$

$\therefore$ Height of cliff hill $= (24+8)m = 32m$

Distance between hill and ship $8\sqrt{3}m.$

43. There are two temples, one on each bank of a river, just opposite to each other. One temple is 50 m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are 30° and 60° respectively. Find the width of the river and the height of the other temple.

Sol:



Height of temple $1(AB) = 50m$

Angle of depression of top of temple 2, $\alpha = 30°$

Angle of depression of bottom of temple 2, $\beta = 60°$

Height of temple $2(CD) = 'h'm$

Width of river $= BD = 'x'm.$ the above data is represents in form of figure as shown

In right triangle if one of $'h'm$ included angle is $\theta,$ then

$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$ here $BD = CX, CD = BX,$

$$\tan\alpha = \frac{AX}{CX}$$

$$\tan 30° = \frac{AX}{CX}$$

$$CX = A \times \sqrt{3}$$

$$\tan\beta = \frac{AB}{BD}$$

$$\tan 60° = \frac{50}{CX}$$

$$CX = \frac{50}{\sqrt{3}}$$

$$AX\left(\sqrt{3}\right) = \frac{50}{\sqrt{3}} \Rightarrow AX = \frac{50}{3}m.$$

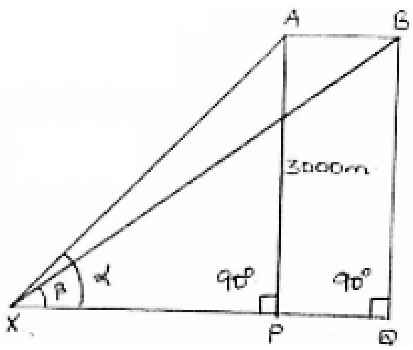
$$CD = XB = AB - AX = 50 - \frac{50}{3} = \frac{100}{3}m$$

Width of river $= \frac{50}{\sqrt{2}} m$

Height of temple 2 $= \frac{100}{3} m$

44. The angle of elevation of an aeroplane from a point on the ground is 45°. After a flight of 15 seconds, the elevation changes to 30°. If the aeroplane is flying at a height of 3000 meters, find the speed of the aeroplane.

Sol:



Let aeroplane travelled from A to B in $15\sec$

Angle of elevation of point A $\alpha = 45°$

Angle of elevation of point B $\beta = 30°$

Height of aeroplane from ground $= 3000$ meters

$= AP = BQ$

Distance travelled in 15 sees $= AB = PQ$

Velocity (or) speed = distance travelled time the above data is represents is form of figure as shown

In right triangle one of the included angle is θ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

$$\tan\alpha = \frac{AP}{XP}$$

$$\tan 45° = \frac{3000}{XP}$$

$$XP = 3000m$$

$$\tan\beta = \frac{BQ}{XQ}$$

$$\tan 30° = \frac{3000}{XQ}$$

$$XQ = 3000\sqrt{3}$$

$PQ = XQ - XP = 3000\left(\sqrt{3}-1\right)m$

Speed $= \frac{PQ}{time} = \frac{3000\left(\sqrt{3}-1\right)}{15} = 200\left(\sqrt{3}-1\right)$

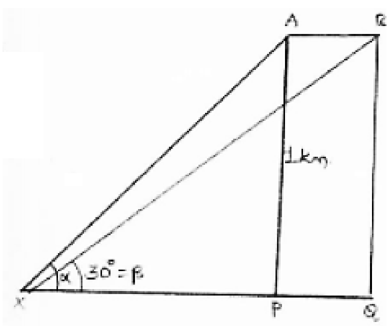
$= 2000 \times 0 \cdot 732$

$= 146 \cdot 4\ m/\sec$

Speed of aeroplane $= 146.4\ m/\sec$

45. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60°. After 10 seconds, its elevation is observed to be 30°. Find the speed of the aeroplane in km/hr.

Sol:



Let aeroplane travelled from A to B in 10 secs

Angle of elevation of point $A = \alpha = 60°$

Angle of elevation of point $B = \beta = 30°$

Height of aeroplane from ground $= 1km = AP = BQ$

Distance travelled in 10 sec $= AB = PQ$

The above data is represent in form of figure as shown

In right triangle if one of the included angle is θ then $\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$

$\tan\alpha = \frac{AP}{PX}$

$\tan 60° = \frac{1}{PX}$

$PX = \frac{1}{\sqrt{3}} km$

$\tan\beta = \frac{BQ}{XQ}$

$\tan 30° = \frac{1}{XQ}$

$XQ = \sqrt{3}km$

$PQ = XQ - PX = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}km. = \frac{2\sqrt{3}}{2}km.$

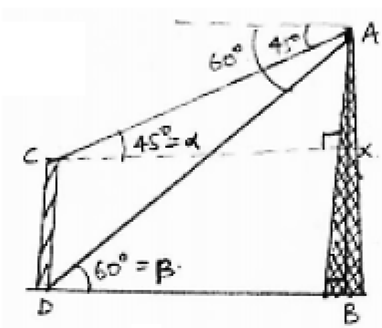
Speed $= \frac{PQ}{time} = \frac{2\sqrt{3}/3km}{\frac{10}{60\times60}hr} = \frac{2\sqrt{3}}{\not{3}} \times 60 \times \not{6}^{2}$

$= 240\sqrt{3}\ km/hr$

Speed of aeroplane $= 240\sqrt{3}\ km/hr$

46. From the top of a 50 m high tower, the angles of depression of the top and bottom of a pole are observed to be 45° and 60° respectively. Find the height of the pole.

Sol:



$AB =$ height of tower $= 50m.$

$CD =$ height of (Pole)

Angle of depression of top of building $\alpha = 45°$

Angle of depression of bottom of building $\beta = 60°$

The above data is represent in the form of figure as shown

In right triangle one of included angle is θ then $\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$

$\tan\alpha = \frac{AX}{CX}$

$\tan 45° = \frac{AX}{CX}$

$AX = CX$

$\tan\beta = \frac{AB}{BD}$

$\tan 60° = \frac{50}{BD}$

$CX = \frac{50}{\sqrt{3}}$

$AX = \frac{50}{3} m = BD$

$CD + AB - AX = 50 - \frac{50}{\sqrt{3}} = \frac{50\left(\sqrt{3}-1\right)}{\sqrt{3}}$

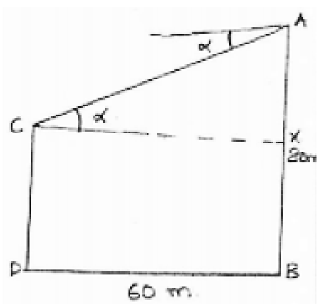
$= \frac{50}{3}\left(3-\sqrt{3}\right)$

Height of building (pole) $= \frac{50}{3}\left(3-\sqrt{3}\right) m.$

Distance between pole and tower $= \frac{50}{\sqrt{3}} m.$

47. The horizontal distance between two trees of different heights is 60 m. The angle of depression of the top of the first tree when seen from the top of the second tree is 45°. If the height of the second tree is 80 m, find the height of the first tree.

Sol:



Distance between trees $= 60m.[80]$

Height of second tree $= 80m[CD]$

Let height of first tree $= 'h' m[AB]$

Angle of depression from second tree top from first tree top $\alpha = 45°$

The above information is represent in form of figure as shown

In right triangle if one of the included angle is 0 their

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

Draw $CX \perp AB, CX = BD = 60n.$

$XB = CD = AB - AX$

$\tan\alpha = \frac{AX}{CX}$

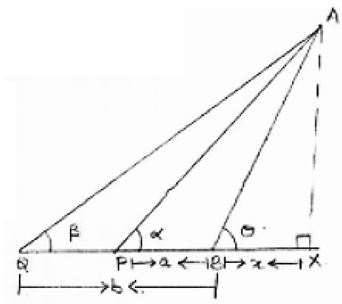
$\tan 45° = \frac{AX}{60} \Rightarrow AX = 60m.$

$XB = CD = AB - AX$

$= 80 - 60$
$= 20m$
Height of second tree $= 80m$
Height of first tree $= 20m$

48. A tree standing on a horizontal plane is leaning towards east. At two points situated at distances a and b exactly due west on it, the angles of elevation of the top are respectively α and β Prove that the height of the op from the ground is $\frac{(b-a)\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$

Sol:



AB be the tree leaning east
From distance $'a'm$ from tree, Angle of elevation be α at point P.
From distance $'b'm$ from tree, Angle of elevation be β at point Q.
The above data is represented in the form of figure as shown in right triangle if one of the included angle is θ then

$$\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$$

Draw $AX \perp QB$ let $BX = 'a'm$

$$\tan\alpha = \frac{AX}{PX}$$
$$\tan\alpha = \frac{AX}{x+a}$$
$$\cot\alpha = \frac{x+a}{Ax}$$
$$x + B = AX\cot \qquad \text{...............(1)}$$
$$\tan\beta = \frac{AX}{QX}$$
$$\tan\beta = \frac{AX}{x+b}$$
$$\cot\beta = \frac{x+b}{AX}$$
$$x + B = AX\cot\beta \qquad \text{...............(2)}$$

(2) and (1) $\Rightarrow (x+b)-(x+a) = AX \cot\beta - AX \cot\alpha$

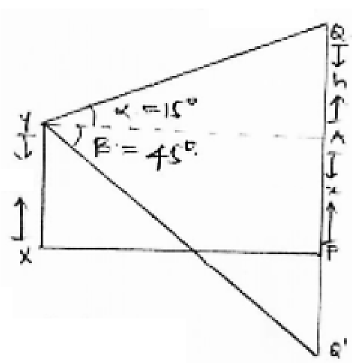
$$\Rightarrow b-a = AX\left[\frac{\tan\alpha - \tan\beta}{\tan\alpha \cdot \tan\beta}\right]$$

$$\Rightarrow AX = \frac{(b-a)\tan\alpha \cdot \tan\beta}{\tan\alpha - \tan\beta}$$

$\therefore$ Height of top from ground $= \dfrac{(b-0)\tan\alpha \cdot \tan\beta}{\tan\alpha - \tan\beta}$

49. The angle of elevation of the top of a vertical tower PQ from a point X on the ground is 60°. At a point Y, 40 m vertically above X, the angle of elevation of the top is 45°. Calculate the height of the tower.
Sol:

50. The angle of elevation of a stationery cloud from a point 2500 m above a lake is 15° and the angle of depression of its reflection in the lake is 45°. What is the height of the cloud above the lake level? (Use tan 15° = 0.268)
Sol:



Let cloud be at height PQ as represented from lake level

From point x, 2500 meters above the lake angle of elevation of top of cloud $\alpha = 15°$

Angle of depression of shadow reflection in water $\beta = 45°$

Here $PQ = PQ'$ draw $AY \perp PQ$

Let $AQ = 'h' m AP = 'x' m.$

$PQ = (h+x)m \quad PQ' = (h+x)m$

The above data is represented in from of figure as shown

In right triangle if one of included angle is θ then $\boxed{\tan\theta = \dfrac{\text{Opposite side}}{\text{Adjacent side}}}$

$$\tan 15° = \frac{AQ}{AY}$$

$\Rightarrow 0\cdot 268 = \dfrac{h}{AY}$

$\Rightarrow AY = \dfrac{h}{0\cdot 268}$ ………(1)

$\tan 45° = \dfrac{AB'}{AY} = \dfrac{AP + PQ'}{AY}$

$\Rightarrow AY = x + (h + x)$

$= h + 2x$

$\Rightarrow AY = h + 2x$ ………..(2)

From (1) and (2) $\dfrac{h}{0\cdot 268} = h + 2x \Rightarrow 3\cdot 131h - h = 2 \times 2500$

$\Rightarrow h = \dfrac{5000}{0\cdot 731} = 1830\cdot 8312$

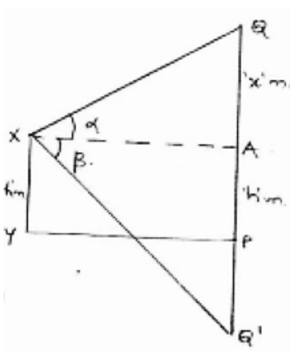
Height of cloud above lake $= h + x$

$= 1830\cdot 8312 + 2500$

$= 4300\cdot 8312\ m$

51. If the angle of elevation of a cloud from a point h meters above a lake is a and the angle of depression of its reflection in the lake be b, prove that the distance of the cloud from the point of observation is $\frac{2\,h \sec\alpha}{\tan\beta - \tan\alpha}$

Sol:



Let x be point $'b'$ meters above lake

Angle of elevation of cloud from $X = \alpha$

Angle of depression of cloud refection in lake $= \beta$

Height of cloud from lake $= PQ$

PQ' be the reflection then $PQ' = PQ$

Draw $XA \perp PQ, AQ = 'x'm \quad AP = XY = 'h'm.$

Distance of cloud from point of observation is XQ

The above data is represented in form of figure as shown

In ΔAQX

$$\tan\alpha = \frac{AQ}{AX}$$

$$\tan\alpha = \frac{x}{AX} \qquad \ldots\ldots\ldots..(1)$$

In $\Delta AXQ'$

$$\tan\beta = \frac{AQ'}{AX}$$

$$\tan\beta = \frac{h+x+h}{AX} \qquad \ldots\ldots\ldots..(2)$$

(2) and (1) $\Rightarrow \tan\beta - \tan\alpha = \frac{2h}{AX} \Rightarrow AX = \frac{2h}{\tan\beta - \tan\alpha}$

In ΔAXQ

$$\cos\alpha = \frac{AX}{XQ} \Rightarrow QX = AX \sec$$

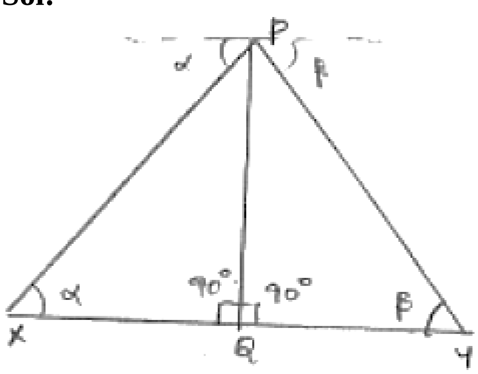
$$\Rightarrow XQ = \frac{2h\sec}{\tan\beta - \tan\alpha}$$

$\therefore$ Distance of cloud from point of observation

$= 2h\sec\alpha / \tan\beta - \tan\alpha$

52. From an aeroplane vertically above a straight horizontal road, the angles of depression of two consecutive mile stones on opposite sides of the aeroplane are observed to be α and β Show that the height in miles of aeroplane above the road is given by $\frac{\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$

Sol:



Let PQ be height of aeroplane from ground x and y be two mile stones on opposite sides of the aeroplane $xy = 1$ mile

Angle of depression of x from $p = \alpha$

Angle of depression of y from $p = \beta$

The above data is represented in form of figure as shown

In right triangle, if one of included angle is θ then $\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$

In $\Delta P \times Q$

$$\tan\alpha = \frac{PQ}{XQ}$$

$$XQ = \frac{PQ}{\tan\alpha}$$

In PQY

$$\tan\beta = \frac{PQ}{QY}$$

$$QY = \frac{PQ}{QY}$$

$$XQ + QY = \frac{PQ}{\tan\alpha} + \frac{PQ}{\tan\beta} \Rightarrow XY = PQ\left[\frac{1}{\tan\alpha} + \frac{1}{\tan\beta}\right]$$

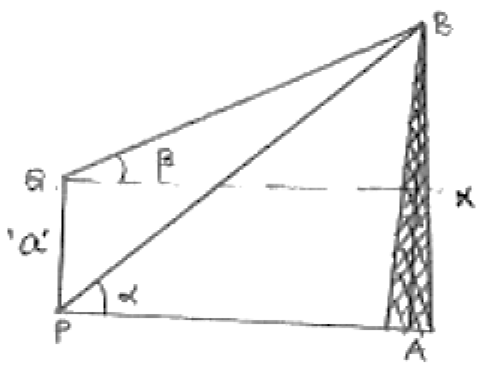
$$\Rightarrow 1 = PQ\left[\frac{\tan\alpha + \tan\beta}{\tan\alpha \cdot \tan\beta}\right]$$

$$\Rightarrow PQ = \frac{\tan\alpha \cdot \tan\beta}{\tan\alpha + \tan\beta}$$

Height of aeroplane $= \dfrac{\tan\alpha \cdot \tan\beta}{\tan\alpha + \tan\beta}$ miles

53. PQ is a post of given height a, and AB is a tower at some distance. If α and β are the angles of elevation of B, the top of the tower, at P and Q respectively. Find the height of the tower and its distance from the post.

Sol:



PQ is part height $= 'a' m$ AB is tower height

Angle of elevation of B from $P = \alpha$

Angle of elevation of B from $Q = \beta$

The above information is represented in form of figure as shown

In right triangle if one of the included angle is θ, then $\boxed{\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}}$

Draw $QX \perp AB, PQ = AK$

In ΔBQX

$$\tan\beta = \frac{BX}{QX}$$

$$\Rightarrow \tan\beta = \frac{AB - AX}{QX}$$

$$\Rightarrow \tan\beta = \frac{AB - a}{QX} \quad \text{..........(1)}$$

In ΔBPA

$$\tan\alpha = \frac{AB}{AP}$$

$$\Rightarrow \tan\beta = \frac{AB}{QX} \quad \text{..........(2)}$$

(1) divided by (2)

$$\Rightarrow \frac{\tan\beta}{\tan\alpha} = \frac{AB - a}{AB} = 1 - \frac{a}{AB}$$

$$\Rightarrow \frac{a}{AB} = 1 - \frac{\tan\beta}{\tan\alpha} = \frac{\tan\alpha - \tan\beta}{\tan\alpha}$$

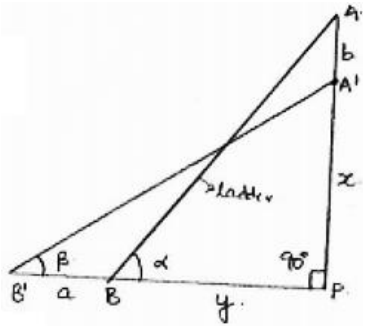
$$\Rightarrow \boxed{AB = \frac{a\tan\alpha}{\tan\alpha - \tan\beta}} Q \times \frac{AB}{\tan\alpha} = \frac{a}{\tan\alpha - \tan\beta}$$

Height of power $= a\tan\alpha\left(\tan\alpha - \tan\beta\right)$

Distance between past and tower $= a\left(\tan\alpha - \tan\beta\right)$

54. A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance a, so that it slides a distance b down the wall making an angle β with the horizontal. Show that $\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$

Sol:



Let AB be ladder initially at an inclination α to ground
When its foot is pulled through distance $'a'$ let $BB' = 'a'm$ and $AA' = 'b'm$
New angle of elevation from $B' = B$ the above information is represented in form of figure as shown
Let $AP \perp$ ground $B'P$ $\quad AB = A'B'$
$A'P = x \qquad BP = y$
In ΔABP

$$\sin\alpha = \frac{AP}{AB} \Rightarrow \sin\alpha = \frac{x+b}{AB} \quad \ldots\ldots\ldots(1)$$

$$\cos\alpha = \frac{BP}{AB} \Rightarrow \cos\alpha = \frac{y}{AB} \quad \ldots\ldots\ldots(2)$$

In $\Delta A'B'P$.

$$\sin\beta = \frac{A'P}{A'B'} \Rightarrow \sin\beta = \frac{x}{AB} \quad \ldots\ldots\ldots(3)$$

$$\cos\beta = \frac{B'P}{A'B'} \Rightarrow \cos\beta = \frac{y+a}{AB} \quad \ldots\ldots\ldots(4)$$

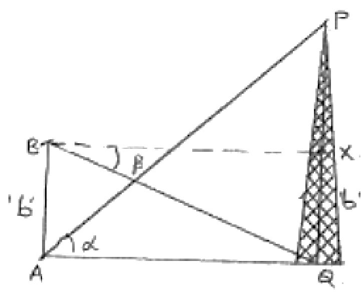
(1) and (3) $\Rightarrow \sin\alpha - \sin\beta = \frac{b}{AB}$

(4) and (2) $\Rightarrow \cos\beta - \cos\alpha = \frac{a}{AB}$

$$\Rightarrow \boxed{\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}}$$

55. A tower subtends an angle α at a point A in the plane of its base and the angle if depression of the foot of the tower at a point b metres just above A is β. Prove that the height of the tower is b tan α cot β

Sol:



Let height of tower be $'h'm = PQ$
Angle of elevation at point A on ground $= \alpha$
Let B be point $'b'm$ above the A.

Angle of depression of foot of tower from $B=\beta$ the above data is represented in ffrom of figure as shown draw $BX \perp PQ$ from figure $QX = 'b'm$

In ΔPBX

$$\tan\alpha = \frac{PQ}{BX\left(AD\right)} \qquad \text{........(1)}$$

In ΔQBX

$$\tan\beta = \frac{QX}{BX} \qquad \text{........(2)}$$

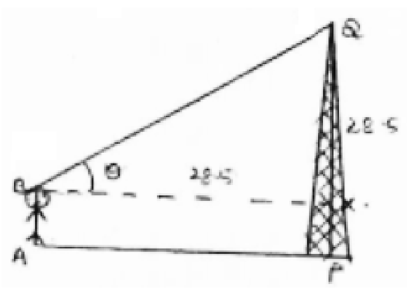
(1) and (2) $\Rightarrow \dfrac{\tan\alpha}{\tan\beta} = \dfrac{PQ}{QX}$

$$\Rightarrow PQ = QX \cdot \frac{\tan\alpha}{\tan\beta} = b \cdot \tan\alpha \cdot \cot\beta$$

$\therefore$ Height of tour $= b \cdot \tan\alpha \cdot \cot\beta$

56. An observer, 1.5 m tall, is 28.5 m away from a tower 30 m high. Determine the angle of elevation of the top of the tower from his eye.

Sol:



Height of observer $= AB = 1\cdot 5m$

Height of tower $= PQ = 30m$

Height of tower above the observe eye $= 30 - 1\cdot 5$

$QX = 28\cdot 5m.$

Distance between tower and observe $XB = 28\cdot 5m.$

θ be angle of elevation of tower top from eye

The above data is represented in form of figure as shown from figure

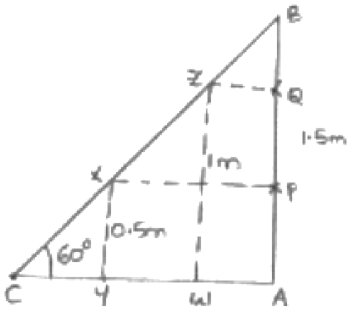
$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$\tan\theta = \frac{QX}{BX} = \frac{28\cdot 5}{28\cdot 5} = 1 \Rightarrow \theta = \tan^{-1}\left(1\right) = 45°$$

Angle of elevation $= 45°$

57. A carpenter makes stools for electricians with a square top of side 0.5 m and at a height of 1.5 m above the ground. Also, each leg is inclined at an angle of 60° to the ground. Find the length of each leg and also the lengths of two steps to be put at equal distances.

Sol:



Let AB be height of stool $= 1\cdot 5m.$

Let $P\ and\ Q$ be equal distance then $AP = 0\cdot 5m, AQ = 1m$ the above information is represented in form of figure as shown

$BC =$ length of leg

$\sin 60° = \frac{AB}{BC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{1\cdot 5}{BC}$

$\Rightarrow BC = \frac{1\cdot 5\times 2}{\sqrt{3}} = \sqrt{3}m.$

Draw $PX \perp AB, QZ \perp AB, XY \perp CA, ZW \perp CA$

$\sin 60° = \frac{XY}{XC}$

$\Rightarrow XC = \frac{0\cdot 5}{\sqrt{3}} \times \sqrt{4}$

$= \left(\frac{\sqrt{3}}{4}\right) \times \frac{8}{3}$

$= \frac{2}{\sqrt{3}}$

$\Rightarrow XC = 1\cdot 1077m.$

$\sin 60° = \frac{ZW}{CZ}$

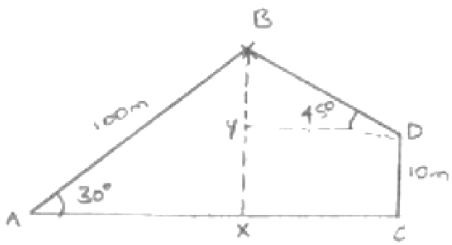
$CZ = \frac{1}{\frac{\sqrt{3}}{2}}$

$= \frac{2}{\sqrt{3}}$

$CZ = 1\cdot 654m.$

58. A boy is standing on the ground and flying a kite with 100 m of string at an elevation of 30°. Another boy is standing on the roof of a 10 m high building and is flying his kite at an elevation of 45°. Both the boys are on opposite sides of both the kites. Find the length of the string that the second boy must have so that the two kites meet.

Sol:



For boy

Length of string $AB = 100m.$

Angle Made by string with ground $= \alpha = 30°$

For boy 2

Height of building $CD = 10m.$

Angle made by string with building top $\beta = 45°$ length of kite thread of boy 2 if both the kites meet must be $'DB'$

The above information is represented in form of figure as shown

Drawn $BX \perp AC, YD \perp BC$

In ΔABX

$$\tan 30° = \frac{BC}{AX}$$

$$\sin 30° = \frac{BX}{AB} \Rightarrow \frac{1}{2} = \frac{BX}{100} \Rightarrow BX = 20m.$$

$$BY = BX - XY = 50 - 10m = 50m.$$

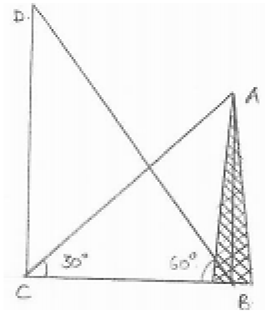
In $\Delta BYD \sin 45° = \frac{BY}{BD}$

$$\frac{1}{\sqrt{2}} = \frac{40}{BD} \Rightarrow BD = 40\sqrt{2}m.$$

Length of thread or string of boy $2 = 40\sqrt{2}m.$

59. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 50 m high, what is the height of the hill?

Sol:



Height of towers $AB = 50m$

Height of hill $CD = 'h'm.$

Angle of elevation of top of hill from of tower $\alpha = 60°.$

Angle of elevation of top of tower from foot of hill $\beta = 30°.$

The above information is represented I form of figure as shown

From figure

In ΔABC

$$\tan 30° = \frac{Opposite\ side}{Adjacent\ side} = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{50}{BC} \Rightarrow BC = 50\sqrt{3}.$$

In ΔBCD

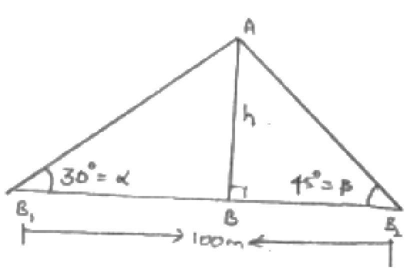
$$\tan 60° = \frac{Opposite\ side}{Adjacent\ side} = \frac{CD}{BC} = \frac{CD}{50\sqrt{3}}$$

$$\sqrt{3} = \frac{CD}{50\sqrt{3}} \Rightarrow CD = 50 \times 3 = 150m$$

Height of hill $= 150m.$

60. Two boats approach a light house in mid-sea from opposite directions. The angles of elevation of the top of the light house from two boats are 30° and 45° respectively. If the distance between two boats is 100 m, find the height of the light house.

Sol:



Let B_1 be boat 1 and B_2 be boat 2.

Height of light house $= 'h'm = AB$

Distance between $B_1B_2 = 100m$

Angle of elevation of A from B_1 $\alpha = 30°$

Angle of elevation of B from B_2 $\beta = 45°$

The above information is represented in the form of figure as shown here

In ΔABB_1

$$\tan 30° = \frac{Opposite\ side}{Adjacent\ side} = \frac{AB}{B_1B}$$

$B_1B = AB\sqrt{3} = h\sqrt{3}$ ………….(1)

In ΔABB_2

$$\tan 30° = \frac{Opposite\ side}{Adjacent\ side} = \frac{AB}{B_1B}$$ ………….(2)

(1) + (2) $\Rightarrow B_1B + BB_2 = h\sqrt{3} + h$

$\Rightarrow B_1B_2 = h\left(\sqrt{3}+1\right)$

$$\Rightarrow h = \frac{B_1B_2}{\sqrt{3}+1} = \frac{100}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{100\left(\sqrt{3}-1\right)}{2} = 50\left(\sqrt{3}-1\right)$$

Height of light house $= 50\left(\sqrt{3}-1\right)$

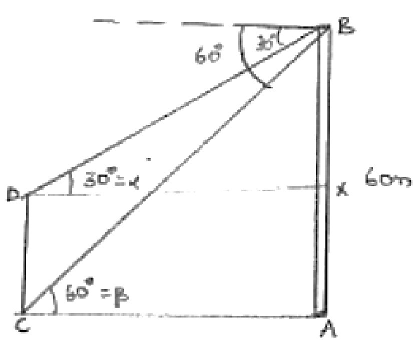
61. From the top of a building AB, 60 m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find

(i) The horizontal distance between AB and CD.

(ii) The height of the lamp post.

(iii) The difference between the heights of the building and the lamp post.

Sol:



Height of building $AB = 60m.$

Height of lamp post $CD = h\,m$

Angle of depression of top of lamp post from top of building $\alpha = 30°$

Angle of depression of bottom of lamp post from top of building $\beta = 60°$

The above information is represented in the form of figure as shown

Draw $DX \perp AB, DX = AC, CD = AX$

In ΔBDX

$$\tan\alpha = \frac{Opposite\ side}{Adjacent\ side} = \frac{BX}{DX}$$

$$\tan 30° = \frac{60 - CD}{DX}$$

$$\frac{1}{\sqrt{3}} = \frac{60 - h}{AC}$$

$$AC = (60 - h)\sqrt{3}m \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

In ΔBCA

$$\tan\beta = \frac{AB}{AC} \Rightarrow \tan 60° = \frac{60}{AC}$$

$$\Rightarrow AC = \frac{60}{\sqrt{3}} = 20\sqrt{3}m \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

From (1) and (2)

$$(60 - h)\sqrt{3} = 20\sqrt{3}$$

$$60 - h = 20$$

$$\Rightarrow h = 40m$$

Height of lamp post $= 40m$

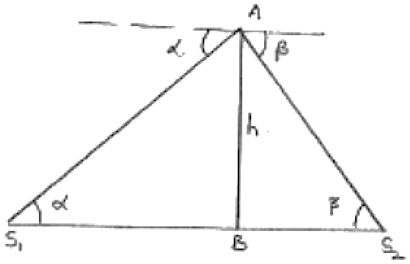
Distance between lamp posts building $AC = 20\sqrt{3}m.$

Difference between heights of building and lamp post

$$= BX = 60 - h = 60 - 40 = 20m$$

62. From the top of a light house, the angles of depression of two ships on the opposite sides of it are observed to be a and 3. If the height of the light house be h meters and the line joining the ships passes through the foot of the light house, show that the distance $\frac{h(\tan\alpha + \tan\beta)}{\tan\alpha\tan\beta}$ metres

Sol:



Height of light house $= 'h'$ meters $= AB$

S_1 *and* S_2 be two ships on opposite sides of light house $= \alpha$

Angle of depression of S_1 from top of light house $= \alpha$

Angle of depression of S_2 from top of light house

Required to prove that

Distance between ships $= \dfrac{h(\tan\alpha + \tan\beta)}{\tan\alpha \cdot \tan\beta}$ meters

The above information is represented in the form of figure as shown

In ΔABS_1

$$\tan\alpha = \frac{Opposite\,side}{Adjacent\,side} = \frac{AB}{S_1B}$$

$$S_1B = \frac{h}{\tan\alpha} \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

In ΔABS_2

$$\tan\beta = \frac{AB}{BS_2} \Rightarrow BS_2 = \frac{h}{\tan\beta} \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

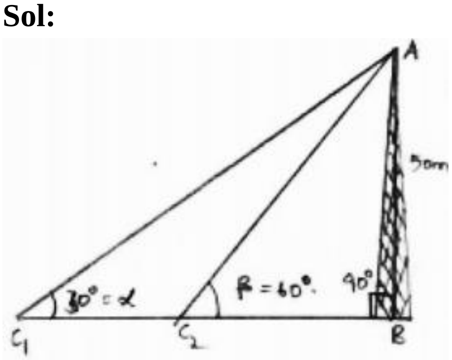
(1) and (2) $\Rightarrow BS_1 + BS_2 = \dfrac{h}{\tan\alpha} + \dfrac{h}{\tan\beta}$

$$\Rightarrow S_1S_2 = h\left\{\frac{1}{\tan\alpha} + \frac{1}{\tan\beta}\right\} = \frac{h(\tan\alpha + \tan\beta)}{\tan\alpha \cdot \tan\beta}$$

$\therefore$ Distance between ships $= \dfrac{h(\tan\alpha + \tan\beta)}{\tan\alpha \cdot \tan\beta}$.meters

63. A straight highway leads to the foot of a tower of height 50 m. From the top of the tower, the angles of depression of two cars standing on the highway are 30° and 60° respectively. What is the distance between the two cars and how far is each car from the tower?

Sol:



Height of towers $AB = 50mts$

C_1 and C_2 be two cars

Angle of depression of C_1 from top of towers $\alpha = 30°$

Angle of depression of C_2 from top of towers $\beta = 60°$

Distance between cars C_1C_2

The above information is represented in form of figure as shown

In ΔABC_2

$$\tan\beta = \frac{Opposite\ side}{Adjacent\ side} = \frac{AB}{BC_2}$$

$$\tan 60° = \frac{50}{BC_1}$$

$$BC_2 = \frac{50}{\sqrt{3}}$$

In ΔABC_1

$$\tan\alpha = \frac{AB}{BC_1}$$

$$\tan 30° = \frac{50}{BC_1} \Rightarrow BC_1 = 50\sqrt{3},$$

$$C_1C_2 = BC_1 - BC_2 = 50\sqrt{3} - \frac{50}{\sqrt{3}} = 50\left(\frac{3-1}{\sqrt{3}}\right) = \frac{100}{\sqrt{3}} = \frac{100}{\sqrt{3}}\sqrt{3}mts.$$

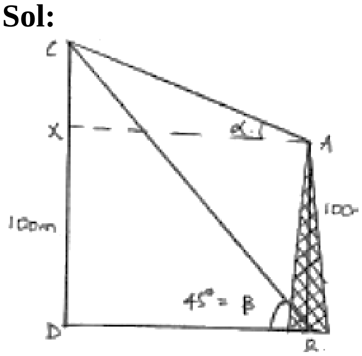
Distance between cars $C_1C_2 = \frac{100}{3}\sqrt{3}\,mts$

Distance of car1 from tower $= 50\sqrt{3}\,mts.$

Distance of car 2 from tower $= \frac{50}{\sqrt{3}}mts$

64. The angles of elevation of the top of a rock from the top and foot of a loo m high tower are respectively 30° and 45°. Find the height of the rock.

Sol:



Height of tour $AB = 100m$

Height of rock $CD = 'h'm$

Angle of elevation of top of root from top of tower $\alpha = 30°$

Angle of elevation of top of root from bottom of tower $\beta = 45°$

The above data is represented in form of figure as shown

Draw $AX \perp CD$

$XD = AB = 100m$

$XA = DB.$

In $\Delta CXA, \quad \tan\alpha = \frac{CX}{AX}$

$\Rightarrow \tan 30° = \frac{CX}{DB}$

$\Rightarrow DB = C \times \sqrt{3}$ …………..(1)

In $\Delta CBD, \qquad \tan\beta = \frac{CD}{DB} = \frac{100 + CX}{DB}$

$\tan 45° = \frac{100+CX}{DB} \Rightarrow DB = 100 + CX$ ……………(2)

From (1) and (2)

$100 + CX = C \times \sqrt{3} \Rightarrow C \times \left(\sqrt{3} - 1\right) = 100$

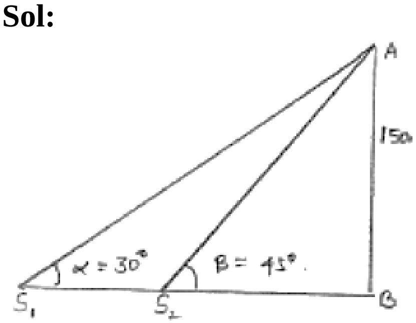
$\Rightarrow CX = \frac{100}{\sqrt{2}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$CX = 50\left(\sqrt{3}+1\right)$

Height of hill $= 100 + 50\left(\sqrt{3}+1\right) = 150\left(3+\sqrt{3}\right) mts.$

65. As observed from the top of a 150 m tall light house, the angles of depression of two ships approaching it are 30° and 45°. If one ship is directly behind the other, find the distance between the two ships

Sol:



Height of light house $AB = 150mts.$

Let S_1 *and* S_2 be two ships approaching each other.

Angle of depression of S_1, $\alpha = 50°$

Angle of depression of S_2, $\beta = 50°$

Distance between ships $= S_1S_2$.

The above data is represented in the form of figure as shown

In ΔABS_2

$$\tan\beta = \frac{AB}{BS_2}$$

$$\tan 45° = \frac{150}{BS_2}$$

$$BS_2 = 150m.$$

In ΔABS_1

$$\tan\alpha = \frac{AB}{BS_1}$$

$$\tan 30° = \frac{150}{BS_1}$$

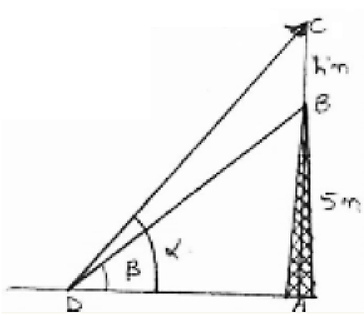
$$BS_1 = 150\sqrt{3}m.$$

$$S_1S_2 = BS_1 - BS_2 = 150\left(\sqrt{3}-1\right)mts$$

Distance between ships $= 150\left(\sqrt{3}-1\right)mts.$

66. A flag-staff stands on the top of a 5 m high tower. From a point on the ground, the angle of elevation of the top of the flag-staff is 60° and from the same point, the angle of elevation of the top of the tower is 45°. Find the height of the flag-staff.

Sol:



Height of tower $= AB = 5m.$

Height of flagstaff $BC = 'h'm$

Angle of elevation of top of flagstaff $a = 60°$

Angle of elevation of bottom of flagstaff $\beta = 45°$

The above data is represented in form of figure as shown

In ΔADB $\tan\beta = \frac{AB}{DA} \Rightarrow \tan 45° = \frac{5}{DA}$

$\Rightarrow DA = 5m.$

In $\Delta ADC, \tan\alpha = \frac{AC}{AD},$

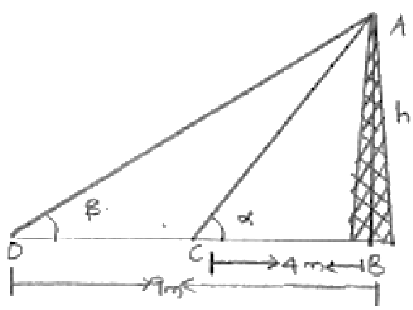
$\tan 60° = \frac{AB+BC}{AD} = \frac{h+5}{5}$

$\sqrt{3} = \frac{h+5}{5}$

$h+5 = 5\sqrt{3} \Rightarrow h = 5\left(\sqrt{3}-1\right) = 5\times 0\cdot 732 = 3\cdot 65$ meters height of flagstaff $= 3\cdot 65$ meters

67. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6m.

Sol:



Height of tower $AB = 'h'$ meters

Let point C be 4 meters from B, Angle of elevation be α given point D be 9 meters from B. Angle of elevation be β. given α, β are complementary, $\alpha + \beta = 90° \Rightarrow \beta = 90° - \alpha$

required to prove that $h = 6$ meters

The above data is represented in the form of figure as shown

In $\Delta ABC, \tan\alpha = \frac{AB}{BC}$

$\tan\alpha = \frac{h}{4}$

$h = 4\tan\alpha$ ………………(1)

In $\Delta ABD, \tan\beta = \frac{AB}{BD} = \frac{h}{9}$

$\tan\left(90-\alpha\right) = \frac{h}{9}$

$h = 4\tan\alpha$ ………………(2)

Multiply (1) and (2) $h\times h = 4\tan\alpha \times 9\cot\alpha$

$= 36\left(\tan\alpha \cdot \cot\alpha\right)$

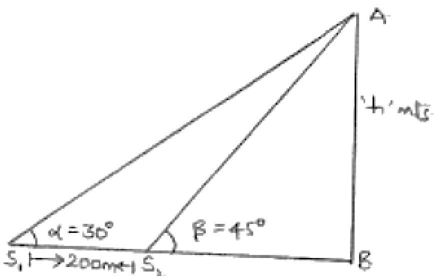
$h^2 = 36$

$h = \sqrt{36} = 6$ meters.

$\therefore$ height of tower $= 6$ meters.

68. The angles of depression of two ships from the top of a light house and on the same side of it are found to be 45° and 30° respectively. If the ships are 200 m apart, find the height of the light house.

Sol:



Height of light house $AB = 'h'$ meters

Let S_1 *and* S_2 be ships distance between ships S_1S_2

Angle of depression of S_1 $[\alpha = 30°]$

Angle of depression of S_2 $[\beta = 45°]$

The above data is represented in form of figure as shown

In ΔABS_2

$$\tan\beta = \frac{AB}{BC_2}$$

$$\tan 45° = \frac{h}{BS_2}$$

$BS_2 = h$ ………..(1)

In ΔABS_1

$$\tan\alpha = \frac{AB}{BS_1}$$

$$\tan 30° = \frac{h}{BS_2}$$

$BS_1 = h\sqrt{3}$ ………..(2)

(2) and (1) $\Rightarrow BS_1 - BS_2 = h\left(\sqrt{3}-1\right)$

$$\Rightarrow 200 = h\left(\sqrt{3}-1\right)$$

$$\Rightarrow h = \frac{200}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{200}{2}\left(\sqrt{3}+1\right) = 100\left(\sqrt{3}+1\right) \text{ meters}$$

$$h = 100(1\cdot 732+1) = 273\cdot 2 \text{ meters}$$

Height of light house $= 273\cdot 2$ meters

Exercise – 13.1

1. The probability that it will rain tomorrow is 0.85. What is the probability that it will not rain tomorrow?

Sol:

Let E be the event of happening of rain

P(E) is given as 0.85

$\bar{E} \longrightarrow not\ happening\ of\ rain$

P($\bar{E}$) = 1 – P(E) = 1 – 0.85 = 0.15

∴ P(not happening of rain) = 0.15

2. A die is thrown. Find the probability of getting:

(i) a prime number

(ii) 2 or 4

(iii) a multiple of 2 or 3

(iv) an even prime number

(v) a number greater than 5

(vi) a number lying between 2 and 6

Sol:

(i) Total no of possible outcomes = 6 {1, 2, 3, 4, 5, 6}

E ⟶ Event of getting a prime no.

No. of favorable outcomes = 3 {2, 3, 5}

$P(E) = \frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

P(E) = $\frac{3}{6} = \frac{1}{2}$

(ii) E ⟶ Event of getting 2 or 4.

No. of favorable outcomes = 2 {2, 4}

Total no.of possible outcomes = 6

Then, P(E) = $\frac{2}{6} = \frac{1}{3}$

(iii) E ⟶ Event of getting a multiple of 2 or 3

No. of favorable outcomes = 4 {2, 3, 4, 6}

Total no.of possible outcomes = 6

Then, P(E) = $\frac{4}{6} = \frac{2}{3}$

(iv) E ⟶ Event of getting an even prime no.

No. of favorable outcomes = 1 {2}

Total no.of possible outcomes = 6 {1, 2, 3, 4, 5, 6}

P(E) = $\frac{1}{6}$

(v) E ⟶ Event of getting a no. greater than 5.

No. of favorable outcomes = 1 {6}

Total no.of possible outcomes = 6

P(E) = $\frac{1}{6}$

(vi) E ⟶ Event of getting a no. lying between 2 and 6.

No. of favorable outcomes = 3 {3, 4, 5}

Total no.of possible outcomes = 6

P(E) = $\frac{3}{6} = \frac{1}{2}$

3. In a simultaneous throw of a pair of dice, find the probability of getting:

(i) 8 as the sum

(ii) a doublet

(iii) a doublet of prime numbers

(iv) a doublet of odd numbers

(v) a sum greater than 9

(vi) an even number on first

(vii) an even number on one and a multiple of 3 on the other

(viii) neither 9 nor 1 1 as the sum of the numbers on the faces

(ix) a sum less than 6

(x) a sum less than 7

(xi) a sum more than 7

(xii) at least once

(xiii) a number other than 5 on any dice.

Sol:

In a throw of pair of dice, total no of possible outcomes = 36 (6×6) which are

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)

(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

(i) Let E be event of getting the sum as 8

No. of favorable outcomes = 5 {(2, 6) (3, 5) (4, 4) (5, 3) (6, 2)}

We know that, Probability P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

P(E) = $\frac{5}{36}$

(ii) E ⟶ event of getting a doublet

No. of favorable outcomes = 5 {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)}

Total no. of possible outcomes = 36

P(E) = $\frac{6}{36} = \frac{1}{6}$

(iii) E ⟶ event of getting a doublet of prime no's

No. of favorable outcomes = 3 {(2, 2) (3, 3) (5, 5)}

Total no. of possible outcomes = 36

P(E) = $\frac{3}{36} = \frac{1}{12}$

(iv) E ⟶ event of getting a doublet of odd no's

No. of favorable outcomes = 3 {(1, 1) (3, 3) (5, 5)}

Total no. of possible outcomes = 36

P(E) = $\frac{3}{36} = \frac{1}{12}$

(v) E ⟶ event of getting a sum greater than 9

No. of favorable outcomes = 6 {(4, 6) (5, 5) (5, 6) (6, 4) (6, 5) (6, 6)}

Total no. of possible outcomes = 36

P(E) = $\frac{6}{36} = \frac{1}{6}$

(vi) E ⟶ event of getting an even no. on first

No. of favorable outcomes = 18 {(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6), (4, 1) , (4, 2) , (4, 3) , (4, 4) , (4, 5) , (4, 6) , (6, 1) , (6, 2) , (6, 3) , (6, 4) , (6, 5) , (6, 6)}

Total no. of possible outcomes = 36

P(E) = $\frac{18}{36} = \frac{1}{2}$

(vii) E ⟶ event of getting an even no. on one and a multiple of 3 on other

No. of favorable outcomes = 11 {(2, 3) (2, 6) (4, 3) (4, 6) (6, 3) (6, 6), (3, 2), (3, 4), (3, 4) , (3, 6) , (6, 2) , (6, 4)}

Total no. of possible outcomes = 36

P(E) = $\frac{11}{36}$

(viii) $\overline{E}$ ⟶ event of getting neither 9 nor 11 as the sum of numbers on faces

E ⟶ getting either 9 or 11 as the sum of no's on faces

No. of favorable outcomes = 6 {(3, 6) (4, 5) (5, 4) (6, 3) (5, 6) (6, 5)}

Total no. of possible outcomes = 36

P(E) = $\frac{6}{36} = \frac{1}{6}$

P$\left(\overline{E}\right) = 1 - P = 1 - \frac{1}{6} = \frac{5}{6}$

(ix) E ⟶ event of getting a sum less than 6

No. of favorable outcomes = 10 {(1, 1) (1, 2) (1, 3) (1, 4) (2, 1) (2, 2) , (2, 3) , (3, 1) , (3, 2) , (4, 1)}

Total no. of possible outcomes = 36

P(E) = $\frac{10}{36} = \frac{5}{18}$

(x) E ⟶ event of getting a sum less than 7

No. of favorable outcomes = 15 {(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (2, 1), (2, 2) (2, 3) (2, 4) (3, 1) (3, 2) (3, 3) (4, 1) (4, 2) (5, 1)}

Total no. of possible outcomes = 36

P(E) = $\frac{15}{36} = \frac{5}{12}$

(xi) E ⟶ event of getting a sum more than 7

No. of favorable outcomes = 15 {(2, 6) (3, 5) (3, 6) (4, 4) (4, 5) (4, 6), (5, 3) (5, 4) (5, 5) (5, 6) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)}

Total no. of possible outcomes = 36

$P(E) = \frac{15}{36} = \frac{5}{12}$

(xii) E ⟶ event of getting a 1 at least once

No. of favorable outcomes = 11 {(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6), (2, 1) (3, 1) (4, 1) (5, 1) (6, 1)}

Total no. of possible outcomes = 36

$P(E) = \frac{11}{36}$

(xiii) E ⟶ event of getting a no other than 5 on any dice

No. of favourable outcomes = 25 {(1, 1) (1, 2) (1, 3) (1, 4) (1, 6) (2, 1), (2, 2) (2, 3) (2, 4) (2, 6) (3, 1) (3, 2) (3, 3) (3, 4) (3, 6) (4, 1) (4, 2) (4, 3) (4, 4) (4, 6) (6, 1) (6, 2) (6, 3) (6, 4) (6, 6)}

Total no. of possible outcomes = 36

$P(E) = \frac{25}{36}$

4. Three coins are tossed together. Find the probability of getting:

(i) exactly two heads
(ii) at least two heads
(iii) at least one head and one tail
(iv) no tails

Sol:

When 3 coins are tossed together,

Total no. of possible outcomes = 8 {HHH, HHT, HTH, HTT, THH,THT, TTH, TTT}

(i) Probability of an event = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

Let E ⟶ event of getting exactly two heads

No. of favourable outcomes = 3 {HHT, HTH, THH}

Total no. of possible outcomes = 8

$P(E) = \frac{3}{8}$

(ii) E ⟶ getting at least 2 Heads

No. of favourable outcomes = 4 {HHH, HHT, HTH, THH}

Total no. of possible outcomes = 8

$P(E) = \frac{4}{8} = \frac{1}{2}$

(iii) E⟶ getting at least one Head & one Tail

No. of favourable outcomes = 6 {HHT, HTH, HTT, THH, THT, TTH}

Total no. of possible outcomes = 8

$P(E) = \frac{6}{8} = \frac{3}{4}$

(iv) E ⟶ getting no tails

No. of favourable outcomes = 1 {HHH}

Total no. of possible outcomes = 8

$P(E) = \frac{1}{8}$

5. What is the probability that an ordinary year has 53 Sundays?

Sol:

Ordinary year has 365 days

365 days = 52 weeks + 1 day

That 1 day may be Sun, Mon, Tue, Wed, Thu, Fri, Sat

Total no. of possible outcomes = 7

Let E ⟶ event of getting 53 Sundays

No. of favourable outcomes = 1 {Sun}

P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{1}{7}$

6. What is the probability that a leap year has 53 Sundays and 53 Mondays?

Sol:

A leap year has 366 days

366 days = 52 weeks + 2 days

That 2 days may be (Sun, Mon) (Mon, Tue) (Tue, Wed) (Wed, Thu) (Thu, Fri) (Fri, Sat) (Sat, Sun)

Let E ⟶ event of getting 53 Sundays & 53 Mondays.

No. of favourable outcomes = 1 {(Sun, Mon)}

Since 52 weeks has 52 Sundays & 52 Mondays & the extra 2 days must be Sunday & Monday.

Total no. of possible outcomes = 7

P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{1}{7}$

7. A and B throw a pair of dice. If A throws 9, find B's chance of throwing a higher number.

Sol:

When a pair of dice are thrown, then total no. of possible outcomes = 6 × 6 = 36, which are

{ (1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)

(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6) }

E ⟶ event of throwing a no. higher than 9.

No. of favourable outcomes = 6 {(4, 6) (5, 5) (6, 4) (5, 6) (6, 5) (6, 6)}

We know that P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

i.e., P(E) = $\frac{6}{36} = \frac{1}{6}$

8. Two unbiased dice are thrown. Find the probability that the total of the numbers on the dice is greater than 10.

Sol:

When a pair of dice are thrown, then total no. of possible outcomes = $6 \times 6 = 36$

let E $\longrightarrow$ event of getting sum on dice greater than 10

then no of favourable outcomes = 3 {(5, 6) (6, 5) (6, 6)}

we know that, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

i.e., P(E) = $\frac{3}{36} = \frac{1}{12}$

9. A card is drawn at random from a pack of 52 cards. Find the probability that card drawn is

(i) a black king
(ii) either a black card or a king
(iii) black and a king
(iv) a jack, queen or a king
(v) neither a heart nor a king
(vi) spade or an ace
(vii) neither an ace nor a king
(viii) Neither a red card nor a queen.
(ix) other than an ace
(x) a ten
(xi) a spade
(xii) a black card
(xiii) the seven of clubs
(xiv) jack
(xv) the ace of spades
(xvi) a queen

Sol:

Total no. of outcomes = 52 {52 cards}

(i) E$\longrightarrow$ event of getting a black king

No of favourable outcomes = 2{king of spades & king of clubs}

We know that, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{2}{52} = \frac{1}{26}$

(ii) E$\longrightarrow$ event of getting either a black card or a king.

No. of favourable outcomes = 26 + 2 {13 spades, 13 clubs, king of hearts & diamonds}

P(E) = $\frac{26+2}{52} = \frac{28}{52} = \frac{7}{13}$

(iii) E$\longrightarrow$ event of getting black & a king.

No. of favourable outcomes = 2 {king of spades & clubs}

P(E) = $\frac{2}{52} = \frac{1}{26}$

(iv) E$\longrightarrow$ event of getting a jack, queen or a king

No. of favourable outcomes = 4 + 4 + 4 = 12 {4 jacks, 4 queens & 4 kings}

P(E) = $\frac{12}{52} = \frac{3}{13}$

(v) E$\longrightarrow$ event of getting neither a heart nor a king.

No. of favourable outcomes = 52 – 13 – 3 = 36 {since we have 13 hearts, 3 kings each of spades, clubs & diamonds}

P(E) = $\frac{36}{52} = \frac{9}{13}$

(vi) E⟶ event of getting spade or an all.
No. of favourable outcomes = 13 + 3 = 16 {13 spades & 3 aces each of hearts, diamonds & clubs}
P(E) = $\frac{16}{52} = \frac{4}{13}$

(vii) E⟶ event of getting neither an ace nor a king.
No. of favourable outcomes = 52 – 4 – 4 = 44 {Since we have 4 aces & 4 kings}
P(E) = $\frac{44}{52} = \frac{11}{13}$

(viii) E⟶ event of getting neither a red card nor a queen.
No. of favourable outcomes = 52 – 26 – 2 = 24 {Since we have 26 red cards of hearts & diamonds & 2 queens each of heart & diamond}
P(E) = $\frac{24}{52} = \frac{6}{13}$

(ix) E⟶ event of getting card other than an ace.
No. of favourable outcomes = 52 – 4 = 48 {Since we have 4 ace cards}
P(E) = $\frac{48}{52} = \frac{12}{13}$

(x) E⟶ event of getting a ten.
No. of favourable outcomes = 4 {10 of spades, clubs, diamonds & hearts}
P(E) = $\frac{4}{52} = \frac{1}{13}$

(xi) E⟶ event of getting a spade.
No. of favourable outcomes = 13 {13 spades}
P(E) = $\frac{13}{52} = \frac{1}{24}$

(xii) E⟶ event of getting a black card.
No. of favourable outcomes = 26 {13 cards of spades & 13 cards of clubs}
P(E) = $\frac{26}{52} = \frac{1}{2}$

(xiii) E⟶ event of getting 7 of clubs.
No. of favourable outcomes = 1 {7 of clubs}
P(E) = $\frac{1}{52}$

(xiv) E⟶ event of getting a jack.
No. of favourable outcomes = 4 {4 jack cards}
P(E) = $\frac{4}{52} = \frac{1}{13}$

(xv) E⟶ event of getting the ace of spades.
No. of favourable outcomes = 1{ace of spades}
P(E) = $\frac{1}{52}$

(xvi) E⟶ event of getting a queen.
No. of favourable outcomes = 4 {4 queens}
P(E) = $\frac{4}{52} = \frac{1}{13}$

(xvii) E⟶ event of getting a heart.

No. of favourable outcomes = 13 {13 hearts}

P(E) = $\frac{13}{52} = \frac{1}{4}$

(xviii) E⟶ event of getting a red card.

No. of favourable outcomes = 26 {13 hearts, 13 diamonds}

P(E) = $\frac{26}{52} = \frac{1}{2}$

10. In a lottery of 50 tickets numbered 1 to 50, one ticket is drawn. Find the probability that the drawn ticket bears a prime number.

Sol:

Total no. of possible outcomes = 50 {1, 2, 3, …. , 50}

E⟶ event of getting a prime no.

No. of favourable outcomes = 15

{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47}

Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

i.e. P(E) = $\frac{15}{50} = \frac{3}{10}$

11. An urn contains 10 red and 8 white balls. One ball is drawn at random. Find the probability that the ball drawn is white.

Sol:

Total no of possible outcomes = 18 {10 red balls, 8 white balls}

E ⟶ event of drawing white ball

No. of favourable outcomes = 8 {8 white balls}

Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

= $\frac{8}{18} = \frac{4}{9}$

12. A bag contains 3 red balls, 5 black balls and 4 white balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is:

(i) White
(ii) Red
(iii) Black
(iv) Not red

Sol:

Total number of possible outcomes = 12 {3 red balls, 5 black balls & 4 white balls}

(i) E ⟶ event of getting white ball

No. of favourable outcomes = 4 {4 white balls}

Probability, P(E) = $\frac{4}{12} = \frac{1}{3}$

(ii) E ⟶ event of getting red ball

No. of favourable outcomes = 5 {3 red balls}

P (E) = $\frac{3}{12} = \frac{1}{4}$

(iii) E ⟶ event of getting black ball
No. of favourable outcomes = 5 {5 black balls}
P (E) = $\frac{5}{12}$

(iv) E ⟶ event of getting red
No. of favourable outcomes = 3 {3 black balls}
P(E) = $\frac{3}{12} = \frac{1}{4}$
($\bar{E}$) ⟶ event of not getting red.
P($\bar{E}$) = 1 – P(E)
= 1 – $\frac{1}{4}$
= $\frac{3}{4}$

13. What is the probability that a number selected from the numbers 1, 2, 3, ..., 15 is a multiple of 4?
Sol:
Total no. possible outcomes = 15 {1, 2, 3, …. , 15}
E ⟶ event of getting a multiple of 4
No. of favourable outcomes = 3 {4, 8, 12}
Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{3}{15} = \frac{1}{5}$

14. A bag contains 6 red, 8 black and 4 white balls. A ball is drawn at random. What is the probability that ball drawn is not black?
Sol:
Total no. of possible outcomes = 18 {6 red, 8 black, 4 white}
Let E ⟶ event of drawing black ball.
No. of favourable outcomes = 8 {8 black balls}
Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{8}{18} = \frac{4}{9}$
$\bar{E}$ ⟶ event of not drawing black ball
$P(\bar{E}) = 1 - P(E)$
$= 1 - \frac{4}{9} = \frac{5}{9}$

15. A bag contains 5 white and 7 red balls. One ball is drawn at random. What is the probability that ball drawn is white?
Sol:
Total no. of possible outcomes = 12 {5 white, 7 red}
E ⟶ event of drawing white ball.
No. of favorable outcomes = 5 {white balls are 5}

Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

P(E) = $\frac{5}{12}$

16. Tickets numbered from 1 to 20 are mixed up and a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 7?

Sol:

Total no. of possible outcomes = 20 {1, 2, 3, …. , 20}

E ⟶ event of drawing ticket with no multiple of 3 or 7

No. of favourable outcomes = 8 which are {3, 6, 9, 12, 15, 18, 7, 14}

Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{8}{20} = \frac{2}{5}$

17. In a lottery there are 10 prizes and 25 blanks. What is the probability of getting a prize?

Sol:

Total no. of possible outcomes = 35 {10 prizes, 25 blanks}

E ⟶ event of getting prize

No. of favourable outcomes = 10 {10 prizes}

Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{10}{35} = \frac{2}{7}$

18. If the probability of winning a game is 0.3, what is the probability of losing it?

Sol:

E ⟶ event of winning a game

P(E) is given as 0.3

$(\bar{E})$ ⟶ event of loosing the game

we know that P(E) + P($\bar{E}$) = 1

$P(\bar{E}) = 1 - P(E)$

= 1 – 0.3 = 0.7

19. A bag contains 5 black, 7 red and 3 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is:

(i) Red (ii) black or white (iii) not black

Sol:

Total no. of possible outcomes = 15

{5 black, 7 red & 3 white balls}

(i) E ⟶ event of drawing red ball

No. of favorable outcomes = 7 {7 red balls}

Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

P(E) = $\frac{7}{15}$

(ii) E ⟶ event of drawing black or white
No. of favourable outcomes = 8 {5 black & 3 white}
P(E) = $\frac{8}{15}$

(iii) E ⟶ event of drawing black ball
No. of favourable outcomes = 5 {5 black balls}
P (E) = $\frac{5}{15} = \frac{1}{3}$
$\bar{E} \longrightarrow event\ of\ not\ drawing\ black\ ball$
P $(\bar{E}) = 1 - P(E)$
$= 1 - \frac{1}{3} = \frac{2}{3}$

20. A bag contains 4 red, 5 black and 6 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is:
(i) White
(ii) Red
(iii) Not black
(iv) Red or white

Sol:
Total no. of possible outcomes = 15 {4 red, 5 black, 6 white balls}

(i) E ⟶ event of drawing white ball.
No. of favourable outcomes = 6 {6 white}
Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$
P(E) = $\frac{6}{15} = \frac{2}{5}$

(ii) E ⟶ event of drawing red ball
No. of favourable outcomes = 4 {4 red balls}
P(E) = $\frac{4}{15}$

(iii) E ⟶ event of drawing black ball
No. of favourable outcomes = 5 {5 black balls}
P(E) = $\frac{5}{15} = \frac{1}{3}$
$\bar{E}$ ⟶ event of not drawing black ball
$P(\bar{E}) = 1 - \frac{1}{3} = \frac{2}{3}$

(iv) E ⟶ event of drawing red or white ball
No. of favourable outcomes = 10 {4 red & 6 white}
P(E) = $\frac{10}{15} = \frac{2}{3}$

21. A black die and a white die are thrown at the same time. Write all the possible outcomes. What is the probability?
(i) that the sum of the two numbers that turn up is 8?
(ii) of obtaining a total of 6?

(iii) of obtaining a total of 10?
(iv) of obtaining the same number on both dice?
(v) of obtaining a total more than 9?
(vi) that the sum of the two numbers appearing on the top of the dice is 13?
(vii) that the sum of the numbers appearing on the top of the dice is less than or equal to 12?

Sol:

Total no. of possible outcomes when 2 dice are thrown = $6 \times 6 = 36$ which are

{ (1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6) }

(i) E ⟶ event of getting sum that turn up is 8
No. of possible outcomes = 36
No. of favourable outcomes = 5 {(2, 6) (3, 5) (4, 4) (5, 3) (6, 2)}
P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{5}{36}$

(ii) Let E ⟶ event of obtaining a total of 6
No. of favourable outcomes = 5
{(1, 5) (2, 4) (3, 3) (4, 2) (5, 1)}
P(E) = $\frac{5}{36}$

(iii) Let E ⟶ event of obtaining a total of 10.
No. of favourable outcomes = 3 {(4, 6) (5, 5) (6, 4)}
P(E) = $\frac{3}{36} = \frac{1}{12}$

(iv) Let E ⟶ event of obtaining the same no. on both dice
No. of favourable outcomes = 6 {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)}
P(E) = $\frac{3}{36} = \frac{1}{12}$

(v) E ⟶ event of obtaining a total more than 9
No. of favourable outcomes = 6 {(4, 6) (5, 5) (6, 4) (5, 6) (6, 5) (6, 6)}
P(E) = $\frac{6}{36} = \frac{1}{6}$

(vi) The maximum sum is 12 (6 on 1^{st} + 6 on 2^{nd})
So, getting a sum of no's appearing on the top of the two dice as 13 is an impossible event.
∴ Probability is 0

(vii) Since, the sum of the no's appearing on top of 2 dice is always less than or equal to 12, it is a sure event.
Probability of sure event is 1.

So, the required probability is 1.

22. One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting:

(i) a king of red suit
(ii) a face card
(iii) a red face card
(iv) a queen of black suit
(v) a jack of hearts
(vi) a spade

Sol:

Total no. of possible outcomes = 52 (52 cards)

(i) E ⟶ event of getting a king of red suit
No. of favourable outcomes = 2 {king heart & king of diamond}
P(E), = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{2}{52} = \frac{1}{26}$

(ii) E ⟶ event of getting face card
No. of favourable outcomes = 12 {4 kings, 4 queens & 4 jacks}
P(E) = $\frac{12}{52} = \frac{3}{13}$

(iii) E ⟶ event of getting red face card
No. favourable outcomes = 6 { kings, queens, jacks of hearts & diamonds}
P(E) = $\frac{6}{26} = \frac{3}{26}$

(iv) E ⟶ event of getting a queen of black suit
No. favourable outcomes = 6 { kings, queens, jacks of hearts & diamonds}
P(E) = $\frac{6}{26} = \frac{3}{26}$

(v) E ⟶ event of getting red face card
No. favourable outcomes = 6 { queen of spades & clubs}
P(E) = $\frac{1}{52}$

(vi) E ⟶ event of getting a spade
No. favourable outcomes = 13 {13 spades}
P(E) = $\frac{13}{52} = \frac{1}{4}$

23. Five cards—ten, jack, queen, king, and an ace of diamonds are shuffled face downwards. One card is picked at random.

(i) What is the probability that the card is a queen?
(ii) If a king is drawn first and put aside, what is the probability that the second card picked up is the ace?

Sol:

Total no. of possible outcomes = 5 {5 cards}

(i) E ⟶ event of drawing queen
No. favourable outcomes = 1 {1 queen card}
P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{1}{5}$

(ii) When king is drawn and put aside, total no. of remaining cards = 4
Total no. of possible outcomes = 4
E ⟶ event of drawing ace card
No. favourable outcomes = 1 {1 ace card}
P(E) = $\frac{1}{4}$

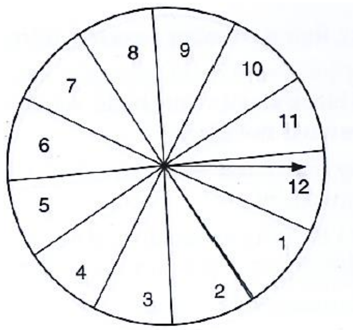
24. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is:
(i) Red
(ii) Black

Sol:
Total no. of possible outcomes = 8 {3 red, 5 black}
(i) Let E ⟶ event of drawing red ball.
No. favourable outcomes = 1 {1 ace card}
P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{3}{8}$
(ii) Let E ⟶ event of drawing black ball.
No. favourable outcomes = 5 {5 black balls}
P(E) = $\frac{5}{8}$

25. A bag contains cards which are numbered from 2 to 90. A card is drawn at random from the bag. Find the probability that it bears.
(i) a two digit number
(ii) a number which is a perfect square

Sol:
Total no. of possible outcomes = 89 {2, 3, 4, …., 90}
(i) Let E ⟶ event of getting a 2 digit no.
No. favourable outcomes = 81 {10, 11, 12, 13, ….., 80}
P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{81}{89}$
(ii) E ⟶ event of getting a no. which is perfect square
No. favourable outcomes = 8 {4, 9, 16, 25, 36, 49, 64, 81}
P(E) = $\frac{8}{89}$

26. A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the number, 1, 2, 3, ..., 12 as shown in Fig. below. What is the probability that it will point to:



(i) 10?

(ii) an odd number?

(iii) a number which is multiple of 3?

(iv) an even number?

Sol:

Total no. of possible outcomes = 12 {1, 2, 3,…., 12}

(i) Let E ⟶ event of pointing 10

No. favourable outcomes = 1 {10}

$P(E) = \frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{1}{12}$

(ii) Let E ⟶ event of pointing at an odd no.

No. favourable outcomes = 6 {1, 3, 5, 7, 9, 11}

$P(E) = \frac{6}{12} = \frac{1}{2}$

(iii) Let E ⟶ event of pointing at a no. multiple of 3

No. favourable outcomes = 4 {3, 6, 9, 12}

$P(E) = \frac{4}{12} = \frac{1}{3}$

(iv) Let E ⟶ event of pointing at an even no.

No. favourable outcomes = 6 {2, 4, 6, 8, 10, 12}

$P(E) = \frac{6}{12} = \frac{1}{2}$

27. Two customers are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another. What is the probability that both will visit the shop on:

(i) the same day? (ii) different days? (iii) consecutive days?

Sol:

Total no. of days to visit the shop = 6 {Mon to Sat}

Total no. possible outcomes = $6 \times 6 = 36$

i.e. two customers can visit the shop in 36 ways

(i) E⟶ event of visiting shop on the same day.

No. of favourable outcomes = 6 which are (M, M) (T, T) (Th, Th) (F, F) (S, S)

Probability, $P(E) = \frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

$P(E) = \frac{6}{36} = \frac{1}{6}$

(ii) E⟶ event of visiting shop on the same day.

E⟶ event of visiting shop on the different days.

In above bit, we calculated P(E) as $\frac{1}{6}$

We know that, P(E) + P($\bar{E}$) = 1

P($\bar{E}$) = 1 – P(E)

= $1 - \frac{1}{6} = \frac{5}{6}$

(iii) E⟶ event of visiting shop on c

No. of favourable outcomes = 6 which are (M, T) (T, W) (W, Th) (Th, F) (F, S)

P(E) = $\frac{5}{36}$

28. In a class, there are 18 girls and 16 boys. The class teacher wants to choose one pupil for class monitor. What she does, she writes the name of each pupil on a card and puts them into a basket and mixes thoroughly. A child is asked to pick one card from the basket. What is the probability that the name written on the card is:

(i) the name of a girl

(ii) the name of a boy

Sol:

Total no. of possible outcomes = 34 (18 girls, 16 boys)

(i) E ⟶ event of getting girl name

No. of favorable outcomes = 18 (18 girls)

Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{18}{34} = \frac{9}{17}$

(ii) E ⟶ event of getting boy name

No. of favorable outcomes = 16 (16 boys)

P(E) = $\frac{16}{34} = \frac{8}{17}$

29. Why is tossing a coin considered to be a fair way of deciding which team should choose ends in a game of cricket?

Sol:

No. of possible outcomes while tossing a coin = 2 {1 head & 1 tail}

Probability = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

P(getting head) = $\frac{1}{2}$

P(getting tail) = $\frac{1}{2}$

Since probability of two events are equal, these are called equally like events.

Hence, tossing a coin is considered to be a fair way of deciding which team should choose ends in a game of cricket.

30. What is the probability that a number selected at random from the number 1,2,2,3,3,3, 4, 4, 4, 4 will be their average?

Sol:

Given no's are 1, 2, 2, 3, 3, 3, 4, 4, 4, 4

Total no. of possible outcomes = 10

Average of the no's = $\frac{sum\ of\ no's}{total\ no's} = \frac{1+2+2+3+3+3+4+4+4+4}{10} = \frac{30}{10} = 3$

E ⟶ event of getting 3

No. of favourable outcomes = 3 {3, 3, 3}

P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

P(E) = $\frac{3}{10}$

31. The faces of a red cube and a yellow cube are numbered from 1 to 6. Both cubes are rolled. What is the probability that the top face of each cube will have the same number?

Sol:

Total no. of outcomes when both cubes are rolled = $6 \times 6 = 36$ which are

{ (1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)

(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6) }

E ⟶ event of getting same no. on each cube

No. of favourable outcomes = 6 which are

{ (1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)}

Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{6}{36} = \frac{1}{6}$

32. The probability of selecting a green marble at random from a jar that contains only green, white and yellow marbles is $\frac{1}{4}$. The probability of selecting a white marble at random from the same jar is $\frac{1}{3}$. If this jar contains 10 yellow marbles. What is the total number of marbles in the jar?

Sol:

Let the no. of green marbles = x

The no. of white marbles = y

No. of yellow marbles = 10

Total no. of possible outcomes = x + y + 10 (total no. of marbles)

Probability P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

Probability (green marble) = $\frac{1}{4} = \frac{x}{x+y+10}$

⇒ x + y + 10 = 4x
⇒ 3x – y – 10 = 0 …..(i)
Probability (white marble) = $\frac{1}{3} = \frac{y}{x+y+10}$
⇒ x + y 10 = 3y
⇒ x – 2y + 10 = 0 ….(ii)
⇒ 3x – 6y + 30 = 0 ….(iii)
Multiplying by 3,
Sub (i) from (iii), we get
−6y + y + 30 + 10 = 0
⇒ −5y + 40 = 0
⇒ 5y = 40
⇒ y = 8
Subs. Y in (i), 3x – 8 – 10 = 0
3x – 18 = 0
x = $\frac{18}{3} = 6$
Total no. of marbles in jar = x + y + 10 = 6 + 8 + 10 = 24

33. There are 30 cards, of same size, in a bag on which numbers 1 to 30 are written. One card is taken out of the bag at random. Find the probability that the number on the selected card is not divisible by 3.

Sol:
Total no. of possible outcomes = 30 {1, 2, 3, … 30}
E ⟶ event of getting no. divisible by 3.
No. of favourable outcomes = 10 {3, 6, 9, 12, 15, 18, 21, 24, 27, 30}
Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$
P(E) = $\frac{10}{30} = \frac{1}{3}$
$\bar{E}$ ⟶ event of getting no. not divisible by 3.
P($\bar{E}$) = 1 – P(E)
= 1 - $\frac{1}{3} = \frac{2}{3}$

34. A bag contains 5 red, 8 white and 7 black balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is
(i) red or white
(ii) not black
(iii) neither white nor black.

Sol:
Total no. of possible outcomes = 20 {5 red, 8 white & 7 black}
(i) E ⟶ event of drawing red or white ball

No. of favourable outcomes = 13 {5 red, 8 white}

Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

P(E) = $\frac{13}{20}$

(ii) Let E ⟶ be event of getting black ball

No. of favourable outcomes = 13 {5 red, 8 white}

P(E) = $\frac{7}{20}$

$(\bar{E})$⟶ event of not getting black ball

$P(\bar{E})$ = 1 – P(E)

$= 1 - \frac{7}{20} = \frac{13}{20}$

(iii) Let E ⟶ be event of getting neither white nor black ball

No. of favourable outcomes = 20 – 8 – 7 = 5 {total balls – no. of white balls – no. of black balls}

P(E) = $\frac{5}{20} = \frac{1}{4}$

35. Find the probability that a number selected from the number 1 to 25 is not a prime number when each of the given numbers is equally likely to be selected.

Sol:

Total no. of possible outcomes = 25 {1, 2, 3, … 25}

E ⟶ event of getting a prime no.

No. of favourable outcomes = 9 {2, 3, 5, 7, 11, 13, 17, 19, 23}

Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{9}{25}$

$(\bar{E}) \longrightarrow event\ of\ not\ getting\ a\ prime\ no.$

$P(\bar{E}) = 1 - P(E) = 1 - \frac{9}{25} = \frac{16}{25}$

36. A bag contains 8 red, 6 white and 4 black balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is

(i) Red or white

(ii) Not black

(iii) Neither white nor black

Sol:

Total no. of possible outcomes = 8 + 6 + 4 = 18 {8 red, 6 white, 4 black}

(i) E ⟶ event of getting red or white ball

No. of favourable outcomes = 4 {4 black balls}

P(E) = $\frac{4}{18} = \frac{2}{9}$

$(\bar{E}) \longrightarrow event\ of\ not\ getting\ black\ ball$

P$(\bar{E}) = 1 - P(E) = 1 - \frac{2}{9} = \frac{7}{9}$

(ii) E ⟶ event of getting neither white nor black.
No. of favourable outcomes = 15 – 6 – 4 = 8 {Total balls – no. of white balls – no. of black balls}
P(E) = $\frac{8}{18} = \frac{4}{9}$

37. Find the probability that a number selected at random from the numbers 1, 2, 3, ..., 35 is a
(i) Prime number (ii) Multiple of 7 (iii) Multiple of 3 or 5
Sol:
Total no. of possible outcomes = 35 {1, 2, 3, …. 35}
(i) E ⟶ event of getting a prime no.
No. of favourable outcomes = 11 {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31}
Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{11}{35}$
(ii) E ⟶ event of getting no. which is multiple of 7
No. of favourable outcomes = 5 {7, 14, 21, 28, 35}
P(E) = $\frac{5}{35} = \frac{1}{7}$
(iii) E ⟶ event of getting no which is multiple of 3 or 5
No. of favourable outcomes = 16 {3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 5, 10, 20, 25, 35}
P(E) = $\frac{16}{35}$

38. From a pack of 52 playing cards Jacks, queens, kings and aces of red colour are removed. From the remaining, a card is drawn at random. Find the probability that the card drawn is
(i) A black queen
(ii) A red card
(iii) A black jack
(iv) a picture card (Jacks, queens and kings are picture cards)
Sol:
Total no. of cards = 52
All jacks, queens & kings, aces of red colour are removed.
Total no. of possible outcomes = 52 – 2 – 2 – 2 – 2 = 44 {remaining cards}
(i) E ⟶ event of getting a black queen
No. of favourable outcomes = 2 {queen of spade & club}
Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$
P(E) = $\frac{2}{44} = \frac{1}{22}$
(ii) E ⟶ event of getting a red card
No. of favourable outcomes = 26 – 8 = 18 {total red cards – jacks, queens, kings, aces of red colour}

P(E) = $\frac{18}{44} = \frac{9}{22}$

(iii) E ⟶ event of getting a black jack

No. of favourable outcomes = 2 {jack of club & spade}

P(E) = $\frac{2}{44} = \frac{1}{22}$

(iv) E ⟶ event of getting a picture card

No. of favourable outcomes = 6 {2 jacks, 2 kings & 2 queens of black colour}

P(E) = $\frac{6}{44} = \frac{3}{22}$

39. A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out

(i) an orange flavoured candy?

(ii) a lemon flavoured candy?

Sol:

(i) The bag contains lemon flavoured candies only. So, the event that malini will take out an orange flavoured candy is an impossible event. Since, probability of impossible event is O, P(an orange flavoured candy) = 0

(ii) The bag contains lemon flavoured candies only. So, the event that malini will take out a lemon flavoured candy is sure event. Since probability of sure event is 1, P(a lemon flavoured candy) = 1

40. It is given that m a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

Sol:

Let E ⟶ event of 2 students having same birthday P(E) is given as 0.992

Let $(\bar{E})$ ⟶ event of 2 students not having same birthday.

We know that, P(E) + P$(\bar{E}) = 1$

P$(\bar{E}) = 1 - P(E)$

= 1 – 0.992

= 0.008

41. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is

(i) red? (ii) not red?

Sol:

Total no. of possible outcomes = 8 {3 red, 5 black}

(i) E ⟶ event of getting red ball.

No. of favourable outcomes = 3 {3 red}

Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

P(E) = $\frac{3}{8}$

(ii) $\bar{E}$ ⟶ event of getting no red ball.

$P(E) + P(\bar{E}) = 1$

$P(\bar{E}) = 1 - P(E)$

$= 1 - \frac{3}{8} = \frac{5}{8}$

42. (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?

(ii) Suppose the bulb drawn in

(a) is not defective and nõt replaced. Now bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

Sol:

Total no. of possible outcomes = 20 {20 bulbs}

(i) E ⟶ be event of getting defective bulb.

No. of favourable outcomes = 4 {4 defective bulbs}

Probability, $P(E) = \frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}} = \frac{4}{20} = \frac{1}{5}$

(ii) Bulb drawn in is not detective & is not replaced remaining bulbs = 15 good + 4 bad bulbs = 19

Total no. of possible outcomes = 19

E ⟶ be event of getting defective

No. of favorable outcomes = 15 (15 good bulbs)

$P(E) = \frac{15}{9}$

43. A box contains 90 discs which are numbered from 1 to 90. If one discs is drawn at random from the box, find the probability that it bears

(i) a two digit number

(ii) a perfect square number

(iii) (iii) a number divisible by 5.

Sol:

Total no. of possible outcomes = 90 {1, 2, 3, … 90}

(i) E ⟶ event of getting 2 digit no.

No. of favourable outcomes = 81 {10, 11, 12, …. 90}

Probability P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

P(E) = $\frac{81}{90}$

(ii) E ⟶ event of getting a perfect square.

No. of favourable outcomes = 9 {1, 4, 9, 16, 25, 26, 49, 64, 81}

P(E) $\frac{9}{90} = \frac{1}{10}$

(iii) E ⟶ event of getting a no. divisible by 5.
No. of favourable outcomes = 18 {5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90}
P(E) = $\frac{18}{90} = \frac{1}{5}$

44. A lot consists of 144 ball pens of which 20 are defective and others good. Nun will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that
(i) She will buy it? (ii) She will not buy it?
Sol:
No. of good pens = 144 – 20 = 24
No. of detective pens = 20
Total no. of possible outcomes = 144 {total no pens}
(i) E ⟶ event of buying pen which is good.
No. of favourable outcomes = 124 {124 good pens}
P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$
P(E) = $\frac{124}{144} = \frac{31}{36}$
(ii) $\bar{E}$ ⟶ event of not buying a pen which is bad P(E) + P($\bar{E}$) = 1
$P(E) + P(\bar{E}) = 1$
P($\bar{E}$) = 1 - P(E)
$= 1 - \frac{31}{36} = \frac{5}{36}$

45. 12 defective pens are accidently mixed with 132 good ones. It is not possible to just look at pen and tell whether or not it is defective. one pen is taken out at random from this lot. Determine the probability that the pen taken out is good one.
Sol:
No. of good pens = 132
No. of defective pens = 12
Total no. of possible outcomes = 12 + 12 {total no of pens}
E ⟶ event of getting a good pen.
No. of favourable outcomes = 132 {132 good pens}
P (E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$
∴ P(E) = $\frac{132}{144} = \frac{66}{72} = \frac{33}{36} = \frac{11}{2}$

46. Five cards — the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.
(i) What is the probability that the card is the queen?

(ii) If the queen is drawn and put a side, what is the probability that the second card picked up is
a. an ace?
b. a queen?

Sol:

Total no. of possible outcomes = 5 {5 cards}

(i) E ⟶ event of getting a good pen.
No. of favourable outcomes = 132 {132 good pens}
P (E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$
∴ P(E) = $\frac{1}{5}$

(ii) If queen is drawn & put aside,
Total no. of remaining cards = 4
(a) E ⟶ event of getting a queen.
No. of favourable outcomes = 1 {1 ace card}
Total no. of possible outcomes = 4 {4 remaining cards}
P(E) = $\frac{1}{4}$
(b) E ⟶ event of getting a good pen.
No. of favourable outcomes = 0 {there is no queen}
P(E) = $\frac{0}{4} = 0$
∵ E is known as impossible event.

47. Harpreet tosses two different coins simultaneously (say, one is of Re 1 and other of Rs 2). What is the probability that he gets at least one head?

Sol:

Total no. of possible outcomes = 4 which are{HT, HH, TT, TH}
E ⟶ event of getting at least one head
No. of favourable outcomes = 3 {HT, HH, TH}
Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$
P(E) = $\frac{3}{4}$

48. Two dice, one blue and one grey, are thrown at the same time. Complete the following table:

Event: 'Sum on two dice'	2	3	4	5	6	7	8	9	10	11	12
Probability											

From the above table a student argues that there are 1 1 possible outcomes 2,3,4,5,6,7, 8, 9, 10, 11 and 12. Therefore, each of them has a probability j-j . Do you agree with this argument?

Sol:

Total no. of possible outcomes when 2 dice are thrown = $6 \times 6 = 36$ which are

{ (1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6) }

E ⟶ event of getting sum on 2 dice as 2

No. of favourable outcomes = 1{(1, 1)}

Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

P(E) = $\frac{1}{36}$

E ⟶ event of getting sum as 3

No. of favourable outcomes = 2 {(1, 2) (2, 1)}

P(E) = $\frac{2}{36}$

E ⟶ event of getting sum as 4

No. of favourable outcomes = 3 {(3, 1) (2, 2) (1, 3)}

P(E) = $\frac{3}{36}$

E ⟶ event of getting sum as 5

No. of favourable outcomes = 4 {(1, 4) (2, 3) (3, 2) (4, 1)}

P(E) = $\frac{4}{36}$

E ⟶ event of getting sum as 6

No. of favourable outcomes = 5 {(1, 5) (2, 4) (3, 3) (4, 2) (5, 1)}

P(E) = $\frac{6}{36}$

E ⟶ event of getting sum as 7

No. of favourable outcomes = 6 {(1, 6) (2, 5) (3, 4) (4, 3) (5, 2) (6, 1)}

P(E) = $\frac{6}{36}$

E ⟶ event of getting sum as 8

No. of favourable outcomes = 5 {(2, 6) (3, 5) (4, 4) (5, 3) (6, 2)}

P(E) = $\frac{5}{36}$

E ⟶ event of getting sum as 9

No. of favourable outcomes = 4 {(3, 6) (4, 5) (5, 4) (6, 3)}

P(E) = $\frac{4}{36}$

E ⟶ event of getting sum as 10

No. of favourable outcomes = 3 {(4, 6) (5, 5) (6, 4)}

P(E) = $\frac{3}{36}$

E ⟶ event of getting sum as 11
No. of favourable outcomes = 2 {(5, 6) (6, 5)}
P(E) = $\frac{2}{36}$
E ⟶ event of getting sum as 12
No. of favourable outcomes = 1 {(6, 6)}
P(E) = $\frac{1}{36}$

Event 'Sum on two dice'	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

No, the outcomes are not equally likely from the above table we see that, there is different probability for different outcome

49. Cards marked with numbers 13, 14, 15, ..., 60 are placed in a box and mixed thoroughly. One card is drawn at random from the box. Find the probability that number on the card drawn is
(i) divisible by 5
(ii) a number is a perfect square
Sol:
Total no. of possible outcomes = 48 {13, 14, 15, …., 60}
(i) E ⟶ event of getting no divisible by 5
No. of favourable outcomes = 10{15, 20, 25, 30, 35, 40, 45, 50 55, 60}
Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$
P(E) = $\frac{10}{48} = \frac{5}{24}$
(ii) E ⟶ event of getting a perfect square.
No. of favourable outcomes = 4 {16, 25, 36, 49}
P(E) = $\frac{4}{48} = \frac{1}{12}$

50. A bag contains 6 red balls and some blue balls. If the probability of drawing a blue ball the bag is twice that of a red ball, find the number of blue balls in the bag.
Sol:
No of red balls = 6
Let no. of blue balls = x
Total no. of possible outcomes = 6 + x(total no. of balls)
P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$
P(blue ball) = 2 P(red ball)
⇒ $\frac{x}{x+6} = \frac{2(6)}{x+6}$
⇒ x = 2(6)
x = 12

∴ No of blue balls = 12

51. A bag contains tickets numbered 11, 12, 13,..., 30. A ticket is taken out from the bag at random. Find the probability that the number on the drawn ticket
(i) is a multiple of 7
(ii) is greater than 15 and a multiple of 5.

Sol:

Total no. of possible outcomes = 20 {11, 12, 13, ….., 30}

(i) E ⟶ event of getting no. which is multiple of 7
No. of favorable outcomes = 3 {14, 21, 28}
Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$
P(E) = $\frac{3}{20}$

(ii) E ⟶ event of getting no. greater than 15 & multiple of 5
No. of favorable outcomes = 3 {14, 21, 28}
P(E) = $\frac{3}{20}$

52. The king, queen and jack of clubs are removed from a deck of 52 playing cards and the remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of
(i) heart
(ii) queen
(iii) clubs.

Sol:

Total no. of remaining cards = 52 – 3 = 49

(i) E⟶ event of getting hearts
No. of favorable outcomes = 3 {4 – 1}
Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$
P(E) = $\frac{13}{49}$

(ii) E ⟶ event of getting queen
No. of favorable outcomes = 3 (4 – 1) {Since queen of clubs is removed}
P(E) = $\frac{3}{49}$

(iii) E ⟶ event of getting clubs
No. of favorable outcomes = 10 (13 – 3) {Since 3 club cards are removed}
P(E) = $\frac{10}{49}$

53. Two dice are thrown simultaneously. What is the probability that:
(i) 5 will not come up on either of them?

(ii) 5 will come up on at least one?
(iii) 5 wifi come up at both dice?

Sol:

Total no. of possible outcomes when 2 dice are thrown = $6 \times 6 = 36$ which are
{ (1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6) }

(i) E ⟶ event of 5 not coming up on either of them
No. of favourable outcomes = 25 which are
{ (1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6) }

Probability, P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

P(E) = $\frac{25}{36}$

(ii) E ⟶ event of 5 coming up at least once {(1, 5) (2, 5) (3, 5) (4, 5) (5, 5) (5, 1) (5, 2) (5, 3) (5, 4) (5, 6) (6, 5)}
P(E) = $\frac{11}{36}$

(iii) E ⟶ event of getting 5 on both dice
No. of favourable outcomes = 1 { (5, 5) }
P(E) = $\frac{1}{36}$

54. Fill in the blanks:
(i) Probability of a sure event is...........
(ii) Probability of an impossible event is...........
(iii) The probability of an event (other than sure and impossible event) lies between……
(iv) Every elementary event associated to a random experiment has .... probability.
(v) Probability of an event A + Probability of event 'not A' —...........
(vi) Sum of the probabilities of each outcome m an experiment is ....

Sol:

(i) 1, ∵ P(sure event) = 1
(ii) 0, ∵ P(impossible event) = 0
(iii) 0 & 1, ∵ O ∠ P(E) ∠ 1

(iv) Equal
(v) 1, ∵ $P(E) + P(\bar{E}) = 1$
(vi) 1

55. Examine each of the following statements and comment:
(i) If two coins are tossed at the same time, there are 3 possible outcomes—two heads, two tails, or one of each. Therefore, for each outcome, the probability of occurrence is 1/3
(ii) (ii) If a die is thrown once, there are two possible outcomes—an odd number or an even number. Therefore, the probability of obtaining an odd number is 1 /2 and the probability of obtaining an even number is $\frac{1}{2}$.

Sol:
(i) Given statement is incorrect. If 2 coins are tossed at the same time,
Total no. of possible outcomes = 4 {HH, HT, TH, TT}
P(HH) = P(HT) = P(TH) = P(TT) = $\frac{1}{4}$ {∵ Probability = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$}
I.e. for each outcome, probability of occurrence is $\frac{1}{4}$
Outcomes can be classified as (2H, 2T, 1H & 1T) P(2H) = $\frac{1}{4}$, P(2T) = $\frac{1}{4}$, P(1H & 1T) = $\frac{2}{4}$
Events are not equally likely because the event 'one head & 1 tail' is twice as likely to occur as remaining two.
(ii) This statement is true
When a die is thrown; total no. of possible outcomes = 6 {1, 2, 3, 4, 5, 6}
These outcomes can be taken as even no. & odd no.
P(even no.) = P(2, 4, 6) = $\frac{3}{6} = \frac{1}{2}$
P(odd no.) = $p(1, 3, 5)$ = $\frac{3}{6} = \frac{1}{2}$
∴ Two outcomes are equally likely

56. A box contains loo red cards, 200 yellow cards and 50 blue cards. If a card is drawn at random from the box, then find the probability that it will be
(i) a blue card
(ii) not a yellow card
(iii) neither yellow nor a blue card.

Sol:
Total no. of possible outcomes = 100 + 200 + 50 = 350 {100 red, 200 yellow & 50 blue}
(i) E ⟶ event of getting blue card.
No. of favourable outcomes = 50 {50 blue cards}
P(E) = $\frac{50}{350} = \frac{1}{7}$

(ii) E ⟶ event of getting yellow card

No. of favourable outcomes = 200 {200 yellow}

P(E) = $\frac{200}{350} = \frac{4}{7}$

$\bar{E}$ ⟶ event of not getting yellow card

$P(\bar{E}) = 1 - P(E)$

$= 1 - \frac{4}{7} = \frac{3}{7}$

(iii) E ⟶ getting neither yellow nor a blue card

No. of favourable outcomes = 350 – 200 – 50 = 100 {removing 200 yellow & 50 blue cards}

P(E) = $\frac{100}{350} = \frac{2}{7}$

57. A number is selected at random from first 50 natural numbers. Find the probability that it is a multiple of 3 and 4.

Sol:

Total no. of possible outcomes = 50 {1, 2, 3 …. 50}

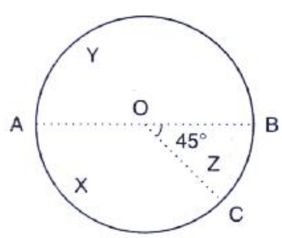
No. of favourable outcomes = 4 {12, 24, 36, 48}

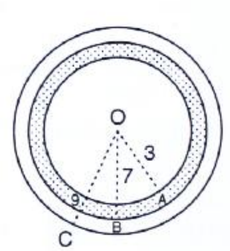
P(E) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of possible outcomes}}$

P(E) = $\frac{4}{50} = \frac{2}{25}$

Exercise – 13.2

1. In the accompanying diagram a fair spinner is placed at the center O of the circle. Diameter AOB and radius OC divide the circle into three regions labelled X, Y and Z. If ∠BOC = 45°. What is the probability that the spinner will land in the region X? (See fig)





Sol:

Given ∠BOC = 45°

∠AOC = 180 – 45 = 135°

Area of circle = πr^2

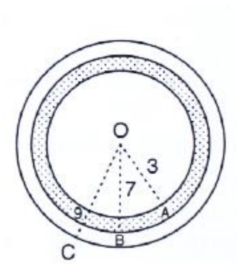
Area of region × = $\frac{\theta}{360°} \times \pi r^2$

$= \frac{135}{360} \times \pi r^2 = \frac{3}{8}\pi r^2$

Probability that the spinner will land in the region

X = $\frac{Area\ of\ region\ x}{total\ area\ of\ circle} = \frac{\frac{3}{8}\pi r^2}{\pi r^2} = \frac{3}{8}$

2. A target shown in Fig. below consists of three concentric circles of radii, 3, 7 and 9 cm respectively. A dart is thrown and lands on the target. What is the probability that the dart will land on the shaded region?



Sol:

1^{st} circle ⟶ with radius 3

2^{nd} circle ⟶ with radius 7

3^{rd} circle ⟶ with radius 9

Area of 1^{st} circle = $(3)^2 = 9\pi$

Area of 2^{nd} circle= $(7)^2 = 49\pi$

Area of 3^{rd} circle = $(9)^2 = 81\pi$

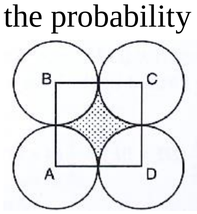
Area of shaded region = Area of 2^{nd} circle – area of 1^{st} circle

$= 49\pi - 9\pi$

$= 40\pi$

Probability that will land on the shaded region = $\frac{area\ of\ shaded\ region}{area\ of\ 3rd\ circle} = \frac{40\pi}{81\pi} = \frac{40}{81}$

3. In below Fig., points A, B, C and D are the centers of four circles that each have a radius of length one unit. If a point is selected at random from the interior of square ABCD. What is the probability that the point will be chosen from the shaded region?



Sol:

Radius of circle = 1cm

Length of side of square = 1 + 1 = 2cm

Area of square = 2 × 2 = $4cm^2$

Area of shaded region = area of square – 4 × area of quadrant

= $4 - 4\left(\frac{1}{4}\right)\pi(1)^2$

= $(4 - \pi)\ cm^2$

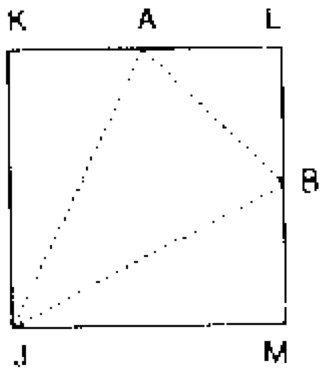
Probability that the point will be chosen from the shaded region = $\frac{Area\ of\ shaded\ region}{Area\ of\ square\ ABCD}$

$= \frac{4-\pi}{4} = 1 - \frac{\pi}{4}$

Since geometrical probability,

P(E) = $\frac{Measure\ of\ specified\ part\ of\ region}{Measure\ of\ the\ whole\ region}$

4. In the Fig. below, JKLM is a square with sides of length 6 units. Points A and B are the mid- points of sides KL and LM respectively. If a point is selected at random from the interior of the square. What is the probability that the point will be chosen from the interior of ΔJAB?



Sol:

Length of side of square JKLM = 6 cm

Area of square JKLM = 6^2 = 36 cm^2

Since A & B are the mid points of KL & LM

KA = AL = LB = LM = 3 cm

Area of Δ AJB = area of square – area of Δ AKJ – area of Δ ALB – area of Δ BMJ

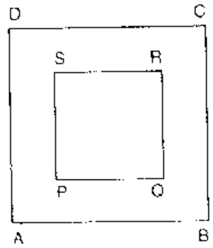
$= 36 - \frac{1}{2} \times 6 \times 3 - \frac{1}{2} \times 6 \times 3$

$= 36 - 9 - 4.5 - 9$

$= 13.5\ sq.units$

Probability that the point will be chosen from the interior of ΔAJB = $\frac{Area\ of\ \Delta AJB}{Area\ of\ square}$

5. In the Fig. below, 13, a square dart board is shown. The length of a side of the larger square is 1.5 times the length of a side of the smaller square. If a dart is thrown and lands on the larger square. What is the probability that it will land in the interior of the smaller square?



Sol:

Let length of side of smaller square = a

Then length of side of bigger square = 1.5a

Area of smaller square = a^2

Area of bigger square = $(1.5)^2a^2 = 2.25a^2$.

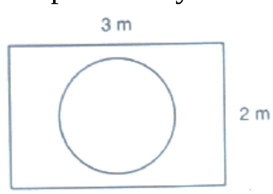
Probability that dart will land in the interior of the smaller square = $\frac{Area\ of\ smaller\ square}{Area\ of\ bigger\ square}$

$= \frac{a^2}{2.25a^2} = \frac{1}{2.25}$

∵ Geometrical probability,

$P(E) = \frac{measure\ of\ specified\ region\ part}{measure\ of\ the\ whole\ region}$

6. Suppose you drop a tie at random on the rectangular region shown in Fig. below. What is the probability that it will land inside the circle with diameter 1 m?



Sol:

Area of circle with radius 0.5 m

A circle = $(0.5)^2 = 0.25\ \pi m^2$

Area of rectangle = $3 \times 2 = 6m^2$

Probability (geometric) = $\frac{measured\ of\ specified\ region\ part}{measure\ of\ whole\ region}$

Probability that tie will land inside the circle with diameter 1m

$= \frac{area\ of\ circle}{area\ of\ rectangle}$

$= \frac{0.25\pi m^2}{6\ m^2}$

$= \frac{1}{4} \times \frac{\pi}{6}$

$= \frac{\pi}{24}$

Exercise 14.1

1. On which axis do the following points lie?
   (i) P(5, 0)
   (ii) Q(0 – 2)
   (iii) R(−4, 0)
   (iv) S(0, 5)

Sol:

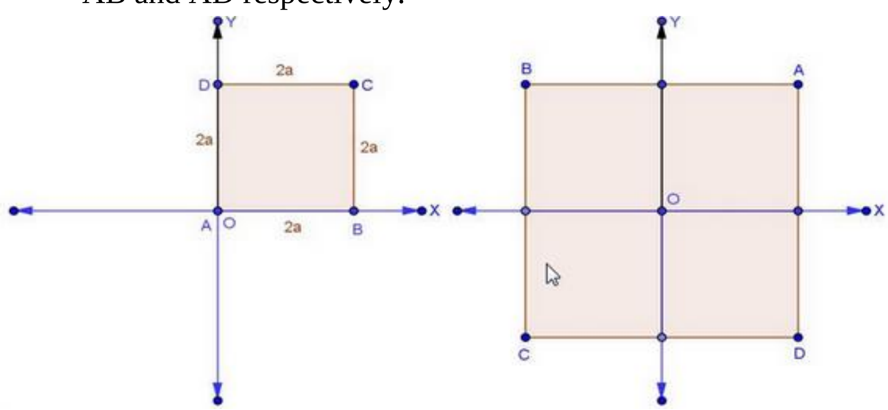
(i) $P(5,0)$ lies on $x-axis$

(ii) $Q(0,-2)$ lies on $y-axis$

(iii) $R(-4,0)$ lies on $x-axis$

(iv) $S(0,5)$ lies on $y-axis$

2. Let ABCD be a square of side 2a. Find the coordinates of the vertices of this square when
   (i) A coincides with the origin and AB and AB and coordinate axes are parallel to the sides AB and AD respectively.
   (ii) The center of the square is at the origin and coordinate axes are parallel to the sides AB and AD respectively.



Sol:

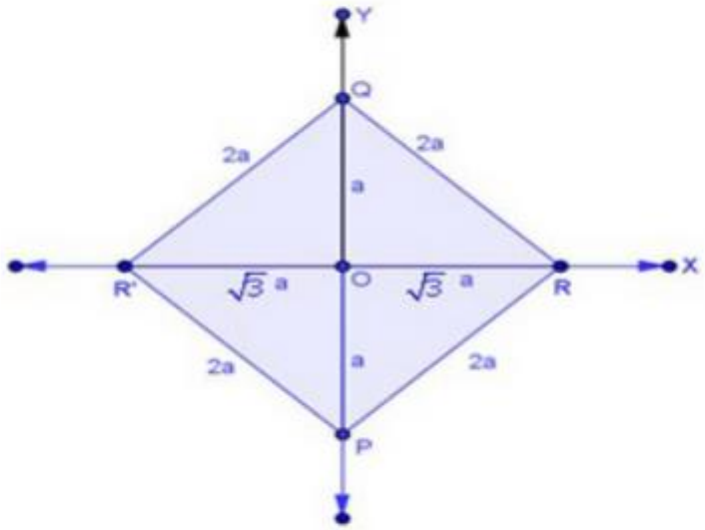
(i) Coordinate of the vertices of the square of side $2a$ are:

$A(0,0), B(2a,0), C(2a,2a)$ *and* $D(0,2a)$

(ii) Coordinate of the vertices of the square of side $2a$ are:

$A(a,a), B(-a,a), C(-a,-a)$ *and* $(a,-a)$

3. The base PQ of two equilateral triangles PQR and PQR' with side 2a lies along y-axis such that the mid-point of PQ is at the origin. Find the coordinates of the vertices R and R' of the triangles.



Sol:

We have two equilateral triangle PQR and PQR' with side $2a$.

O is the mid-point of PQ.

In ΔQOR, $\angle QOR = 90°$

Hence, by Pythagoras theorem

$OR^2 + OQ^2 = QR^2$

$OR^2 = (2a)^2 - (a)^2$

$OR^2 = 3a^2$

$OR = \sqrt{(3)}a$

Coordinates of vertex R is $(\sqrt{3}a, 0)$ and coordinate of vertex R' is $(-\sqrt{3}a, 0)$

Exercise 14.2

1. Find the distance between the following pair of points:

(i) $(-6, 7)$ and $(-1, -5)$

(ii) $(a+b, b+c)$ and $(a-b, c-b)$

(iii) $(a\sin\alpha, -b\cos\alpha)$ and $(-a\cos\alpha, b\sin\alpha)$

(iv) $(a, 0)$ and $(0, b)$

Sol:

(i) We have $P(-6,7)$ and $Q(-1,-5)$

Here,

$x_1 = -6, y_1 = 7$ and

$x_2 = -1, y_2 = -5$

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$PQ = \sqrt{[-1-(-6)]^2 + (-5-7)^2}$

$PQ = \sqrt{(-1+6)^2 + (-5-7)^2}$

$PQ = \sqrt{(5)^2 + (-12)^2}$

$PQ = \sqrt{25+144}$

$PQ = \sqrt{169}$

$PQ = 13$

(ii) we have $P(a+b, b+c)$ and $Q(a-b, c-b)$ here,

$x_1 = a+b, y_1 = b+c$ and $x_2 = a-b, y_2 = c-b$

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$PQ = \sqrt{[a-b-(a+b)]^2 + (c-b-(b+c))^2}$

$PQ = \sqrt{(a-b-a-b)^2 + (c-b-b-c)^2}$

$PQ = \sqrt{(-2b)^2 + (-2b)^2}$

$PQ = \sqrt{4b^2 + 4b^2}$

$PQ = \sqrt{8b^2}$

$PQ = \sqrt{4 \times 2b^2}$

$PQ = 2\sqrt{2}b$

(iii) we have $P(a\sin\alpha, -b\cos\alpha)$ and $Q(-a\cos\alpha, b\sin\alpha)$ here

$x_1 = a\sin\alpha, y_1 = -b\cos\alpha$ and

$x_2 = -a\cos\alpha, y_2 = b\sin\alpha$

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$PQ = \sqrt{(-a\cos\alpha - a\sin\alpha)^2 + [-b\sin\alpha - (-b\cos\alpha)]^2}$

$$PQ=\sqrt{(-a\cos\alpha)^2+(-a\sin\alpha)^2+2(-a\cos\alpha)(-a\sin\alpha)+(b\sin\alpha)^2+(-b\cos\alpha)^2-2(b\sin\alpha)(-b\cos\alpha)}$$

$$PQ=\sqrt{a^2\cos^2\alpha+a^2\sin^2\alpha+2a^2\cos\alpha\sin\alpha+b^2\sin^2\alpha+b^2\cos^2\alpha+2b^2\sin\alpha\cos\alpha}$$

$$PQ=\sqrt{a^2\left(\cos^2\alpha+\sin^2\alpha\right)+2a^2\cos\alpha\sin\alpha+b^2\left(\sin^2\alpha+\cos^2\alpha\right)+2b^2\sin\alpha\cos\alpha}$$

$$PQ=\sqrt{a^2\times1+2a^2\cos\alpha\sin\alpha+b^2\times1+2b^2\sin\alpha\cos\alpha} \qquad \left[\because \sin^2\alpha+\cos^2\alpha=1\right]$$

$$PQ=\sqrt{a^2+b^2+2a^2\cos\alpha\sin\alpha+2b^2\sin\alpha\cos\alpha}$$

$$PQ=\sqrt{\left(a^2+b^2\right)+2\cos\alpha\sin\alpha\left(a^2+b^2\right)}$$

$$PQ=\sqrt{\left(a^2+b^2\right)\left(1+2\cos\alpha\sin\alpha\right)}$$

(iv) We have $P(a,0)$ and $Q(0,b)$

Here,

$x_1=a, y_1=0, x_2=0, y_2=b,$

$$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$PQ=\sqrt{(0-a)^2+(b-0)^2}$$

$$PQ=\sqrt{(-a)^2+(b)^2}$$

$$PQ=\sqrt{a^2+b^2}$$

2. Find the value of a when the distance between the points (3, a) and (4, 1) is $\sqrt{10}$.

Sol:

We have $P(3,a)$ and $Q(4,1)$

Here,

$x_1=3, y_1=a$

$x_2=4, y_2=1$

$PQ=\sqrt{10}$

$$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\Rightarrow \sqrt{10}=\sqrt{(4-3)^2+(1-a)^2}$$

$$\Rightarrow \sqrt{10}=\sqrt{(1)^2+(1-a)^2}$$

$$\Rightarrow \sqrt{10}=\sqrt{1+1+a^2-2a} \qquad \left[\because (a-b)^2=a^2+b^2-2ab\right]$$

$$\Rightarrow \sqrt{10}=\sqrt{2+a^2-2a}$$

Squaring both sides

$\Rightarrow \left(\sqrt{10}\right)^2 = \left(\sqrt{2+a^2-2a}\right)^2$

$\Rightarrow 10 = 2 + a^2 - 2a$

$\Rightarrow a^2 - 2a + 2 - 10 = 0$

$\Rightarrow a^2 - 2a - 8 = 0$

Splitting the middle team.

$\Rightarrow a^2 - 4a + 2a - 8 = 0$

$\Rightarrow a(a-4) + 2(a-4) = 0$

$\Rightarrow (a-4)(a+2) = 0$

$\Rightarrow a = 4, a = -2$

3. If the points (2, 1) and (1, -2) are equidistant from the point (x, y) from (-3, 0) as well as from (3, 0) are 4.

Sol:

We have $P(2,1)$ and $Q(1,-2)$ and $R(X,Y)$

Also, $PR = QR$

$PR = \sqrt{(x-2)^2 + (y-1)^2}$

$\Rightarrow PR = \sqrt{x^2 + (2)^2 - 2xx \times 2 + y^2 + (1)^2 - 2 \times y \times 1}$

$\Rightarrow PR = \sqrt{x^2 + 4 - 4x + y^2 + 1 - 2y}$

$\Rightarrow PR = \sqrt{x^2 + 5 - 4x + y^2 - 2y}$

$QR = \sqrt{(x-1)^2 + (y+2)^2}$

$\Rightarrow PR = \sqrt{x^2 + 1 - 2x + y^2 + 4 + 4y}$

$\Rightarrow PR = \sqrt{x^2 + 5 - 2x + y^2 + 4y}$

$\because PR = QR$

$\Rightarrow \sqrt{x^2 + 5 - 4x + y^2 - 2y} = \sqrt{x^2 + 5 - 2x + y^2 + 4y}$

$\Rightarrow x^2 + 5 - 4x + y^2 - 2y = x^2 + 5 - 2x + y^2 + 4y$

$\Rightarrow x^2 + 5 - 4x + y^2 - 2y = x^2 + 5 - 2x + y^2 + 4y$

$\Rightarrow -4x + 2x - 2y - 4y = 0$

$\Rightarrow -2x - 6y = 0$

$\Rightarrow -2(x + 3y) = 0$

$\Rightarrow x + 3y = \dfrac{0}{-2}$

$\Rightarrow x + 3y = 0$

Hence proved.

4. Find the values of x, y if the distances of the point (x, y) from (−3, 0) as well as from (3,0) are 4.

Sol:

We have $P(x, y), Q(-3, 0)$ and $R(3, 0)$

$PQ = \sqrt{(x+3)^2 + (y-0)^2}$

$\Rightarrow 4 = \sqrt{x^2 + 9 + 6x + y^2}$

Squaring both sides

$\Rightarrow (4)^2 = \left(\sqrt{x^2 + 9 + 6x + y^2}\right)^2$

$\Rightarrow 16 = x^2 + 9 + 6x + y^2$

$\Rightarrow x^2 + y^2 = 16 - 9 - 6x$

$\Rightarrow x^2 + y^2 = 7 - 6x$ …………..(1)

$PR = \left(\sqrt{(x-3)^2 + (y-0)^2}\right)$

$\Rightarrow 4 = \sqrt{x^2 + 9 - 6x + y^2}$

Squaring both sides

$(4)^2 = \left(\sqrt{x^2 + 9 - 6x + y^2}\right)^2$

$\Rightarrow 16 = x^2 + 9 - 6x + y^2$

$\Rightarrow x^2 + y^2 = 16 - 9 + 6x$

$\Rightarrow x^2 + y^2 = 7 + 6x$ …………...(2)

Equating (1) and (2)

$7 - 6x = 7 + 6x$

$\Rightarrow 7 - 7 = 6x + 6x$

$\Rightarrow 0 = 12x$

$\Rightarrow x = 12$

Equating (1) and (2)

$7 - 6x = 7 + 6x$

$\Rightarrow 7 - 7 = 6x + 6x$

$\Rightarrow 0 = 12x$

$\Rightarrow x = 12$

Substituting the value of $x = 0$ in (2)

$x^2 + y^2 = 7 + 6x$

$0 + y^2 = 7 + 6 \times 0$

$y^2 = 7$

$y = \pm\sqrt{7}$

5. The length of a line segment is of 10 units and the coordinates of one end-point are (2,−3). If the abscissa of the other end is 10, find the ordinate of the other end.

Sol:

Let two ordinate of the other end R be Y

$\therefore$ Coordinates of other end R are $(10, y)$ i.e., $R(10, y)$

Distance $PR = 10$ [given]

$PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow 10 = \sqrt{(10-2)^2 + (y+3)^2}$

$\Rightarrow 10 = \sqrt{8^2 + y^2 + 9 + 6y}$

$\Rightarrow 10 = \sqrt{64 + y^2 + 9 + 6y}$

$= 10 = \sqrt{73 + y^2 + 6y}$

Squaring both sides

$(10)^2 = \left(\sqrt{73 + y^2 + 6y}\right)^2$

$\Rightarrow 100 = 73 + y^2 + 6y$

$\Rightarrow y^2 + 6y + 73 - 100 = 0$

$\Rightarrow y^2 + 6y - 27 = 0$

Splitting the middle term

$y^2 + 9y - 3y - 27 = 0$

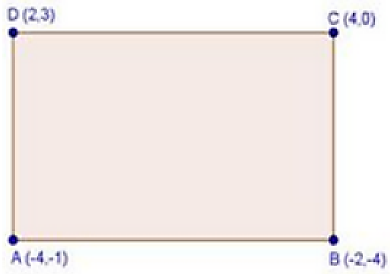
$\Rightarrow y^2 + 9y - 3y - 27 = 0$

$\Rightarrow y(y+9) - 3(y+9) = 0$

$\Rightarrow (y+9)(y-3) = 0$

$\Rightarrow y = -9, y = 3$

6. Show that the points (−2, − 4), (4, 0) and (2, 3) are the vertices points of are the vertices points of a rectangle.



Sol:

Let $A(-4,-1), B(-2,-4), C(4,0)$ *and* $D(2,3)$ be the given points

Now,

$AB = \sqrt{(-2+4)^2 + (-4+1)^2}$

$\Rightarrow AB = \sqrt{(2)^2 + (-3)^2}$

$\Rightarrow AB = \sqrt{4+9}$

$\Rightarrow AB = \sqrt{13}$

$CD = \sqrt{(4-2)^2 + (0-3)^2}$

$\Rightarrow CD = \sqrt{(2)^2 + (-3)^2}$

$\Rightarrow CD = \sqrt{4+9}$

$\Rightarrow CD = \sqrt{13}$

$BC = \sqrt{(4+2)^2 + (0+4)^2}$

$\Rightarrow BC = \sqrt{(6)^2 + (4)^2}$

$\Rightarrow BC = \sqrt{36+16}$

$\Rightarrow BC + \sqrt{52}$

$AD = \sqrt{(-4-2)^2 + (-1-3)^2}$

$\Rightarrow AD = \sqrt{(-6)^2 + (-4)^2}$

$\Rightarrow AD = \sqrt{36+16}$

$\Rightarrow AD = \sqrt{52}$

$\therefore$ $AB = CD$ and $AD = BC \Rightarrow ABCD$ is a parallelogram

Now,

$AC = \sqrt{(4+4)^2 + (0+1)^2}$

$\Rightarrow AC = \sqrt{(8)^2 + (1)^2}$

$\Rightarrow AC = \sqrt{64+1}$

$\Rightarrow AC = \sqrt{65}$

$BD = \sqrt{(2+2)^2 + (3+4)^2}$

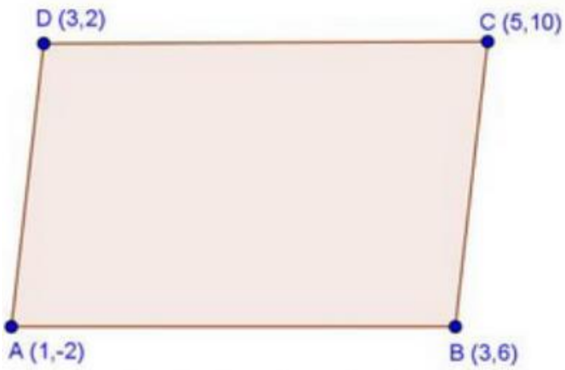
$\Rightarrow BD = \sqrt{(4)^2 + (7)^2}$

$\Rightarrow BD = \sqrt{16+49}$

$\Rightarrow BD = \sqrt{65}$

Since the diagonals of parallelogram $ABCD$ are equal i.e., $AC = BD$
Hence, $ABCD$ is a rectangle

7. Show that the points A(1, −2), B(3, 6), C(5, 10) and D(3, 2) are the vertices of a parallelogram



Sol:

Let $A(1,-2), B(3,6), C(5,10), D(3,2)$ be the given points

$AB = \sqrt{(3-1)^2 + (6+2)^2}$

$\Rightarrow AB = \sqrt{(2)^2 + (8)^2}$

$\Rightarrow AB = \sqrt{4+64}$

$\Rightarrow AB = \sqrt{68}$

$CD = \sqrt{(5-3)^2 + (10-2)^2}$

$\Rightarrow CD = \sqrt{(2)^2 + (8)^2}$

$\Rightarrow CD = \sqrt{4+64}$

$\Rightarrow CD = \sqrt{68}$

$AD = \sqrt{(3-1)^2 + (2+2)^2}$

$\Rightarrow AD = \sqrt{(2)^2 + (4)^2}$

$\Rightarrow AD = \sqrt{4+16}$

$\Rightarrow AD = \sqrt{20}$

$BC = \sqrt{(5-3)^2 + (10-6)^2}$

$\Rightarrow BC = \sqrt{(2)^2 + (4)^2}$

$\Rightarrow BC = \sqrt{4+16}$

$\Rightarrow BC = \sqrt{20}$

$\therefore \quad AB = CD$ and $AD = BC$

Since opposite sides of a parallelogram are equal

Hence, $ABCD$ is a parallelogram

8. Prove that the points A (1, 7), B (4, 2), C (−1, −1) and D (−4, 4) are the vertices of a square.

Sol:

Let $A(1,7), B(4,2), C(-1,-1)$ and $D(-4,4)$ be the given point. One way of showing that $ABCD$ is a square is to use the property that all its sides should be equal and both its diagonals should also be equal

Now,

$AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{9+25} = \sqrt{34}$

$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{25+9} = \sqrt{34}$

$CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$

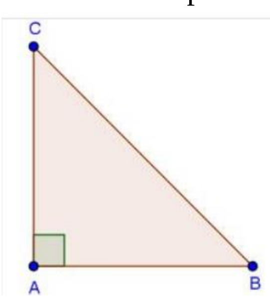
$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{25+9} = \sqrt{34}$

$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{4+64} = \sqrt{68}$

$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68}$

Since, $AB = BC = CD = DA$ and $AC = BD$, all the four sides of the quadrilateral $ABCD$ are equal and its diagonals AC and BD are also equal. Therefore, $ABCD$ is a square

9. Prove that the points (3, 0) (6, 4) and (-1, 3) are vertices of a right angled isosceles triangle.



Sol:

Let $A(3,0), B(6,4)$ and $C(-1,3)$ be the given points

$AB = \sqrt{(6-3)^2 + (4-0)^2}$

$\Rightarrow AB = \sqrt{(3)^2 + (4)^2}$

$\Rightarrow AB = \sqrt{9+16}$

$\Rightarrow AB = \sqrt{25}$

$BC = \sqrt{(-1-6)^2 + (3-4)^2}$

$\Rightarrow BC = \sqrt{(-7)^2 + (-1)^2}$

$\Rightarrow BC = \sqrt{49+1}$

$\Rightarrow BC = \sqrt{50}$

$AC = \sqrt{(-1-3)^2 + (3-0)^2}$

$\Rightarrow AC = \sqrt{(-4)^2 + (3)^2}$

$\Rightarrow A\sqrt{16+9}$

$\Rightarrow AC = \sqrt{25}$

$AB^2 = \left(\sqrt{25}\right)^2$

$\Rightarrow AB^2 = 25$

$AC^2 = 25$

$BC^2 = \left(\sqrt{50}\right)^2$

$BC^2 = 50$

Since $AB^2 + AC^2 = BC^2$ and $AB = AC$

$\therefore ABC$ is a right angled isosceles triangle

10. Prove that (2, —2), (—2, 1) and (5, 2) are the vertices of a right angled triangle. Find the area of the triangle and the length of the hypotenuse.

Sol:

Let $A(2,-2), B(-2,1)$ and $C(5,2)$ be the given points

$AB = \sqrt{(-2-2)^2 + (1+2)^2}$

$\Rightarrow AB = \sqrt{(-4)^2 + (3)^2}$

$\Rightarrow AB + \sqrt{16-9}$

$\Rightarrow AB=\sqrt{25}$

$BC=\sqrt{(5+2)^3+(2-1)^2}$

$\Rightarrow B\sqrt{(7)^2+(1)^2}$

$\Rightarrow BC=\sqrt{49+1}$

$\Rightarrow BC=\sqrt{50}$

$AC=\sqrt{(5-2)^2+(2+2)^2}$

$\Rightarrow AC=\sqrt{(3)^2+(4)^2}$

$\Rightarrow AC=\sqrt{9+16}$

$\Rightarrow AC=\sqrt{25}$

$AB^2=\left(\sqrt{25}\right)^2$

$\Rightarrow AB^2=25$

$BC^2=\left(\sqrt{50}\right)^2$

$\Rightarrow BC^2=50$

Since, $AB^2+AC^2=BC^2$

$\therefore ABC$ is a right angled triangle.

Length of the hypotenuse $BC=\sqrt{50}=5\sqrt{2}$

Area of $\Delta ABC=\frac{1}{2}\times AB\times AC$

$=\frac{1}{2}\times\sqrt{25}\times\sqrt{25}$

$=\frac{25}{2}$ square units.

11. Prove that the points (2 a, 4 a), (2 a, 6 a) arid (2a + $\sqrt{3}a$, 5a) are the vertices of an equilateral triangle.

Sol:

Let $A(2a,4a),B(2a,6a)$ and $C\left(2a+\sqrt{3}a,5a\right)$ be the given points

$AB=\sqrt{(2a-2a)^2+(6a-4a)^2}$

$\Rightarrow AB=\sqrt{(0)^2+(2a)^2}$

$\Rightarrow AB=\sqrt{4a^2}$

$\Rightarrow AB=2a$

$BC = \sqrt{\left(2a + \sqrt{3}a - 2a\right)^2 + \left(5a - 6a\right)^2}$

$\Rightarrow BC = \sqrt{\left(\sqrt{3}a\right)^2 + \left(-a\right)^2}$

$\Rightarrow BC = \sqrt{3a^2 + a^2}$

$\Rightarrow BC = \sqrt{4a^2}$

$\Rightarrow BC = 2a$

$AC = \sqrt{\left(2a + \sqrt{3}a - 2a\right)^2 + \left(5a - 4a\right)^2}$

$\Rightarrow AC = \sqrt{\left(\sqrt{3}a\right)^2 + \left(a\right)^2}$

$\Rightarrow AC = \sqrt{3a^2 + a^2}$

$\Rightarrow AC = \sqrt{4a^2}$

$\Rightarrow AC = 2a$

Since, $AB = BC = AC$

$\therefore$ ABC is an equilateral triangle

12. Prove that the points (2, 3), (-4, -6) and (1, 3/2) do not form a triangle.

Sol:

Let $A(2,3), B(-4,-6)$ and $C(1,3/2)$ be the given points

$AB = \sqrt{(-4-2)^2 + (-6-3)^2}$

$\Rightarrow AB = \sqrt{(-6)^2 + (-9)^2}$

$\Rightarrow AB = \sqrt{36+81}$

$\Rightarrow AB = \sqrt{117}$

$BC = \sqrt{(1+4)^2 + \left(\frac{3}{2}+6\right)^2}$

$\Rightarrow BC = \sqrt{(5)^2 + \left(\frac{15}{2}\right)^2}$

$\Rightarrow BC = \sqrt{25 + \frac{225}{4}}$

$\Rightarrow BC = \sqrt{\frac{325}{4}}$

$\Rightarrow BC = \sqrt{8125}$

$$AC = \sqrt{(2-1)^2 + \left(3-\frac{3}{2}\right)^2}$$

$$\Rightarrow AC = \sqrt{(1)^2 + \left(\frac{3}{2}\right)^2}$$

$$\Rightarrow AC = \sqrt{1+\frac{9}{4}}$$

$$\Rightarrow AC = \sqrt{\frac{13}{4}}$$

$$\Rightarrow AC = \sqrt{3.25}$$

We know that for a triangle sum of two sides is greater than the third side

Here $AC + BC$ is not greater than AB.

$\therefore ABC$ is not triangle

13. An equilateral triangle has two vertices are (2, -1), (3, 4), (-2, 3) and (-3, -2), find the coordinates of the third vertex.

Sol:

Let $A(3,4), B(-2,3)$ and $C(x, y)$ be the three vertices of the equilateral triangle then,

$$AB^2 = BC^2 = CA^2$$

$$AB = \sqrt{(-2-3)^2 + (3-4)^2} = \sqrt{(-5)^2 + (-1)^2} = \sqrt{25+1} = \sqrt{26}$$

$$BC = \sqrt{(x+2)^2 + (y-3)^2} = \sqrt{x^2 + 4 + 4x + y^2 + 9 - 6y} = \sqrt{x^2 + y^2 - 6x - 8y + 25}$$

$$CA = \sqrt{(x-3)^2 + (y-4)^2} = \sqrt{x^2 + 9 - 6x + y^2 + 16 - 8y} = \sqrt{x^2 + y^2 - 6x - 8y + 25}$$

Now, $AB^2 = BC^2$

$$\Rightarrow x^2 + y^2 + 4x - 6y + 13 = 26$$

$$\Rightarrow x^2 + y^2 + 4x - 6y - 13 = 0 \qquad \text{.......(i)}$$

$$AB^2 = CA^2$$

$$\Rightarrow 26 - x^2 + y^2 - 6x - 8y + 25$$

$$\Rightarrow x^2 + y^2 - 6x - 8y - 1 = 0 \qquad \text{.......(ii)}$$

Subtracting (ii) from (i) we get,

$$10x + 2y - 12 = 0$$

$$\Rightarrow 5x + y = 6 \qquad \text{.......(iii)}$$

$$\Rightarrow 5x = 6 - y$$

$$\Rightarrow x = \frac{6-y}{5}$$

Subtracting $x=\frac{6-y}{5}$ in (i) we get

$$\left(\frac{6-y}{5}\right)^2+y^2+4\left(\frac{6-y}{5}\right)-6y-13=0$$

$$\Rightarrow \frac{(6-y)^2}{25}+y^2+\frac{24-4y}{5}-6y-13=0$$

$$\Rightarrow \frac{36+y^2-12y}{25}+y^2+\frac{24-4y}{5}-6y-13=0$$

$$\Rightarrow \frac{36+y^2-12y+25y^2+120-20y-150-13\times25}{25}=0$$

$$\Rightarrow 26y^2-32y+6-325=0$$

$$\Rightarrow 26y^2-32y-319=0$$

$$D=b^2-4ac$$

$$D=(-32)^2-4\times26\times(-319)=1024+33176=34200$$

$$\therefore \quad y=\frac{-b\pm\sqrt{D}}{2a}=\frac{-(-32)\pm\sqrt{34200}}{2\times26}$$

$$\therefore y=\frac{32+185}{52}=\frac{217}{52} \text{ or } y=\frac{32-185}{52}=\frac{-153}{52}$$

Substituting $y=\frac{217}{52}$ in (iii)

$$5x+\frac{217}{52}=6$$

$$5x=6-\frac{217}{52}=\frac{95}{52}$$

$$x=\frac{19}{52}$$

Again substituting $y=\frac{-153}{52}$ in (iii)

$$5x-\frac{153}{52}=6$$

$$5x=6+\frac{153}{52}=\frac{465}{52}$$

$$x=\frac{93}{52}$$

Therefore, the coordinates of the third vertex are $\left(\frac{19}{52},\frac{217}{52}\right)$ or $\left(\frac{93}{52},\frac{-153}{52}\right)$

14. Show that the quadrilateral whose vertices are (2, -1) (3, 4), (-2, 3) and (-3, -2) is a rhombus.

Sol:

Let $A(2,-1), B(3,4), C(-2,3)$ and $D(-3,-2)$

$AB = \sqrt{(3-2)^2 + (4+1)^2} = \sqrt{(1)^2 + (5)^2} = \sqrt{1+25} = \sqrt{26}$

$BC = \sqrt{(-2-3)^2 + (3-4)^2} = \sqrt{(-5)^2 + (-1)^2} = \sqrt{25+1} = \sqrt{26}$

$CD = \sqrt{(-3+2)^2 + (-2-3)^2} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{1+25} = \sqrt{26}$

$AD = \sqrt{(-3-2)^2 + (-2+1)^2} = \sqrt{(-5)^2 + (-1)^2} = \sqrt{25+1} = \sqrt{26}$

Since $AB = BC = CD = AD$

$\therefore$ $ABCD$ is a rhombus

15. Two vertices of an isosceles triangle are (2, 0) and (2, 5). Find the third vertex if the length of the equal sides is 3.

Sol:

Two vertices of an isosceles triangle are $A(2,0)$ and $B(2,5)$. Let $C(x, y)$ be the third vertex

$AB = \sqrt{(2-2)^2 + (5-0)^2} = \sqrt{(0)^2 + (5)^2} = \sqrt{25} = 5$

$BC = \sqrt{(x-2)^2 + (y-5)^2} = \sqrt{x^2 + 4 - 4x + y^2 + 25 - 10y} = \sqrt{x^2 - 4x + y^2 - 10y + 29}$

$AC = \sqrt{(x-2)^2 + (y-0)^2} = \sqrt{x^2 + 4 - 4x + y^2}$

Also we are given that

$AC - BC = 3$

$\Rightarrow AC^2 = BC^2 = 9$

$\Rightarrow x^2 + 4 - 4x + y^2 = x^2 - 4x + y^2 - 10y + 29$

$\Rightarrow 10y = 25$

$\Rightarrow y = \frac{25}{10} = \frac{5}{2}$

$AC^2 = 9$

$x^2 + 4 - 4x + y^2 = 9$

$x^2 + 4 - 4x + (2.5)^2 = 9$

$x^2 + 4 - 4x + 6.25 = 9$

$x^2 - 4x + 1.25 = 0$

$D = (-4)^2 - 4 \times 1 \times 1.25$

$D = 16 - 5$

$D = 11$

$$x = \frac{-(-4) + \sqrt{11}}{2 \times 1} = \frac{4 + 3.31}{2} = \frac{7.31}{2} = 3.65$$

Or $x = \frac{-(-4) - \sqrt{11}}{2} = \frac{4 - \sqrt{11}}{2} = \frac{4 - 3.31}{2} = 0.35$

The third vertex is $(3.65, 2.5)$ or $(0.35, 2.5)$

16. Which point on x-axis is equidistant from (5, 9) and (-4, 6)?

Sol:

Let $A(5,9)$ and $B(-4,6)$ be the given points.

Let $C(x,0)$ be the point on $x - axis$

Now,

$AC = \sqrt{(x-5)^2 + (0-9)^2}$

$\Rightarrow AC = \sqrt{x^2 + 25 - 10x + (-9)^2}$

$\Rightarrow AC = \sqrt{x^2 - 10x + 25 + 81}$

$\Rightarrow AC = \sqrt{x^2 - 10x + 106}$

$BC = \sqrt{(x+4)^2 + (0-6)^2}$

$\Rightarrow BC = \sqrt{x^2 + 16 + 8d + (-6)^2}$

$\Rightarrow BC = \sqrt{x^2 + 8x + 16 + 36}$

$\Rightarrow BC = \sqrt{x^2 + 8x + 5x}$

Since $AC = BC$

Or, $AC^2 = BC^2$

$x^2 - 10x + 106 = x^2 + 8x + 52$

$\Rightarrow -10x + 106 = 8x + 52$

$\Rightarrow -10x - 8x = 52 - 106$

$\Rightarrow -18x = -54$

$\Rightarrow x = \frac{54}{18}$

$\Rightarrow x = 3$

Hence the points on x-axis is $(3,0)$.

17. Prove that the points (— 2, 5), (0, 1) and (2, — 3) are collinear.

Sol:

Let $A(-2,5), B(0,1)$ and $C(2,-3)$ be the given points

$AB = \sqrt{(0+2)^2 + (1-5)^2}$

$\Rightarrow AB = \sqrt{4+(-4)^2}$

$\Rightarrow AB = \sqrt{4+16}$

$\Rightarrow AB = \sqrt{20}$

$\Rightarrow AB = 2\sqrt{5}$

$BC = \sqrt{(2-0)^2 + (-3-1)^2}$

$\Rightarrow BC = \sqrt{(2)^2 + (-4)^2}$

$\Rightarrow BC = \sqrt{4+16}$

$\Rightarrow BC = \sqrt{20}$

$\Rightarrow BX = 2\sqrt{5}$

$AC = \sqrt{(2+2)^2 + (-3-5)^2}$

$\Rightarrow AC = \sqrt{(4)^2 + (-8)^2}$

$\Rightarrow AC = \sqrt{16+64}$

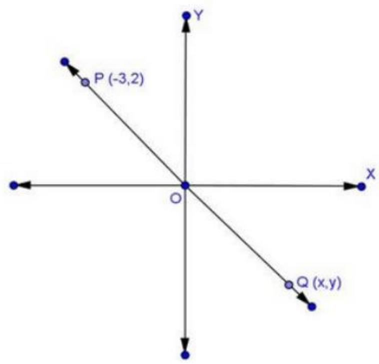
$\Rightarrow AC = \sqrt{80}$

$\Rightarrow AC = 4\sqrt{5}$

Since $AB + BC = AC$

Hence $A(-2,5), B(0,1),$ and $C(2,-3)$ are collinear

18. The coordinates of the point P are (—3, 2). Find the coordinates of the point Q which lies on the line joining P and origin such that OP = OQ.



Sol:

Let the coordinates of Q be (x, y)

Since Q lies on the line joining P and O (origin) and $OP = OQ$

By mid-point theorem

$\frac{(x-3)}{2} = 0$ and $\frac{(y+2)}{2} = 0$

$\therefore x = 3, y = -2$

Hence coordinates of points Q are $(3, -2)$

19. Which point on y-axis is equidistant from (2, 3) and (—4, 1)?

Sol:

$A(2,3)$ and $B(-4,1)$ are the given points.

Let $C(0, y)$ be the points are $y - axis$

$AC = \sqrt{(0-2)^2 + (y-3)^2}$

$\Rightarrow AC = \sqrt{4 + y^2 + 9 - 6y}$

$\Rightarrow AC = \sqrt{y^2 - 6y + 13}$

$BC = \sqrt{(0+4)^2 + (y-1)^2}$

$\Rightarrow BC = \sqrt{16 + y^2 + 1 - 2y}$

$\Rightarrow BC = \sqrt{y^2 - 2y + 17}$

Since $AC = BC$

$AC^2 = BC^2$

$y^2 - 6y + 13 = y^2 - 2y + 17$

$\Rightarrow -6y + 2y = 17 - 13$

$\Rightarrow -4y = 4$

$\Rightarrow y = -1$

$\therefore$ The point on $y - axis$ is $(0, -1)$

20. The three vertices of a parallelogram are (3, 4), (3, 8) and (9, 8). Find the fourth vertex.

Sol:

Let $A(3,4), B(3,8)$ and $C(9,8)$ be the given points

Let the forth vertex be $D(x, y)$

$AB = \sqrt{(3-3)^2 + (8-4)^2}$

$\Rightarrow AB = \sqrt{0 + (4)^2}$

$\Rightarrow AB = \sqrt{16}$
$\Rightarrow AB = 4$
$BC = \sqrt{(9-3)^2 + (8-8)^2}$
$\Rightarrow BC = \sqrt{(6)^2 + 0}$
$\Rightarrow BC = \sqrt{36}$
$\Rightarrow BC = B$
$CD = \sqrt{(x-9)^2 + (y-8)^2}$
$\Rightarrow CD = \sqrt{x^2 + (9^2) - 18x + y^2 + (8^2) - 16y}$
$\Rightarrow CD = \sqrt{x^2 + 81 - 18x + y^2 + 64 - 16y}$
$\Rightarrow CD = \sqrt{x^2 - 18x + y^2 - 16y + 145}$
$AD = \sqrt{(x-3)^2 + (y-4)^2}$
$\Rightarrow AD = \sqrt{x^2 + 9 - 6x + y^2 + 16 - 8y}$
$\Rightarrow AD = \sqrt{x^2 - 6x + y^2 - 8y + 25}$

Since ABCD is a parallelogram and opposite sides of a parallelogram are equal

$AB = CD$ and $AD = BC$
$AB = CD$
$AB^2 = CD^2$
$\Rightarrow x^2 - 18x + y^2 - 16y + 145 = 16$
$\Rightarrow x^2 - 18x + y^2 - 16y + 145 - 16 = 0$
$\Rightarrow x^2 - 18x + y^2 - 16y + 129 = 0$ ………..(1)
$BC = AD$
$BC^2 = AD^2$
$x^2 - 6x + y^2 - 8y + 25 = 36$
$\Rightarrow x^2 - 6x + y^2 - 8y + 25 - 36 = 0$
$\Rightarrow x^2 - 6x + y^2 - 8y - 11 = 0$ ………..(2)
$x = 9, y = 4$

The fourth vertex is $D(9,4)$

21. Find the circumcenter of the triangle whose vertices are (—2, —3), (— 1, 0), (7, — 6).

Sol:

Circumcenter of a triangle is the point of intersection of all the three perpendicular bisectors of the sides of triangle. So, the vertices of the triangle lie on the circumference of the circle.

Let the coordinates of the circumcenter of the triangle be (x, y)

$\therefore (x, y)$ will the equidistant from the vertices of the triangle.

Using distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, it is obtained:

$D_1 = \sqrt{(x+2)^2 + (y+3)^2}$

$\Rightarrow D_1 = \sqrt{x^2 + 4 + 4x + y^2 + 9 + 6y}$ (Taking points (x, y) and $(-2, -3)$)

$\Rightarrow D_1 = \sqrt{x^2 + y^2 + 4x + 6y + 13}$

$D^2 = \sqrt{(x+1)^2 + (y-0)^2}$ (Taking points (x, y) and $(-1, 0)$)

$\Rightarrow D_2 = \sqrt{x^2 + 1 + 2x + y^2}$

$D_3 = \sqrt{(x-7)^2 + (y+6)^2}$ (Taking points (x, y) and $(7, -6)$)

$\Rightarrow D_3 = \sqrt{x^2 + 49 - 14x + y^2 + 36 + 12y}$

$\Rightarrow D_3 = \sqrt{x^2 + y^2 - 14x + 12y + 85}$

As (x, y) is equidistant from all the three vertices

So, $D_1 = D_2 = D_3$

$D_1 = D_2$

$\therefore \sqrt{x^2 + y^2 + 4x + 6y + 13} = \sqrt{x^2 + 1 + 2x + y^2}$

$\Rightarrow x^2 + y^2 + 4x + 6y + 13 = x^2 + 1 + 2x + y^2$

$\Rightarrow 4x + 6y - 2x = 1 - 13$

$\Rightarrow 2x + 6y = -12$

$\Rightarrow x + 3y = -6$ ……….(1)

$D_2 = D_3$

$\therefore \sqrt{x^2 + 1 + 2x + y^2} = \sqrt{x^2 + y^2 - 14x + 12y + 85}$

$\Rightarrow x^2 + 1 + 2x + y^2 = x^2 + y^2 - 14x + 12y + 85$

$\Rightarrow 2x + 14x - 12y = 85 - 1$

$\Rightarrow 16x - 12y = 84$

$\Rightarrow 4x - 3y = 21$ ……….(2)

Adding equations (1) and (2):

$x + 3y + 4x - 3y = -6 + 21$

$\therefore 5x = 15$

$\Rightarrow x = \frac{15}{3}$

$\Rightarrow x = 3$

When $x = 3,$ we get

$y = \frac{4(3) - 21}{3}$ [Using (2)]

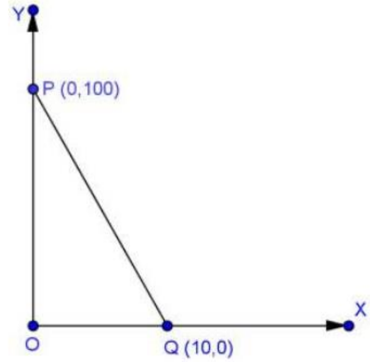
$\Rightarrow y = \frac{12 - 21}{3}$

$\Rightarrow y = -\frac{9}{3}$

$\Rightarrow y = -3$

$\therefore (3, -3)$ are the coordinates of the circumcenter of the triangle

22. Find the angle subtended at the origin by the line segment whose end points are (0, 100) and (10, 0).



Sol:

Let the point $P(0,100)$ and $Q(10,0)$ be the given points.

$\therefore$ The angle subtended by the line segment PQ at the origin O is $90°$.

23. Find the centre of the circle passing through (5, — 8), (2, — 9) and (2, 1).

Sol:

Let the center of the circle be $O(x, y)$

Since radii of the circle is constant

Hence, distance of O from $A(5, -8), B(2, -9)$ and $C\ (2,1)$ will be constant and equal

$\therefore OA^2 = OB^2 = OC^2$

$(x-5)^2 + (y+B)^2 = (x-2)^2 + (y+9)^2$

$x^2 + 25 - 10x + y^2 + 64 + 16y = x^2 + 4 - 4x + y^2 + 81 + 18y$

$-6x-2y+4=0$

$3x+y-2=0$

$y=2-3x$ …………(i)

Also, $OB^2=OC^2$

$(x-2)^2+(y+9)^2+(y-1)^2$

$y^2+81+18y=y^2+1-2y$

$80+20y=0$

$y=-4$

Substituting y in (i)

$-4=2-3x$

$3x=6$

$x=2$

Hence center of circle $(2,-4)$

24. Find the value of k, if the point P (0, 2) is equidistant from (3, k) and (k, 5).

Sol:

Let the point $P(0,2)$ is equidistant from $A(3,k)$ and $(k,5)$

$PA=PB$

$PA^2=PB^2$

$(3-0)^2+(k-2)^2=(k-0)^2+(5-2)^2$

$\Rightarrow 9+k^2+4-4k=k^2+9.$

$\Rightarrow 9+k^2+4-4k-k^2-9=0$

$\Rightarrow 4-4k=0$

$\Rightarrow -4k=-4$

$\Rightarrow k=1$

25. If two opposite vertices of a square are (5, 4) and (1, —6), find the coordinates of its remaining two vertices.

Sol:

Let $ABCD$ be a square and let $A(5,4)$ and $C(1,-6)$ be the given points.

Let (x,y) be the coordinates of B.

$AB=BC$

$AB^2=BC^2$

$(x-5)^2+(y-4)^2=(x-1)^2+(y+6)^2$

$\Rightarrow x^2+25-10x+y^2+16-8y=x^2+1-2x+y^2+36+12y$

$\Rightarrow x^2-10x+y^2-8y-x^2+2x-y^2-12y=1+36-25-16$

$\Rightarrow -8x-20y=-4$

$\Rightarrow -8x=20y-4$

$\Rightarrow x=\dfrac{20y-4}{-8}$

$\Rightarrow x=\dfrac{4(5y-1)}{-8}$

$\Rightarrow x=\dfrac{5y-1}{-2}$

$\Rightarrow x=\dfrac{1-5y}{2}$ ………..(1)

In right triangle ABC

$AB^2+BC^2=AC^2$

$(x-5)^2+(y-4)^2+(x-1)^2+(y+6)^2=(5-1)^2+(4+6)^2$

$\Rightarrow x^2+25-10x+y^2+16-8y+x^2+1-2x+y^2+36+12y=16+100$

$\Rightarrow 2x^2+2y^2-12x+4y=116-78$

$\Rightarrow 2x^2+2y^2-12+4y=38$

$\Rightarrow x^2+y^2-6x+2y=19$

$\Rightarrow x^2+y^2-6x+2y-19=0$ ………..(2)

Substituting the value of x form (1) in (2), we get

$\left(\dfrac{1-5y}{2}\right)^2+y^2-6\left(\dfrac{1-5y}{2}\right)+2y-19=0$

$\Rightarrow \dfrac{(1-5y)^2}{4}+y^2-3(1-5y)+2y-19=0$

$\Rightarrow \dfrac{1+25y^2-10y}{4}+y^2-3+15y+2y-19=0$

$\Rightarrow \dfrac{1+25y^2-10y+4y^2-12+60y+8y-76}{4}=0$

$\Rightarrow 29y^2+58y-87=0$

$\Rightarrow y^2+2y-3=0$

$\Rightarrow y^2+3y-y-3=0$

$\Rightarrow y(y+3)-1(y+3)=0$

$\Rightarrow (y+3)(y-1)=0$

$\Rightarrow y=-3, y=1$

Substituting $y=-3$ and $y=1$ in equation (1), we get

$$x=\frac{1-5(-3)}{2}$$

$$\Rightarrow x=\frac{1+15}{2}$$

$$\Rightarrow x=8$$

$$x=\frac{1-5(1)}{2}$$

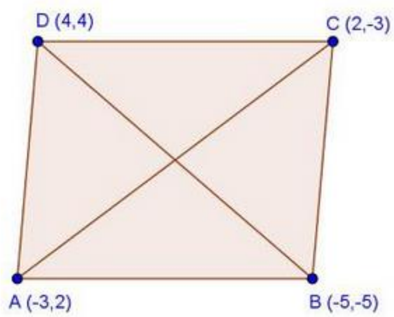
$$\Rightarrow x=\frac{1-5}{2}$$

$$\Rightarrow x=\frac{-4}{2}$$

$$\Rightarrow x=-2$$

Hence, the required vertices of the square are $(-2,1)$ and $(8,-3)$.

26. Show that the points (—3, 2), (—5, —5), (2, —3) and (4, 4) are the vertices of a rhombus. Find the area of this rhombus.



Sol:

$A(-3,2), B(-5,-5), C(2,-3)$ and $D(4,4)$ be the given points.

$$AB=\sqrt{(-5+3)^2+(-5-2)^2}$$

$$\Rightarrow AB=\sqrt{(2)^2+(-7)^2}$$

$$\Rightarrow AB=\sqrt{4+49}$$

$$\Rightarrow AB=\sqrt{53}$$

$$BC=\sqrt{(2+5)^2+(-5-2)^2}$$

$$\Rightarrow BC=\sqrt{(7)^2+(2)^2}$$

$$\Rightarrow BC=\sqrt{49+4}$$

$$\Rightarrow BC=\sqrt{53}$$

$CD=\sqrt{(4-2)^2+(4+3)^2}$

$\Rightarrow CD=\sqrt{(2)^2+(7)^2}$

$\Rightarrow CD=\sqrt{4+49}$

$\Rightarrow CD=\sqrt{53}$

$AD=\sqrt{(4+3)^2+(4-2)^2}$

$\Rightarrow AD=\sqrt{(7)^2+(2)^2}$

$\Rightarrow AD=\sqrt{49+4}$

$\Rightarrow AD=\sqrt{53}$

$AC=\sqrt{(2+3)^2+(-3-2)^2}$

$\Rightarrow AC=\sqrt{(5)^2+(-5)^2}$

$\Rightarrow AC=\sqrt{25+25}$

$\Rightarrow AC=\sqrt{50}$

$BD=\sqrt{(4+5)^2+(4+5)^2}$

$\Rightarrow BD=\sqrt{(9)^2+(9)^2}$

$\Rightarrow BD=\sqrt{81+81}$

$\Rightarrow BD=\sqrt{162}$

Since $AB=BC=CD=AD$ and diagonals $AC\neq BD$

$\therefore ABCD$ is a rhombus

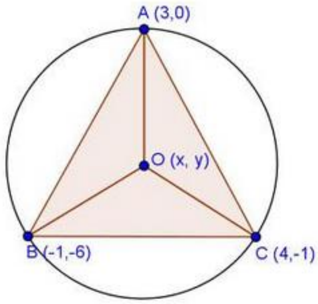
Area of rhombus $ABCD=\frac{1}{2}\times AC\times BD$

$=\frac{1}{2}\times\sqrt{50}\times\sqrt{162}$

$=\frac{1}{2}\times 90$

$=45$ sq. units

27. Find the coordinates of the circumcenter of the triangle whose vertices are (3, 0), (—1, —6) and (4,—1). Also, find its circumradius.



Sol:

Let $A(3,0), B(-1,-6)$ and $C(4,-1)$ be the given points.

Let $O(x, y)$ be the circumcenter of the triangle

$OA = OB = OC$

$OA^2 = OB^2$

$(x-3)^2 + (y-0)^2 = (x+1)^2 + (y+6)^2$

$\Rightarrow x^2 + 9 - 6x + y^2 = x^2 + 1 + 2x + y^2 + 36 + 12y$

$\Rightarrow x^2 - 6x + y^2 - x^2 - 2x - y^2 - 12y = 1 + 36 - 9$

$\Rightarrow -8x - 12y = 28$

$\Rightarrow -2x - 3y = 7$

$\Rightarrow 2x + 3y = -7$ ....(1)

Again

$OB^2 = OC^2$

$(x+1)^2 + (y+6)^2 = (x-4)^2 + (y+1)^2$

$\Rightarrow x^2 + 1 + 2x + y^2 + 36 + 12y = x^2 + 16 - 8x + y^2 + 1 + 2y$

$\Rightarrow x^2 + 2x + y^2 + 12y - x^2 + 8x - y^2 - 2y = 16 + 1 - 1 - 36$

$\Rightarrow 10x + 10y = -20$

$\Rightarrow x + y = -2$ ....(2)

Solving (1) and (2), we get

$x = 1, y = -3$

Hence circumcenter of the triangle is $(1, -3)$

Circum radius $= \sqrt{(1+1)^2 + (-3+6)^2}$

$= \sqrt{(2)^2 + (3)^2}$

$= \sqrt{4+9}$

$\sqrt{13}$ units

28. Find a point on the x-axis which is equidistant from the points (7, 6) and (—3, 4).

Sol:

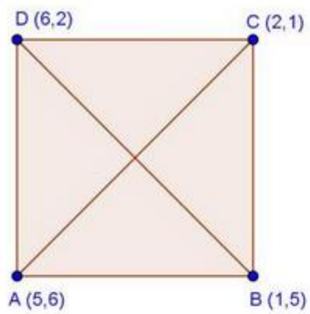
Let $A(7,6)$ and $B(-3,4)$ be the given points.

Let $P(x.0)$ be the point on $x-axis$ such that $PA = PB$

$$PA = PB$$
$$PA^2 = PB^2$$
$$(x-7)^2+(0-6)^2=(x+3)^2+(0-4)^2$$
$$\Rightarrow x^2+49-14x+36=x^2+9+6x+16$$
$$\Rightarrow x^2-14-x^2-6x=9+16-36-49$$
$$\Rightarrow -20x=-60$$
$$\Rightarrow x=3$$

$\therefore$ The point on $x-axis$ is $(3,0)$.

29. (i) Show that the points A(5, 6), B (1, 5), C(2, 1)and D(6, 2) are the vertices of a square.
(ii) Prove that the points A (2, 3), B (—2, 2), C (—1, —2), and D (3, —1) are the vertices of a square ABCD.



Sol:

$A(5,6), B(1,5), C(2,1)$ and $D(6,2)$ are the given points

$$AB=\sqrt{(5-1)^2+(6-5)^2}$$
$$\Rightarrow AB=\sqrt{(4)^2+(1)^2}$$
$$\Rightarrow AB=\sqrt{16+1}$$
$$\Rightarrow AB=\sqrt{17}$$
$$BC=\sqrt{(1-2)^2+(5-1)^2}$$
$$\Rightarrow BC=\sqrt{(-1)^2+(4)^2}$$
$$\Rightarrow BC=\sqrt{1+16}$$
$$\Rightarrow BC=\sqrt{17}$$

$CD = \sqrt{(6-2)^2 + (2-1)^2}$
$\Rightarrow CD = \sqrt{(4)^2 + (1)^2}$
$\Rightarrow CD = \sqrt{16+1}$
$\Rightarrow CD = \sqrt{17}$
$AD = \sqrt{(6-5)^2 + (2-6)^2}$
$\Rightarrow AD = \sqrt{(1)^2 + (-4)^2}$
$\Rightarrow AD = \sqrt{1+16}$
$\Rightarrow AD = \sqrt{17}$
$AC = \sqrt{(5-2)^2 + (6-1)^2}$
$\Rightarrow AC = \sqrt{(3)^2 + (5)^2}$
$\Rightarrow AC = \sqrt{9+25}$
$\Rightarrow AC = \sqrt{34}$
$BD = \sqrt{(6-1)^2 + (2-5)^2}$
$\Rightarrow BD = \sqrt{(5)^2 + (-3)^2}$
$\Rightarrow BD = \sqrt{25+9}$
$\Rightarrow BD = \sqrt{34}$
Since $AB = BC = CD = AD$ and diagonals $AC = BD$
$\therefore ABCD$ is a square

30. Find the point on x-axis which is equidistant from the points (—2, 5) and (2,—3).
Sol:
Let $A(-2,5)$ and $(2,-3)$ be the given points.
Let $(x,0)$ be the point on $x-axis$
Such that $PA = PB$
$PA = PB$
$PA^2 = PB^2$
$(x+2)^2 + (0-5)^2 = (x-2)^2 + (0+3)^2$
$\Rightarrow x^2 + 4 + 4x + 25 = x^2 + 4 - 4x + 9$
$\Rightarrow x^2 + 4x + x^2 + 4x = 4 + 9 - 4 - 25$
$\Rightarrow 8x = -16$
$\Rightarrow x = -2$

$\therefore$ The point on $x-axis$ is $(-2,0)$

31. Find the value of x such that PQ = QR where the coordinates of P, Q and R are (6,—1), (1, 3) and (x, 8) respectively.

Sol:

$P(6,-1), Q(1,3)$ and $R(x,8)$ are the given points.

$PQ = QR$

$PQ^2 = QR^2$

$\Rightarrow (6-1)^2 + (-1-3)^2 = (x-1)^2 + (8-3)^2$

$\Rightarrow (5)^2 + (-4)^2 = x^2 + 1 - 2x + (5)^2$

$\Rightarrow 25 + 16 = x^2 + 1 - 2x + 25$

$\Rightarrow 41 = x^2 - 2x + 26$

$\Rightarrow x^2 - 2x + 26 - 41 = 0$

$\Rightarrow x^2 - 2x - 15 = 0$

$\Rightarrow x^2 - 5x + 3x - 15 = 0$

$\Rightarrow x(x-5) + 3(x-5) = 0$

$\Rightarrow (x+3)(x-5) = 0$

$\Rightarrow x = -3$ or $x = 5$

32. Prove that the points (0, 0), (5, 5) and (—5, 5) are the vertices of a right isosceles triangle.

Sol:

Let $A(0,0), B(5,5)$ and $C(-5,5)$ be the given points

$AB = \sqrt{(5-0)^2 + (5-0)^2}$

$\Rightarrow AB = \sqrt{25+25}$

$\Rightarrow AB = \sqrt{50}$

$BC = \sqrt{(5+5)^2 + (5-5)^2}$

$\Rightarrow BC = \sqrt{(10)^2 + 0}$

$\Rightarrow BC = \sqrt{100}$

$AC = \sqrt{(0+5)^2 + (0-5)^2}$

$\Rightarrow AC = \sqrt{25+25}$

$\Rightarrow AC = \sqrt{50}$

$AB^2 = 50$

$BC^2 = 100$
$AC^2 = 50$
$\Rightarrow AB^2 + AC^2 = BC^2$
Since, $AB = AC$ and $AB^2 + AC^2 = BC^2$
$\therefore ABC$ is a right isosceles triangle

33. If the point P(x, y) is equidistant from the points A(5, 1) and B (1, 5), prove that x = y.
Sol:
Since $P(x, y)$ is equidistant from $A(5,1)$ and $B(1,5)$
$AP = BP$
Hence, $AP^2 = BP^2$
$(x-5)^2 + (y-1)^2 = (x-1)^2 + (y-5)^2$
$x^2 + 25 - 10x + y^2 + 1 - 2y = x^2 + 1 - 2x + y^2 + 25 - 10y$
$-10x + 2x = -10y + 2y$
$-8x = -8y$
$x = y$
Hence, proved.

34. If Q (0, 1) is equidistant from P (5, —3) and R (x, 6), find the values of x. Also, find the distances QR and PR
Sol:
Given $Q(0,1)$ is equidistant from $P(-5,-3)$ and $R(x,6)$ so $PQ = QR$
$\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(0-x)^2 + (1-6)^2}$
$\sqrt{(5)^2 + (-4)^2} = \sqrt{(-x)^2 + (-5)^2}$
$\sqrt{25+16} = \sqrt{x^2 + 25}$
$41 = x^2 + 25$
$16 = x^2$
$x = \pm 4$
So, point R is $(4,6)$ or $(-4,6)$
When point R is $(4,6)$
$PR = \sqrt{(5-4)^2 + (-3-6)^2} = \sqrt{1^2 + (-9)} = \sqrt{1+81} = \sqrt{82}$
$QR = \sqrt{(0-4)^2 + (1-6)^2} = \sqrt{(-4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$
When point R is $(-4,6)$

$$PR=\sqrt{\left(5-\left(-4\right)\right)^2+\left(-3-6\right)^2}=\sqrt{\left(9\right)^2+\left(-9\right)^2}=\sqrt{81+81}=9\sqrt{2}$$

$$QR=\sqrt{\left(0\left(-4\right)\right)^2+\left(1-6\right)^2}=\sqrt{\left(4\right)^2+\left(-5\right)^2}=\sqrt{16+25}=\sqrt{41}$$

35. Find the values of y for which the distance between the points P (2, —3) and Q (10, y) is 10 units

Sol:

Given that distance between $(2,-3)$ and $(10,y)$ is 10

Therefore using distance formula $\sqrt{\left(2-10\right)^2+\left(-3-y\right)^2}=10$

$\sqrt{\left(-8\right)^2+\left(3+y\right)^2}=10$

$64+\left(y-3\right)^2=100$

$\left(y+3\right)^2=36$

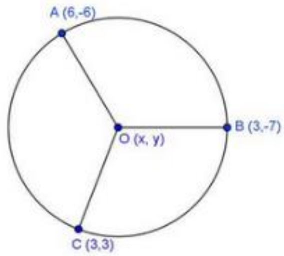
$y+3=\pm 6$

$y+3=6$ or $y+3=-6$

Therefore $y=3$ or -9

36. Find the centre of the circle passing through (6, —6), (3, —7) and (3, 3)

Sol:



Let $O(x,y)$ be the center of the circle passing through $A(6,-6), B(3,-7)$ and $C(3,3)$

$OA=OB=OC$

$OA^2=OB^2$

$\left(x-6\right)^2+\left(y+6\right)^2=\left(x-3\right)^2+\left(y-7\right)^2$

$\Rightarrow x^2+36-12x+y^2+36+12y=x^2+9-6x+y^2+49+14y$

$\Rightarrow x^2+36-12x+y^2+36+12y-x^2-9+6x-y^2-49-14y=0$

$\Rightarrow -6x-2y=-36-36+9+49$

$\Rightarrow -6x-2y=-14$ ....(1)

$OB^2=OC^2$

$(x-3)^2+(y+7)^2=(x-3)^2+(y-3)^2$

$\Rightarrow x^2+9-6x+y^2+49+14y=x^2+9-6x+y^2+9-6y$

$\Rightarrow x^2-6x+y^2+14y-x^2+6x-y^2+6y=9+9-9-49$

$\Rightarrow 20y=-40$

$\Rightarrow y=-2$

Substituting $y=-2$ in (1)

$-6x-2(-2)=-14$

$\Rightarrow -6x+4=-14$

$\Rightarrow -6x=-14-4$

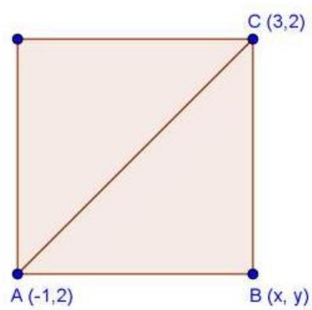
$\Rightarrow -6x=-18$

$\Rightarrow x=3$

$\therefore$ The centre of the circle is $(3,-2)$

37. Two opposite vertices of a square are (—1, 2) and (3, 2). Find the coordinates of other two vertices.

Sol:



Let $ABCD$ be a square and let $A(-1,2)$ and $(3,2)$ be the opposite vertices and let $B(x,y)$ be the unknown vertex.

$AB=BC$

$AB^2=BC^2$

$(x+1)^2+(y-2)^2=(x-3)^2+(y-2)^2$

$\Rightarrow x^2+1+2x+y^2+4-4y=x^2+9-6x+y^2+4-4y$

$\Rightarrow x^2+2x+y^2-4y-x^2+6x-y^2+4y=9+4-1-4$

$\Rightarrow 8x=8$

$\Rightarrow x=1$ ………(1)

In right triangle ABC

$AB^2+BC^2=AC^2$

$\Rightarrow (x+1)^2 + (y-2)^2 + (x-3)^2 + (y-2)^2 = (3+1)^2 + (2-2)^2$

$\Rightarrow x^2 + 1 + 2x + y^2 + 4 - 4y + x^2 + 9 - 6x + y^2 + 4 - 4y = 16$

$\Rightarrow 2x^2 + 2y^2 - 4x - 8y = 16 - 1 - 4 - 9 - 4$

$\Rightarrow 2x^2 + 2y^2 - 4x - 8y = -2$ ....(2)

Substituting $x = 1$ from (1) and (2)

$2(1)^2 + 2y^2 - 4(1) - 8y = -2$

$\Rightarrow 2 + 2y^2 - 4 - 8y = -2$

$\Rightarrow 2y^2 - 8y - 2 + 2 = 0$

$\Rightarrow 2y^2 - 8y = 0$

$\Rightarrow 2y(y-4) = 0$

$\Rightarrow y = 0,$ or $y = 4$

Hence the required vertices of the square are $(1,0)$ and $(1,4)$

38. Name the quadrilateral formed, if any, by the following points, and give reasons for your answers:

(i) A (—1, — 2), B (1, 0), C (—1, 2), D (—3, 0)
(ii) A (—3, 5), B (3, 1), C (0, 3), D (—1, — 4)
(iii) A (4, 5), B (7, 6), C (4, 3), D (1, 2)

Sol:

(i) Let, $A = (-1,-2), B = (1,0), C = (-1,2,), D(-3,0)$

$AB = \sqrt{(-1-1)^2 + (-2-0)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$BC = \sqrt{(1-(-1))^2 + (0-2)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$CD = \sqrt{(-1-(-3))^2 + (2-0)^2} = \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$AD = \sqrt{(-1-(-3))^2 + (-2-0)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

Diagonal $AC - \sqrt{(-1-(-1))^2 + (-2-2)^2} = \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$

Diagonal $BD - \sqrt{(1-(-3))^2 + (0-0)^2} = \sqrt{(4)^2 + 0^2} = \sqrt{16} = 4$

Here, all sides of this quadrilateral are of same length and also diagonals are of same length. So, given points are vertices of a square

(ii) Let, $A = (-3,5), B = (3,1), C = (0,3), D = (-1,-4)$

$AB = \sqrt{(-3-3)^2 + (5-1)^2} = \sqrt{(-6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

$BC=\sqrt{(3-0)^2+(1-3)^2}=\sqrt{(3)^2+(-2)^2}=\sqrt{9+4}=\sqrt{13}$

$CD=\sqrt{(0-(-1))^2+(3-(-4))^2}=\sqrt{(1)^2+(7)^2}=\sqrt{1+49}=\sqrt{50}=5\sqrt{2}$

$AD=\sqrt{(-3-(-1))^2+(5-(-4))^2}=\sqrt{(-2)^2+(9)^2}=\sqrt{4+81}=\sqrt{85}$

Here, all sides of this quadrilateral are of different length . So, we can say that it is only a general quadrilateral not specific like square, rectangle etc.

(iii) Let, $A=(4,5),B=(7,6),C=(4,3),D=(1,4)$

$AB=\sqrt{(4-7)^2+(5-6)^2}=\sqrt{(-3)^2+(-1)^2}=\sqrt{9+1}=\sqrt{10}$

$BC=\sqrt{(7-4)^2+(6-3)^2}=\sqrt{(3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}$

$CD=\sqrt{(4-1)^2+(3-2)^2}=\sqrt{(3)^2+(1)^2}=\sqrt{9+1}=\sqrt{10}$

$AD=\sqrt{(4-1)^2+(5-2)^2}=\sqrt{(3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}$

Diagonal $AC-\sqrt{(4-4)^2+(5-3)^2}=\sqrt{(0)^2+(2)^2}=\sqrt{0+4}=2$

Diagonal $BD-\sqrt{(7-1)^2+(6-2)^2}=\sqrt{(6)^2+(4)^2}=\sqrt{36+16}=\sqrt{52}=13\sqrt{2}$

Here, opposite sides of this quadrilateral are of same length but diagonals are different length . So, given points are vertices of a parallelogram.

39. Find the equation of the perpendicular bisector of the line segment joining points (7,1) and (3, 5).

Sol:

Bisector passes through midpoint

Midpoint of $(7,1)$ and $(3,5)=\left[\frac{(7+3)}{2},\frac{(1+5)}{2}\right]=(5,3)$

Perpendicular bisector has slope that is negative reciprocal of line segment joining points $(7,1)$ and $(3,5)$

Slope of line segment $=\left(\frac{5-1}{3-7}\right)=\frac{4}{(-4)}=-1$

Perpendicular bisector has slope = 1 and passes through point $(4,4)$

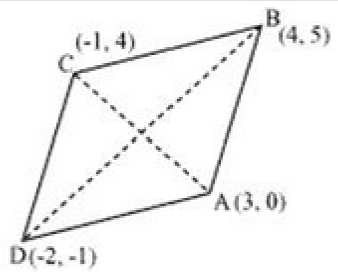
Use point slope form

$y-3=1(x-5)$

$y=x\cdot 2$

40. Prove that the points (3, 0), (4, 5), (—1, 4) and (—2, —1), taken in order, form a rhombus. Also, find its area.

Sol:



Let the given vertices be $A(3,0), B(4,5), C(-1,4)$ and $D(-2,-1)$

Length of $AB = \sqrt{(4-3)^2+(5-0)^2} = \sqrt{1+25} = \sqrt{26}$

Length of $BC = \sqrt{(-1-4)^2+(4-5)^2} = \sqrt{25+1} = \sqrt{26}$

Length of $CD = \sqrt{(-2+1)^2+(-1-4)^2} = \sqrt{1+25} = \sqrt{26}$

Length of $DA = \sqrt{(3+2)^2+(0+1)^2} = \sqrt{25+1} = \sqrt{26}$

Length of diagonal $AC = \sqrt{\left[3-(-1)^2+(0-4)^2\right]}$

$= \sqrt{16+16} = 4\sqrt{2}$

Length of diagonal $BD = \sqrt{\left[4-(-2)^2\right]+\left[5-(-1)\right]^2}$

$= \sqrt{36+36} = 6\sqrt{2}$

Here all sides of the quadrilateral ABCD are off same lengths but the diagonals are of different lengths

So, ABCD is a rhombus.

Therefore area of rhombus $ABCD = \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$

$= 24$ square units

41. In the seating arrangement of desks in a classroom three students Rohini, Sandhya and Bina are seated at A (3, 1), B (6, 4) and C (8, 6). Do you think they are seated in a line?

Sol:

Let $A(3,1), B(6,4)$ and $C(8,6)$ be thee given points

$AB = \sqrt{(6-3)^2+(4-1)^2}$

$\Rightarrow AB = \sqrt{(3)^2+(3)^2}$

$\Rightarrow AB = \sqrt{9+9}$

$\Rightarrow AB = \sqrt{18}$

$\Rightarrow AB = 3\sqrt{2}$

$BC = \sqrt{(8-6)^2 + (6-4)^2}$

$\Rightarrow BC = \sqrt{(2)^2 + (2)^2}$

$\Rightarrow BC = \sqrt{4+4}$

$\Rightarrow BC = \sqrt{8}$

$\Rightarrow BC = 2\sqrt{2}$

$AC = \sqrt{(8-3)^2 + (6-1)^2}$

$\Rightarrow AC = \sqrt{(5)^2 + (5)^2}$

$\Rightarrow AC = \sqrt{25+25}$

$\Rightarrow AC = \sqrt{50}$

$\Rightarrow AC = 5\sqrt{2}$

Since, $AB + BC = AC$

Points A, B, C are collinear

Hence, Rohini, Sandhya and Bina are seated in a line

42. Find a point on y-axis which is equidistant from the points (5, — 2) and (— 3, 2).

Sol:

Let $A(5,-2)$ and $B(-3,2)$ be the given points,

Let $P(0, y)$ be the point on $y-axis$

$PA = PB$

$PA^2 = PB^2$

$(0-5)^2 + (y+2)^2 = (0+3)^2 + (y-2)^2$

$\Rightarrow 25 + y^2 + 4 + 4y = 9 + y^2 + 4 - 4y$

$\Rightarrow y^2 + 4y - y^2 + 4y = 9 + 4 - 4 - 25$

$\Rightarrow 8y = -16$

$\Rightarrow y = -2$

43. Find a relation between x and y such that the point (x, y) is equidistant from the points (3, 6) and (—3, 4).

Sol:

Point (x, y) is equidistant form $(3,6)$ and $(-3,4)$

Therefore $\sqrt{(x-3)^2+(y-6)^2}=\sqrt{(x-(-3))^2+(y-4)^2}$

$\sqrt{(x-3)^2+(y-6)^2}=\sqrt{(x+3)^2+(y-4)^2}$

$(x-3)^2+(y-6)^2=(x+3)^2+(y-4)^2$

$x^2+9-6x+y^2+36-12y=x^2+9+6x+y^2+16-8y$

$36-16=6x+6x+12y-8y$

$20=12x+4y$

$3x+y=5$

44. If a point A (0, 2) is equidistant from the points B (3, p) and C (p, 5), then find the value of p.

Sol:

$A(0,2), B(3,P)$ and $C(p,5)$ are given points

It is given that $AB=AC$

$\therefore AB^2=AC^2$

$(3-0)^2+(p-2)^2=(p-0)^2+(5-2)^2$

$9+p^2+4-4p=p^2+9$

$4-4p=0$

$p=1$

Exercise 14.3

1. Find the coordinates of the point which divides the line segment joining (— 1, 3) and (4, — 7) internally in the ratio 3 : 4.

Sol:

Let $P(x, y)$ be the required point.

$x=\frac{mx_2+nx_1}{m+n}$

$y=\frac{my_2+ny_1}{m+n}$

Here, $x_1=-1$

$y_1=3$

$x_2=4$

$y_2=-7$

$m:n=3:4$

$$x=\frac{3\times4+4\times(-1)}{3+4}3$$

$$x=\frac{12-4}{7}$$

$$x=\frac{8}{7}$$

$$y=\frac{3\times(-7)+4\times3}{3+4}$$

$$y=\frac{-21+12}{7}$$

$$y=\frac{-9}{7}$$

$\therefore$ The coordinates of P are $\left(\frac{8}{7},\frac{-9}{7}\right)$

2. Find the points of trisection of the line segment joining the points:

(i) (5, —6) and (— 7, 5),

(ii) (3, —2) and (—3, —4)

(iii) (2, —2) and (—7, 4).

Sol:

(i) Let P and Q be the point of trisection of AB i.e., $AP=PQ=QB$

$(5,-6)$ $(-7,5)$

Therefore, P divides AB internally in the ratio of 1:2, thereby applying section formula, the coordinates of P will be

$$\left(\frac{1(-7)+2(5)}{1+2}\right),\left(\frac{1(5)+2(-6)}{1+2}\right) i.e., \left(1,\frac{-7}{3}\right)$$

Now, Q also divides AB internally in the ratio of 2:1 there its coordinates are

$$\left(\frac{2(-7)+1(5)}{2+1}\right),\frac{2(5)+1(-6)}{2+1} i.e., \left(-3,\frac{4}{3}\right)$$

(ii)

Let P, Q be the point of tri section of AB i.e.,

$AP=PQ=QB$

$(3,-2)$ $(-3,-4)$

Therefore, P divides AB internally in the ratio of 1:2
Hence by applying section formula, Coordinates of P are

$$\left(\left(\frac{1(-3)+2(3)}{1+2}\right),\frac{1(-4)+1(-2)}{1+2}\right) i.e., \left(1,\frac{-8}{3}\right)$$

Now, Q also divides as internally in the ratio of 2:1
So, the coordinates of Q are

$$\left(\left(\frac{2(-3)+1(3)}{2+1}\right),\frac{2(-4)+1(-2)}{2+1}\right) i.e., \left(-1,\frac{-10}{3}\right)$$

Let P and Q be the points of trisection of AB i.e., $AP = PQ = OQ$

Therefore, P divides AB internally in the ratio 1 : 2. Therefore, the coordinates of P, by applying the section formula, are

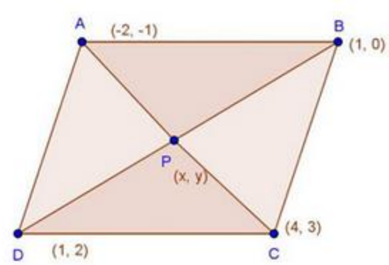
$$\left(\left(\frac{1(-7)+2(2)}{(1+2)}\right),\left(\frac{1(4)+2(-2)}{(1+2)}\right)\right), i.e., (-1,0)$$

Now, Q also divides AB internally in the ration 2 : 1. So, the coordinates of Q are

$$\left(\frac{2(-7)+1(2)}{2+1},\frac{2(4)+1(2)}{2+1}\right), i.e., (-4,2)$$

3. Find the coordinates of the point where the diagonals of the parallelogram formed by joining the points (—2, —1), (1, 0), (4, 3) and (1, 2) meet.

Sol:



Let P(x, y) be the given points.

We know that diagonals of a parallelogram bisect each other.

$x = \frac{-2+4}{2}$

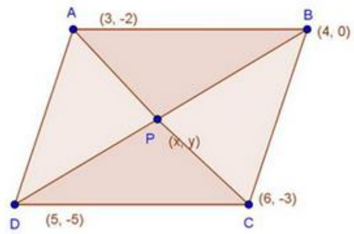
$\Rightarrow x = \frac{2}{2} = 1$

$y = \frac{-1+3}{2} = \frac{2}{2} = 1$

$\therefore$ Coordinates of P are $(1,1)$

4. Prove that the points (3, —2), (4, 0), (6, —3) and (5, —5) are the vertices of a parallelogram.

Sol:



Let $P(x, y)$ be the point of intersection of diagonals *AC* and 80 of *ABCD*.

$x = \frac{3+6}{2} = \frac{9}{2}$

$y = \frac{-2-3}{2} = \frac{-5}{2}$

Mid – point of $AC = \left(\frac{9}{5}, \frac{-5}{2}\right)$

Again,

$x = \frac{5+4}{2} = \frac{9}{2}$

$y = \frac{-5+0}{2} = \frac{-5}{2}$

Mid – point of $BD = \left(\frac{9}{2}, -\frac{5}{2}\right)$

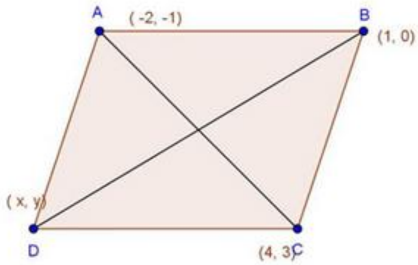
Here mid-point of AC – Mid - point of BD i.e, diagonals AC and BD bisect each other.

We know that diagonals of a parallelogram bisect each other

$\therefore$ *ABCD* is a parallelogram.

5. Three consecutive vertices of a parallelogram are (—2, —1), (1, 0) and (4, 3). Find the fourth vertex.

Sol:



Let $A(-2,-1), B(1,0), C(4,3)$ and $D(x,y)$ be the vertices of a parallelogram *ABCD* taken in order.

Since the diagonals of a parallelogram bisect each other.

$\therefore$ Coordinates of the mid - point of *AC* = Coordinates of the mid-point of *BD.*

$\Rightarrow \frac{-2+4}{2} = \frac{1+x}{2}$

$\Rightarrow \frac{2}{2} = \frac{x+1}{2}$

$\Rightarrow 1 = \frac{x+1}{2}$

$\Rightarrow x+1 = 2$

$\Rightarrow x = 1$

And, $\frac{-1+3}{2} = \frac{y+0}{2}$

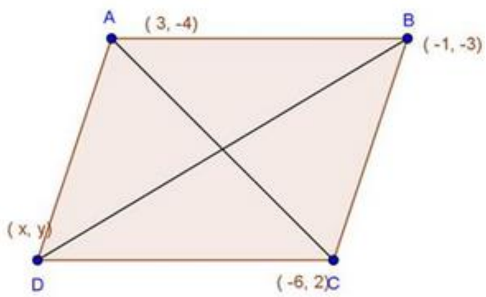
$\Rightarrow \frac{2}{2} = \frac{y}{2}$

$\Rightarrow y = 2$

Hence, fourth vertex of the parallelogram is $(1,2)$

6. The points (3, —4) and (—6, 2) are the extremities of a diagonal of a parallelogram. If the third vertex is (—1, —3). Find the coordinates of the fourth vertex.

Sol:



Let $A(3,-4)$ and $C(-6,-2)$ be the extremities of diagonal *AC* and $B(-1,-3)$, $D(x,y)$ be the extremities of diagonal *BD.*

Since the diagonals of a parallelogram bisect each other.

$\therefore$ Coordinates of mid-point of *AC* = Coordinates of mid point of *BD.*

$\Rightarrow \frac{3-6}{2} = \frac{x-1}{2}$

$\Rightarrow \frac{-3}{2} = \frac{x-1}{2}$

$\Rightarrow x = -2$

And, $\frac{-4+2}{2} = \frac{y-3}{2}$

$\Rightarrow \frac{-2}{2} = \frac{y-3}{2}$

$\Rightarrow y = 1$

Hence, fourth vertex of parallelogram is $(-2,1)$

7. Find the ratio in which the point (2, y) divides the line segment joining the points A (— 2, 2) and B (3, 7). Also, find the value of y.

Sol:

Let the point $P(2,y)$ divide the line segment joining the points $A(-2,2)$ and $B(3,7)$ in the ratio $K:1$

Then, the coordinates of *P* are

$\left[\frac{3k+(-2)\times 1}{k+1}, \frac{7k+2\times 1}{k1}\right]$

$= \left[\frac{3k-2}{k+1}, \frac{7k+2}{k+1}\right]$

But the coordinates of *P* are given as $(2,y)$

$\therefore \frac{3k-2}{k+1} = 2$

$\Rightarrow 3k - 2 = 2k + 2$

$\Rightarrow 3k - 2k = 2 + 2$

$\Rightarrow k = 4$

$\frac{7k+2}{k+1} = y$

Putting the value of k, we get

$\frac{7\times 4+2}{4+1} = y$

$\frac{30}{5} = y$

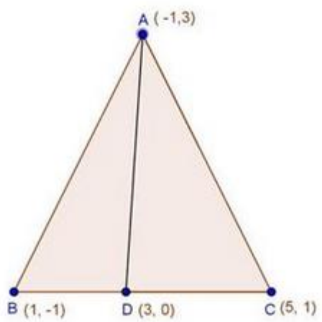
$6 = y$

i.e., $y = 6$

Hence the ratio is $4:1$ and $y = 6$.

8. If A (—1, 3), B (1, —1) and C (5, 1) are the vertices of a triangle ABC, find the length of the median through A.

Sol:



Let $A(1,3), B(1,-1)$ and $C(5,1)$ be the vertices of triangle ABC and let AD be the median through A.

Since, AD is the median, D is the mid-point of BC

$\therefore$ Coordinates of D are $\left(\frac{1+5}{2}, \frac{-1+1}{2}\right) = (3,0)$

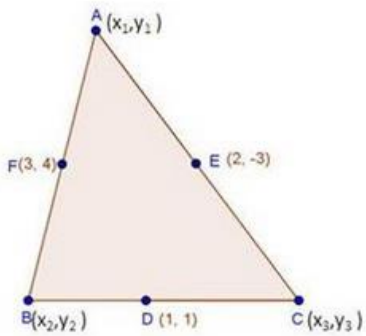
Length of median $AD = \sqrt{(3+1)^2 + (0-3)^2}$

$= \sqrt{(4)^2 + (-3)^2}$

$= \sqrt{16+9}$
$= \sqrt{25}$
$= 5$ units.

9. If the coordinates of the mid-points of the sides of a triangle are (1, 1), (2, —3) and (3, 4), find the vertices of the triangle.

Sol:



Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC.

Let $D(1,1), E(2,-3)$ and $F(3,4)$ be the mid-points of sides BC, CA and AB respectively.

Since, D is the mid-point of BC.

$\therefore \frac{x_2 + x_3}{2} = 1$ and $\frac{y_2 + y_3}{2} = 1$

$\Rightarrow x_2 + x_3 = 2$ and $y_2 + y_3 = 2$ ………..(i)

Similarly E and F are the mid-points of CA and AB respectively.

$\therefore \frac{x_1 + x_3}{2} = 2$ and $\frac{y_1 + y_3}{2} = -3$

$\Rightarrow x_1 + x_3 = 4$ and $y_1 + y_3 = 6$ ………..(ii)

And, $\frac{x_1 + x_2}{2} = 3$ and $\frac{y_1 + y_2}{2} = 4$

$\Rightarrow x_1 + x_2 = 6$ and $y_1 + y_2 = 8$ ………..(iii)

From (i), (ii) and (iii) we get

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = 2 + 4 + 6$ and

$y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 2 + (-6) + 8$

$\Rightarrow 2(x_1 + x_2 + x_3) = 12$ and $2(y_1 + y_2 + y_3) = 4$

$x_1 + x_2 + x_3 = 6$ and $y_1 + y_2 + y_3 = 2$ ………..(iv)

From (i) and (iv) we get

$x_1 + 2 = 6$ and $y_1 + 2 = 2$

$\Rightarrow x_1 = 6 - 2$ and $\Rightarrow y_2 = 2 - 2\,2$

$\Rightarrow x_1 = 4 \Rightarrow y_1 = 0$

So the coordinates of A are $(4,0)$

From (ii) and (iv) we get

$x_2 + 4 = 6$ and $y_2 + (-6) = 2$

$\Rightarrow x_2 = 2 \Rightarrow y_2 - 6 = 2 \Rightarrow y_2 = 8$

So the coordinates of B are $(2,8)$

From (iii) and (iv) we get

$6 + x_3 = 6$ and $8 + y_3 = 2$

$\Rightarrow x_3 = 6 - 6 \Rightarrow y_3 = 2 - 8$

$\Rightarrow x_3 = 0$ and $y_3 = -6$

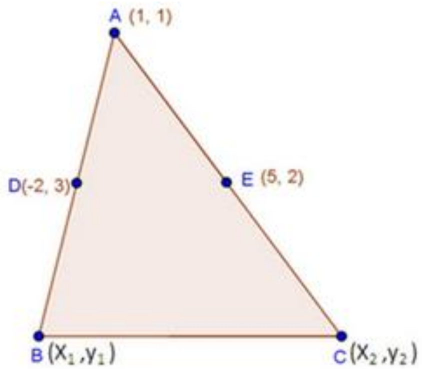
So the coordinates of C are $(0,-6)$

Hence, the vertices of triangle ABC are:

$A(4,0), B(2,8)$ and $C(0,-6)$

10. If a vertex of a triangle be (1, 1) and the middle points of the sides through it be (—2, 3) and (5, 2), find the other vertices.

Sol:



Let $A(1,1)$, be the given vertex

And, $D(-2,3), E(5,2)$ be the mid-point of AB and AC respectively,

Now, since D and E are the midpoints of AB and AC

$\frac{x_1 + 1}{2} = -2, \frac{y_1 + 1}{2} = 3$

$\Rightarrow x_1 + 1 = -4 \Rightarrow y_1 + 1 = 6$

$\Rightarrow x_1 = -5 \Rightarrow y_1 = 5$

So, the coordinates of B are $(-5,5)$

And, $\frac{x_2+1}{2}=5, \quad \frac{y_2+1}{2}=2$

$\Rightarrow x_2+1=10 \Rightarrow y_2+1=4$

$\Rightarrow x_2=9 \Rightarrow y_2=3$

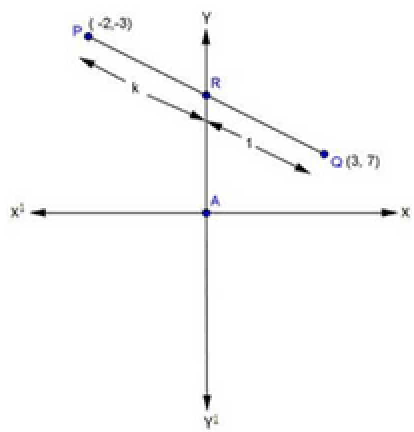
So the coordinates of C are $(9,3)$

Hence, the over vertices are $B(-5,5)$ and $C(9,3)$

11. (i) In what ratio is the line segment joining the points (—2, —3) and (3, 7) divided by the y-axis? Also, find the coordinates of the point of division.

(ii) In what ratio is the line segment joining (—3, —1) and (—8, —9) divided at the point $(-5, -\frac{21}{5})$?

Sol:



Suppose $y-axis$ divides PQ in the ration $K:1$ at R

Then, the coordinates of the point of division are:

$$R\left[\frac{3k+(-2)\times 1}{k+1}, \frac{7k+(-3)\times 1}{k+1}\right]$$

$$=R\left[\frac{3k-2}{k+1}, \frac{7k-3}{k+1}\right]$$

Since, R lies on $y-axis$ and $x-$coordinate of every point on y-axis is zero

$$\therefore \frac{3k-2}{k+1}=0$$

$\Rightarrow 3k-2=0$

$\Rightarrow 3k=2$

$$\Rightarrow k=\frac{2}{3}$$

Hence, the required ratio is $\frac{2}{3}:1$

i.e., 2 : 3

Putting $k=\frac{2}{3}$ in the coordinates of R

We get, $(0,1)$

Let the point P divide AB in the ratio $K:1$

Then, the coordinates of P are $\left[\frac{-8k-3}{k+1},\frac{-9k-1}{k+1}\right]$

But the coordinates of P are given as $\left(-5,\frac{-21}{5}\right)$

$\therefore \frac{-8k-3}{k+1}=-5$

$\Rightarrow -8k-3=-5k-5$

$\Rightarrow -8k+5k=-5+3$

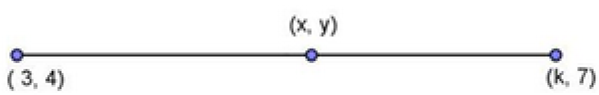
$\Rightarrow -3k=-2$

$\Rightarrow k=\frac{2}{3}$

Hence, the point P divides AB in the ratio $\frac{2}{3}:1\Rightarrow 2:3$

12. If the mid-point of the line joining (3, 4) and (k, 7) is (x, y) and 2 x + 2 y + 1 = 0. find the value of k.

Sol:



Since, (x,y) is the mid-point

$x=\frac{3+k}{2}, y=\frac{4+7}{2}=\frac{11}{2}$

Again,

$2x+2y+1=0$

$\Rightarrow 2\times\frac{(3\times k)}{2}+2\times\frac{11}{2}+1=0$

$\Rightarrow 3+k+11+1=0$

$\Rightarrow 3+k+12=0$
$\Rightarrow k+15=0$
$\Rightarrow k=-15$

13. Determine the ratio in which the straight line x — y — 2 = 0 divides the line segment joining (3, —1) and (8, 9).

Sol:

Suppose the line $x-y-2=0$ divides the line segment joining $A(3,-1)$ and $B(8,9)$ in the ratio $K:1$ at point P. Then the coordinates of P are

$$\left(\frac{8k+3}{k+1},\frac{9k-1}{k+1}\right)$$

But P lies on $x-y-2=0$

$$\therefore \frac{8k+3}{k+1}-\frac{9k-1}{k+1}-2=0$$
$$\Rightarrow \frac{8k+3}{k+1}-\frac{9k-1}{k+1}=2$$
$$\Rightarrow \frac{8k+3-9k+1}{k+1}=2$$
$$\Rightarrow -k+4=2k+2$$
$$\Rightarrow -k-2k=2-4$$
$$\Rightarrow -3k=-2 \Rightarrow k=\frac{2}{3}$$

So, the required ratio is $2:3$

14. Find the ratio in which the line segment joining (—2, —3) and (5, 6) is divided by
(i) x-axis
(ii) y-axis.
Also, find the coordinates of the point of division in each case.

Sol:

(i) Suppose $x-axis$ divides AB in the ratio $K:1$ at point P

Then, the coordinates of the point of division of division are

$$P\left[\frac{5k-2}{k+1},\frac{6k-3}{k+1}\right]$$

Since, P lies on x-axis, and y-coordinates of every point on x-axis is zero.

$$\therefore \frac{6k-3}{k+1}=0$$
$$\Rightarrow 6k-3=0$$
$$\Rightarrow 6k=3$$

$\Rightarrow k = \frac{3}{6} \Rightarrow k = \frac{1}{2}$

Hence, the required ratio is $1:2$

Putting $k = \frac{1}{2}$ in the coordinates of P

We find that its coordinates are $\left(\frac{1}{3}, 0\right)$.

(ii) Suppose y-axis divides AB in the ratio $k:1$ at point Q.

Then, the coordinates of the point of division are

$Q\left[\frac{5k-2}{k+1}, \frac{6k-3}{k+1}\right]$

Since, Q lies on $y-axis$ and x-coordinates of every point on y-axis is zero.

$\therefore \frac{5k-2}{k+1} = 0$

$\Rightarrow 5k - 2 = 0$

$\Rightarrow k = \frac{2}{5}$

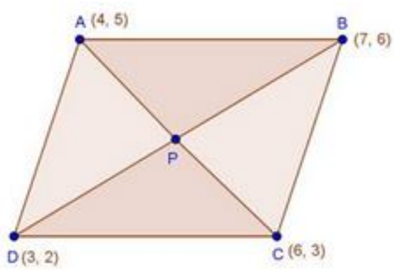
Hence, the required ratio is $\frac{2}{5}:1 = 2:5$

Putting $k = \frac{2}{5}$ in thee coordinates of Q.

We find that the coordinates are $\left(0, \frac{-3}{7}\right)$

15. Prove that the points (4, 5), (7, 6), (6, 3), (3, 2) are the vertices of a parallelogram. Is it a rectangle.

Sol:



Let $A(4,5), B(7,6), C(6,3)$ and $D(3,2)$ be the given points.

And, P the points of intersection of AC and BD.

Coordinates of the mid-point of AC are $\left(\frac{4+6}{2}, \frac{5+3}{2}\right) = (5,4)$

Coordinates of the mid-point of BD are $\left(\frac{7+3}{2},\frac{6+2}{2}\right)=(5,4)$

Thus, AC and BD have the same mid-point.
Hence, $ABCD$ is a parallelogram
Now, we shall see whether $ABCD$ is a rectangle.
We have,

$$AC=\sqrt{(6-4)^2+(3-5)^2}$$

$$\Rightarrow AC=\sqrt{4+4}$$

$$\Rightarrow AC=\sqrt{8}$$

And, $BD+\sqrt{(7-3)^2+(6-2)^2}$

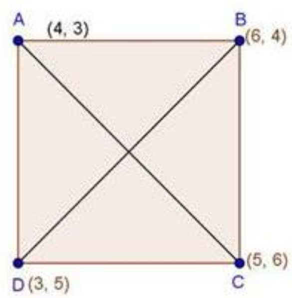
$$\Rightarrow BD=\sqrt{16+16}$$

$$\Rightarrow BD=\sqrt{32}$$

Since, $AC\neq BD$
So, $ABCD$ is not a rectangle

16. Prove that (4, 3), (6, 4), (5, 6) and (3, 5) are the angular points of a square.

Sol:



Let $A(4,3),B(6,4),C(5,6)$ and $D(3,5)$ be the given points.

Coordinates of the mid-point of AC are $\left(\frac{4+5}{2},\frac{3+6}{2}\right)=\left(\frac{9}{2},\frac{9}{2}\right)$

Coordinates of the mid-point of BD are $\left(\frac{6+3}{2},\frac{4+5}{2}\right)=\left(\frac{9}{2},\frac{9}{2}\right)$

AC and BD have the same mid-point

$\therefore$ $ABCD$ is a parallelogram

Now,

$$AB=\sqrt{(6-4)^2+(4-3)^2}$$

$$\Rightarrow AB=\sqrt{4+1}$$

$$\Rightarrow AB=\sqrt{5}$$

And, $BC = \sqrt{(6-5)^2 + (4-6)^2}$

$\Rightarrow BC = \sqrt{1+4}$

$\Rightarrow BC = \sqrt{5}$

$\therefore \quad AB = BC$

So, ABCD is a parallelogram whose adjacent sides are equal

$\therefore \quad ABCD$ is a rhombus

We have,

$AC = \sqrt{(5-4)^2 + (6-3)^2}$

$\Rightarrow AC = \sqrt{10}$

$BD = \sqrt{(6-3)^2 + (4-5)^2}$

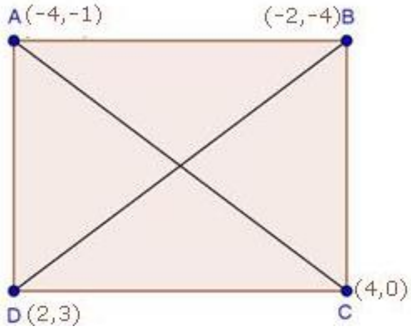
$\Rightarrow BD = \sqrt{10}$

$AC = BD$

Hence, $ABCD$ is a square

17. Prove that the points (—4, —1), (—2, —4), (4, 0) and (2, 3) are the vertices of a rectangle.

Sol:



Let $A(-4,-1), B(-2,-4), C(4,0)$ and $D(2,3)$ be the given points

Coordinates of the mid-point of AC are $\left(\frac{-4+4}{2}, \frac{-1+0}{2}\right) = \left(0, \frac{-1}{2}\right)$

Coordinates of the mid-point of BD are $\left(\frac{-2+2}{2}, \frac{-4+3}{2}\right) = \left(0, \frac{-1}{2}\right)$

Thus AC and BD have the same mid-point

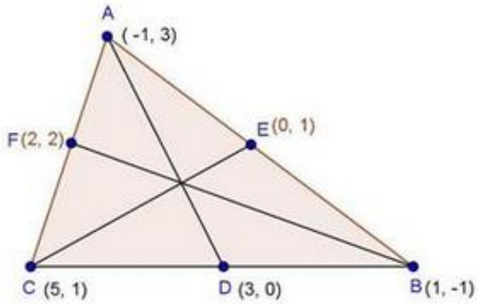
$AC = \sqrt{(4+4)^2 + (0+1)^2} = \sqrt{65}$

$BD = \sqrt{(-2-2)^2 + (-4-3)^2} = \sqrt{65}$

Hence $ABCD$ is a rectangle

18. Find the lengths of the medians of a triangle whose vertices are A (−1, 3), B (1, −1) and C (5,1).

Sol:



Let AD, BF and CE be the medians of $\Delta ABCD$

Coordinates of D are $\left(\frac{5+1}{2}, \frac{1-1}{2}\right) = (3,0)$

Coordinates of E are $\left(\frac{-1+1}{2}, \frac{3-1}{2}\right) = (0,1)$

Coordinates of F are $\left(\frac{5-1}{2}, \frac{1+3}{2}\right) = (2,2)$

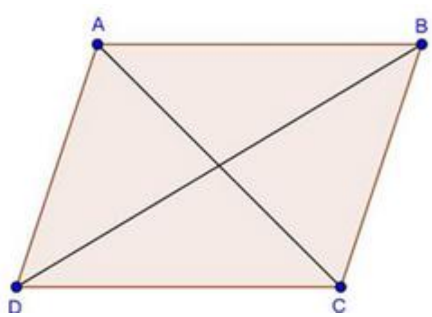
Length of $AD = \sqrt{(-1-3)^2 + (3-0)^2} = 5$ units

Length of $BF = \sqrt{(2-1)^2 + (2+1)^2} = \sqrt{10}$ units

Length of $CE = \sqrt{(5-0)^2 + (1-1)^2} = 5$ units

19. Three vertices of a parallelogram are (a + b, a — b), (2a + b, 2a — b), (a — b, a + b). Find the fourth vertex.

Sol:



Let $A(a+b, a-b), B(2a+b, 2a-b), C(a-b, a+b)$ and (x, y) be the given points

Since, the diagonals of a parallelogram bisect each other

$\therefore$ Coordinates of the midpoint of $AC =$ Coordinates of the midpoint of BD

$$\left(\frac{a+b+a-b}{2},\frac{a-b+a+b}{2}\right)=\left(\frac{2a+b+x}{2},\frac{2a-b+y}{2}\right)$$

$$\Rightarrow (a,a)=\left(\frac{2a+b+x}{2},\frac{2a-b+y}{2}\right)$$

$$\Rightarrow \frac{2a+b+x}{2}=a \text{ and } \frac{2a-b+y}{2}=a$$

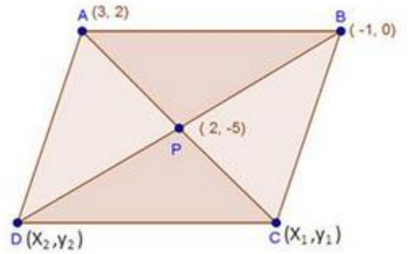
$$\Rightarrow 2a+b+x=2a \Rightarrow 2a-b+y=2a$$

$$\Rightarrow x=-b \Rightarrow y=b$$

Hence, the fourth vertex is $(-b,b)$.

20. If two vertices of a parallelogram are (3, 2), (—1, 0) and the diagonals cut at (2, —5), find the other vertices of the parallelogram.

Sol:



Let $A(3,2), B(-1,0), C(x_1,y_1)$ and $D(x_2,y_2)$ be the given points.

Since, the diagonals of parallelogram bisect each other.

Coordinates of the midpoint of $AC =$ Coordinates of the midpoint of BD

$$\left(\frac{x_1+3}{2},\frac{y_1+2}{2}\right)=\left(\frac{x_2-1}{2},\frac{y_2+0}{2}\right)$$

But $\frac{x_1+3}{2}=2, \frac{y_1+2}{2}=-5$

$$\Rightarrow x_1+3=4 \Rightarrow y_1=-10-2$$

$$\Rightarrow x_1=1 \Rightarrow y_1=-12$$

And, $\frac{x_2-1}{2}=2$

$$\Rightarrow x_2-1=4$$

$$\Rightarrow x_2=5$$

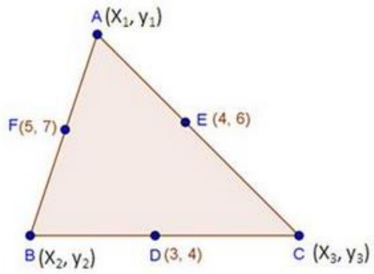
$$\frac{y_2+0}{2}=-5$$

$$y_2=-10$$

Hence, the other vertices of parallelogram are $(1,-12)$ and $(5,-10)$.

21. If the coordinates of the mid-points of the sides of a triangle are (3, 4), (4, 6) and (5, 7), find its vertices

Sol:



Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC

Let $D(3,4), E(4,6)$ and $F(5,7)$ be the midpoints of BC, CA and AB.

Since, D is the midpoint of BC

$\therefore \frac{x_2 + x_3}{2} = 3$ and $\frac{y_2 + y_3}{2} = 4$

$\Rightarrow x_2 + x_3 = 6$ and $y_2 + y_3 = 8$ ……….(i)

Since, E is the midpoint of CA

$\therefore \frac{x_1 + x_3}{2} = 4$ and $\frac{y_1 + y_3}{2} = 6$

$\therefore x_1 + x_3 = 8$ and $y_1 + y_3 = 12$ ……….(ii)

Since F is the mid-point of AB

$\frac{x_1 + x_2}{2} = 5$ and $\frac{y_1 + y_2}{2} = 7$

$\Rightarrow x_1 + x_2 = 10$ and $y_1 + y_3 = 14$ ……….(iii)

From (i), (ii) and (iii), we get

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = 6 + 8 + 10$

$x_1 + x_2 + x_3 = 12$ ……….(iv)

And $y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 8 + 12 + 14$

$y_1 + y_2 + y_3 = 17$ ……….(iv)

From (i) and (iv)

$x_1 + 6 = 12, y_1 + 8 = 17$

$x_1 = 6, y_1 = 9$

From (ii) and (iv)

$x_2 + 8 = 12, y_2 + 12 = 17$

$x_2 = 4, y_2 = 5$

From (iii) and (iv)

$x_3 + 10 = 12, y_3 + 14 = 17$

$x_3 = 2, y_3 = 3$

Hence the vertices of triangle ABC are $(6,9);(4,5);(2,3)$

22. The line segment joining the points P (3, 3) and Q (6, — 6) is bisected at the points A and B such that A is nearer to P. If A also lies on the line given by 2x + y + k = 0, find the value of k.

Sol:

We are given PQ is the line segment, A and B are the points of trisection of PQ.

We know that $PA:QA = \ 1:2$

So, the coordinates of A are

$$\left(\frac{6\times1+3\times2}{2+1}, \frac{-6\times1+3\times2}{2+1}\right)$$

$$=\frac{12}{3},0$$

$$=(4,0)$$

Since, A lies on the line

$2x + y + k = 0$

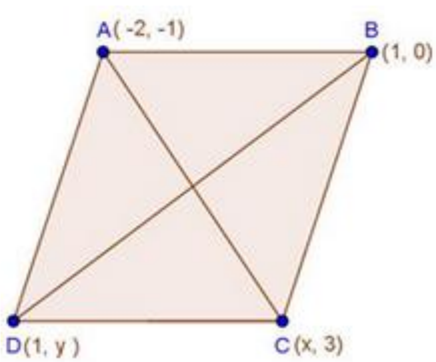
$\Rightarrow 2\times4+0+k=0$

$\Rightarrow 8+k=0$

$\Rightarrow 8+k=-8$

23. If the points (—2 , —1), (1, 0), (x, 3) and (1, y) form a parallelogram, find the values of x and y.

Sol:



Let $A(-2,-1), B(1,0), C(x,3)$ and $D(1,y)$ be the given points.

We know that diagonals of a parallelogram bisect each other
$\therefore$ Coordinates of the mid-point of $AC =$ Coordinates of the mid-point of BD

$$\left(\frac{x-2}{2}, \frac{3-1}{2}\right) = \left(\frac{1+1}{2}, \frac{y+0}{2}\right)$$

$$\Rightarrow \left(\frac{x-2}{2}, 1\right) = \left(1, \frac{y}{2}\right)$$

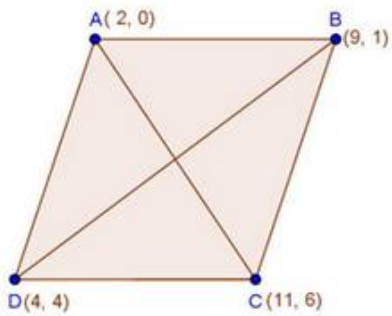
$$\Rightarrow \frac{x-2}{2} = 1 \text{ and } \frac{y}{2} = 1$$

$$\Rightarrow x - 2 = 2 \Rightarrow y = 2$$

$$\Rightarrow x = 4 \Rightarrow y = 2$$

24. The points A (2, 0), B (9, 1), C (11, 6) and D (4, 4) are the vertices of a quadrilateral ABCD. Determine whether ABCD is a rhombus or not.

Sol:



Let $A(2,0), B(9,1), C(11,6)$ and $D(4,4)$ be the given points.

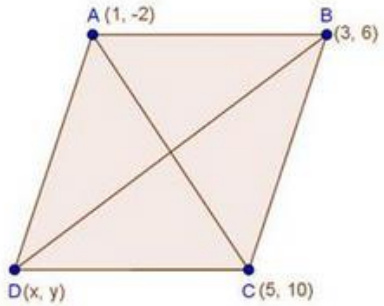
Coordinates of midpoint AC are $\left(\frac{11+2}{2}, \frac{6+0}{2}\right) = \left(\frac{13}{2}, 3\right)$

Coordinates of midpoint BD are $\left(\frac{9+4}{2}, \frac{1+4}{2}\right) = \left(\frac{13}{2}, \frac{5}{2}\right)$

Since, coordinates of mid-point of $AC \neq$ coordinates of mid-point of BD.
So, $ABCD$, is not a parallelogram. Hence, it is not a rhombus.

25. If three consecutive vertices of a parallelogram are (1, —2), (3, 6) and (5, 10), find its fourth vertex.

Sol:



Let $A(1,-2), B(3,6), C(5,10)$ and $D(x,y)$ be the given points taken in order.

Since, diagonals of parallelogram bisect each other

Coordinates of mid-point of $AC =$ Coordinates of midpoint of BD

$$\left(\frac{5+1}{2},\frac{10-2}{2}\right)=\left(\frac{x+3}{2},\frac{y+6}{2}\right)$$

$$\Rightarrow (3,4)=\frac{x+3}{2},\frac{y+6}{2}$$

$$\Rightarrow \frac{x+3}{2}=3 \text{ and } \frac{y+6}{2}=4$$

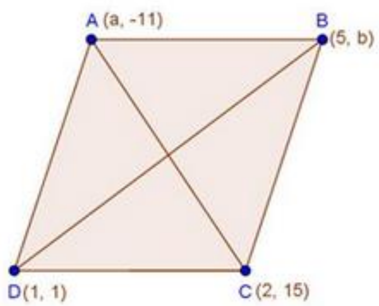
$$\Rightarrow x+3=6 \qquad \Rightarrow y+6=8$$

$$\Rightarrow x=3 \qquad \Rightarrow y=2$$

Hence, the fourth vertex is $(3,2)$.

26. If the points A (a, —11), B (5, b), C (2, 15) and D (1, 1) are the vertices of a parallelogram ABCD, find the values of a and b.

Sol:



Let $A(a,-11), B(5,b), C(2,15)$ and $D(1,1)$ be the given points.

We know that diagonals of parallelogram bisect each other.

$\therefore$ Coordinates of mid-point of $AC =$ Coordinates of mid-point of BD

$$\left(\frac{a+2}{2},\frac{15-11}{2}\right)=\left(\frac{5+1}{2},\frac{b+1}{2}\right)$$

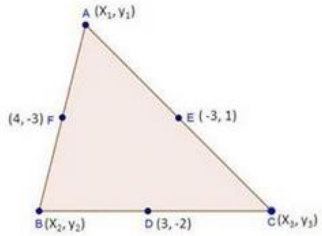
$\Rightarrow \frac{a+2}{2}=3$ and $\frac{b+1}{2}=2$

$\Rightarrow a+2=6 \quad \Rightarrow b+1=4$

$\Rightarrow a=4 \quad \Rightarrow b=3$

27. If the coordinates of the mid-points of the sides of a triangle be (3, —2), (—3, 1) and (4, —3), then find the coordinates of its vertices.

Sol:



Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC

Let $D(3,-2), E(-3,1)$ and $F(4,-3)$ be the midpoint of sides BC, CA and AB respectively

Since, D is the midpoint of BC

$\therefore \frac{x_2+x_3}{2}=3$ and $\frac{y_2+y_3}{2}=-2$

$\Rightarrow x_2+x_3=6$ and $y_2+y_3=-4$ …….(i)

Similarly, E and F are the midpoint of CA and AB respectively.

$\therefore \frac{x_1+x_3}{2}=-3$ and $\frac{y_1+y_3}{2}=1$

$\Rightarrow x_1+x_3=-6$ and $y_1+y_3=2$ …….(ii)

And,

$\therefore \frac{x_1+x_2}{2}=4$ and $\frac{y_1+y_2}{2}=-3$

$\Rightarrow x_1+x_2=8$ and $y_1+y_2=-6$ …….(iii)

From (i), (ii) and (iii), we have

$x_2+x_3+x_1+x_3+x_1+x_2=6+(-6)+8$ and

$y_2+y_3+y_1+y_3+y_1+y_2=-4+2-6$

$\Rightarrow 2(x_1+x_2+x_3)=8$ and $2(y_1+y_2+y_3)=-8$

$\Rightarrow x_1+x_2+x_3=4$ and $y_1+y_2+y_3=-4$ …….(iv)

From (i) and (iv)

$x_1+6=4$ and $y_1-4=-4$

$\Rightarrow x_1 = -2 \qquad \Rightarrow y_1 = 0$

So, the coordinates of A are $(-2,0)$

From (ii) and (iv)

$x_2 - 6 = 4$ and $y_2 + 2 = -4$

$\Rightarrow x_2 = 10 \Rightarrow y_2 = -6$

So, the coordinates of B are $(10,-6)$

From (iii) and (iv)

$x_3 + 8 = 4$ and $y_3 - 6 = -4$

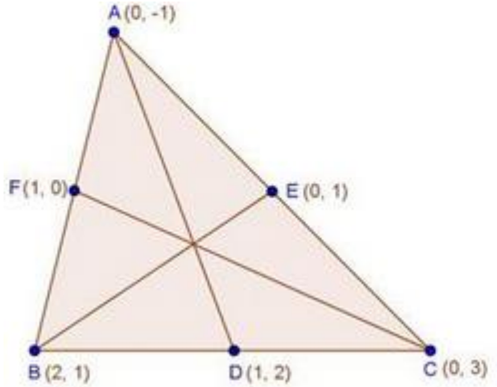
$\Rightarrow x_3 = -4 \qquad \Rightarrow y_3 = 2$

So, the coordinates of C are $(-4,2)$

Hence, the vertices of ΔABC are $A(-2,0), B(10,6)$ and $C(-4,2)$.

28. Find the lengths of the medians of a ΔABC having vertices at A (0,—1), B (2, 1) and C (0, 3).

Sol:



Let $A(0,1), B(2,1)$ and $C(0,3)$ be the given points

Let AD, BE and CF be the medians

Coordinates of D are $\left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1,2)$

Coordinates of E are $\left(\frac{0}{2}, \frac{3-1}{2}\right) = (0,1)$

Coordinates of F are $\left(\frac{2+0}{2}, \frac{1-1}{2}\right) = (1,0)$

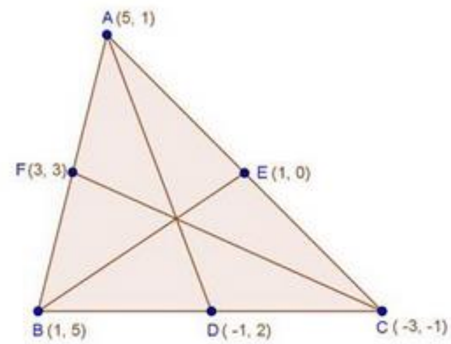
Length of median $AD = \sqrt{(1-0)^2 + (2+1)^2} = \sqrt{10}$ units

Length of median $BE = \sqrt{(2-0)^2 + (1-1)^2} = 2$ units

Length of median $CF = \sqrt{(1-0)^2 + (0-3)^2} = \sqrt{10}$ units

29. Find the lengths of the medians of a ΔABC having vertices at A (5, 1), B (1, 5), and C (−3,-1).

Sol:



Let $A(5,1), B(1,5)$ and $C(-3,-1)$ be vertices of ΔABC

Let AD, BE and CF be the medians

Coordinates of D are $\left(\frac{1-3}{2}, \frac{5-1}{2}\right) = (-1, 2)$

Coordinates of E are $\left(\frac{5-3}{2}, \frac{1-1}{2}\right) = (1, 0)$

Coordinates of F are $\left(\frac{5+1}{2}, \frac{1+5}{2}\right) = (3, 3)$

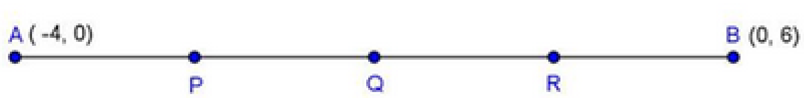
Length of median $AD = \sqrt{(5+1)^2 + (1-2)^2} = \sqrt{37}$ units

Length of median $BE = \sqrt{(1-1)^2 + (5-0)^2} = 5$ units

Length of median $CF = \sqrt{(3+3)^2 + (3+1)^2} = 2\sqrt{13} = \sqrt{52}$ units

30. Find the coordinates of the points which divide the line segment joining the points (−4, 0) and (0, 6) in four equal parts.

Sol:



Let $A(-4,0)$ and $B(0,6)$ be the given points.

And, Let P, Q, R be the points which divide AB in four equal points..

We know $AP : PB = 1:3$

$\therefore$ Coordinates of P are

$$\left(\frac{1\times0+3(-4)}{1+3},\frac{1\times6+3\times0}{1+3}\right)$$

$$=\left(-3,\frac{3}{2}\right)$$

We know that Q is midpoint of AB

$\therefore$ Coordinates of Q are

$$\left(\frac{3\times0+1\times(-4)}{3+1},\frac{3\times6+1\times0}{3+1}\right)$$

$$=\left(-1,\frac{9}{2}\right)$$

31. Show. that the mid-point of the line segment joining the points (5, 7) and (3, 9) is also the mid-point of the line segment joining the points (8, 6) and (0, 10).

Sol:

Let $A(5,7),B(3,9),C(8,6)$ and $D(0,,10)$ be the given points

Coordinates of the mid-point of $AB\ are\left(\frac{5+3}{2},\frac{7+9}{2}\right)=(4,8)$

Coordinates of the mid-point of $CD\ are\left(\frac{8+0}{2},\frac{6+10}{2}\right)=(4,8)$

Hence, the midpoints of AB = midpoint of CD.

32. Find the distance of the point (1, 2) from the mid-point of the line segment joining the points (6, 8) and (2, 4).

Sol:

Let $P(1,2),A(6,8)$ and $B(2,4)$ be thee given points.

Coordinates of midpoint of the line segment joining $A(6,8)$ and $B(2,4)$ are

$$Q\left(\frac{6+2}{2},\frac{8+4}{2}\right)=Q(4,6)$$

Now, distance $PQ=\sqrt{(4-1)^2+(6-2)^2}$

$\Rightarrow PQ=\sqrt{9+16}$

$\Rightarrow PQ=\sqrt{25}$

$\Rightarrow PQ=5$

Hence, the distant $=5$ units

33. If A and B are (1, 4) and (5, 2) respectively, find the coordinates of P when $\frac{AP}{BP} = \frac{3}{4}$

Sol:

Let $A(1,4)$ and $B(5,2)$ be the given points.

We know that $\frac{AP}{BP} = \frac{3}{4}$

Or, $AP : BP = 3 : 4$

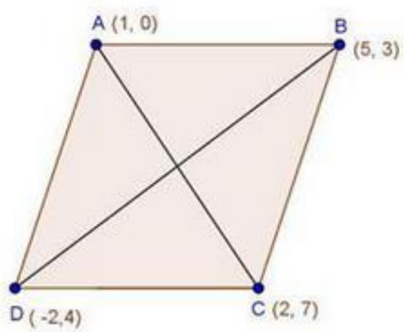
Coordinates of P are

$$\left(\frac{3\times5+4\times1}{3+4}, \frac{3\times2+4\times4}{3+4}\right)$$

$$=\left(\frac{19}{7}, \frac{22}{7}\right)$$

34. Show that the points A (1, 0), B (5, 3), C (2, 7) and D (—2, 4) are the vertices of a parallelogram.

Sol:



Let $A(1,0), B(5,3), C(2,7)$ and $D(-2,4)$ be the given points

Coordinates of the midpoint of AC are $\left(\frac{1+2}{2}, \frac{0+7}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$

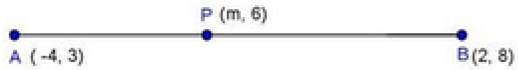
Coordinates of the midpoint of BD are $\left(\frac{5-2}{2}, \frac{3+4}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$

Since, coordinates of midpoint of $AC =$ coordinates off midpoint of BD

$\therefore ABCD$ is a parallelogram as we know diagonals of parallelogram bisect each other.

35. Determine the ratio in which the point P (m, 6) divides the join of A(—4, 3) and B(2, 8). Also, find the value of m.

Sol:



Let $P(m,6)$ divides the join of $A(-4,3)$ and $B(2,8)$ in the ratio $K:1$

Then, the coordinates of P are

$$\left(\frac{2k+1\times(-4)}{k+1},\frac{8k+1\times 3}{k+1}\right)$$

$$=\left(\frac{2k-4}{k+1},\frac{8k+3}{k+1}\right)$$

But, $\frac{8k+3}{k+1}=6$

$\Rightarrow 8k+3=6k+6$

$\Rightarrow 8k-6k=3$

$\Rightarrow k=\frac{3}{2}$

Hence, P divides AB in the ratio $3:2$

Again,

$$\frac{2k-4}{k+1}=m$$

Substituting $k=\frac{3}{2}$, we get

$$\frac{2\times\frac{3}{2}-4}{\frac{3}{2}+1}=m$$

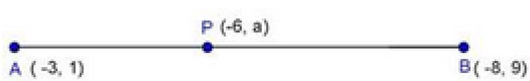
$$\Rightarrow \frac{-1}{\frac{5}{2}}=m$$

$$\Rightarrow \frac{-2}{5}=m$$

$$\therefore m=\frac{-2}{5}$$

36. Determine the ratio in which the point (—6, a) divides the join of A(—3, 1) and B(—8, 9). Also find the value of a.

Sol:



Let $P(-6,a)$ divides the join of $A(-3,1)$ and $B(-8,9)$ in the ratio $k:1$

Then, the coordinates of P are

$$\left(\frac{-8k-3}{k+1},\frac{9k+1}{k+1}\right)$$

But, $\frac{-8k-3}{k+1}=-6$

$\Rightarrow -8k-3=-6k-6$

$\Rightarrow -8k+6k=-6+3$

$\Rightarrow -2k=-3$

$\Rightarrow k=\frac{3}{2}$

Hence, P divides AB in the ratio $3:2$

Again

$\frac{9k+1}{k+1}=a$

Substituting $k=\frac{3}{2}$

We get,

$$\frac{9\times\frac{3}{2}+1}{\frac{3}{2}+1}=a$$

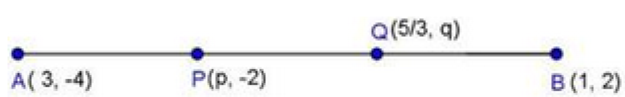
$$\Rightarrow \frac{\frac{29}{2}}{\frac{5}{2}}=a$$

$$\Rightarrow \frac{29}{5}=a$$

$$\therefore a=\frac{29}{5}$$

37. The line segment joining the points (3, —4) and (1, 2) is trisected at the points P and Q . If the coordinates of P and Q are (p, —2) and ($\frac{5}{3}$, q) respectively. Find the values of p and q.

Sol:



We have $P(p,-2)$ and $Q\left(\frac{5}{3},q\right)$ are the points of trisection of the line segment joining $A(3,-4)$ *and* $B(1,2)$

We know $AP:PB=1:2$

$\therefore$ Coordinates of P are

$$\left(\frac{1\times1+2\times3}{1+2},\frac{1\times2+2\times(-4)}{1+2}\right)$$

$$=\left(\frac{7}{3},-2\right)$$

Hence, $P=\frac{7}{3}$

Again we know that $AQ:QB=2:1$

$\therefore$ Coordinates of Q are

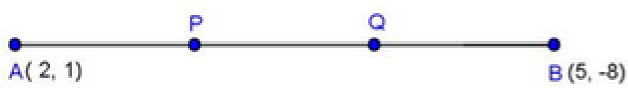
$$\left(\frac{2\times1+1\times3}{2+1},\frac{2\times2+1\times(-4)}{2+1}\right)$$

$$=\left(\frac{5}{3},0\right)$$

Hence, $q=0$

38. The line joining the points (2, 1) and (5,—8) is trisected at the points P and Q. If point P lies on the line 2x — y + k = 0. Find the value of k.

Sol:



Since, P is the point of trisection of the line segment joining the point $A(2,1)$ and $B(5,-8)$

We have $AP:PB=1:2$

$\therefore$ Coordinates of the point P are

$$\left(\frac{1\times5+2\times2}{1+2},\frac{1\times(-8)+2\times1}{1+2}\right)$$

$$=(3,-2)$$

But, P lies on the line

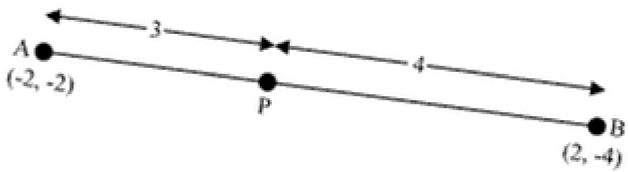
$$2x-y+k=0$$

$$\Rightarrow 2\times3-(-2)+k=0$$

$$\Rightarrow 6+2+k=0$$

$\Rightarrow 8 + k = 0$
$\Rightarrow k = -8$

39. If A and B are two points having coordinates (—2, —2) and (2, —4) respectively, find the coordinates of P such that AP = AB.
Sol:



The Coordinates of point A and B are $(-2, -2)$ and $(2, -4)$ respectively

Since $AP = \frac{3}{7} AB$

Therefore $AP : PB = 3 : 4$

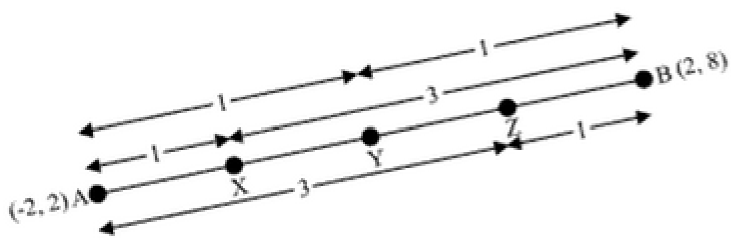
So, point P divides the line segment AB in a ratio $3 : 4$.

Coordinates of $P = \left(\frac{3 \times 2 + 4 \times (-2)}{3 + 4}, \frac{3 \times (-4) + 4 \times (-2)}{3 + 4} \right)$

$= \left(\frac{6 - 8}{7}, \frac{-12 - 8}{7} \right)$

$= \left(\frac{-2}{7}, \frac{20}{7} \right)$

40. Find the coordinates of the points which divide the line segment joining A (—2, 2) and B (2, 8) into four equal parts.
Sol:



From the figure we have points X, Y, Z are dividing the line segment in a ratio $1 : 3, 1 : 1, 3 : 1$ respectively.

Coordinates of $X = \left(\frac{1 + 2 + 3 \times (-2)}{1 + 3}, \frac{1 \times 8 + 3 \times 2}{1 + 3} \right)$

$=\left(-1,\frac{7}{2}\right)$

Coordinates of $Y=\left(\frac{2+(-2)}{2},\frac{2+8}{2}\right)$

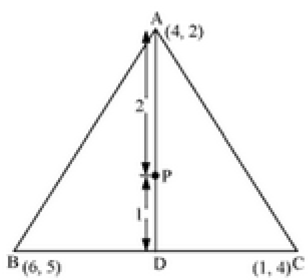
$=(0,5)$

Coordinates of $Z=\left(\frac{3\times 2+1\times(-2)}{3+1},\frac{3\times 8+1\times 2}{3+1}\right)$

$=\left(1,\frac{13}{2}\right)$

41. A (4, 2), B (6, 5) and C (1, 4) are the vertices of ΔABC.
(i) The median from A meets BC in D. Find the coordinates of the point D.
(ii) Find the coordinates of point P on AD such that AP : PD = 2 : 1.
(iii) Find the coordinates of the points Q and R on medians BE and CF respectively such that BQ : QE = 2 : l and CR : RF 2 : 1.
(iv) What do you observe?

Sol:



(i) Median *AD* of the triangle will divide the side *BC* in two equals parts. So D is the midpoint of side *BC*.

Coordinates of D $=\left(\frac{6+1}{2},\frac{5+4}{2}\right)=\left(\frac{7}{2},\frac{9}{2}\right)$

(ii) Point P divides the side AD in a ratio 2:1

Coordinates of P $=\left(\frac{2\times\frac{7}{6}+1\times 4}{2+1},\frac{2\times\frac{9}{2}+1\times 2}{2+1}\right)$

$=\left(\frac{11}{3},\frac{11}{3}\right)$

(iii) Median BE of the triangle will divide the side AC in two equal parts. So E is the midpoint of side AC.

Coordinates of $E = \left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$

Point Q divides the side BE in a ratio 2:1

Coordinates of Q $= \left(\frac{2\times\frac{5}{2}+1\times 6}{2+1}, \frac{2\times 3+1\times 5}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

Median CF of the triangle will divide the side AB in two equal parts. So F is the midpoint of side AB.

Coordinates of F $= \left(\frac{4+6}{2}, \frac{2+1}{2}\right) = \left(5, \frac{7}{2}\right)$

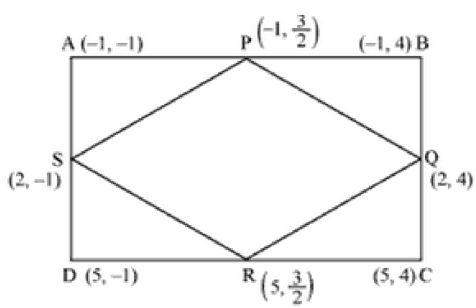
Point R divides the side CF in a ratio 2:1.

Coordinates of R $= \left(\frac{2\times 5+1\times 1}{2+1}, \frac{2\times\frac{7}{2}+1\times 4}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

(iv) Now we may observe that coordinates of point P, Q, R are same. So, all these are representing same point on the plane i.e. centroid of the triangle.

42. ABCD is a rectangle formed by joining the points A (—1, —1), B (—1, 4), C (5, 4) and D (5, −1). P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square? a rectangle? or a rhombus? Justify your answer.

Sol:



Length of $PQ = \sqrt{(-1-2)^2 + \left(\frac{3}{2}-4\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

Length of $QR = \sqrt{(2-5)^2 + \left(4-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

Length of $RS = \sqrt{(5-2)^2 + \left(\frac{3}{2}+1\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

Length of $SP = \sqrt{(2+1)^2 + \left(-1-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

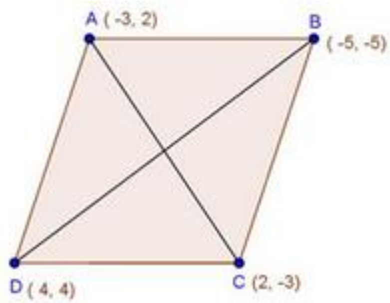
Length of $PR\sqrt{(-1-5)^2 + \left(\frac{3}{2}-\frac{3}{2}\right)^2} = 6$

Length of $QS\sqrt{(2-2)^2 + (4+1)^2} = 5$

Here all sides of given quadrilateral is of same measure but the diagonals are of different lengths. So, PQRS is a rhombus.

43. Show that A(—3, 2), B(—5,—5), C(2,—3) and D(4,4)are the vertices of a rhombus.

Sol:



Let $A(-3,2), B(-5,-5), C(2,-3)$ and $D(4,4)$ be the given points

Coordinates of the midpoint of $AC\ are \left(\frac{-3+2}{2}, \frac{2-3}{2}\right) = \left(\frac{-1}{2}, \frac{-1}{2}\right)$

Coordinates of the midpoint of $BD\ are \left(\frac{-5+4}{2}, \frac{-5+4}{2}\right) = \left(\frac{-1}{2}, \frac{-1}{2}\right)$

Thus, AC and BD have the same midpoint

Hence, $ABCD$ is a parallelogram

Now, $AB = \sqrt{(-5+3)^2 + (-5-2)^2}$

$\Rightarrow AB = \sqrt{4+49}$

$\Rightarrow AB = \sqrt{53}$

Now, $BC = \sqrt{(-5-2)^2 + (-5+3)^2}$

$\Rightarrow BC = \sqrt{49+4}$

$\Rightarrow BC = \sqrt{53}$

$\therefore AB = BC$

So, $ABCD$ is a parallelogram whose adjacent sides are equal.

Hence, $ABCD$ is a rhombus.

44. Find the ratio in which the y-axis divides the line segment joining the points (5, —6) and (—1, —4). Also, find the coordinates of the point of division.

Sol:

Let $P(5,-6)$ and $Q(-1,-4)$ be the given points.

Let y-axis divide PQ in the ratio $k:1$

Then, the coordinates of the point of division are

$R\left[\frac{-k+5}{k+1},\frac{-4k-6}{k+1}\right]$

Since, R lies on y-axis and x-coordinates of every point on y-axis is zero

$\therefore \frac{-k+5}{k+1}=0$

$\Rightarrow -k+5=0$

$\Rightarrow k=5$

Hence, the required ratio is $5:1$

Putting $k=5$ in the coordinates of R, we get

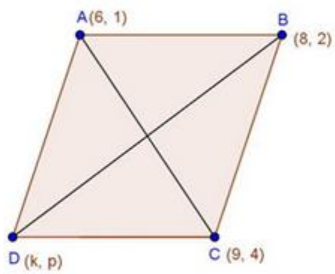
$\left(\frac{-5+5}{5+1},\frac{-4\times5-6}{5+1}\right)$

$=\left(0,\frac{-13}{3}\right)$

Hence, the coordinates of the point of division are $\left(0,-\frac{13}{3}\right)$.

45. If the points A (6, 1), B (8, 2), C (9, 4) and D (k, p) are the vertices of a parallelogram taken in order, then find the values of k and p.

Sol:



Let $A(6,1),B(8,2),C(9,4)$ and $D(k,p)$ be the given points.

Since, $ABCD$ is a parallelogram

Coordinates of midpoint of $AC=$Coordinates of the midpoints of BD

$\Rightarrow\left(\frac{6+9}{2},\frac{1+4}{2}\right)=\left(\frac{8+k}{2},\frac{2+p}{2}\right)$

$\Rightarrow\left(\frac{15}{2},\frac{5}{2}\right)=\left(\frac{8+k}{2},\frac{2+p}{2}\right)$

$\Rightarrow\frac{8+k}{2}=\frac{15}{2}$ and $\frac{2+p}{2}=\frac{5}{2}$

$\Rightarrow 8+k=15 \quad \Rightarrow 2+p=5$

$\Rightarrow k=7 \quad \Rightarrow p=3$

46. In what ratio does the point (—4, 6) divide the line segment joining the points A (--6, 10) and B (3, -8)?

Sol:

Let $(-4,6)$

Divide AB internally in the ratio $k:1$ using thee section formula, we get

$(-4,6)=\left(\frac{3k-6}{k+1},\frac{-8k+10}{k+1}\right)$ ……..(2)

So, $-4=\frac{3k-6}{k+1}$

i.e., $-4k-4=3k-6$

i.e., $7k=2$

i.e., $k:1=2:7$

You c check for the y-coordinate also. So, the point $(-4,6)$ divides the line segment j joining the points $A(-6,10)$ and $B(3,-8)$ in the ratio 2: 7

47. Find the coordinates of a point A, where AB is a diameter of the circle whose centre is (2, —3) and B is (1, 4).

Sol:

Let coordinates of point A be (x,y)

Mid-point of diameter AB is center of circle $(2,-3)$

$(2,-3)=\left(\frac{x+1}{2},\frac{y+4}{2}\right)$

$\frac{x+1}{2}=2 \; and \; \frac{y+4}{2}=-3$

$x+1=4$ and $y+4=-6$

$x=3$ and $y=-10$

Therefore coordinates of A are $(3,10)$

48. A point P divides the line segment joining the points A (3, — 5) and B (— 4, 8) such that $\frac{AP}{PB} = \frac{k}{1}$. If P lies on the line x + y = 0, then find the value of y.

Sol:

Given points are $A(3,-5)$ and $B(-4,8)$

P divides AB in the ratio $k:1$,

Using the section formula, we have:

Coordinate of point P are $\{(-4k+3/k+1)(8k-5/k+1)\}$

Now it is given, that P lies on the line $x+y=0$

Therefore

$-4k+3/k+1+8k-5/k+1=0$

$\Rightarrow -4k+3+8k-5=0$

$\Rightarrow 4k-2=0$

$\Rightarrow k=2/4$

$\Rightarrow k=1/2$

Thus, the value of k is $\frac{1}{2}$.

Exercise 14.4

1. Find the centroid of the triangle whose vertices are:

(i) $(1,4),(-1,-1)$ *and* $(3,-2)$

Sol:

We know that the coordinates of the centroid of a triangle whose vertices are

$(x_1,y_1),(x_2,y_2),(x_3,y_3)$ are

$$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

So, the coordinates of the centroid of a triangle whose vertices are

$(1,4),(-1,-1)$ *and* $(3,-2)$ are $\left(\frac{1-1+3}{3},\frac{4-1-2}{3}\right)$

$$=\left(1,\frac{1}{3}\right)$$

2. Two vertices of a triangle are (1, 2), (3, 5) and its centroid is at the origin. Find the coordinates of the third vertex.

Sol:

Let the coordinates of the third vertex bee (x, y), Then

Coordinates of centroid of triangle are

$$\left(\frac{x+1+3}{3}, \frac{y+2+5}{3}\right)$$

We have centroid is at origin $(0,0)$

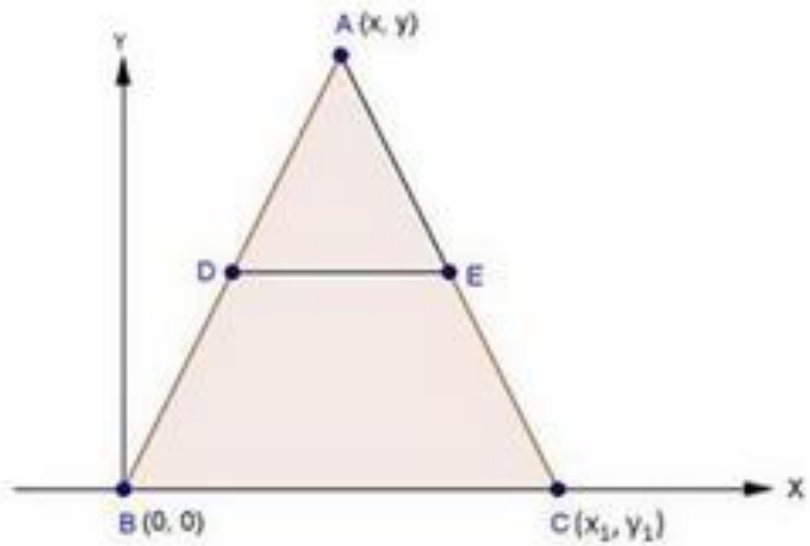
$$\therefore \frac{x+1+3}{3} = 0 \text{ and } \frac{y+2+5}{3} = 0$$

$\Rightarrow x+4=0 \qquad \Rightarrow y+7=0$

$\Rightarrow x=-4 \qquad \Rightarrow y=-7$

3. Prove analytically that the line segment joining the middle points of two sides of a triangle is equal to half of the third side.

Sol:



Let ABC be a triangle such that BC is along x-axis

Coordinates of A, B and C are $(x, y), (0,0)$ and (x_1, y_1)

D and E are the mid-points of AB and AC respectively

Coordinates of D are $\left(\frac{x+0}{2}, \frac{y+0}{2}\right)$

$$= \left(\frac{x}{2}, \frac{y}{2}\right)$$

Coordinates of E are $\left(\frac{x+x_1}{2}, \frac{y+y_1}{2}\right)$

Length of $BC = \sqrt{x_1^2 + y_1^2}$

Length of DE $= \sqrt{\left(\frac{x+x_1}{2}-\frac{x}{2}\right)^2+\left(\frac{x+y_1}{2}-\frac{y}{2}\right)^2}$

$= \sqrt{\left(\frac{x_1}{2}\right)^2+\left(\frac{y_1}{2}\right)^2}$

$= \sqrt{\frac{x_1^2}{4}+\frac{y_1^2}{4}}$

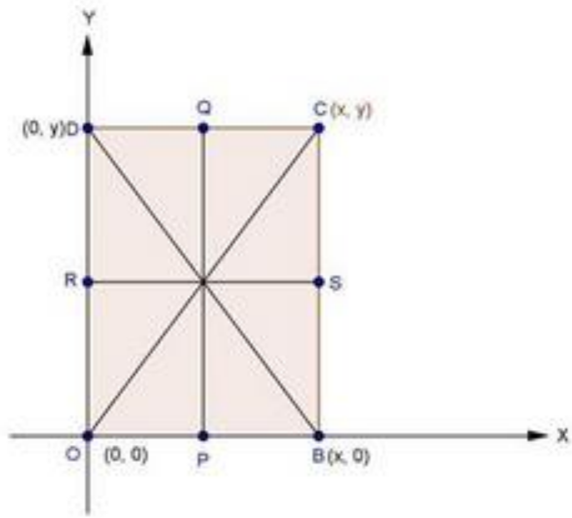
$= \sqrt{\frac{1}{4}\left(x_1^2+y_1^2\right)}$

$= \frac{1}{2}\sqrt{x_1^2+y_1^2}$

$= \frac{1}{2}BC$

4. Prove that the lines joining the middle points of the opposite sides of a quadrilateral and the join of the middle points of its diagonals meet in a point and bisect one another.

Sol:



Let $OBCD$ be the quadrilateral P,Q,R,S be the midpoint off OB,CD,OD and BC.

Let the coordinates of O,B,C,D are $(0,0),(x,0),(x,y)$ and $(0,y)$

Coordinates of P are $\left(\frac{x}{2},0\right)$

Coordinates of Q are $\left(\frac{x}{2},y\right)$

Coordinates of R are $\left(0,\frac{y}{2}\right)$

Coordinates of S are $\left(x, \frac{y}{2}\right)$

Coordinates of midpoint of PQ are

$$\left[\frac{\frac{x}{2}+\frac{x}{2}}{2}, \frac{0+y}{2}\right]=\left(\frac{x}{2}, \frac{y}{2}\right)$$

Coordinates of midpoint of RS are $\left[\frac{(0+x)}{2}, \frac{\frac{y}{2}+\frac{y}{2}}{2}\right]=\left[\frac{x}{2}, \frac{y}{2}\right]$

Since, the coordinates of the mid-point of $PQ =$ coordinates of mid-point of RS

$\therefore PQ$ and RS bisect each other

5. If G be the centroid of a triangle ABC and P be any other point in the plane, prove that $PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + 3GP^2$.

Sol:

Let $A(0,0), B(a,0)$, and $C(c,d)$ are the o-ordinates of triangle ABC

Hence, $G\left[\frac{c+0+a}{3}, \frac{d}{3}\right]$

i.e., $G\left[\frac{a+c}{3}, \frac{d}{3}\right]$

let $P(x,y)$

To prove:

$$PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + 3GP^2$$

Or, $PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + GP^2 + GP^2 + GP^2$

Or, $PA^2 - GP^2 + PB^2 - GP^2 + PC^2 + GP^2 = GA^2 + GB^2 + GC^2$

Proof:

$$PA^2 = x^2 + y^2$$

$$GP^2 = \left(x - \frac{a+c}{3}\right)^2 + \left(y - \frac{d}{3}\right)^2$$

$$PB^2 = (x-a)^2 + y^2$$

$$PC^2 = (x-c)^2 + (y-d)^2$$

L.H.S

$$= x^2 + y^2 - \left[x^2 + \left(\frac{a+c}{3}\right)^2 + 2x\frac{(a+c)}{3} + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right] + (x-a)^2 + y^2$$

$$-\left[x^2 + \left(\frac{a+c}{3}\right)^2 - 2x\left(\frac{a+c}{3}\right) + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right] + (x-c)^2 + (y-d)^2$$

$$-\left[x^2 + \left(\frac{a+c}{3}\right)^2 - 2x\left(\frac{a+c}{3}\right) + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right]$$

$$= x^2 + y + x^2 + x^2 + a^2 - 2ax + y^2 + x^2 + c^2 - 2xc + y^2 + d^2 - 2yd - 3$$

$$\left[x^2 + \left(\frac{a+c}{3}\right)^2 - 2x\left(\frac{a+c}{3}\right) + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right]$$

$$= \cancel{3x^2} + \cancel{3y^2} + a^2 + c^2 + d^2 - 2ax - 2xc - 2yd - \cancel{3x^2} - \frac{(a+c)^2}{3} + 2x(a+c) - \cancel{3y^2} - \frac{d^2}{3} + 2yd$$

$$= a^2 + c^2 + d^2 - \cancel{2ax} - \cancel{2xc} - \cancel{2yd} - \frac{a^2 + c^2 + 2ac}{3} + \cancel{2ax} + \cancel{2cx} - \frac{d^2}{3} + \cancel{2yd}$$

$$= \frac{3a^2 + 3c^2 + 3d^2 - a^2 - c^2 - 2ac - d^2}{3} = \frac{2a^2 + 2c^2 + 2d^2 - 2ac}{3} = L.H.S$$

Solving R.H.S

$$GA^2 + GB^2 + GC^2$$

$$GA^2 = \left(\frac{a+c}{3}\right)^2 + \left(\frac{d}{3}\right)^2 = \frac{a^2 + c^2 + 2ac}{9} + \frac{d^2}{9}$$

$$GC^2 = \left(\frac{a+c}{3} = a\right)^2 + \left(\frac{d}{3}\right)^2 = \left(\frac{c-2a}{3}\right)^2 + \left(\frac{d}{3}\right)^2$$

$$= \frac{a^2 + 4c^2 - 4ca}{9} + \frac{4d^2}{9}$$

$$GB^2 = \left(\frac{a+c}{3} - a\right)^2 + \left(\frac{d}{3}\right)^2 = \left(\frac{c-2a}{3}\right)^2 + \left(\frac{d}{3}\right)^2$$

$$= \frac{c^2 + 4a^2 - 4ac}{9} + \frac{d^2}{9}$$

$$GA^2 + GB^2 + GC^2 = \frac{a^2 + c^2 + 2ac}{9} + \frac{d^2}{9} + \frac{a^2 + 4c^2 - 4ac}{9} + \frac{4d^2}{9} + \frac{c^2 + 4a^2 - 4ac}{9} + \frac{d^2}{9}$$

$$= \frac{a^2 + c^2 + 2ac + d^2 + a^2 + 4c^2 - 4ac + 4d^2 + c^2 + 4a^2 - 4ac + d^2}{9}$$

$$= \frac{6a^2 + 6c^2 + 6d^2 + 6ac}{9} = \frac{2a^2 + 2c^2 + 2d^2 + 2ac}{3}$$

$$\therefore L.H.S = R.H.S$$

6. If G be the centroid of a triangle ABC, prove that:
$AB^2 + BC^2 + CA^2 = 3\left(GA^2 + GB^2 + GC^2\right)$

Sol:

Let $A(b,c), B(0,0)$ and $C(a,0)$ be the coordinates of ΔABC

Then coordinates of centroid are $G\left[\frac{a+b}{3}, \frac{c}{3}\right]$

To prove:

$AB^2 + BC^2 + CA^2 = 3\left(GA^2 + GB^2 + GC^2\right)$

Solving L.H.S

$AB^2 + BC^2 + CA^2$

$= b^2 + c^2 + a^2 (a-b)^2 + c^2$

$= b^2 + c^2 + a^2 + a^2 + b^2 - 2ab + c^2$

$= 2a^2 + 2b^2 + 2c^2 - 2ab$

Solving R.H.S

$$3\left[\left(\frac{a+b}{3} - b\right)^2 + \left(c - \frac{c}{3}\right)^2 + \left(\frac{a+b}{3}\right)^2 + \left(\frac{c}{3}\right)^2 + \left(\frac{a+b}{2} - a\right)^2 + \left(\frac{c}{3}\right)^2\right]$$

$$= 3\left[\left(\frac{a-2b}{3}\right)^2 + \left(\frac{2c}{3}\right)^2 + \left(\frac{a+b}{3}\right)^2 + \left(\frac{c}{3}\right)^2 + \left(\frac{b-2a}{3}\right)^2 + \left(\frac{c}{3}\right)^2\right]$$

$$= 3\left[\frac{a^2+4b^2-4ab}{9} + \frac{4c^2}{9} + \frac{a^2+b^2+2ab}{9} + \frac{c^2}{9} + \frac{b^2+4a^2-4ab}{9} + \frac{c^2}{9}\right]$$

$$= 3\left[\frac{a^2+4b^2-4ab+4c^2+a^2+b^2+2ab+c^2+b^2+4a^2-4ab+c^2}{9}\right]$$

$$= 3\left[\frac{6a^2+6b^2+6c^2-6ab}{9}\right]$$

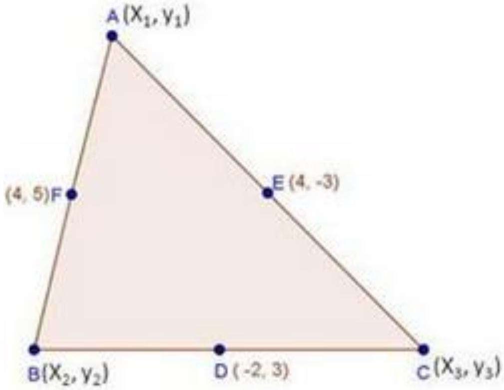
$$= \cancel{3}\times 3\left[\frac{2a^2+2b^2+2c^2-2ab}{\cancel{9}}\right]$$

$= 2a^2 + 2b^2 + 2c^2 - 2ab$

$\therefore$ L.H.S = R.H.S proved

7. If (—2, 3), (4, —3) and (4, 5) are the mid-points of the sides of a triangle, find the coordinates of its centroid.

Sol:



Let $A(x_1, y_2), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC

Let $D(-2,3), E(4,-3)$ and $F(4,5)$ be the midpoints of sides BC, CA and AB respectively

Since, D is the midpoint of BC

$\frac{x_2 + x_3}{2} = -2$ and $\frac{y_2 + y_3}{2} = 3$

$\Rightarrow x_2 + x_3 = -4$ and $y_2 + y_3 = 6$ ....(i)

And, $\frac{x_1 + x_3}{2} = 4$ and $\frac{y_1 + y_3}{2} = -3$

$\Rightarrow x_1 + x_2 = 8$ and $y_1 + y_3 = -6$ ....(ii)

And, $\frac{x_1 + x_2}{2} = 4$ and $\frac{y_1 + y_2}{2} = 5$

$\Rightarrow x_1 + x_2 = 8$ and $y_1 + y_3 = 10$ ....(iii)

From (i), (ii) and (iii), we get

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = -4 + 8 + 8$ and

$y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 6 - 6 + 10$

$\Rightarrow 2(x_1 + x_2 + x_3) = 12$ and $2(y_1 + y_2 + y_3) = 10$

$\Rightarrow x_1 + x_2 + x_3 = 6$ and $y_1 + y_2 + y_3 = 5$ ....(iv)

From (i) and (iv), we get

$x_1 - 4 = 6$ and $y_1 + 6 = 5$

$\Rightarrow x_1 = 10$ $\Rightarrow y_1 = -1$

So, the coordinates of A are $(10, -1)$

From (ii) and (iv)

$x_2 + 8 = 6$ and $y_2 - 6 = 5$

$\Rightarrow x_2 = -2$ $\Rightarrow y_2 = 11$

So, the coordinates of B are $(-2, 11)$

From (iii) and (iv)

$x_3 + 8 = 6 \text{ and } y_3 + 10 = 5$

$\Rightarrow x_3 = -2 \qquad \Rightarrow y_3 = -5$

So, the coordinates of C are $(-2, -5)$

$\therefore$ The vertices of ΔABC are $A(10, -1), B(-2, 11)$ and $C(-2, -5)$

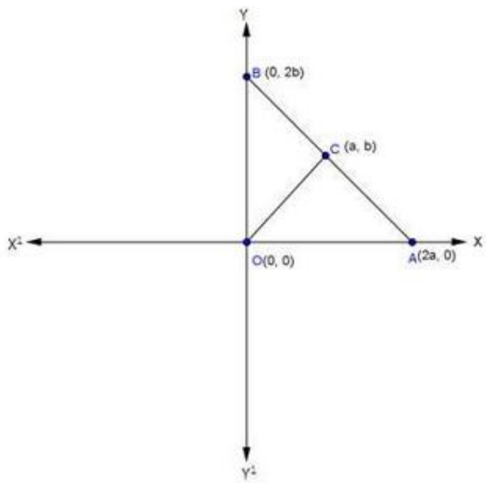
Hence, coordinates of the centroid of ΔABC are

$$\left(\frac{10-2-2}{3}, \frac{-1+11-5}{3}\right)$$

$$= \left(2, \frac{5}{3}\right)$$

8. In below Fig. a right triangle BOA is given. C is the mid-point of the hypotenuse AB. Show that it is equidistant from the vertices O, A and B.

Sol:



Given a right triangle *BOA* with vertices $B(0, 2b), 0(0, 0)$ and $A(2a, 0)$

Since, C is the midpoint of AB

$\therefore$ coordinates of C are $\left(\frac{2a+0}{2}, \frac{0+2b}{2}\right)$

$= (a, b)$

Now, $CO = \sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$

$CA=\sqrt{(2a-a)^2+(0-b)^2}=\sqrt{a^2+b^2}$

$CB=\sqrt{(a-0)^2+(b-2b)^2}=\sqrt{a^2+b^2}$

Since, $CO=CA=CB.$

$\therefore C$ is equidistant from $O, A\ and\ B.$

9. Find the third vertex of a triangle, if two of its vertices are at (-3, 1) and (0, -2) and the centroid is at the origin

Sol:

Let the coordinates of the third vertex be (x, y), Then

Coordinates of centroid of triangle are

$$\left(\frac{x-3+0}{3},\frac{y+1-2}{3}\right)=\left(\frac{x-3}{2},\frac{y-1}{3}\right)$$

We have centroid is at origin $(0,0)$

$$\therefore \frac{x-3}{3}=0\ \ and\ \ \frac{y-1}{3}=0$$

$\Rightarrow x-3=0 \qquad \Rightarrow y-1=0$

$\Rightarrow x=3 \qquad \Rightarrow y=1$

Hence, the coordinates of the third vertex are $(3,1)$.

10. A (3, 2) and B (−2, 1) are two vertices of a triangle ABC whose centroid G has the coordinates $\left(\frac{5}{11},\frac{1}{3}\right)$. Find the coordinates of the third vertex C of the triangle.

Sol:

Let the third vertex be $C(x, y)$

Two vertices $A(3,2)$ and $B(-2,1)$

Coordinates of centroid of triangle are

$$\left(\frac{x+3-2}{3},\frac{y+2+1}{3}\right)$$

But the centroid of the triangle are $\left(\frac{5}{3},-\frac{1}{3}\right)$

$$\therefore \frac{x+3-2}{3}=\frac{5}{3}\text{ and }\frac{y+2+1}{3}=-\frac{1}{3}$$

$\Rightarrow \frac{x+1}{3}=\frac{5}{3} \qquad \Rightarrow \frac{y+3}{3}=-\frac{1}{3}$

$\Rightarrow x+1=5 \qquad \Rightarrow y+3=-1$

$\Rightarrow x=4 \qquad \Rightarrow y=-4$

Hence, the third vertex of the triangle is $C(4,-4)$

Exercise 14.5

1. Find the area of a triangle whose vertices are

(i) $(6,3),(-3,5)$ and $(4,-2)$

(ii) $\left[\left(at_1^2,2at_1\right),\left(at_2^2,2at_2\right)\left(at_3^2,2at_3\right)\right]$

(iii) $(a,c+a),(a,c)\, and\, (-a,c-a)$

Sol:

(i) Area of a triangle is given by

$$\frac{1}{2}\left[x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1+y_2\right)\right]$$

Here, $x_1=6, y_1=3, x_2=-3, y_2=5, x_3=4, y_3=-2$

Let $A(6,3), B(-3,5)$ and $C(4,-2)$ be the given points

Area of $\Delta ABC=\frac{1}{2}\left[6(5+2)+(-3)(-2-3)+4(3-5)\right]$

$$=\frac{1}{2}\left[6\times7-3\times(-5)+4(-2)\right]$$

$$=\frac{1}{2}[42+15-8]$$

$$=\frac{49}{2}\, sq.units$$

(ii) Let $A=\left(x_1,y_1\right)=\left(at_1^2,2at_1\right)$

$B=\left(x_2,y_2\right)=\left(at_2^2,2at_2\right)$

$=\left(x_3,y_3\right)=\left(at_3^2,2at_3\right)$ be the given points.

The area of ΔABC

$$=\frac{1}{2}\left[at_1^2\left(2at_2-2at_3\right)+at_2^2\left(2at_3-2at_1\right)+at_3^2\left(2at_1-2at_2\right)\right]$$

$$=\frac{1}{2}\left[2a^2t_1^2t_2-2a^2t_1^2t_3+2a^2t_2^2t_3-2a^2t_2^2t_1+2a^2t_3^2t_1-2a^2t_3^2t_2\right]$$

$$=\frac{1}{2}\times2\left[a^2t_1^2\left(t_2-t_3\right)+a^2t_2^2\left(t_3-t_1\right)+a^2+t_3^2\left(t_1-t_2\right)\right]$$

$$=a^2\left[t_1^2\left(t_2-t_3\right)+t_2^2\left(t_3-t_1\right)+t_3^2\left(t_1-t_2\right)\right]$$

(iii) Let $A=\left(x_1,y_1\right)=(a,c+a)$

$B=\left(x_2,y_2\right)=(a,c)$

$C=(x_3,y_3)=(-a,c-a)$ be the given points

The area of ΔABC

$$=\frac{1}{2}\left[a\left(c-\{c-a\}\right)+a\left(c-a-(c+a)\right)+(-a)(c+a-a)\right]$$

$$=\frac{1}{2}\left[a(c-c+a)+a(c-a-c-a)-a(c+a-c)\right]$$

$$=\frac{1}{2}\left[a\times a+ax(-2a)-a\times a\right]$$

$$=\frac{1}{2}\left[a^2-2a^2-a^2\right]$$

$$=\frac{1}{2}\times(-2a)^2$$

$$=-a^2$$

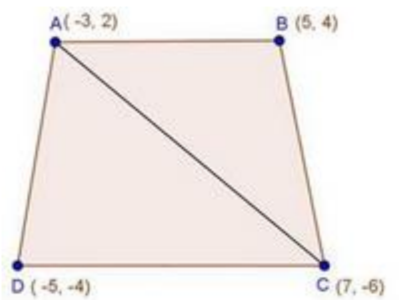
2. Find the area of the quadrilaterals, the coordinates of whose vertices are

(i) (—3, 2), (5, 4), (7, — 6) and (—5, — 4)

(ii) (1, 2), (6, 2), (5, 3) and (3, 4)

(iii) (—4, — 2), (—3, — 5), (3, — 2), (2, 3)

Sol:



Let $A(-3,2), B(5,4), C(7,-6)$ and $D(-5,-4)$ be the given points.

Area of ΔABC

$$=\frac{1}{2}\left[-3(4+6)+5(-6-2)+7(2-4)\right]$$

$$=\frac{1}{2}\left[-3\times10+5\times(-8)+7(-2)\right]$$

$$=\frac{1}{2}[-30-40-14]$$

$$=-42$$

But area cannot be negative

$\therefore$ Area of $\Delta ADC=42$ square units

Area of ΔADC

$=\frac{1}{2}\left[-3\left(-6+4\right)+7\left(-4-2\right)+\left(-5\right)\left(2+6\right)\right]$

$=\frac{1}{2}\left[-3\left(-2\right)+7\left(-6\right)-5\times 8\right]$

$=\frac{1}{2}\left[6-42-40\right]$

$=\frac{1}{2}\times -76$

$=-38$

But area cannot be negative

$\therefore$ Area of $\Delta ADC=38$ square units

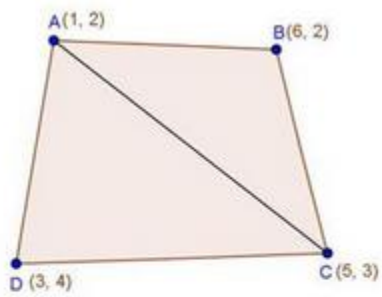
Now, area of quadrilateral $ABCD$

$=Ar.\ of\ ABC+Ar\ of\ ADC$

$=\left(42+38\right)$

$=80$ square. units

(i)



Let $A\left(1,2\right), B\left(6,2\right), C\left(5,3\right)$ and $\left(3,4\right)$ be the given points

Area of ΔABC

$=\frac{1}{2}\left[1\left(2-3\right)+6\left(3-2\right)+5\left(2-2\right)\right]$

$=\frac{1}{2}\left[-1+6\times\left(1\right)+0\right]$

$=\frac{1}{2}\left[-1+6\right]$

$=\frac{5}{2}$

Area of ΔADC

$=\frac{1}{2}\left[1\left(3-4\right)+5\left(4-2\right)+3\left(2-3\right)\right]$

$=\frac{1}{2}\left[-1\times 5\times 2+3\left(-1\right)\right]$

$=\frac{1}{2}[-1+10-3]$

$=\frac{1}{2}[6]$

$=3$

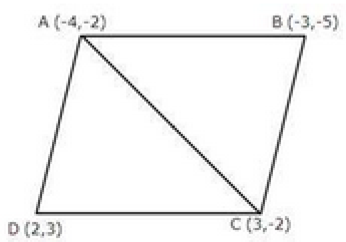
Now, Area of quadrilateral $ABCD$

$=$ Area of $ABC+$ Area of ADC

$=\left(\frac{5}{2}+3\right) sq.\ units$

$=\frac{11}{2}\ sq.\ units$

(ii)



Let $A(-4,-2), B(-3,-5), C(3,-2)$ and $D(2,3)$ be the given points

Area of $\Delta ABC=\frac{1}{2}\left|(-4)(-5+2)-3(-2+2)+3(-2+5)\right|$

$=\frac{1}{2}\left|(-4)(-3)-3(0)+3(3)\right|$

$=\frac{21}{2}$

Area of $\Delta ACD=\frac{1}{2}\left|(-4)(3+2)+2(-2+2)+3(-2-3)\right|$

$=\frac{1}{2}\left|-4(5)+2(0)+3(-5)\right|=\frac{-35}{2}$

But area can't be negative, hence area of $\Delta ADC=\frac{35}{2}$

Now, area of quadrilateral $(ABCD)=ar(\Delta ABC)+ar(\Delta ADC)$

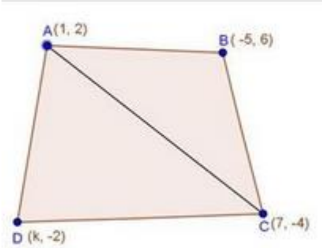
Area (quadrilateral $ABCD$) $=\frac{21}{2}+\frac{35}{2}$

Area (quadrilateral $ABCD$) $=\frac{56}{2}$

Area (quadrilateral $ABCD$) $=28$ square. Units

3. The four vertices of a quadrilateral are (1, 2), (−5, 6), (7, −4) and (k, −2) taken in order. If the area of the quadrilateral is zero, find the value of k.

Sol:



Let $A(1,2), B(-5,6), C(7,-4)$ and $(k,-2)$ be the given points.

Area of ΔABC

$= \frac{1}{2}[1(6+4)+(-5)(-4-2)+7(2-6)]$

$= \frac{1}{2}[10+30-28]$

$= \frac{1}{2}\times 12$

$= 6$

Area of ΔADC

$= \frac{1}{2}[1(-4+2)+7(-2-2)+k(2+4)]$

$= \frac{1}{2}[-2+7\times(-4)+k\times 6]$

$= \frac{1}{2}[-2-28+6k]$

$= \frac{1}{2}[-30+6k]$

$= -15+3k$

$= 3k-15$

Area of quadrilateral $ABCD$

= Area of $ABC +$ Area of ADC

$= (6+3k-15)$

But are of quadrilateral $= 0$ (given)

$\therefore 6+3k-15=0$

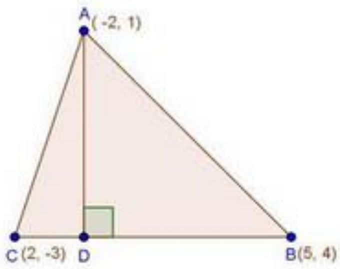
$\Rightarrow 3k = 15-6$

$\Rightarrow 3k = 9$

$\Rightarrow k = 3$

4. The vertices of ΔABC are (—2, 1), (5, 4) and (2, —3) respectively. Find the area of the triangle and the length of the altitude through A.

Sol:



Let $A(-2,1), B(5,4)$ and $C(2,-3)$ be the vertices of $\Delta ABC.$

Let AD be the altitude through $A.$

Area of ΔABC

$$=\frac{1}{2}\left[-2(4+3)+5(-3-1)+2(1-4)\right]$$

$$=\frac{1}{2}\left[-14-20-6\right]$$

$$=\frac{1}{2}\times -40$$

$$=-20$$

But area cannot be negative

$\therefore$ Area of $\Delta ABC = 20$ square units

Now, $BC=\sqrt{(5-2)^2+(4+3)^2}$

$$\Rightarrow BC=\sqrt{(3)^2+(7)^2}$$

$$\Rightarrow BC=\sqrt{58}$$

We know that area of Δ

$$=\frac{1}{2}\times Base\times Altitude$$

$$\therefore 20=\frac{1}{2}\times\sqrt{58}\times AD$$

$$\Rightarrow AD=\frac{40}{\sqrt{58}}$$

$\therefore$ Length of the altitude $AD=\dfrac{40}{\sqrt{58}}$

5. Show that the following sets of points are collinear.

(a) (2, 5), (4, 6) and (8, 8) (b) (1, —1), (2, 1) and (4, 5)

Sol:

(a) Let $A(2,5), B(4,6)$ and $C(8,8)$ be the given points

Area of ΔABC

$=\frac{1}{2}[2(6-8)+4(8-5)+8(5-6)]$

$=\frac{1}{2}[2\times(-2)+4\times3+8\times(-1)]$

$=\frac{1}{2}[-4+12-8]$

$=\frac{1}{2}\times0$

$=0$

Since, area of $\Delta ABC=0$

$\therefore (2,5),(4,6)$ and $(8,8)$ are collinear.

(b) Let $A(1,-1), B(2,1)$ and $C(4,5)$ be the given points

Area of ΔABC

$=\frac{1}{2}[1(1-5)+2(5+1)+4(-1-1)]$

$=\frac{1}{2}[-4+12-8]$

$=\frac{1}{2}\times0$

$=0$

Since, area of $\Delta ABC=0$

$\therefore$ The points $(1,-1),(2,1)$ and $(4,5)$ are collinear

6. Prove that the points (a, 0), (0, b) and (1, 1) are collinear if, $\frac{1}{a}+\frac{1}{b}=1$.

Sol:

Let $A(a,0), B(0,b)$ and $C(1,1)$ be the given points

Area of ΔABC

$=\frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$

$=\frac{1}{2}\{a(b-1)+0(1-0)+1(0-b)\}$

$=\frac{1}{2}\{ab-a+0-b\}$

$=\frac{1}{2}\{ab-a-b\}$

$=\frac{1}{2}\{ab-(a+b)\}$

$=\frac{1}{2}\{ab-ab\}$ $\left[\because \frac{1}{a}+\frac{1}{b}=1\right]$

$\Rightarrow \frac{a+b}{ab}=1$

$\Rightarrow a+b=ab$

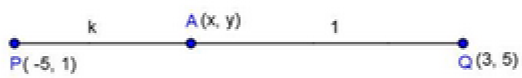
$=\frac{1}{2}\times 0$

$=0$

Hence, $A(a,0), B(0,b)$ and $(1,1)$ are collinear if $\frac{1}{a}+\frac{1}{b}=1.$

7. The point A divides the join of P (—5, 1) and Q (3, 5) in the ratio k : 1. Find the two values of k for which the area of ΔABC where B is (1, 5) and C (7, —2) is equal to 2 units.

Sol:



Let $A(x,y)$ divides the join of $P(-5,1)$ and $(3,5)$ in the ratio $k:1$

$x=\frac{3k-5}{k+1}, y=\frac{5k+1}{k+1}$

Area of ΔABC with $A\left(\frac{3k-5}{k+1},\frac{5k+1}{k+1}\right)B(1,5)$ and $C(7,-2)$

$=\frac{1}{2}\left\{\frac{3k-5}{k+1}(5-2)+1\left(2-\frac{5k+1}{k+1}\right)+7\left(\frac{5k+1}{k+1}=5\right)\right\}$

$=\frac{1}{2}\left\{\frac{3k-5}{k+1}\times 7+\frac{-7k-3}{k+1}+\frac{-4}{k+1}\right\}$

$=\frac{1}{2}\left\{\frac{21k-35}{k+1}+\frac{-7k-3}{k+1}+\frac{-4}{k+1}\right\}$

$=\frac{1}{2}\left\{\frac{21k-35-7k-3-4}{k+1}\right\}$

$=\frac{1}{2}\left\{\frac{14k-42}{k+1}\right\}$

$=\frac{14k-42}{2(k+1)}$

But area of $\Delta ABC = 2$ given,

$\Rightarrow \frac{14k-42}{2(k+1)} = 2$

$\Rightarrow 14k-42 = 4(k+1)$

$\Rightarrow 14k-42 = 4k+4$

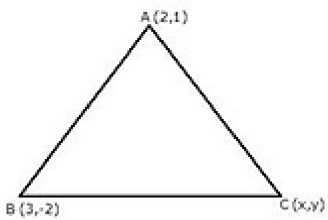
$\Rightarrow 14k-4k = 4+42$

$\Rightarrow 10k = 46$

$\Rightarrow k = \frac{46}{10} = \frac{23}{5}$

8. The area of a triangle is 5. Two of its vertices are (2, 1) and (3, -2). Third vertex lies on y = x + 3. Find the third vertex.

Sol:



Let $A(2,1), B(3,-2)$ be the vertices of Δ

And $C(x, y)$ be the third vertex

Area of $\Delta ABC = \frac{1}{2}\left|2(-2-y)+3(y-1)+x(1+2)\right|$

$=\frac{1}{2}\left|-4-2y+3y-3+3x\right|$

$=\frac{1}{2}\left|3x+y-7\right|$

But it is given that area of $\Delta ABC = 5$

$\therefore 5 = \frac{\pm 1}{2}\left[3x+y-7\right]$

$\pm 10 = 3x+y-7$

$3x+y=17$ or $3x+y=-3$ (i)

But it is given that third vertices lies on $y = x+3$

Hence subsisting value of y in (i)

$3x+x+3=17$ *or* $3x+x+3=-3$

$4x = 14 \quad or \quad 4x = -6$

$x = \frac{7}{2} \quad or \quad x = \frac{-3}{2}$

$y = \frac{7}{2} + 3 \quad or \quad y = \frac{-3}{2} + 3$

$y = \frac{13}{2} \quad or \quad y = \frac{3}{2}$

Hence coordinates of c will be $\left(\frac{7}{2}, \frac{13}{2}\right)$ *or* $\left(\frac{-3}{2}, \frac{3}{2}\right)$

9. If $a \neq b \neq c$, prove that the points $(a, a^2), (b, b^2), (c, c^2)$ can never be collinear.

Sol:

Let $A(a, a^2)\, B(b, b^2)$ *and* (c, c^2) be the given points.

$\therefore$ Area of ΔABC

$= \frac{1}{2}\left\{a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)\right\}$

$= \frac{1}{2}\left\{ab^2 - ac^2 + bc^2 - ba^2 + ca^2 - cb^2\right\}$

$= \frac{1}{2} \times 0$

$= 0 \quad [if \ a = b = c]$

i.e., the points are collinear if $a = b = c$

Hence, the points can never be collinear if $a \neq b \neq c$.

10. Four points A(6, 3), B (-3, 5), C(4, -2) and D(x, 3x) are given in such a way that $\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$, find *x*.

Sol:

Area of $\Delta DBC = \frac{1}{2}\left\{x(5+2) + (-3)(-2-3x) + 4(3x-5)\right\}$

$= \frac{1}{2}\left\{7\times + (6+9x) + 12x - 20\right\}$

$= \frac{1}{2}\left\{28x - 14\right\}$

Area of $\Delta ABC = \frac{1}{2}\left\{6(5+2) + (-3)(-2-3) + 4(3-5)\right\}$

$= \frac{1}{2}\left\{42 + 15 - 8\right\}$

$=\frac{1}{2}\times 49$

Given

$\frac{\Delta DBC}{\Delta ABC}=\frac{1}{2}$

$\Rightarrow \frac{\frac{1}{2}(28x-14)}{\frac{1}{2}\times 49}=\frac{1}{2}$

$\Rightarrow \frac{28x-14}{49}=\frac{1}{2}$

$\Rightarrow 2(28x-14)=49$

$\Rightarrow 56x-28=49$

$\Rightarrow 56x=77$

$\Rightarrow x=\frac{77}{56}$

$\Rightarrow x=\frac{11}{8}$

11. For what value of a point (a, 1), (1, -1) and (11, 4) are collinear?

Sol:

Let $A(a,1), B(1,-1)$ and $C(11,4)$ be the given points

Area of ΔABC

$=\frac{1}{2}\{a(-1-4)+1(4-1)+11(1+1)\}$

$=\frac{1}{2}\{-5+3+22\}$

$=\frac{1}{2}\{-5a+25\}$

For the points to be collinear

Area of $\Delta ABC=0$

$=\frac{1}{2}\{-5a+25\}=0$

$\Rightarrow -5a+25=0$

$\Rightarrow -5a=-25$

$\Rightarrow a=5$

12. Prove that the points (a, b), (a_1, b_1) and $(a-a_1, b-b_1)$ are collinear if $ab_1=a_1b$

Sol:

Let $A(a,b), B(a_1,b_1)$ and $C(a-a_1, b-b_1)$ be the given points.

Area of ΔABC

$$=\frac{1}{2}\left\{a\left[b_1-(b-b_1)+a_1(b-b_1-b)+(a-a_1)(b-b_1)\right]\right\}$$

$$=\frac{1}{2}\left\{a(b_1-b+b_1)+a_1(-b)_1+ab-ab_1-a_1b+a_1b_1\right\}$$

$$=\frac{1}{2}\left\{ab_1-ab+ab_1-a_1b_1+ab-ab_1-a_1b+a_1b_1\right\}$$

$$=\frac{1}{2}\left\{ab_1=a_1b\right\}$$

$$=\frac{1}{2}\times 0=0 \qquad \left[if\ \ ab_1=a_1b\right]$$

Hence, the points are collinear if $ab_1=a_1b.$

Exercise 15.1

1. Find the circumference and area of circle of radius 4.2 cm

Sol:

Radius (r) = 4.2 cm

Circumference = 2 × r

$= 2 \times \frac{22}{7} \times 4.2$

$= \left(\frac{44}{10} \times 6\right) = \frac{264}{10}$

$= 26.4\ cm$

Area = $\pi r^2 = \frac{22}{7} \times 4.2 \times 4.2$

$= \frac{22\times6\times42}{10\times10} = \frac{5544}{100} = 55.44 cm^2$

2. Find the circumference of a circle whose area is 301.84 cm^2.

Sol:

Area of circle = 301.84 cm^2.

Let radius = r cm

Area of circle = πr^2

$\pi r^2 = 301.84$

$\frac{22}{7} \times r^2 = 301.84$

$r^2 = \frac{301.84\times7}{22} = \left(\sqrt{7\times7}\right)^{\frac{1}{2}} \times 13.75$

$r = \sqrt{13.72\ \times 7} = \sqrt{7\times7\times1.96} = 7 \times 1.4 = 9.8\ cm$

Radius = r = 9.8 cm

Circumference = 2 × r = $2 \times \frac{22}{7} \times 9.8$

$= 44 \times 1.4$

$= 61.6\ cm$

3. Find the area of circle whose circumference is 44 cm.

Sol:

Circumference = 44 cm

Let radius = r cm

Circumference = 2 × r = 44 cm

$2 \times \frac{22}{7} \times r = 44$

$r = \frac{44\times7}{2\times22} = 7cm$

radius = 7 cm

Area of circle = πr^2

$= \frac{22}{7} \times 7 \times 7 = (22 \times 7) = 154 cm^2$

4. The circumference of a circle exceeds diameter by 16.8 cm. Find the circumference of circle.

Sol:

Let radius of circle = r cms

Diameter(d) = $2 \times radius = 2r$

Circumference (c) = $2\pi r$

Given circumference exceeds diameter by 16.8cm

C = d + 16.8

$\Rightarrow 2\pi r = 2r + 16.8$

$\Rightarrow 2r(\pi - 1) = 16.8$

$\Rightarrow 2r \times \left(\frac{22}{7} - 1\right) = 16.8$

$\Rightarrow 2r \times \frac{15}{7} = 16.8$

$\Rightarrow r = \frac{16.8 \times 7}{30} = 5.6 \times 0.7$

$\Rightarrow r = 3.92$ cms

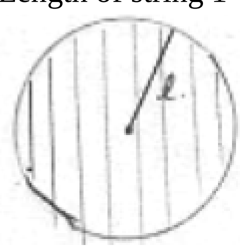
Circumference $= 2\pi r = 2 \times \frac{22}{7} \times 3.92$

$= \frac{2464}{100} = 24.64\ cms$

5. A horse is tied to a pole with 28m long string. Find the area where the horse can graze.

Sol:

Length of string 1 = 28m



Area it can graze is area of circle with radius equal to length of string

Area = πl^2

$= \frac{22}{7} \times 28 \times 28$

$= 88 \times 28$

$= 2464\ cm^2$

$\therefore$ area grazed by horse = $2464 cm^2$.

6. A steel wire when bent is the form of square encloses an area of 12 cm^2. If the same wire is bent in form of circle. Find the area of circle.

Sol:

Let side of square = s and length of wire be l. As wire is bent into square

l = perimeter of square = 4s.

Area of square = 121cm^2 = s^2.

S = $\sqrt{121} = 11cm$

∴ length of wire l = 4(11) = 44cm

As wire is bent into circle (let radius be r)

Length of wire = circumference

44 = 2πr

$\frac{22}{7} \times 2 \times r = 44 \Rightarrow r = \frac{44\times7}{2\times22} = 7cm$

$Area\ of\ circle = \pi r^2$

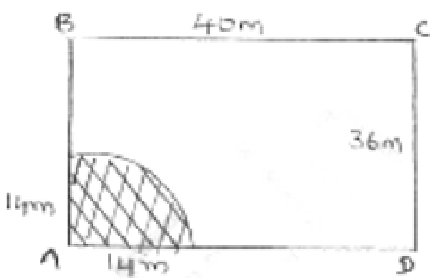
$= \frac{22}{7} \times 7 \times 7$

$= 22 \times 7$

$= 154cm^2$

7. A horse is placed for grazing inside a rectangular field 40m by 36m and is tethered to one corner by a rope 14m long. Over how much area can it graze.

Sol:



The fig shows rectangular field ABCD at corner A, a horse is tied with rope length = 14m. The area it can graze is represented A as shaded region= area of quadrant with (radius = length) of string

Area = $\frac{1}{4} \times (area\ of\ circle) = \pi r^2$

= $\frac{1}{4} \times \frac{22}{7} \times 14 \times 14$

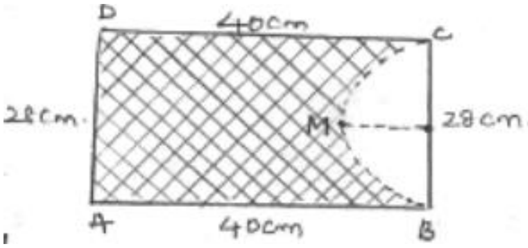
= (22 × 7)

= 154 m^2.

Area it can graze = 154m^2.

8. A sheet of paper is in the form of rectangle ABCD in which AB = 40cm and AD = 28 cm. A semicircular portion with BC as diameter is cut off. Find the area of remaining paper.

Sol:



Given sheet of paper ABCD

AB = 40 cm, AD = 28 cm

⇒ CD = 40 cm, BC = 28 cm [since ABCD is rectangle]

Semicircle be represented as BMC with BC as diameter

Radius = $\frac{1}{2} \times BC = \frac{1}{2} \times 28 = 14cms$

Area of remaining (shaded region) = (area of rectangle) – (area of semicircle)

= $(AB \times BC) - \left(\frac{1}{2}\pi r^2\right)$

= $(40 \times 28) - \left(\frac{1}{7} \times \frac{22}{7} \times 14 \times 14\right)$

= $1120 - 308$

= 812 cm^2.

9. The circumference of two circles are in ratio 2:3. Find the ratio of their areas

Sol:

Let radius of two circles be r_1 and r_2 then their circumferences will be $2\pi r_1$: $2\pi r_2$

= $r_1 : r_2$

But circumference ratio is given as 2 : 3

$r_1 : r_2 = 2 : 3$

Ratio of areas = $\pi {r_2}^2 : \pi {r_2}^2$

$= \left(\frac{r_1}{r_2}\right)^2$

$= \left(\frac{2}{3}\right)^2$

$= \frac{4}{9}$

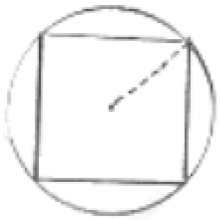
$= 4 : 9$

$\therefore ratio\ of\ areas = 4 : 9$

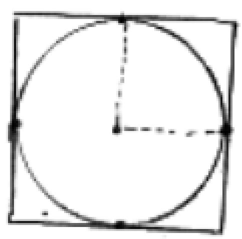
10. The side of a square is 10 cm. find the area of circumscribed and inscribed circles.

Sol:

Circumscribed circle

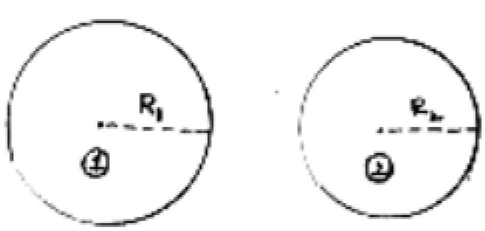


Radius = $\frac{1}{2}(diagonal\ of\ square)$
$= \frac{1}{2} \times \sqrt{2}$ side
$= \frac{1}{2} \times \sqrt{2} \times 10$
$= 5\sqrt{2}$ cm
Area = πr^2
$= \frac{22}{7} \times 25 \times 2$
$= \frac{1100}{7} cm^2$
Inscribed circle



Radius = $\frac{1}{2}$ (sides)
$= \frac{1}{2} \times 10$
= 5 cm
Area = πr^2
$= \frac{22}{7} \times 5 \times 5$
$= \frac{550}{7} cm^2$

11. The sum of the radii of two circles is 140 cm and the difference of their circumferences in 88 cm. Find the diameters of the circles.
Sol:



Let radius of circles be $r_1 and\ r_2$
Given sum of radius = 140cm

$r_1 + r_2 = 140$ …..(i)

$difference\ in\ circumgerences = 88\ cm$

$2 \times r_1 - 2\pi r_2 = 88$

$2 \times \frac{22}{7}(r_1 - r_2)\ 88$

$r_1 - r_2 = \frac{88\times7}{2\times22} = 14$

$r_1 = r_2 + 14$ ….(ii)

$(ii) in\ (i) \Rightarrow\ r_2 + r_2 + 14 = 140$

$\Rightarrow 2r_2 = 126$

$\Rightarrow r_2 = \frac{126}{2} = 63cms$

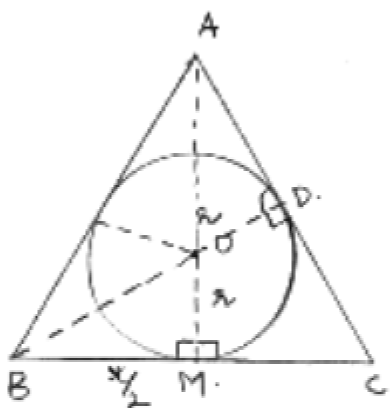
$r_2 = 63cms\ in\ (ii)\ r_1 = 63 + 14 = 77\ cms$

Diameter of circle (i) = $2r_1 = 2 \times 77 = 154\ cms$

Diameter of circle (ii) = $2r_2 = 2 \times 63 = 126\ cms$

12. The area of circle, inscribed in equilateral triangle is 154 cms^2. Find the perimeter of triangle.

Sol:



Let circle inscribed in equilateral triangle

Be with centre O and radius 'r'

Area of circle = πr^2

But given that area = 154 cm2.

$\pi r^2 = 154$

$\frac{22}{7} \times r^2 = 154$

$r^2 = 7 \times 7$

r = 7cms

Radius of circle = 7cms

From fig. at point M, BC side is tangent at point M, BM ⊥ OM. In equilateral triangle, the perpendicular from vertex divides the side into two halves

BM = $\frac{1}{2}BC = \frac{1}{2}(side = x) = \frac{x}{2}$

ΔBMO is right triangle, by Pythagoras theorem

$OB^2 = BM^2 + MO^2$

$OB = \sqrt{r^2 + \frac{x^2}{4}} = \sqrt{49 + \frac{x^2}{4}} \qquad OD = r$

Altitude BD = $\frac{\sqrt{3}}{2}\ (side) = \frac{\sqrt{3}}{2}x = OB + OD$

BD – OD = OB

$\Rightarrow \frac{\sqrt{3}}{2}x - r = \sqrt{49 + \frac{x^2}{4}}$

$\Rightarrow \frac{\sqrt{3}}{2}x - 7 = \sqrt{49 + \frac{x^2}{4}}$

$\Rightarrow \left(\frac{\sqrt{3}}{2}x - 7\right)^2 = \left(\sqrt{\frac{x^2}{4} + 49}\right)^2$

$\Rightarrow \frac{3}{4}x^2 - 7\sqrt{3}x + 49 = \frac{x^2}{4} + 49$

$\Rightarrow \frac{x}{2} = 7\sqrt{3} \Rightarrow x = 14\sqrt{3}\ cm$

Perimeter = 3x = $3 \times 14\sqrt{3}$

= $42\sqrt{3}$ cms

13. A field is in the form of circle. A fence is to be erected around the field. The cost of fencing would to Rs. 2640 at rate of Rs.12 per metre. Then the field is to be thoroughs ploughed at cost of Rs. 0.50 per m^2. What is amount required to plough the field?

Sol:

Given

Total cost of fencing the circular field = Rs. 2640

Cost per metre fencing = Rs 12

Total cost of fencing = circumference × cost per fencing

$\Rightarrow$ 2640 = circumference × 12

$\Rightarrow$ circumference $= \frac{2640}{12} = 220m$

Let radius of field be r m

Circumference = $2\pi r$ m

$2\pi r = 220$

$2 \times \frac{22}{7} \times r = 220$

$r = \frac{70}{2} = 35m$

Area of field = πr^2

$= \frac{22}{7} \times 35 \times 35$

= 3850 m^2.

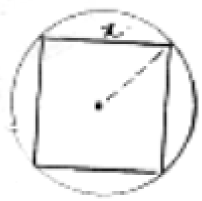
Cost of ploughing per m^2 land = Rs. 0.50

Cost of ploughing 3850 m^2 land $= \frac{1}{2} \times 3850$

= Rs. 1925.

14. If a square is inscribed in a circle, find the ratio of areas of the circle and the square.

Sol:



Let side of square be x cms inscribed in a circle.

Radius of circle (r) = $\frac{1}{2}(diagonal\ of\ square)$

$= \frac{1}{2}\left(\sqrt{2}x\right)$

$= \frac{x}{\sqrt{2}}$

Area of square = $(\text{side})^2 = x^2$

Area of circle = πr^2

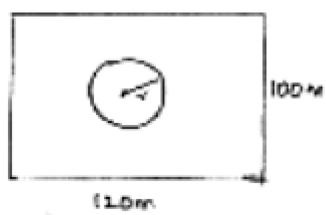
$= \pi\left(\frac{x}{\sqrt{2}}\right)^2$

$= \frac{\pi x^2}{2}$

$\frac{area\ of\ circle}{area\ of\ square} = \frac{\frac{\pi}{2}x^2}{x^2} = \frac{\pi}{2} = \pi : 2$

15. A park is in the form of rectangle 120m × 100m. At the centre of park there is a circular lawn. The area of park excluding lawn is 8700m^2. Find the radius of circular lawn.

Sol:



Dimensions of rectangular park length = 120m

Breadth = 100m

Area of park = l × b

$= 120 \times 100 = 12000m^2$.

$Let\ radius\ of\ circular\ lawn\ be\ r$

Area of circular lawn $= \pi r^2$

Area of remaining park excluding lawn = (area of park) – (area of circular lawn)

⇒ $8700 = 12000 - \pi r^2$

⇒ $\pi r^2 = 12000 - 8700 = 3300$

$\Rightarrow \frac{22}{7} \times r^2 = 3300$
$\Rightarrow r^2 = 150 \times 7 = 1050$
$\Rightarrow$ r = $\sqrt{1050} = 5\sqrt{42}\ metres$
∴ radius of circular lawn = $5\sqrt{42}$ metres.

16. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having its are equal to the sum of the areas of two circles.
Sol:
Radius of circles are 8cm and 6 cm
Area of circle with radius 8 cm = $\pi(8)^2 = 64\pi cm^2$
Area of circle with radius 6cm = $\pi(6)^2 = 36\pi\ cm^2$
Areas sum = $64\pi + 36\pi = 100\pi$ cm^2
Radius of circle be x cm
Area = πx^2
$\pi x^2 = 100\pi$
$x^2 = 100 \Rightarrow x = \sqrt{100} = 10cm$

17. The radii of two circles are 19cm and 9 cm respectively. Find the radius and area of the circle which has circumferences is equal to sum of circumference of two circles.
Sol:
Radius of 1st circle = 19cm
Radius of 2nd circle = 9 cm
Circumference of 1st circle = 2(19) = 38π cm
Circumference of 2nd circle = 2π (9) = 18π cm
Let radius of required circle = R cm
Circumference of required circle = $2\pi R = c_1 + c_2$
$2\pi R = 38\pi + 18\pi$
$2\pi R = 56\pi$
R = 28 cms
Area of required circle = πr^2
$= \frac{22}{7} \times 28 \times 28$
$= 2464\ cm^2$

18. A car travels 1 km distance in which each wheel makes 450 complete revolutions. Find the radius of wheel.
Sol:
Let radius of wheel = ‘r’ m

Circumference of wheel = $(2\pi r)$m.
No. of revolutions = 450
Distance for 450 revolutions = $450 \times 2\pi r = 900\pi r\ m$
But distance travelled = 1000 m.
900πr = 1000
r = 10000 $9\pi \times 100$
$= \frac{10}{9\pi} m$
$= \frac{1000}{9\pi} cms$
$radius\ (r) = \frac{1000}{9\pi}\ cms$

19. The area enclosed between the concentric circles is 770cm^2. If the radius of inner circle.
Sol:
Radius of outer circle = $21cm$

Radius of inner circle = R_2
Area between concentric circles = area of outer circle – area of inner circle
⇒ 770 = $\frac{22}{7}(21^2 - {R_2}^2)$
⇒ $21^2 - {R_2}^2 = 35 \times 7 = 245$
⇒ 441 – 245 = ${R_2}^2$
⇒ $R_2 = \sqrt{196} = 14\ cm$
Radius of inner circle = 14cm.

Exercise 15.2

1. Find in terms of x the length of the arc that subtends an angle of 30°, at the centre of circle of radius 4 cm.

Sol:

Length of arc = $\frac{\theta}{360°} \times 2\pi r$

Radius = r = 4 cm

θ = angle subtended at centre = 30°

Arc length = $\frac{30°}{360°} \times 2 \times (7)$

= $\frac{2\pi}{3}$ cm

2. Find the angle subtended at the centre of circle of radius 5cm by an arc of length $\left(\frac{5\pi}{3}\right) cm$

Sol:

Radius (r) = 5 cm

θ = angle subtended at centre (degrees)

Length of Arc = $\frac{\theta}{360°} \times 2\pi$r cm

But arc length = $\frac{5\pi}{3} cm$

$\frac{\theta}{360°} \times 2\pi \times 5 = \frac{5\pi}{3}$

$\theta = \frac{360° \times \pi}{3 \times 2\pi} = 60°$

∴ Angle subtended at centre = 60°

3. An arc of length 20π cm subtends an angle of 144° at centre of circle. Find the radius of the circle.

Sol:

Length of arc = 20πcm

Let radius = 'r' cm

O = angle subtended at centre = 144°

Length of arc = $\frac{\theta}{360°} \times 2\pi r$

= $\frac{144}{360} \times 2\pi r = \frac{4\pi}{5} r$

$= \frac{4\pi}{5} r = 20\pi$

$r = \frac{20\pi \times 5}{4\pi} = 25\ cms$

4. An arc of length 15 cm subtends an angle of 45° at the centre of a circle. Find in terms of π, radius of the circle.

Sol:

Length of arc = 15cm

θ = angle subtended at centre = 45°

Let radius = r cm

arc length = $\frac{\theta}{360°} \times 2\pi r$

= $\frac{45°}{360°} \times 2\pi r$

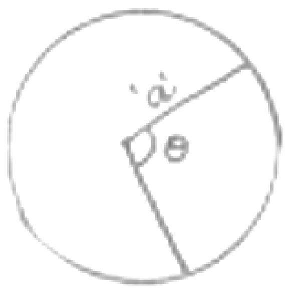
$\frac{45}{360} \times 2\pi r = 15$

$r = \frac{15 \times 360}{45 \times 2\pi} = \frac{60}{\pi}$ cms

$Radius = \frac{60}{\pi} cms$

5. Find the angle subtended at the centre of circle of radius ‘a’ cm by an arc of length $\frac{a\pi}{4}\ cm$

Sol:



Length of arc = $\frac{a\pi}{4}\ cm$

Radius r = ‘a’ cm

θ = angle subtended at centre

arc length = $\frac{\theta}{360°} \times 2\pi r$

= $\frac{\theta}{360°} \times 2\pi a$

$\therefore \frac{\theta}{360°} \times 2\pi a = \frac{a\pi}{4}$

$\Rightarrow \theta = \frac{9\pi \times 360°}{4 \times 2\pi a} = 45°$

6. A sector of circle of radius 4cm contains an angle of 30°. Find the area of sector

Sol:

Radius = 4 cm = r

Angle subtended at centre = θ = 30°

Area of sector (shaded region)

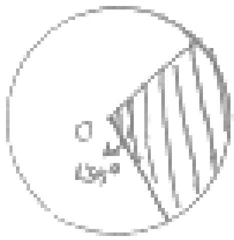
$= \frac{\theta}{360°} \times \pi r^2$

$= \frac{30}{360} \times \frac{22}{7} \times 4 \times 4$

$= \frac{88}{21} cm^2$

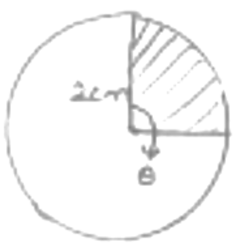
$\therefore area\ of\ required\ sector = \frac{88}{21} cm^2$

7. A sector of a circle of radius 8cm contains the angle of 135°. Find the area of sector.
Sol:
Radius (r) = 8cm



θ = angle subtended at centre = 135°
Area of sector = $\frac{x}{360°} \times \pi r^2$
$= \frac{135}{360} \times \frac{22}{7} \times 8 \times 8$
$= \frac{528}{7} cm^2$

8. The area of sector of circle of radius 2cm is πcm^2. Find the angle contained by the sector.
Sol:



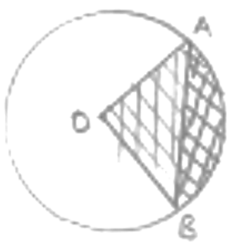
Area of sector = $\pi\ cm^2$
Radius of circle = $2cm$
Let θ = angle subtended by arc at centre
Area of sector = $\frac{\theta}{360°} \times \pi r^2$
$= \frac{\theta}{360°} \times \pi \times 2 \times 2$
$= \frac{\pi\theta}{90°}$
$\frac{\pi\theta}{90°} = \pi \Rightarrow\ \theta = 90°$

9. The area of sector of circle of radius 5cm is 5π cm^2. Find the angle contained by the sector.
Sol:

Area of sector = 5π cm^2.
Radius (r) = 5cm
Let θ = angle subtended at centre area of sector = $\frac{\theta}{360°} \times \pi r^2$
$= \frac{\theta}{360} \times \pi \times 5 \times 5 = \frac{5\pi\theta}{72°}$
$= \frac{5\pi\theta}{72°} = 5\pi$
$\Rightarrow \theta = 72°$

10. AB is a chord of circle with centre O and radius 4cm. AB is length of 4cm. Find the area of sector of the circle formed by chord AB

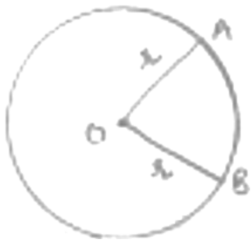
Sol:



AB is chord AB = 4cm
OA = OB = 4cm
OAB is equilateral triangle ∠AOB = 60°
Area of sector (formed by chord [shaded region]) = (area of sector)
= $\frac{\theta}{360°} \times \pi r^2 = \frac{60}{360} \times \pi \times 4 \times 4 = \frac{8\pi}{3} cm^2$

11. In a circle of radius 35 cm, an arc subtends an angle of 72° at the centre. Find the length of arc and area of sector

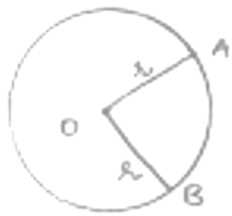
Sol:



Radius (r) = 35 cm
θ = angle subtended at centre = 72°
Length of arc = $\frac{\theta}{360°} \times 2\pi r$
= $\frac{72}{360} \times 2 \times \frac{22}{7} \times 35$
= $2 \times 22 = 44 cms$
Area of sector = $\frac{\theta}{360°} \times \pi r^2$

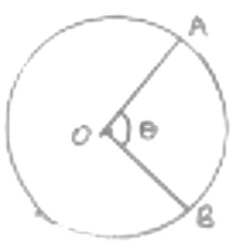
$= \frac{72}{360} \times \frac{22}{7} \times 35 \times 35$
$= (35 \times 22) = 770\ cm^2$

12. The perimeter of a sector of circle of radius 5.7m is 27.2 m. Find the area of sector.
Sol:



Radius = OA = OB (From fig) = r
= 5.7 m
Perimeter = 27.2 m
Let angle subtended at centre = θ
Perimeter = $\left(\frac{\theta}{360°} \times 2\pi r\right) + OA + OB$
$= \frac{\theta}{360°} \times 2(5.7) \times \pi + 2(5.7)$
$= \frac{2\pi(5.7)\theta}{360°} + 11.4$
$= \frac{\pi(5.7)\theta}{180°} + 11.4 = 27.2$
$= \frac{\pi(5.7)\theta}{180°} = 15.8$
$Area\ of\ sector = \frac{\theta}{360°} \times \pi r^2$
$= \frac{158.8}{360} \times \frac{22}{7} \times 5.7 \times 5.7$
$= 45.048\ cm^2$

13. The perimeter of certain sector of circle of radius 5.6 m is 27.2 m. Find the area of sector.
Sol:



θ = angle subtended at centre
Radius (r) = 5.6m = OA $\pm$ OB
Perimeter of sector = 27.2 m
(AB arc length) + OA + OB = 27.2
$\Rightarrow \left(\frac{\theta}{360°} \times 2\pi r\right) + 5.6 + 5.6 \pm 27.2$

$\Rightarrow \frac{5.6\,\pi\,\theta}{180°} + 11.2 = 27.2$

$\Rightarrow 5.6 \times \frac{22}{7} \times \theta = 16 \times 180$

$\Rightarrow \theta = \frac{16\times180}{0.8\times22} = 163.64°$

Area of sector = $\frac{\theta}{360°} \times \pi r^2 = \frac{163.64°}{360°} \times \frac{22}{7} \times 5.6 \times 5.6$

= $\frac{163.64}{180} \times 11 \times 0.8 \times 5.6$

= $44.8\ cm^2$

14. A sector is cut-off from a circle of radius 21 cm the angle of sector is 120°. Find the length of its arc and its area.

Sol:

Radius of circle (r) = 21 cm

θ = angle subtended at centre = 120°

Length of its arc = $\frac{\theta}{360°} \times 2\pi r$

= $\frac{120}{360} \times 2 \times \frac{22}{7} \times 21$

= 44 cms

Area of sector = $\frac{\theta}{360°} \times \pi r^2$

= $\frac{120}{360} \times \frac{22}{7} \times 21 \times 21$

$= (22 \times 21)$

$= 462\ cm^2$

$Length\ of\ arc = 44\ cm$

Area of sector = 462 cm^2

15. The minute hand of a clock is $\sqrt{21}\ cm$ long. Find area described by the minute hand on the face of clock between 7 am and 7:05 am

Sol:

Radius of minute hand (r) = $\sqrt{21}\ cm$
For 1hr = 60 min, minute hand completes one revolution = 360°
60 min = 360°
1 min = 6°
From 7 am to 7:05 am it is 5 min angle subtended = $5 \times 6° = 30° = \theta$
Area described = $\frac{\theta}{360°} \times \pi r^2$
$= \frac{30}{360} \times \frac{22}{7} \times 21$
$= \frac{22}{4} = 5.5cm^2$

16. The minute hand of clock is10cm long. Find the area of the face of the clock described by the minute hand between 8am and 8:25 am
Sol:

Radius of minute hand (r) = 10 cm
For 1 hr = 60 min, minute hand completes one revolution = 360°
60 min = 360°
1 min = 6°
From 8 am to 8:25 am it is 25 min angle subtended = 6° × 25 = 150° = θ
Area described = $\frac{\theta}{360°} \times \pi r^2$
$= \frac{150}{360} \times \frac{22}{7} \times 10 \times 10$
$= \frac{250 \times 11}{3}$
$= \frac{2750}{3}\ cm^2$

17. A sector of 56° cut out from a circle contains area of 4.4 cm^2. Find the radius of the circle
Sol:
Angle subtended by sector at centre θ = 56°
Let radius be 'x' cm
Area of sector = $\frac{\theta}{360°} \times \pi r^2$
= $\frac{56}{360} \times \frac{22}{7} \times r^2$
= $\frac{22}{45} r^2$
But area of sector = 4.4cm^2 = $\frac{44}{10} cm^2$

$\frac{22}{45} r^2 = \frac{44}{10}$

$\Rightarrow r^2 = \frac{45\times44}{22\times10} = 9$

$\Rightarrow r = \sqrt{9}$

$= 3\ cm$

∴ radius (r) = 3cm

18. In circle of radius 6cm, chord of length 10 cm makes an angle of 110° at the centre of circle find

(i) Circumference of the circle

(ii) Area of the circle

(iii) Length of arc

(iv) The area of sector

Sol:

(i) Radius of circle (r) = 6 cm

Angle subtended at the centre = 110°

Circumference of the circle = 2πr

$= 2 \times \frac{22}{7} \times 6$

$= \frac{264}{7}\ cm$

(ii) Area of circle = $\pi r^2 = \frac{22}{7} \times 6 \times 6$

$= \frac{792}{7}\ cm^2$

(iii) Length of arc = $\frac{\theta}{360°} \times 2\pi r$

$= \frac{110}{360} \times 2 \times \frac{22}{7} \times 6$

$= \frac{232}{21}\ cm$

(iv) Area of sector = $\frac{\theta}{360°} \times \pi r^2$

$= \frac{110}{360} \times \frac{22}{7} \times 6 \times 6$

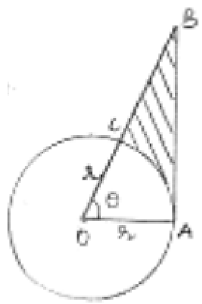
$= \frac{232}{7}\ cm^2$

19. Below fig shows a sector of a circle, centre O. containing an angle $\theta°$. Prove that

(i) Perimeter of shaded region is $r\left(\tan\ \theta + \sec\theta + \frac{\pi\theta}{180} - 1\right)$

(ii) Area of shaded region is $\frac{r^2}{2}\left(\tan\theta - \frac{\pi\theta}{180}\right)$

Sol:



Given angle subtended at centre of circle = θ

∠OAB = 90° [At joint of contact, tangent is perpendicular to radius]

OAB is right angle triangle

Cos $\theta = \frac{adj.side}{hypotenuse} = \frac{r}{OB} \Rightarrow OB = r\sec\theta \ldots\ldots(i)$

tan $\theta = \frac{opp.side}{adj.\ side} = \frac{AB}{r} \Rightarrow AB = r\tan\theta \ldots\ldots.(ii)$

Perimeter of shaded region = AB + BC + (CA arc)

$= r\tan\theta + (OB - OC) + \frac{\theta}{360°} \times 2\pi r$

$= r\tan\theta + r\sec\theta - r + \frac{\pi\,\theta\,r}{180°}$

$= r\left(\tan\theta + \sec\theta + \frac{\pi\theta}{180°} - 1\right)$

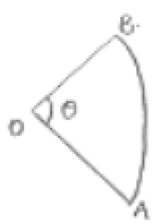
Area of shaded region = (area of triangle) – (area of sector)

$= \left(\frac{1}{2} \times OA \times AB\right) - \frac{\theta}{360°} \times \pi r^2$

$= \frac{1}{2} \times r \times r\tan\theta - \frac{r^2}{2}\left[\frac{\theta}{180°} \times \pi\right]$

$= \frac{r^2}{2}\left[\tan\theta - \frac{\pi\theta}{180}\right]$

20. The diagram shows a sector of circle of radius ‘r’ can containing an angle θ. The area of sector is A cm^2 and perimeter of sector is 50 cm. Prove that



(i) $\theta = \frac{360}{\pi}\left(\frac{25}{r} - 1\right)$

(ii) A = 25r – r^2

Sol:

(i) Radius of circle = ‘r’ cm

Angle subtended at centre = θ

Perimeter = OA + OB + (AB arc)

$= r + r + \frac{\theta}{360°} \times 2\pi r = 2r + 2r\left[\frac{\pi\theta}{360°}\right]$

But perimeter given as 50

$50 = 2r\left[1 + \frac{\pi\theta}{360°}\right]$

$\Rightarrow \frac{\pi\theta}{360°} = \frac{50}{2r} - 1$

$\Rightarrow \theta = \frac{360°}{\pi}\left[\frac{25}{r} - 1\right]$ .....(i)

(ii) Area of sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{\frac{360°}{\pi}\left(\frac{25}{r} - 1\right)}{360°} \times \pi r^2$

$= \frac{25}{r} \times r^2 - r^2$

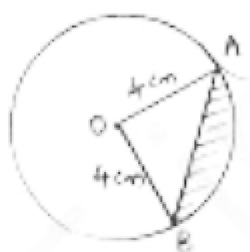
$= 25r - r^2$

$\Rightarrow A = 25r - r^2$ .....(ii)

Exercise 15.3

1. AB is a chord of a circle with centre O and radius 4cm. AB is length 4cm and divides circle into two segments. Find the area of minor segment

Sol:



Radius of circle r = 4cm = OA = OB

Length of chord AB = 4cm

OAB is equilateral triangle ∠AOB = 60° ⟶ θ

Angle subtended at centre θ = 60°

Area of segment (shaded region) = (area of sector) – (area of ΔAOB)

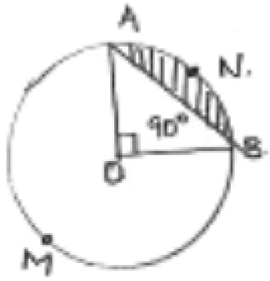
$= \frac{\theta}{360°} \times \pi r^2 = \frac{\sqrt{3}}{4}(side)^2$

$= \frac{60}{360} \times \frac{22}{7} \times 4 \times 4 = \frac{\sqrt{2}}{4} \times 4 \times 4$

$= \frac{176}{3} - 4\sqrt{3} = 58.67 - 6.92 = 51.75\ cm^2$

2. A chord of circle of radius 14cm makes a right angle at the centre. Find the areas of minor and major segments of the circle.

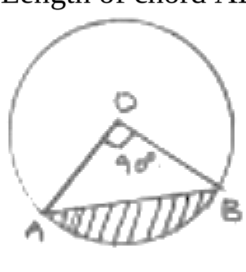
Sol:



Radius (r) = 14cm
θ = 90°
= OA = OB
Area of minor segment (ANB)
= $(area\ of\ ANB\ sector) - (area\ of\ \Delta AOB)$
= $\frac{\theta}{360°} \times \pi r^2 - \frac{1}{2} \times OA \times OB$
= $\frac{90}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{2} \times 14 \times 14$
= $154 - 98 = 56cm^2$
Area of major segment (other than shaded)
= area of circle – area of segment ANB
= $\pi r^2 - 56$
= $\frac{22}{7} \times 14 \times 14 - 56$
= 616 – 56
= 560 cm^2.

3. A chord 10 cm long is drawn in a circle whose radius is $5\sqrt{2}$ cm. Find the area of both segments

Sol:
Given radius = r = $5\sqrt{2}$ cm = OA = OB
Length of chord AB = 10cm



In ΔOAB, OA = OB = $5\sqrt{2}\ cm$ AB = 10cm
$OA^2 + OB^2 = \left(5\sqrt{2}\right)^2 + \left(5\sqrt{2}\right)^2 = 50 + 50 = 100 = (AB)^2$
Pythagoras theorem is satisfied OAB is right triangle
θ = angle subtended by chord = ∠AOB = 90°
Area of segment (minor) = shaded region
= $area\ of\ sector - area\ of\ \Delta OAB$

$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times OA \times OB$

$= \frac{90}{360} \times \frac{22}{7}\left(5\sqrt{2}\right)^2 - \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2}$

$= \frac{275}{7} - 25 - \frac{100}{7}\ cm^2$

Area of major segment = (area of circle) – (area of minor segment)

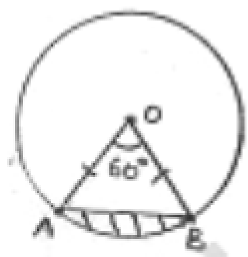
$= \pi r^2 2 - \frac{100}{7}$

$= \frac{22}{7} \times \left(5\sqrt{2}\right)^2 - \frac{100}{7}$

$= \frac{1100}{7} - \frac{100}{7} = \frac{1000}{7}\ cm^2$

4. A chord AB of circle, of radius 14cm makes an angle of 60° at the centre. Find the area of minor segment of circle.

Sol:



Given radius (r) = 14cm = OA = OB

θ = angle at centre = 60°

In ΔAOB, ∠A = ∠B [angles opposite to equal sides OA and OB] = x

By angle sum property ∠A + ∠B + ∠O = 180°

x + x + 60° = 180° ⇒ 2x = 120° ⇒ x = 60°

All angles are 60°, OAB is equilateral OA = OB = AB

Area of segment = area of sector – area Δle OAB

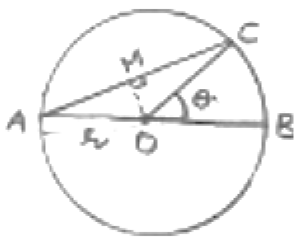
$= \frac{\theta}{360°} \times \pi r^2 - \frac{\sqrt{3}}{4} \times (-AB)^2$

$= \frac{60}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{\sqrt{3}}{4} \times 14 \times 14$

$= \frac{308}{3} - 49\sqrt{3} = \frac{308-147\sqrt{3}}{3}\ cm^2$

5. AB is the diameter of a circle, centre O. C is a point on the circumference such that ∠COB = θ. The area of the minor segment cutoff by AC is equal to twice the area of sector BOC. Prove that $\sin\frac{\theta}{2} . \cos\frac{\theta}{2} = \pi\left(\frac{1}{2} - \frac{\theta}{120°}\right)$

Sol:



Given AB is diameter of circle with centre O

$\angle COB = \theta$

Area of sector BOC = $\frac{\theta}{360^\circ} \times \pi r^2$

Area of segment cut off, by AC = (area of sector) – (area of ΔAOC)

$\angle AOC = 180 - \theta$ [∠AOC and ∠BOC form linear pair]

Area of sector $= \frac{(180-\theta)}{360^\circ} \times \pi r^2 = \frac{\pi r^2}{2} - \frac{\pi\theta r^2}{360^\circ}$

In ΔAOC, drop a perpendicular AM, this bisects ∠AOC and side AC.

Now, In ΔAMO, sin∠AOM = $\frac{AM}{DA} \Rightarrow \sin\left(\frac{180-\theta}{2}\right) = \frac{AM}{R}$

$\Rightarrow$ AM = R $\sin\left(90 - \frac{\theta}{2}\right) = R.\cos\frac{\theta}{2}$

cos ∠ADM = $\frac{OM}{OA} \Rightarrow \cos\left(90 - \frac{\theta}{2}\right) = \frac{OM}{Y} \Rightarrow OM = R.Sin\frac{\theta}{2}$

Area of segment = $\frac{\pi r^2}{2} - \frac{\pi\theta r^2}{360^\circ} - \frac{1}{2}(AC \times OM)$ $[AC = 2\ AM]$

$= \frac{\pi r^2}{2} - \frac{\pi\theta r^2}{360^\circ} - \frac{1}{2} \times \left(2\ R\cos\frac{\theta}{2}\ R\sin\frac{\theta}{2}\right)$

$= r^2\left[\frac{\pi}{2} - \frac{\pi\theta}{360^\circ} - \cos\frac{\theta}{2}\sin\frac{\theta}{2}\right]$

Area of segment by AC = 2 (Area of sector BDC)

$r^2\left[\frac{\pi}{2} - \frac{\pi\theta}{360^\circ} - \cos\frac{\theta}{2} . \sin\frac{\theta}{2}\right] = 2r^2\left[\frac{\pi\theta}{360^\circ}\right]$

$\cos\frac{\theta}{2} . \sin\frac{\theta}{2} = \frac{\pi}{2} - \frac{\pi\theta}{360} - \frac{2\pi\theta}{360^\circ}$

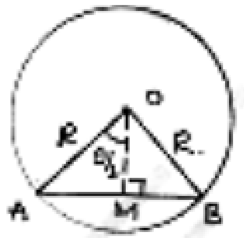
$= \frac{\pi}{2} - \frac{\pi\theta}{360^\circ}[1 + 2]$

$= \frac{\pi}{2} - \frac{\pi\theta}{360^\circ} = \pi\left(\frac{1}{2} - \frac{\theta}{120^\circ}\right)$

$\cos\frac{\theta}{2} . \sin\frac{\theta}{2} = \pi\left(\frac{1}{2} - \frac{\theta}{120^\circ}\right)$

6. A chord of a circle subtends an angle θ at the centre of circle. The area of the minor segment cut off by the chord is one eighth of the area of circle. Prove that 8 $\sin\frac{\theta}{2} . \cos\frac{\theta}{2} + \pi = \frac{\pi\theta}{45}$

Sol:



Let radius of circle = r

Area of circle = πr^2

AB is a chord, OA, OB are joined drop OM ⊥ AB. This OM bisects AB as well as ∠AOB.

$\angle AOM = \angle MOB = \frac{1}{2}(0) = \frac{\theta}{2}$ AB = 2AM

In ΔAOM, ∠AMO = 90°

$\text{Sin}\frac{\theta}{2} = \frac{AM}{AD} \Rightarrow AM = R.\sin\frac{\theta}{2}$ $AB = 2R\sin\frac{\theta}{2}$

$\text{Cos}\frac{\theta}{2} = \frac{OM}{AD} \Rightarrow OM = R\cos\frac{\theta}{2}$

Area of segment cut off by AB = (area of sector) – (area of triangles)

$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times AB \times OM$

$= r^2\left[\frac{\pi\theta}{360°} - \frac{1}{2}.2r sin\frac{\theta}{2} .R\cos\frac{\theta}{2}\right]$

$= R^2\left[\frac{\pi\theta}{360°} - \sin\frac{\theta}{2}.\cos\frac{\theta}{2}\right]$

Area of segment $= \frac{1}{2}(area\ of\ circle)$

$r^2\left[\frac{\pi\theta}{360} - \sin\frac{\theta}{2}.\cos\frac{\theta}{2}\right] = \frac{1}{8}\pi r^2$

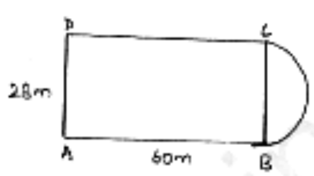
$\frac{8\pi\theta}{360°} - 8\sin\frac{\theta}{2}.\cos\frac{\theta}{2} = \pi$

$8\sin\frac{\theta}{2}.\cos\frac{\theta}{2} + \pi = \frac{\pi\theta}{45}$

Exercise 15.4

1. A plot is in the form of rectangle ABCD having semi-circle on BC. If AB = 60m and BC = 28m, find the area of plot.

Sol:



Given AB = 60m = DC [length]

BC = 28m = AD [breadth]

Radius of semicircle r = $\frac{1}{2} \times BC = 14m$

Area of semicircle r = $\frac{1}{2} \times BC = 14m$

Area of plot = (Area of rectangle ABCD) + (area of semicircle)

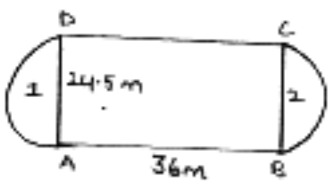
= $(length \ \times breadth) + \frac{1}{2}\pi r^2$

= $(60 \times 28) + \left[\frac{1}{2} \times \frac{22}{7} \times 14 \times 14\right]$

= $1680 + 308 = 1988m^2$

2. A playground has the shape of rectangle, with two semicircles on its smaller sides as diameters, added to its outside. If the sides of rectangle are 36m and 24.5m. find the area of playground.

Sol:



Let rectangular play area be ABCD

AB = CD = 36m [length]

AD = BC = 24.5 m [breadth]

Radius of the semicircle = $\frac{1}{2}(BC) = R$

= $\frac{1}{2} \times (24.5) = 12.25cm$

Area of playground = (Area of rectangle) + 2(Area of semicircle)

= $(AB \times BC) + \left(\frac{1}{2}\pi r^2\right)2$

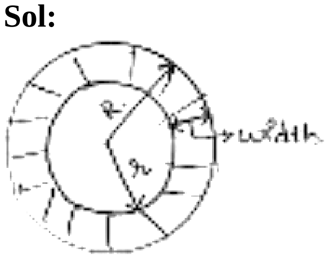
$= (36 \times 24.5) + \left(\frac{1}{2} \times \frac{22}{7} \times 12.25 \times 12.25\right)2$

$= 882 + 471.625$

$= 1353.625 \ m^2$

3. The outer circumference of a circular race track is 528m. The track is everywhere 14m wide. Calculate the cost of leveling the track at rate of 50 paise per square metre.

Sol:



Let inner radius = r width(d) = 14m

Outer radius = R

Outer circumference of track = 2 πr

∴ 2 $\pi r = 528$

$2 \times \frac{22}{7} \times R = 528 \Rightarrow R = \frac{528 \times 7}{2 \times 22} = 84\ m$

Inner radius r = R – d = 84 – 14 = 70m

Area of track = (area of outer circle) – (area of inner circles)

= $\pi R^2 - \pi r^2$

= $\pi(R^2 - r^2) = \frac{22}{7}(84^2 - 70^2)$

$= \frac{22}{7}(84 + 70)(84 - 70) = \frac{22}{7} \times 154 \times 14$

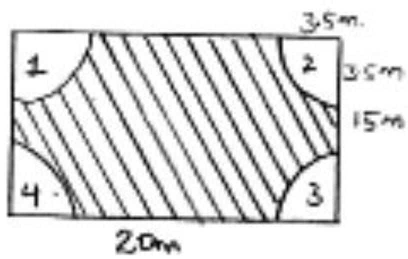
= $6776\ m^2$

Cost of leveling $m^2 = Rs. 0.50$

Total cost of leveling track = 6776 $\times \frac{1}{2} = Rs. 3388$

4. A rectangular piece is 20m long and 15m wide from its four corners, quadrants of 3.5m radius have been cut. Find the area of remaining part.

Sol:



Length of rectangular piece l = 20m

Breadth of rectangular piece b = 15m

Radius of each quadrant r = 3.5m

Area of rectangular piece = (length × breadth) = 20 × 15 = $300m^2$.

Area of quadrant each = $\frac{1}{4}(area\ of\ circle\ with\ radius\ 3.5m)$

= $\frac{1}{4} \times \pi r^2$

= $\frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 = \frac{38.5}{4} m^2$

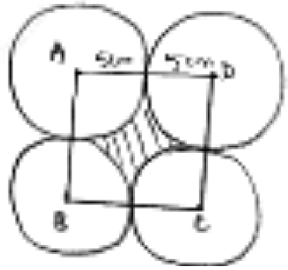
Area of remaining part = [area of rectangular piece] – 4[area of each quadrant]

= 300 – 4 [$\frac{385}{4}$] = 300 – 38.5

= $261.5m^2$

5. Four equal circles, each of radius 5 cm touch each other as shown in fig. Find the area included between them.

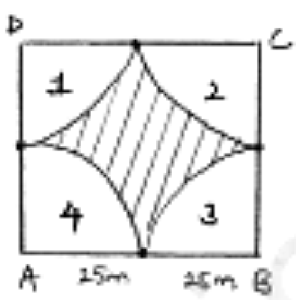
Sol:



Area required shaded = (area of square ABCD) – (Area of 4 quadrant)
Side of square = 5cm + 5cm
= 10cm
Area of square = side × side
= $10cm \times 10cm = 100cm^2$
Area of quadrant = $\frac{1}{4}(area\ of\ circle\ with\ radius\ 5\ cm)$
= $\frac{1}{4} \times \pi r^2$
= $\frac{1}{4} \times \frac{22}{7} \times 5 \times 5 = (25 \times 3.14)\frac{1}{4}\ cm^2$
Area included between circles = (area of square) – 4(area of quadrant)
= 100 – $\left(\frac{1}{4} \times 25 \times 2.14\right)$
= 100 – 78.5
= 21.5cm^2

6. Four cows are tethered at four corners of a square plot of side 50m, so that' they just cant reach one another. What area will be left ungrazed.

Sol:



Side of square plot (s) = 50m
Area grazed by four cows is area of sectors represented by 1, 2, 3 and 4.
Radius of each quadrant = 25m = r.
Area of square plot = $s^2 = 50^2 = 2500m^2$
Area of each quadrant = $\frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 25 \times 25 = (625 \times 3.14) \times \frac{1}{4}$
Area of ungrazed land = (area of square plot) – 4(area of quadrant)
= $2500 - 4\left(\frac{1}{4} \times 3.14 \times 625\right)$
= 2500 – 1962.5 = 537.5 m^2

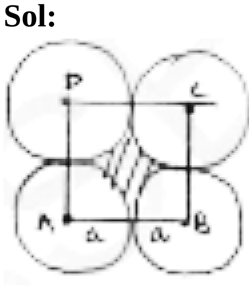
7. A road which is 7m wide surrounds a circular park whose circumference is 352m. Find the area of road.

Sol:

Outer radius of road = R
Inner radius of road = r
Width of park road = d
R = 2 + d
Circumference of road (outer) = 2πR
2πR = 352 [from problem given]
$2 \times \frac{22}{7} \times R = 352$
R = $\frac{352 \times 7}{2 \times 22} = 56m.$
Inner radius = R – d = 56 – 7 = 49 m
Area of road = (area of circle with radius 56m) – (area of circle with radius 49m)
= $\pi R^2 - \pi r^2$
= $\frac{22}{7}(56^2 - 49^2) = \frac{22}{7}(56 - 49)\ (56 + 49)$
= $\frac{22}{7} \times 7 \times 105 = 2310m^2$

8. Four equal circles each of radius a, touch each other. Show that area between them is $\frac{6}{7}a^2$

Sol:

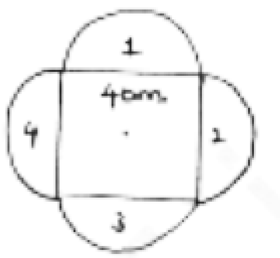


Let circles be with centres A, B, C, D
Join A, B, C and D then ABCD is square formed with side = (a + a) = 2a
Radius = a
Area between circles = area of square – 4(area of quadrant)
(shaded region)
= $(2a)^2 - 4\left(\frac{1}{4}\ area\ of\ circle\ with\ radius\ 'a'\right)$

= $4a^2 - 4\left(\frac{1}{4}\right) \times a^2$
= $a^2(4-\pi)$
= $a^2\left(4-\frac{22}{7}\right)$
= $\left(\frac{28-22}{7}\right)a^2 = \frac{6}{7}\ a^2$
∴ Area between circles = $\frac{6}{7}\ a^2$.

9. A square water tank has its side equal to 40m, there are 4 semicircular flower beds grassy plots all around it. Find the cost of turfing the plot at Rs 1.25/sq.m

Sol:



Side of water tank = 40m
Grassy plot is semicircular with radius = $\frac{side}{2} = \frac{40}{2} = 20m = r$
Area of grassy plot = 4(area od semicircular grassy plot with radius 20m)
= $4\left[\frac{1}{2}\ (area\ of\ circle\ with\ radius)\right]$
= $4 \times \frac{1}{2} \times \pi(20)^2$
= $2 \times 20 \times 20 \times \pi = 800\pi\ m^2$.
Cost of turfing $1m^2$ = Rs. 1.25
Total cost of turfing the grassy plot around tank
= $800\pi\ \times 1.25$
= 1000π
= 1000×3.14
= Rs. 3140.

Exercise 16.1

1. How many balls each of radius 1cm can be made from a solid sphere of lead of radius 8cm?

Sol:

Given that a solid sphere f radius $(r_1) = 8cm$

With this sphere we have to make spherical balls of radius $(r_2) = 1cm$

Since we don't know no of balls let us assume that no of balls formed be 'n'

We know that

$$\boxed{Volume\ of\ \ sphere = \frac{4}{3}\pi r^2}$$

Volume of solid sphere should be equal to sum of volumes of n spherical balls

$$n \times \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi r^3$$

$$n = \frac{\frac{4}{3}\pi(8)^3}{\frac{4}{3}\pi(1)^3}$$

$$n = 8^3$$

$$\boxed{n = 512}$$

$\therefore$ hence 512 no of balls can be made of radius 1cm from a solid sphere of radius $8cm$

2. How many spherical bullets each of 5cm in diameter can be cast from a rectangular block of metal $11dm \times 1m \times 5dm$?

Sol:

Given that a metallic block which is rectangular of diameter $11dm \times 1m \times 5dm$

Given that diameter of each bullet is 5cm

$$\boxed{Volume\ of\ \ sphere = \frac{4}{3}\pi r^2}$$

Dimensions of rectangular block = $11dm \times 1m \times 5dm$

Since we know that $1dm = 10^{-1}m$

$11 \times 10^{-1} \times 1 \times 5 \times 10^{-1} = 55 \times 10^{-2} m^3$ ....(1)

Diameter of each bullet $= 5cm$

Radius of bullet $(r) = \frac{d}{2} = \frac{5}{2} = 2 \cdot 5cm$

$= 25 \times 10^{-2} m$

So volume $=\frac{4}{3}\pi\left(25\times10^{-2}\right)^3$

Volume of rectangular block should be equal sum of volumes of n spherical bullets

Let no of bullets be ‘n’

Equating (1) and (2)

$55\times10^{-2}=n=\frac{4}{3}\pi\left(25\times10^{-2}\right)^3$

$$\frac{55\times10^{-2}}{\frac{4}{3}\times\frac{22}{7}\left(25\times10^{-2}\right)^3}=n$$

$n=8400$

$\boxed{\therefore\ No\ of\ bullets\ found\ were\ 8400}$

3. A spherical ball of radius 3cm is melted and recast into three spherical balls. The radii of the two of balls are $1\cdot9cm$ and $2cm$. Determine the diameter of the third ball?

Sol:

Given that a spherical ball of radius 3cm

We know that Volume of a sphere $=\frac{4}{3}\pi r^2$

So its volume $(v)=\frac{4}{3}\pi(3)^2$

Given that ball is melted and recast into three spherical balls

Radii of first ball $(v_1)=\frac{4}{3}\pi(1\cdot5)^3$

Radii of second ball $(v_2)=\frac{4}{3}\pi(2)^3$

Radii of third ball ____________?

Volume of third ball $=\frac{4}{3}\pi r^3=v_3$

Volume of spherical ball is equal to volume of 3 small spherical balls

$\Rightarrow\frac{4}{3}\pi r^2+\frac{4}{3}\pi(1\cdot5)^3+\frac{4}{3}\pi(2)^3=\frac{4}{3}\pi(3)^3$

$\Rightarrow r^2+(1\cdot5)^3+(2)^3=(3)^3$

$\Rightarrow r^3=3^3-1\cdot5^3-2^3$

$\Rightarrow r=(15\cdot6)\frac{1}{3}$

$\Rightarrow r=2\cdot5cm$

Diameter $(d) = 2r = 2\times 2\cdot 5 = 5cm$

$\boxed{\therefore\ Diameter\ of\ third\ ball\ = 5cm.}$

4. $2\cdot 2$ Cubic dm of grass is to be drawn into a cylinder wire $0\cdot 25cm$ in diameter. Find the length of wire?

Sol:

Given that $2\cdot 2dm^3$ of grass is to be drawn into a cylindrical wire $0.25cm$ in diameter

Given diameter of cylindrical wire $= 0.25cm$

Radius of wire $(r) = \frac{d}{2} = \frac{0\cdot 25}{2} = 0\cdot 125cm$

$= 0\cdot 125\times 10^{-2} m.$

We have to find length of wire?

Let length of wire be 'h' $(\because 1cm = 10^{-2} m)$

$\boxed{Volume\ of\ Cylinder = \pi r^2 h}$

Volume of brass of $2\cdot 2dm^3$ is equal to volume of cylindrical wire

$\frac{22}{7}\left(0\cdot 125\times 10^{-2}\right)h = 2\cdot 2\times 10^{-3}$

$\Rightarrow h = \frac{2\cdot 2\times 10^{-3}\times 7}{22\left(0\cdot 125\times 10^{-2}\right)^2}$

$\Rightarrow h = 448m$

$\boxed{\therefore\ Length\ of\ cylindrical\ wire = 448m}$

5. What length of a solid cylinder 2cm in diameter must be taken to recast into a hollow cylinder of length 16cm, external diameter 20cm and thickness $2\cdot 5mm$?

Sol:

Given that diameter of solid cylinder $= 2cm$

Given that solid cylinder is recast to hollow cylinder

Length of hollow cylinder $= 16cm$

External diameter = 20cm

Thickness $= 2\cdot 5mm = 0\cdot 25cm$

$\boxed{Volume\ of\ \ solid\ \ cylinder = \pi r^2 h}$

Radius of cylinder $= 1cm$

So volume of solid cylinder $= \pi (1)^2\ h$ ……(i)

Let length of solid cylinder be h

$\boxed{Volume\ of\ \ hollow\ cylinder = \pi h\left(R^2 - r^2\right)}$

Thickness $= R - r$

$0\cdot 25 = 10 - r$

$\Rightarrow$ Internal radius $= 9\cdot 75cm$

So volume of hollow cylinder $= \pi \times 16\left(100 - 95\cdot 0625\right)$ ….(2)

Volume of solid cylinder is equal to volume of hollow cylinder.

(1) = (2)

Equating equations (1) and (2)

$\pi\left(1\right)^2 h = \pi \times 16\left(100 - 95\cdot 06\right)$

$\frac{22}{7}\left(1\right)^2 \times h = \frac{22}{7} \times 16\left(4\cdot 94\right)$

$\boxed{h = 79\cdot 04cm}$

$\therefore$ Length of solid cylinder $= 79cm$

6. A cylindrical vessel having diameter equal to its height is full of water which is poured into two identical cylindrical vessels with diameter 42cm and height 21cm which are filled completely. Find the diameter of cylindrical vessel?

Sol:

Given that diameter is equal to height of a cylinder

So $h = 2r$

$\boxed{Volume\ of\ \ cylinder = \pi r^2 h}$

So volume $= \pi r^2\left(2r\right)$

$= 2\pi r^3$

Volume of each vessel $= \pi r^2 h$

Diameter $= 42cm$

Height $= 21cm$

Diameter $\left(d\right) = 2r$

$2r = 42$

$r = 21$

$\therefore$ Radius $= 21cm$

Volume of vessel $= \pi\left(21\right)^2 \times 21$ ……….(2)

Since volumes are equal

Equating (1) and (2)

$\Rightarrow 2\pi r^3 = \pi\left(21\right)^2 \times 21 \times 2$ ($\because$ 2 identical vessels)

$\Rightarrow r^3 = \frac{\pi(21)^2 \times 21 \times 2}{2 \times \pi}$

$\Rightarrow r^3 = (21)^3$

$\Rightarrow r = 21 \Rightarrow \boxed{d = 42cm}$

$\therefore$ Radius of cylindrical vessel $= 21cm$

Diameter of cylindrical vessel $= 42cm.$

7. 50 circular plates each of diameter 14cm and thickness $0 \cdot 5cm$ are placed one above other to form a right circular cylinder. Find its total surface area?

Sol:

Given that 50 circular plates each with diameter $= 14cm$

Radius of circular plates $(r) = 7cm$

Thickness of plates $= 0 \cdot 5$

Since these plates are placed one above other so total thickness of plates $= 0 \cdot 5 \times 50$ $= 25cm.$

$\boxed{Total\ surface\ area\ of\ a\, cylinder = 2\pi rh + 2\pi r^2}$

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 7(25 + 7)$

$\boxed{T.S.A = 1408cm^2}$

$\therefore$ Total surface area of circular plates is $1408cm^2$

8. 25 circular plates each of radius $10 \cdot 5cm$ and thickness $1 \cdot 6cm$ are placed one above the other to form a solid circular cylinder. Find the curved surface area and volume of cylinder so formed?

Sol:

Given that 25 circular plates each with radius (r) = 10.5cm

Thickness $= 1 \cdot 6cm$

Since plates are placed one above other so its height becomes $= 1 \cdot 6 \times 25 = 40cm$

$\boxed{Volume\ of\ cylinder = \pi r^2 h}$

$= \pi (10 \cdot 5)^2 \times 40$

$= 13860cm^3$

$\boxed{Curved\ surface\ area\ of\ a\ cylinder = 2\pi rh}$

$= 2 \times \pi \times 10 \cdot 5 \times 40$

$= 2 \times \frac{22}{7} \times 10 \cdot 5 \times 40$

$= 2640 cm^2$

$\therefore$ Volume of cylinder $= 13860 cm^3$

Curved surface area of a cylinder $= 2640 cm^2$

9. A path 2m wide surrounds a circular pond of diameter 4cm. how many cubic meters of gravel are required to grave the path to a depth of 20cm

Sol:

Diameter of circular pond $= 40m$

Radius of pond(r) $= 20m.$

Thickness $= 2m$

Depth $= 20cm = 0 \cdot 2m$

Since it is viewed as a hollow cylinder

$\boxed{Thickness\ (t) = R - r}$

$2 = R - r$

$2 = R - 20$

$R = 22m$

$\boxed{\therefore Volume\ of\ hollow\ cylinder = \pi\left(R^2 - r^2\right)h}$

$= \pi\left(22^2 - 20^2\right)h$

$= \pi\left(22^2 - 20^2\right) \times 0 \cdot 2$

$= \pi(84) \times 0.2$

$\boxed{\therefore Volume\ of\ hollow\ cylinder = 52 \cdot \pi m^3}$

$\therefore 52 \cdot 77 m^3$ of gravel is required to have path to a depth of 20cm.

10. A 16m deep well with diameter $3 \cdot 5m$ is dug up and the earth from it is spread evenly to form a platform $27 \cdot 5m$ by 7m. Find height of platform?

Sol:

Let as assume well is a solid right circular cylinder

Radius of cylinder $(r) = \frac{3 \cdot 5}{2} = 1 \cdot 75m$

Height (or) depth of well $= 16m.$

$\boxed{Volume\ of\ right\ circular\ cylinder = \pi r^2 h}$

$= \frac{22}{7} \times (1 \cdot 75)^2 \times 16$ ……..(1)

Given that length of platform $(l) = 27.5m$

Breath of platform $(b) = 7cm$

Let height of platform be xm

$\boxed{Volume\ of\ rec\tan gle = lbh}$

$= 27\cdot5 \times 7 \times x = 192\cdot5x$ ………(2)

Since well is spread evenly to form platform

So equating (1) and (2)

$V_1 = V_2$

$\Rightarrow \frac{22}{7}(1\cdot75)^2 \times 16 = 192\cdot5x$

$\Rightarrow x = 0\cdot8m$

$\boxed{\therefore\ Height\ of\ \ platform(h) = 80cm.}$

11. A well of diameter 2m is dug14m deep. The earth taken out of it is spread evenly all around it to form an embankment of height 40cm. Find width of the embankment?

Sol:

Let us assume well as a solid circular cylinder

Radius of circular cylinder $= \frac{2}{2} = 1m$

Height (or) depth of well $= 14m$

$\boxed{Volume\ of\ \ solid\ \ circular\ \ cylindeer = \pi r^2 h}$

$= \pi(1)^2 14$ …..(1)

Given that height of embankment (h) = 40cm

Let width of embankment be 'x' m

Volume of embankment $= \pi r^2 h$

$= \pi\left(\left(1 + x^2\right) - 1\right)^2 \times 0\cdot4$ …..(2)

Since well is spread evenly to form embankment so their volumes will be same so equating (1) and (2)

$\Rightarrow \pi(1)^2 \times 14 = \pi\left((1 + x)^2 - 1\right)^2 \times 0\cdot4$

$\Rightarrow x = 5m$

$\boxed{\therefore Width\ of\ \ embankment\ \ of\ (x) = 5m}$

12. Find the volume of the largest right circular cone that can be cut out of a cube where edge is 9cm_____?

Sol:

Given that side of cube =9cm

Given that largest cone is curved from cube

Diameter of base of cone = side of cube

$\Rightarrow 2x = 9$

$\Rightarrow r = \frac{9}{2} cm$

Height of cone = side of cube

$\Rightarrow$Height of cone (h) = 9cm

$$\boxed{Volume\ of\ l\arg est\ cone = \frac{1}{3}\pi r^2 h}$$

$= \frac{1}{3} \times \pi t \left(\frac{9}{2}\right)^2 \times 9$

$= \frac{\pi}{12} \times 9^3$

$= 190 \cdot 92 cm^3$

$\therefore$ Volume of largest cone $(v) = 190 \cdot 92 cm^3$

13. A cylindrical bucket, 32 cm high and 18cm of radius of the base, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Sol:

36cm, 43.27 cm

14. Rain water, which falls on a flat rectangular surface of length 6cm and breath 4m is transferred into a cylindrical vessel of internal radius 10cm. What will be the height of water in the cylindrical vessel if a rainfall of 1cm has fallen____?

Sol:

Given length of rectangular surface $= 6cm$

Breath of rectangular surface $= 4cm$

Height (h) $1cm$

$$\boxed{Volume\ of\ a\ flat\ rec\tan gular\ surface = lbh}$$

$= 6000 \times 400 \times 1$

Volume $= 240000 cm^3$ __________(1)

Given radius of cylindrical vessel $= 20cm$

Let height off cylindrical vessel be h_1

Since rains are transferred to cylindrical vessel.
So equating (1) with (2)

$$\boxed{Volume\ of\ cylindrical\ vessel = \pi r_1^2 h_1}$$

$$= \frac{22}{7}(20)^2 \times h_1 \qquad \_\_\_\_\_\_\_\_\_\_(2)$$

$$24000 = \frac{22}{7}(20)^2 \times h_1$$

$$\Rightarrow \boxed{h_1 = 190 \cdot 9cm}$$

$\therefore$ height of water in cylindrical vessel $= 190 \cdot 9cms$

15. A conical flask is full of water. The flask has base radius r and height h. the water is proved into a cylindrical flask off base radius one. Find the height of water in the cylindrical flask?

Sol:

Given base radius of conical flask be r
Height of conical flask is h

$$\boxed{Volume\ of\ cone = \frac{1}{3}\pi r^2 h}$$

So its volume $= \frac{1}{3}\pi r^2 h$ _________(1)

Given base radius of cylindrical flask is ms.
Let height of flask be h_1

$$\boxed{Volume\ of\ cylinder = \pi r^2 h_1}$$

So its volume $= \frac{22}{7}(mr)^2 h_1$ __________(2)

Since water in conical flask is poured in cylindrical flask their volumes are same
(1) = (2)

$$\Rightarrow \frac{1}{3}\pi r^2 h = \pi (mr)^2 \times h_1$$

$$\Rightarrow \boxed{h_1 = \frac{h}{3m^2}}$$

$\therefore$ Height of water in cylindrical flask $= \frac{h}{3m^2}$

16. A rectangular tank 15m long and 11m broad is required to receive entire liquid contents from a full cylindrical tank of internal diameter 21m and length 5m. Find least height of tank that will serve purpose______?

Sol:

Given length of rectangular tank = 15m
Breath of rectangular tank = 11m
Let height of rectangular tank be h

$$\boxed{Volume\ of\ rec\tan gular\ \tan k = lbh}$$

Volume $= 15 \times 11 \times h$ ______(1)

Given radius of cylindrical tank $(r) = \frac{21}{2} m$

Length/height of tank = 5m

$$\boxed{Volume\ of\ cylindrical\ \tan k = \pi r^2 h}$$

$= \pi \left(\frac{21}{2}\right)^2 \times 5$ _______(2)

Since volumes are equal
Equating (1) and (2)

$$15 \times 11 \times h = \pi \left(\frac{21}{2}\right)^2 \times 5$$

$$\Rightarrow h = \frac{\frac{22}{7} \times \left(\frac{21}{2}\right)^2 \times 5}{15 \times 11}$$

$$\Rightarrow \boxed{h = 10 \cdot 5m}$$

$\therefore$ Height of tank $= 10 \cdot 5m.$

17. A hemisphere tool of internal radius 9cm is full of liquid. This liquid is to be filled into cylindrical shaped small bottles each of diameter 3cm and height 4cm. how many bottles are necessary to empty the bowl.

Sol:

Given that internal radius of hemisphere bowl $= 90m$

$$\boxed{Volume\ of\ hemisphere = \frac{4}{3} \pi r^3}$$

$= \frac{2}{3} \times \pi (9)^3$ ______(1)

Given diameter of cylindrical bottle $= 3cm$

Radius $= \frac{3}{2} cm$

Height $= 4cm$

$$\boxed{Volume\ of\ cylindrical = \pi r^2 h}$$

$= \pi\left(\frac{3}{2}\right)^2 \times 4$ ______(2)

Volume of hemisphere bowl is equal to volume sum of n cylindrical bottles

(1) = (2)

$\frac{2}{3}\pi(9)^3 = \pi\left(\frac{3}{2}\right)^2 \times 4 \times n$

$\Rightarrow n = \frac{\frac{2}{3}\pi(9)^3}{\pi\left(\frac{3}{2}\right)^2 \times 4}$

$\Rightarrow \boxed{n = 54}$

$\therefore$ No of bottles necessary to empty the bottle $= 54$.

18. The diameters of the internal and external surfaces of a hollow spherical shell are 6 cm and 10 cm respectively. If it is melted and recast and recast into a solid cylinder of diameter 14 cm, find the height of the cylinder.

Sol:

Internal diameter of hollow spherical shell $= 6cm$

Internal radius of hollow spherical shell $= \frac{6}{3} = 3cm$

External diameter of hollow spherical shell $= 10\,cm$

External radius of hollow spherical shell $= \frac{10}{2} = 5\,cm$

Diameter of cylinder $= 14\ cm$

Radius of cylinder $= \frac{14}{2} = 7cm$

Let height of cylinder $= xcm$

According to the question

Volume of cylinder = Volume of spherical shell

$\Rightarrow \pi(7)^2 x \times = \frac{4}{3}\pi\left(5^3 - 3^3\right)$

$\Rightarrow 49x \times = \frac{4}{3}(125 - 27)$

$\Rightarrow 49x \times = \frac{4}{3} \times 98$

$x = \frac{4 \times 98}{3 \times 49} = \frac{8}{3}cm$

$\therefore$ Height off cylinder $=\frac{8}{3}cm$

19. A hollow sphere of internal and external diameter 4cm and 8cm is melted into a cone of base diameter 8cm. Calculate height of cone?

Sol:

Given internal diameter of hollow sphere (r) $=4cm$

External diameter $(R)=8cm$

$$\boxed{Volume\ of\ \ hollow\ sphere=\frac{4}{3}\pi\left(R^2-r^2\right)}$$

$$=\frac{4}{3}\pi\left(B^2-4^2\right) \qquad \_\_\_\_(1)$$

Given diameter of cone $=8cm$

Radius of cone $=4cm$

Let height of cone be h

$$\boxed{Volume\ of\ \ cone=\frac{1}{3}\pi r^2 h}$$

$$=\frac{1}{3}\times\pi(4)^2 h \qquad \_\_\_\_(2)$$

Since hollow sphere is melted into a cone so there volumes are equal

(1) = (2)

$$\Rightarrow\frac{4}{3}\pi(64-16)=\frac{1}{3}\pi(4)^2 h$$

$$\Rightarrow\frac{\frac{4}{3}\pi(48)}{\frac{1}{3}\pi(16)}=h$$

$$\Rightarrow\boxed{h=12cm}$$

$\therefore$ Height of cone $=12cm$

20. A cylindrical tube of radius 12cm contains water to a depth of 20cm. A spherical ball is dropped into the tube and the level of the water is raised by $6\cdot 75cm.$ Find the radius of the ball___?

Sol:

Given that radius of a cylindrical tube (r) $=12cm$

Level of water raised in tube $(h)=6\cdot 75cm$

$$\boxed{Volume\ of\ \ cylinder=\pi r^2 h}$$

$= \pi(12)^2 \times 6\cdot 75cm^3$

$= \frac{22}{7}(12)^2\, 6\cdot 25cm^3$ ………(1)

Let ‘r’ be radius of a spherical ball

$\boxed{Volume\ of\ \ sphere = \frac{4}{3}\pi r^3}$ ………(2)

To find radius of spherical balls

Equating (1) and (2)

$\pi \times (12)^2 \times 6\cdot 75 = \frac{4}{3}\pi r^3$

$r^3 = \frac{\pi \times (12)^2 \times 6\cdot 75}{\frac{4}{3}\times \pi}$

$r^3 = 729$

$r^3 = 9^3$

$\boxed{r = 9cm}$

$\therefore$ Radius of spherical ball $(r) = 9cm$

21. 500 persons have to dip in a rectangular tank which is 80m long and 50m broad. What is the rise in the level of water in the tank, if the average displacement of water by a person is $0\cdot 04m^3$ ____?

Sol:

Given that length of a rectangular tank (r) = 80m

Breath of a rectangular tank (b) $= 50m$

Total displacement of water in rectangular tank

By 500 persons $= 500 \times 0\cdot 04m^3$

$= 20m^3$ ______(1)

Let depth of rectangular tank be h

$\boxed{Volume\ of\ \ rectangular\ \ \tan k = lbh}$

$= 80 \times 50 \times hm^3$ ______(2)

Equating (1) and (2)

$\Rightarrow 20 = 80 \times 50 \times h$

$\Rightarrow 20 = 4000h$

$\Rightarrow \frac{20}{4000} = h$

$\Rightarrow h = 0\cdot 005m$

$\boxed{h = 0\cdot 5cm}$

$\therefore$ Rise in level of water in tank $(h) = 0\cdot 05cm.$

22. A cylindrical jar of radius 6cm contains oil. Iron sphere each of radius $1\cdot 5cm$ are immersed in the oil. How many spheres are necessary to raise level of the oil by two centimetress?

Sol:

Given that radius of a cylindrical jar $(r) = 6cm$

Depth/height of cylindrical jar $(h) = 2cm$

Let no of balls be 'n'

$\boxed{Volume\ of\ a\ cylinder = \pi r^2 h}$

$$V_1 = \frac{22}{7} \times (6)^2 \times 2cm^3 \qquad \ldots\ldots\ldots(1)$$

Radius of sphere $1\cdot 5cm$

$\boxed{So\ volume\ of\ sphere = \frac{4}{3}\pi r^3}$

$$V_2 = \frac{4}{3} \times \frac{22}{7}(1\cdot 5)^3\ cm^3 \qquad \ldots\ldots\ldots(2)$$

Volume of cylindrical jar is equal to sum of volume of n spheres

Equating (1) and (2)

$$\frac{22}{7} \times (6)^2 \times 2 = n \times \frac{4}{3} \times \frac{22}{4}(1\cdot 5)^3$$

$$n = \frac{v_1}{v_2} \Rightarrow n = \frac{\frac{22}{7} \times (6)^2 \times 2}{\frac{4}{3} \times \frac{22}{7}(1\cdot 5)^3}$$

$\boxed{n = 16}$

$\therefore$ No of spherical balls $(n) = 16$

23. A hollow sphere of internal and external radii 2cm and 4cm is melted into a cone of basse radius 4cm. find the height and slant height of the cone______?

Sol:

Given that internal radii of hollow sphere (r) = 2cm

External radii of hollow sphere $(R) = 4cm$

$\boxed{Volume\ of\ hollow\ sphere = \frac{4}{3}\pi\left(R^2 - r^2\right)}$

$v_1 = \frac{4}{3} \times \pi\left(4^2 - 2^2\right)$ ....(1)

Given that sphere is melted into a cone

Base radius of cone $= 4cm$

Let slant height of cone be l

Let height of cone be h

$l^2 = r^2 + h^2$

$l^2 = 16 + h^2$ ....(3)

$\boxed{Volume\ of\ \ cone = \frac{1}{3}\pi r^2 h}$

$v_2 = \frac{1}{3}\pi(4)^2 h$ ....(2)

$v_1 = v_2$ Equating (1) and (2)

$\frac{4}{3}\pi\left(4^2 - 2^2\right) = \frac{1}{3}\pi(4)^2 h$

$\frac{\frac{4}{3}\pi(16-4)}{\frac{1}{3}\pi(16)} = h$

$h = 14cm$

Substituting 'h' value in (2)

$l^2 = 16 + h^2$

$l^2 = 16 + 14^2$

$l^2 = 16 + 196$

$\boxed{l = 14 \cdot 56cm}$

$\therefore$ Slant height of cone $= 14 \cdot 56cm$

24. The internal and external diameters of a hollow hemisphere vessel are $21cm$ and $25 \cdot 2cm.$ The cost of painting $1cm^2$ of the surface is 10paise. Find total cost to paint the vessel all over______?

Sol:

Given that internal diameter of hollow hemisphere $(r) = \frac{21}{2}cm = 10 \cdot 5cm$

External diameter $(R) = \frac{25 \cdot 2}{2} = 12 \cdot 6cm$

Total surface area of hollow hemisphere

$= 2\pi R^2 + 2\pi r^2 + \pi\left(R^2 - r^2\right)$

$= 2\pi(12\cdot6)^2 + 2\pi(10\cdot5)^2 + \pi(12\cdot6^2 - 10\cdot5^2)$

$= 997\cdot51 + 692\cdot72 + 152\cdot39$

$= 1843\cdot38 cm^2$

Given that cost of painting $1cm^2$ of surface $= 10\,ps$

Total cost for painting $1843\cdot38 cm^2$

$= 1843\cdot38 \times 10\,ps$

$= 184\cdot338\, Rs.$

$\therefore$ Total cot to paint vessel all over $= 184\cdot338\, Rs.$

25. A cylindrical tube of radius 12cm contains water to a depth of 20cm. A spherical ball of radius 9cm is dropped into the tube and thus level of water is raised by hcm. What is the value of h_____?

Sol:

Given that radius of cylindrical tube $(r_1) = 12cm$

Let height of cylindrical tube (h)

$\boxed{Volume\ of\ a\ cylinder = \pi r_1^2 h}$

$v_1 = \pi(12)^2 \times h$ ……(1)

Given spherical ball radius $(r_2) = 9cm$

$\boxed{Volume\ of\ sphere = \frac{4}{3}\pi r_2^3}$

$v_2 = \frac{4}{3} \times \pi \times 9^3$ ……(2)

Equating (1) and (2)

$v_1 = v_2$

$\pi(12)^2 \times h = \frac{4}{3} \times \pi \times 9^3$

$h = \frac{\frac{4}{3} \times \pi \times 9^3}{\pi(12)^2}$

$h = 6\cdot75cm$

Level of water raised in tube (h) $= 6\cdot75cm$

26. The difference between outer and inner curved surface areas of a hollow right circular cylinder 14cm long is $88cm^2$. If the volume of metal used in making cylinder is $176cm^3$. find the outer and inner diameters of the cylinder____?

Sol:

Given height of a hollow cylinder = 14cm
Let internal and external radii of hollow
Cylinder be 'r' and R
Given that difference between inner and outer
Curved surface $= 88cm^2$
Curved surface area of cylinder (hollow)
$= 2\pi(R-r)h\ cm^2$
$\Rightarrow 88 = 2\pi(R-r)h$
$\Rightarrow 88 = 2\pi(R-r)14$
$\Rightarrow R-r = 1$ ……(1)
Volume of cylinder (hollow) $= \pi(R^2-r^2)h\ cm^3$
Given volume of a cylinder $= 176cm^3$
$\Rightarrow \pi(R^2-r^2)h = 176$
$\Rightarrow \pi(R^2-r^2)\times 14 = 176$
$\Rightarrow R^2-r^2 = 4$
$\Rightarrow (R+r)(R-r) = 4$
$\Rightarrow R+r = 4$ ……(2)
$R-r = 1$
$R+r = 4$
$2R \quad = 5$
$2R = 5 \Rightarrow \boxed{R = \frac{5}{2} = 2\cdot 5cm}$
Substituting 'R' value in (1)
$\Rightarrow R-r = 1$
$\Rightarrow 2\cdot 5 - r = 1$
$\Rightarrow 2\cdot 5 - 1 = r$
$\Rightarrow \boxed{r = 1\cdot 5cm}$
$\therefore$ Internal radii of hollow cylinder $= 1\cdot 5cm$
External radii of hollow cylinder $= 2\cdot 5cm$

27. Prove that the surface area of a sphere is equal to the curved surface area of the circumference cylinder__?
Sol:
Let radius of a sphere be r
$\boxed{Curved\ surface\ area\ of\ sphere = 4\pi r^2}$

$S_1 = 4\pi r^2$

Let radius of cylinder be $'r'cm$

Height of cylinder be $'2r'cm$

$\boxed{Curved\ surface\ area\ of\ cylinder = 2\pi rh}$

$S_2 = 2\pi r(2r) = 4\pi r^2$

S_1 and S_2 are equal. Hence proved

So curved surface area of sphere = surface area of cylinder

28. The diameter of a metallic sphere is equal to 9cm. it is melted and drawn into a long wire of diameter 2mm having uniform cross-section. Find the length of the wire?

Sol:

Given diameter of a sphere (d) $= 9cm$

Radius (r) $= \frac{9}{2} = 4\cdot 5cm$

$\boxed{Volume\ of\ a\ sphere = \frac{4}{3}\pi r^3}$

$V_1 = \frac{4}{3} \times \pi \times 4\cdot 5^3 = 381\cdot 70cm^3$ ......(1)

Since metallic sphere is melted and made into a cylindrical wire

$\boxed{Volume\ of\ a\ cylinder = \pi r^2 h}$

Given radius of cylindrical wire $(r) = \frac{2mm}{2}$

$= 1mm = 0\cdot 1cm$

$V_2 = \pi(0\cdot 1)^2 h$ ......(2)

Equating (1) and (2)

$V_1 = V_2$

$\Rightarrow 381\cdot 703 = \pi(0\cdot 1)^2 h$

$\Rightarrow \boxed{h = 12150cm}$

$\therefore$ Length of wire (h) $= 12150cm$

29. An iron spherical ball has been melted and recast into smaller balls of equal size. If the radius of each of the smaller balls is $\frac{1}{4}$ of the radius of the original ball, how many such balls are made? Compare the surface area, of all the smaller balls combined together with that of the original ball.

Sol:

Given that radius of each of smaller ball $= \frac{1}{4}$ Radius of original ball.

Let radius of smaller ball be r.

Radius of bigger ball be $4r$

Volume of big spherical ball $= \frac{4}{3}\pi r^3 \quad (\because r = 4r)$

$V_1 = \frac{4}{3}\pi(4r)^3$ ……(1)

$\boxed{Volume\ of\ each\ small\ ball = \frac{4}{3}\pi r^3}$

$V_2 = \frac{4}{3}\pi r^3$ ……(2)

Let no of balls be $'n'$

$n = \frac{V_1}{V_2}$

$\Rightarrow n = \frac{\frac{4}{3}\pi(4r)^3}{\frac{4}{3}\pi(r)^3}$

$\Rightarrow n = 4^3 = 64$

$\boxed{\therefore No\ of\ small\ balls = 64}$

Curved surface area of sphere $= 4\pi r^2$

Surface area of big ball $(S_1) = 4\pi(4r)^2$ …….(3)

Surface area of each small ball $(S_1) = 4\pi r^2$

Total surface area of 64 small balls

$(S_2) = 64 \times 4\pi r^2$ …….(4)

By combining (3) and (4)

$\Rightarrow \frac{S_2}{3} = 4$

$\Rightarrow \boxed{S_2 = 4s}$

$\therefore$ Total surface area of small balls is equal to 4 times surface area of big ball.

30. A tent of height 77dm is in the form a right circular cylinder of diameter 36m and height 44dm surmounted by a right circular cone. Find the cost of canvas at Rs.3.50 per m^2 ?

Sol:

Given that height of a tent = 77dm

Height of cone $= 44dm$

Height of a tent without cone $=77-44=33dm$
$=3\cdot 3m$
Given diameter of cylinder (d) = 36m
Radius $(r)=\frac{36}{2}=18m$
Let 'l' be slant height of cone
$l^2=r^2+h^2$
$l^2=18^2+3\cdot 3^2$
$l^2=324+10\cdot 89$
$l^2=334\cdot 89$
$l=18\cdot 3$
Slant height of cone $l=18\cdot 3$
Curved surface area of cylinder $(S_1)=2\pi rh$
$=2\times\pi\times 18\times 4\cdot 4m^2$ …….(1)
Curved surface area of cone $(S_2)=\pi rl$
$=\pi\times 18\times 18\cdot 3m^2$ …….(2)
Total curved surface of tent $=S_1+S_2$
T.C.S.A $=S_1+S_2$
$=1532\cdot 46m^2$
Given cost canvas per $m^2=Rs\ 3\cdot 50$
Total cost of canvas per $1532\cdot 46\times 3\cdot 50$
$=1532\cdot 46\times 3\cdot 50$
$=5363\cdot 61$
$\therefore$ Total cost of canvas $=Rs\ 5363.61$

31. Metal spheres each of radius 2cm are packed into a rectangular box of internal dimension $16cm\times 8cm\times 8cm$ when 16 spheres are packed the box is filled with preservative liquid. Find volume of this liquid?

Sol:
Given radius of metal spheres $=2cm$

$$\boxed{Volume\ of\ sphere(v)=\frac{4}{3}\pi r^3}$$

So volume of each metallic sphere $=\frac{4}{3}\pi(2)^3\ cm^3$
Total volume of 16 spheres $(v_1)=16\times\frac{4}{3}\pi(2)^3\ cm^3$ …(1)
Volume of rectangular box $=lbh$

$V_2 = 16\times 8\times 8 cm^3$ …(2)

Subtracting (2) – (1) we get volume of liquid

$\Rightarrow V_2 - V_1 =$ Volume off liquid

$\Rightarrow 16\times 8\times 8 - \frac{4}{3}\pi(2)^3 \times 16$

$\Rightarrow 1024 - 536\cdot 16 = 488 cm^3$

$\therefore$ Hence volume of liquid $= 488 cm^3$

32. The largest sphere is to be curved out of a right circular of radius 7cm and height 14cm. find volume of sphere?

Sol:

Given radius of cylinder $(r) = 7cm$

Height of cylinder $(h) = 14cm$

Largest sphere is curved out from cylinder

Thus diameter of sphere = diameter of cylinder

Diameter of sphere $(d) = 2\times 7 = 14cm$

Volume of a sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\times\pi(7)^3$

$= \frac{1372\pi}{3}$

$= 1436\cdot 75 cm^3$

$\therefore$ Volume of sphere $= 1436\cdot 75 cm^3$

33. A copper sphere of radius 3cm is melted and recast into a right circular cone of height 3cm. find radius of base of cone?

Sol:

Given radius of sphere $= 3cm$

Volume of a sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\times\pi\times 3^3 cm^3$ …..(1)

Given sphere is melted and recast into a right circular cone

Given height of circular cone $= 3cm.$

Volume of right circular cone $= \pi r^2 h \times \frac{1}{3}$

$= \frac{\pi}{3}(r)^2 \times 3cm^2$ .....(1)

Equating 1 and 2 we get

$\frac{4}{3}\pi \times 3^3 = \frac{1}{3}\pi(r)^2 \times 3$

$r^2 = \frac{\frac{4}{3}\pi \times 3^3}{\pi}$

$r^2 = 36cm$

$\boxed{r = 6cm}$

$\therefore$ Radius of base of cone $(r) = 6cm$

34. A vessel in the shape of cuboid contains some water. If these identical spheres are immersed in the water, the level of water is increased by 2cm. if the area of base of cuboid is $160cm^2$ and its height 12cm, determine radius of any of spheres?

Sol:

Given that area of cuboid $= 160cm^2$

Level of water increased in vessel $= 2cm$

Volume of a vessel $= 160 \times 2cm^3$ ......(1)

Volume of each sphere $= \frac{4}{3}\pi r^3 cm^3$

Total volume of 3 spheres $= 3 \times \frac{4}{3}\pi r^3 cm^3$ ......(2)

Equating (1) and (2) ($\because$ Volumes are equal $V_1 = V_2$)

$160 \times 2 = 3 \times \frac{4}{3}\pi r^3$

$r^3 = \frac{160 \times 2}{3 \times \frac{4}{3}\pi}$

$r^3 = \frac{320}{4\pi}$

$\boxed{r = 2 \cdot 94cm}$

$\therefore$ Radius of sphere $= 2 \cdot 94cm$

35. A copper rod of diameter 1cm and length 8cm is drawn into a wire of length 18m of uniform thickness. Find thickness of wire?

Sol:

Given diameter of copper rod $(d_1) = 1cm$

Radius $(r_1) = \frac{1}{2} = 0\cdot 5cm$

Length of copper rod $(h_1) = 8cm$

$\boxed{Volume\ of\ \ cylinder = \pi r_1^2 h_1}$

$V_1 = \pi(0\cdot 5)^2 \times 8cm^3$ ……(1)

$V_2 = \pi r_2^2 h_2$

Length of wire $(h_2) = 18m = 1800cm$

$V_2 = \pi r_2^2 (1800) cm^3$ ..….(2)

Equating (1) and (2)

$V_1 = V_2$

$\pi(0\cdot 5)^2 \times 8 = \pi r_2^2 (1800)$

$\frac{\pi(0\cdot 5)^2 \times 8}{\pi(1800)} = r_2^2$

$\boxed{r_2 = 0\cdot 033cm}$

$\therefore$ Radius thickness of wire $= 0\cdot 033cm.$

36. The diameters of internal and external surfaces of hollow spherical shell are 10cm and 6cm respectively. If it is melted and recast into a solid cylinder of length of $2\frac{2}{3}$ cm, find the diameter of the cylinder.

Sol:

Given diameter of internal surfaces of a hollow spherical shell $= 10cm$

Radius $(r) = \frac{10}{2} = 5cm.$

External radii $(R) = \frac{6}{2} = 3cm$

$\boxed{Volume\ of\ \ a\ \ spherica\ \ shell\ (hollow) = \frac{4}{3}\pi\left(R^2 - r^2\right)}$

$V_1 = \frac{4}{3}\pi\left(5^2 - 3^2\right)cm^3$ ………(1)

Given length of solid cylinder $(h) = \frac{8}{3}$

Let radius of solid cylinder be $'r'$

$\boxed{Volume\ of\ \ a\ cylinder = \pi r^2 h}$

$V_2 = \pi r^2 \left(\frac{8}{3}\right) cm^3$ ....(2)

$V_1 = V_2$

Equating (1) and (2)

$\Rightarrow \frac{4}{3}\pi(25-9) = \pi r^2 \left(\frac{8}{3}\right)$

$\Rightarrow \frac{\frac{4}{3}\pi(16)}{\pi\left(\frac{8}{3}\right)} = r^2$

$\Rightarrow r^2 = 49cm$

$\Rightarrow r = 7cm$

$d = 2r = 14cm$

$\boxed{\therefore Diameter\ \ of\ \ cylinder\ = 14cm}$

37. A right angled triangle whose sides are 3 cm, 4 cm and 5 cm is revolved about the sides containing the right angle in two days. Find the difference in volumes of the two cones so formed. Also, find their curved surfaces.

Sol:

(i) Given that radius of cone $(r_1) = 4cm$

Height of cone $(h_1) = 3cm$

Slant height of cone $(l_1) = 5cm$

Volume of cone $(V_1) = \frac{1}{3}\pi r_1^2 h_1$

$= \frac{1}{3}\pi(4)^2(3) = 16\pi cm^3$

(ii) Given radius of second cone $(r_2) = 3cm$

Height of cone $(h_2) = 4cm$

Slant height of cone $(l_2) = 5cm$

Volume of cone $(V_2) = \frac{1}{3} r_2^2 h_2$

$= \frac{1}{3}\pi(3)^2(4) = 12\pi cm^3$

Difference in volumes of two cones $(V) = V_1 - V_2$

$V = 16\pi - 12\pi$

$V = 4\pi cm^3$

Curved surface area of first cone $(S_1) = \pi r_1 l_1$

$S_1 = \pi(4)(5) = 20\pi cm^2$

Curved surface area of first cone $(S_1) = \pi r_1 l_1$

$S_1 = \pi(4)(5) = 20\pi cm^2$

Curved surface area of second cone $(S_2) = \pi r_2 l_2$

$S_1 = \pi(3)(5) = 15\pi cm^2$

$S_1 = 20\pi cm^2 S_2 = 15\pi cm^2$

38. How many coins $1 \cdot 75cm$ in diameter and 2mm thick must be melted to form a cuboid $11cm \times 10cm \times 75cm$ ___?

Sol:

Given that dimensions of a cuboid $11cm \times 10cm \times 75cm$

So its volume $(V_1) = 11cm \times 10cm \times 7cm$

$= 11 \times 10 \times 7cm^3$ …….(1)

Given diameter (d) $= 1 \cdot 75cm$

Radius $(r) = \frac{d}{2} = \frac{1 \cdot 75}{2} = 0 \cdot 875cm$

Thickness $(h) = 2mm = 0 \cdot 2cm$

$\boxed{Volume \quad of \;\; a\, cylinder = \pi r^2 h}$

$V_2 = \pi(0 \cdot 875)^2 (0 \cdot 2) cm^3$ …….(2)

$V_1 = V_2 \times n$

Since volume of a cuboid is equal to sum of n volume of 'n' coins

$n = \frac{V_1}{V_2}$

$n = no$ of coins

$n = \frac{11 \times 10 \times 7}{\pi(0 \cdot 875)^2 (0 \cdot 2)}$

$\boxed{n = 1600}$

$\therefore$ No of coins $(n) = 1600$,

39. A well with inner radius 4m is dug 14m deep earth taken out of it has been spread evenly all around a width of 3m it to form an embankment. Find the height of the embankment?

Sol:

Given that inner radius of a well $(a) = 4m$

Depth of a well $(h) = 14m$

$\boxed{Volume\ of\ a\ cylinder = \pi r^2 h}$

$V_1 = \pi (4)^2 \times 14 cm^3$ ……….(1)

Given well is spread evenly to form an embankment

Width of an embankment $= 3m$

Outer radii of a well $(R) = 4 + 3 = 7m.$

$\boxed{Volume\ of\ a\ hollow\ cylinder = \pi \left(R^2 - r^2\right) \times hm^3}$

$V_2 = \pi \left(7^2 - 4^2\right) \times hm^3$ ………..(2)

Equating (1) and (2)

$V_1 = V_2$

$\Rightarrow \pi (4)^2 \times 14 = \pi (49 - 16) \times h$

$\Rightarrow h = \frac{\pi (4)^2 \times 14}{\pi (33)}$

$\boxed{h = 6 \cdot 78m}$

40. Water in a canal $1.5m$ wide and 6m deep is flowering with a speed of $10km/hr.$ how much area will it irrigate in 30 minutes if 8cm of standing water is desired?

Sol:

Given that water is flowering with a speed $= 10km/hr$

In 30 minutes length of flowering standing water $= 10 \times \frac{30}{60} km$

$= 5km = 5000m.$

Volume of flowering water in 30 minutes

$V = 5000 \times width \times depth\ m^3$

Given width of canal $= 1 \cdot 5m$

Depth of canal $= 6m$

$V = 5000 \times 1 \cdot 5 \times 6m^3$

$\boxed{V = 45000m^3}$

Irrigating area in 30 minutes if 8cm of standing water is desired $= \frac{45000}{0 \cdot 08}$

$= \frac{45000}{0 \cdot 08} = 562500 m^2$

$\boxed{\therefore \textit{Irrigated area in } 30 \text{ minutes} = 562500 m^2}$

41. A farmer runs a pipe of internal diameter 20 cm from the canal into a cylindrical tank in his field which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Sol:

$\frac{9}{8}\ m$

42. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it to a width of 4 m to form an embankment. Find the height of the embankment.

Sol:

Given diameter of well = 3m

Radius of well $= \frac{3}{2} m = 4$

Depth of well $(b) = 14m$

With of embankment $= 4m$

$\therefore$ Radius of outer surface of embankment $= 4 + \frac{3}{2} = \frac{11}{2} m$

Let height of embankment $= hm$

$\boxed{\textit{Volume of embankment}\left(V_1\right) = \pi\left(r_2^2 - r_1^2\right)h}$

($\because$ it is viewed as a hollow cylinder)

$V_1 = \pi\left(\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)\right)^2 \times h - m^3$ …..(1)

$\boxed{\textit{Volume of earth dugout } \left(V_2\right) = \pi r_1^2 h}$

$V_2 = \pi\left(\frac{3}{2}\right)^2 \times 14\ m^3$ …..(2)

Given that volumes (1) and (2) are equal

So $V_1 = V_2$

$\Rightarrow \left(\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right) \times h = \pi\left(\frac{3}{2}\right)^2 \times 14$

$\Rightarrow \left(\frac{121}{4} - \frac{9}{4}\right)h = \frac{9}{4} \times 14$

$\Rightarrow \boxed{h = \frac{9}{8}m}$

$\therefore$ Height of embankment $(h) = \frac{9}{8}m.$

43. The surface area of a solid metallic sphere is 616 cm2. It is melted and recast into a cone of height 28 cm. Find the diameter of the base of the cone so formed (Use it = $\frac{22}{7}$)

Sol:

Given height of cone $(h) = 28cm$

Given surface area of Sphere $= 616cm^2$

We know surface area of sphere $= 4\pi r^2$

$\Rightarrow 4\pi r^2 = 616$

$\Rightarrow r^2 = \frac{616 \times 7}{4 \times 22}$

$\Rightarrow r^2 = 49$

$\Rightarrow \boxed{r = 7cm}$

$\therefore$ Radius of sphere $(r) = 7cm$

Let r_1 be radius of cone

Given volume of cone $=$ Volume of sphere

$\boxed{Volume\ of\ cone = \frac{1}{3}\pi(r^2)h}$

$V_1 = \frac{1}{3}\pi(r_1)^2 \times 28cm^3$ ………..(1)

$\boxed{Volume\ of\ sphere = (V_2) = \frac{4}{3}\pi r^3}$

$V_2 = \frac{4}{3}\pi(7)^3\ cm^3$ ………..(1)

(1) = (2) $\Rightarrow V_1 = V_2$

$\Rightarrow \frac{1}{3}\pi(r_1)^2 \times 28 = \frac{4}{3}\pi(7)^3$

$\Rightarrow r_1^2 = 49$

$r_1 = 7cm$

Radius of cone $(r_1) = 7cm$

$\boxed{Diameter\ of\ base\ of\ cone(d_1) = 2 \times 7 = 14cm}$

44. The difference between the outer and inner curved surface areas of a hollow right circular cylinder 14cm long is $88cm^2$. If the volume of metal used in making cylinder is $176cm^3$ find outer and inner diameters of the cylinder?

Sol:

Given height of a hollow cylinder $=14cm$

Let internal and external radii of hollow

Cylinder be 'r' an 'R'

Given that difference between inner and outer curved surface $=88cm^2$

$$\boxed{Curved\ surface\ area\ of\ hollow\ cylinder = 2\pi(R-r)h}$$

$\Rightarrow 88 = 2\pi(R-0)h$

$\Rightarrow 88 = 2\pi(R-r)14$

$\Rightarrow R-r=1$ ……..(1)

$$\boxed{Volume\ of\ hollow\ cylinder = \pi\left(R^2-r^2\right)h\ cm^3}$$

Given volume of cylinder $=176cm^3$

$\Rightarrow \pi\left(R^2-r^2\right)h=176$

$\Rightarrow \pi\left(R^2-r^2\right)\times 14=176$

$\Rightarrow R^2-r^2=4$

$\Rightarrow (R+r)(R-r)=4$

$\Rightarrow R+4=4$ ……..(2)

By using (1) and (2) equations and solving we get

$R-r=1$ ..(1)

$R+r=4$ ..(2)

$2R=5$

$\Rightarrow \boxed{R=\frac{5}{2}=2\cdot 5cm}$

Substituting 'R' value in (1)

$\Rightarrow R-r=1$

$\Rightarrow 2\cdot 5-r=1$

$\Rightarrow 2\cdot 5-1=r$

$\Rightarrow \boxed{r=1\cdot 5cm}$

External radii of hollow cylinder $(R)=2\cdot 5cm$

Internal radii of hollow cylinder $(r)=1\cdot 5cm$

45. The volume of a hemisphere is $2425\frac{1}{2}cm^3$. Find its curved surface area?

Sol:

Given that volume of a hemisphere $= 2424\frac{1}{2}cm^3$

Volume of a hemisphere $= \frac{2}{3}\pi r^3$

$\Rightarrow \frac{2}{3}\pi r^3 = 2425\frac{1}{2}$

$\Rightarrow \frac{2}{3}\pi r^3 = \frac{4841}{2}$

$\Rightarrow r^3 = \frac{4851\times 3}{2\times 2\times \pi}$

$\Rightarrow r^3 = \frac{4851\times 3}{4\pi}$

r^3

$r = 10\cdot 50cm$

$\therefore$ Radius of hemisphere $= 10\cdot 5cm$

Curved surface area of hemisphere $= 2\pi r^2$

$= 2\pi(10\cdot 5)^2$

$= 692\cdot 72$

$\Rightarrow 693xm^2$

$\therefore$ curved surface area off hemisphere $= 693cm^2$

46. A cylindrical bucket 32cm high and with radius of base 18cm is filled with sand. This bucket is emptied out on the ground and a conical heap of sand is formed. If the height of the conical heap of sand is formed. If the height of the conical heap is 24cm. find the radius and slant height of the heap?

Sol:

Given that height of cylindrical bucket $(h) = 32cm$

Radius $(r) = 18cm$

Volume of cylinder $= \pi r^2 h$

$= \frac{22}{7}(18)^2 \times 32cm^3$ ………(1)

Given height of conical heap $= 24cm$

Let radius of conical heap be r_1

Slant height of conical heap be l_1

$\Rightarrow l_1^2 = r_1^2 + h_1^2$

$\Rightarrow r_1^2 = l_1^2 + h_1^2$

$\Rightarrow r_1^2 = l_1^2 - (24)^2$ ………..(2)

Volume of cone $= \frac{1}{3}\pi r^2 h$

So its volume $= \frac{1}{3}\pi \Rightarrow r_1^2 h_1$

$= \frac{1}{3} \times \frac{22}{7} \times r_1^2 \times 24$

$= \frac{22}{7} \times r_1^2 \times 8cm^3$ ………….(3)

So equating (1) and (3)

(1) = (3)

$\Rightarrow \frac{22}{7}(18)^2 \times 32 = \frac{22}{7} \times r_1^2 \times 8$

$\Rightarrow \frac{(18)^2 \times 32}{8} = r_1^2$

$\Rightarrow r_1^2 = 1296$

$\Rightarrow \boxed{r_1 = 36cm}$

Radius of conical heap is 36cm

Substituting r_1 in (2)

$\Rightarrow r_1^2 = l_1^2 - (24)^2$

$\Rightarrow 1296 = l_1^2 - 576$

$\Rightarrow 1296 + 576 = l_1^2$

$\Rightarrow 1872 = l_1^2$

$\Rightarrow \boxed{l_1 = 43 \cdot 26cm}$

$\therefore$ Slant height of conical heap $= 43 \cdot 26cm$

Exercise 16.2

47. A tent is in the form of a right circular cylinder surmounted by a cone. The diameter of cylinder is 24 m. The height of the cylindrical portion is 11 m while the vertex of the cone is 16 m above the ground. Find the area of canvas required for the tent.

Sol:

Given diameter of cylinder $24m$

Radius $(r) = \frac{24}{2} = 12m$

Given height of cylindrical part $(h_1) = 11m$

$\therefore$ Height of cone part $(h_2) = 5m$

Vertex of cone above ground $= 11 + 5 = 16m$

Curved surface area of cone $(S_1) = \pi rl$

$= \frac{22}{7} \times 12 \times l$

Let l be slant height of cone

$\Rightarrow l = \sqrt{r^2 + h_2^2}$

$\Rightarrow l = \sqrt{12^2 + 5^2} = 13m$

$l = 13m$

$\therefore$ Curved surface area of cone $(5) = \frac{22}{7} \times 12 \times 13m^2$ ……….(1)

Curved surface area of cylinder $(S_2) = 2\pi rh$

$S_2 = 2\pi(12)(11)m^2$ ………..(2)

To find area of canvas required for tent

$S = S_1 + S_2 = (1) + (2)$

$S = \frac{22}{7} \times 12 \times 13 + 2\pi(12)(11)$

$S = 490 + 829 \cdot 38$

$S = 1320m^2$

$\therefore$ Total canvas required for tent $(S) = 1320m^2$

48. A rocket is in the form of a circular cylinder closed at the lower end with a cone of the same radius attached to the top. The cylinder is of radius $2 \cdot 5m$ and height 21m and the cone has a slant height 8m. Calculate total surface area and volume of the rocket?

Sol:

Given radius of cylinder $(a) = 2 \cdot 5m$

Height of cylinder $(h) = 21m$

Slant height of cylinder $(l) = 8m$

Curved surface area of cone $(S_1) = \pi rl$

$S_1 = \pi(2 \cdot 5)(8)cm^2$ ………(1)

Curbed surface area of a cone $= 2\pi rh + \pi r^2$

$S_2 = 2\pi(2\cdot5)(21) + \pi(2\cdot5)^2\ cm^2$ ……….(2)

$\therefore$ Total curved surface area = (1) + (2)

$S = S_1 + S_2$

$S = \pi(2\cdot5)(8) + 2\pi(2\cdot5)(21) + \pi(2\cdot5)^2$

$S = 62\cdot831 + 329\cdot86 + 19\cdot63$

$S = 412\cdot3m^2$

$\therefore$ Total curved surface area $= 412\cdot3m^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

$V_1 = \frac{1}{3}\times\pi(2\cdot5)^2 h\ \ cm^3$ ………...(3)

Let 'h' be height of cone

$l^2 = r^2 + h^2$

$\Rightarrow l^2 - r^2 = h^2$

$\Rightarrow h = \sqrt{l^2 - r^2}$

$\Rightarrow h = \sqrt{8^2 - 25^2}$

$\Rightarrow h = 23\cdot685m$

Subtracting 'h' value in (3)

Volume of a cone $(V_1) = \frac{1}{3}\times\pi(2\cdot5)^2(23\cdot685)cm^2$ ………(4)

Volume of a cylinder $(V_2) = \pi r^2 h$

$= \pi(2\cdot5)^2\, 21m^3$ ………..(5)

Total volume = (4) + (5)

$V = V_1 + V_2$

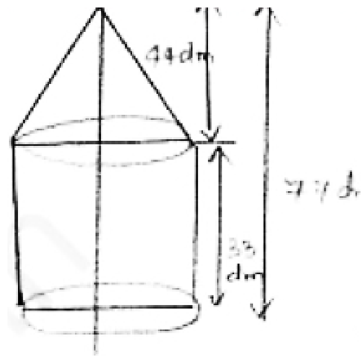
$\Rightarrow V = \frac{1}{3}\times\pi(2\cdot5)^2(23\cdot685) + \pi(2\cdot5)^2 = 1$

$\Rightarrow V = 461\cdot84m^2$

Total volume $(V) = 461\cdot84m^2$

49. A tent of height 77 dm is in the form of a right circular cylinder of diameter 36 m and height 44 dm surmounted by a right circular cone. Find the cost of the canvas at Rs. 350 per m^2 (Use it = $\frac{22}{7}$).

Sol:



Given that height of a tent $= 77dm$

Height of a surmounted cone $= 44dm$

Height of cylinder part $= 77 - 44$

$= 33dm = 3.3m$

Given diameter of cylinder (d) $= 26m$

Radius $(r) = \frac{36}{2} = 18m.$

Let 'l' be slant height of cone

$\Rightarrow l^2 = r^2 + h^2$

$\Rightarrow l^2 = 18^2 + 3 \cdot 3^2$

$\Rightarrow l^2 = 824 + 10 \cdot 89$

$\Rightarrow l = 18 \cdot 3$

$\therefore$ Slant height of cone $(1) = 18 \cdot 3$

Curved surface area of cylinder $(S_1) = 2\pi rh$

$= 2 \times \pi \times 18 \times 4 \cdot 4m^2$ ....(1)

Curved surface area of cone $(S_2) = \pi rh$

$= \pi \times 18 \times 18 \cdot 3m^2$ ....(2)

Total curved surface of tent $= S_1 + S_2$

$S = S_1 + S_2$

$S = 1532 \cdot 46m^2$

$\therefore$ Total curved surface area $(S) = 12 = 1532 \cdot 46m^2$

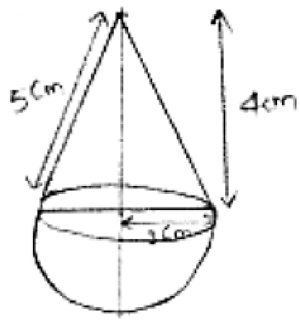
50. A toy is in the form of a cone surmounted on a hemisphere. The diameter of the base and the height of cone are 6cm and 4cm. determine surface area of toy?

Sol:

Given height of cone $(h) = 4cm$

Diameter of cone $(d) = 6cm$

$\therefore$ Radius (r) $= \frac{6}{2} = 3cm$



Let ‘l’ be slant height of cone

$l = \sqrt{r^2 + h^2}$

$= \sqrt{3^2 + 4^2} = 5cm$

$l = 5cm$

$\therefore$ Slant height of cone $(l) = 5cm.$

Curved surface area of cone $(S_1) = \pi rl$

$S_1 = \pi(3)(5) = 47 \cdot 1cm^2$

Curved surface area of hemisphere $(S_2) = 2\pi r^2$

$S_2 = 2\pi(3)^2 = 56 \cdot 52cm^2$

$\therefore$ Total surface area $(s) = 6_1 + S_2$

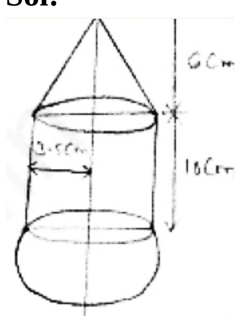
$= 47 \cdot 1 + 56 \cdot 52$

$= 103 \cdot 62cm^2$

$\therefore$ Curved surface area of toy $= 103 \cdot 62cm^2$

51. A solid is in the form of a right circular cylinder, with a hemisphere at one end and a cone at the other end. The radius of the common base is 3.5 cm and the heights of the cylindrical and conical portions are 10 cm. and 6 cm, respectively. Find the total surface area of the solid. (Use n = $\frac{22}{7}$)

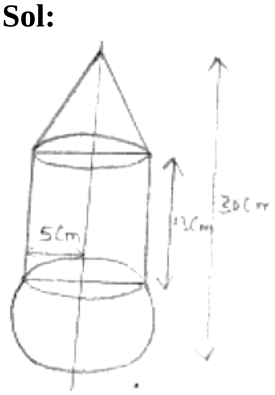
Sol:



Given radius of common base $= 3\cdot 5cm$
Height of cylindrical part $(h) = 10cm$
Height of conical part $(h) = 6cm$
Let $'l'$ be slant height of cone
$l = \sqrt{r^2 + h^2}$
$l = \sqrt{(3\cdot 5)^2 + 6^2}$
$l = 48\cdot 25cm$
Curved surface area of cone $(S_1) = \pi rl$
$= \pi(3\cdot 5)(48\cdot 25)$
$= 76\cdot 408cm^2$
Curved surface area of cylinder $(S_2) = 2\pi rh$
$= 2\pi(3\cdot 5)(10)$
$= 220cm^2$
Curved surface area of hemisphere $(S) = S_1 + S_2 + S_3$
$= 76\cdot 408 + 220 + 77$
$= 373\cdot 408cm^2$
$\therefore$ Total surface area of solid $(S) = 373\cdot 408cm^2$
Cost of canvas per $m^2 = Rs\ 3\cdot 50$
Cost of canvas for $1532\cdot 46m^2 = 1532\cdot 46 \times 3\cdot 50$
$= 5363\cdot 61Rs$
$\therefore$ Cost of canvas required for tent $= Rs\ 5363\cdot 61pr$

52. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The radius and height of the cylindrical part are 5 cm and 13 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Find the surface area of the toy if the total height of the toy is 30 cm.

Sol:



$S_1 = 2\pi(2)(13)$

$S_1 = 408 \cdot 2cm^2$

Curved surface area of cone $(S_2) = \pi rl$

Let l be slant height of cone

$l = \sqrt{r^2 + h^2}$

$h = 30 - 13 - 5 = 12cn$

$\Rightarrow l = \sqrt{12^2 + 5^2} = 13cm$

$l = 13cm$

$\therefore$ Curved surface area of cone $(S_2) = \pi(5)(13)$

$= 204 \cdot 1cm^2$

Curved surface area of hemisphere $(S_3) = 2\pi r^2$

$= 2\pi(5)^2$

$= 2\pi(25) = 50\pi = 157cm^2$

$S_3 = 157cm^2$

Total curved surface area $(S) = S_1 + S_2 + S_3$

$S = 408 \cdot 2 + 204 \cdot 1 + 157$

$S = 769 \cdot 3cm^2$

$\therefore$ Surface area of toy $(S) = 769.3cm^2$

53. A cylindrical tube of radius 5cm and length $9 \cdot 8cm$ is full of water. A solid in form of a right circular cone mounted on a hemisphere is immersed in tube. If radius of hemisphere is immersed in tube if the radius of hemisphere is $3 \cdot 5cm$ and height of the cone outside hemisphere is 5cm. find volume of water left in the tube?

Sol:

Given radius of cylindrical tube $(r) = 5cm.$

Height of cylindrical tube $(h) = 9 \cdot 8cm$

Volume of cylinder $= \pi r^2 h$

$V_1 = \pi(5)^2(9 \cdot 8) = 770cm^3$

Given radius of hemisphere $(r) = 3 \cdot 5cm$

Height of cone $(h) = 5cm$

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \pi (3 \cdot 5)^3 = 89 \cdot 79 cm^3$

Volume of cone $= \frac{1}{3} \pi r^2 h$

$= \frac{\pi}{3} (3 \cdot 5)^2 5 = 64 \cdot 14 cm^3$

Volume of cone + volume of hemisphere $(V_2) = 39 \cdot 79 + 64 \cdot 14 = 154 cm^3$

54. A circular tent has cylindrical shape surmounted by a conical roof. The radius of cylindrical base is $20m.$ The height of cylindrical and conical portions are $4 \cdot 2m$ and $2 \cdot 1m.$ Find the volume of the tent?

Sol:

Given radius of cylindrical base $= 20m$

Height of cylindrical part $(h) = 4 \cdot 2m.$

Volume of cylindrical $= \pi r^2 h_1$

$V_1 = \pi (20)^2 \, 4 \cdot 2 = 5280 m^3$

Volume of cone $= \frac{1}{3} \pi r^2 h_2$

Height of conical part $(h_2) = 2 \cdot 1m$

$V_2 = \frac{\pi}{3} (20)^2 (2 \cdot 1) = 880 m^3$

Volume of tent $(v) = V_1 + V_2$

$V = 5280 + 880$

$V = 6160 m^3$

$\therefore$ Volume of tent $(v) = V_1 + V_2$

$V = 5280 + 880$

$V = 6160 m^3$

$\therefore$ Volume of tent $(v) = 6160 m^3$

55. A petrol tank is a cylinder of base diameter 21cm and length 18cm fitted with conical ends each of axis 9cm. determine capacity of the tank?

Sol:

Given base diameter of cylinder $= 21cm$

Radius $(r) = \frac{21}{2} = 11 \cdot 5 cm$

Height of cylindrical part $(h) = 18cm$

Height of conical part $(h_2) = 9cn$

Volume of cylinder $= \pi r^2 h_1$

$V_1 = \pi(11\cdot5)^2 18 = 7474\cdot77cm^3$

Volume of cone $= \dfrac{1}{3}\pi r^2 h_2$ ($\because$ 2 conical end)

$V_2 = \dfrac{1}{3}\pi(11\cdot5)^2(9)\times2$

$V_2 = \dfrac{1}{3}\pi(1190\cdot25) = 2492\cdot25cm^3$

Volume of tank = volume of cylinder + volume of cone

$V = V_1 + V_2$

$V = 7474\cdot77 + 2492\cdot85$

$V = 9966\cdot36cm^3$

Volume of water left in tube = Volume of cylinder – Volume of hemisphere and cone

$V = V_1 - V_2$

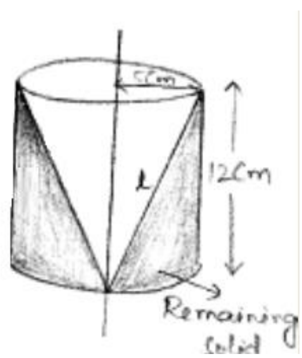
$= 770 - 154$

$= 616cm^3$

$\therefore$ Volume of water left in tube $= 616cm^3$

56. A conical hole is drilled in a circular cylinder of height 12cm and base radius 5cm. The height and base radius of the cone are also the same. Find the whole surface and volume of the remaining cylinder?

Sol:



Given base radius of cylinder $(r) = 5cm$

Height of cylinder $(h) = 12cm$

Let 'l' be slant height of cone

$l = \sqrt{r^2 + h^2}$

$= \sqrt{5^2 + 12^2}$

$l = 13cm$

$\therefore$ Height and base radius of cone and cylinder are same

Total surface area of remaining part $(s) = 2\pi rh + \pi r^2 + \pi rl$

$= 2\pi(5)(12) + \pi(5)^2 + \pi(5)(13)$

T.S.A $= 210\pi cm^2$

Volume of remaining part = Volume of cylinder – Volume of cone

$\Rightarrow V = \pi r^2 h - \frac{1}{3}\pi r^2 h$

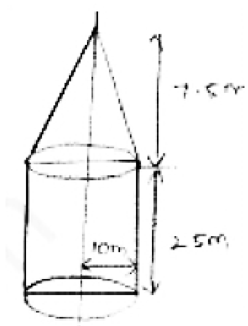
$\Rightarrow V = \pi(5)^2(12) - \frac{1}{3}\pi(5)^2(12)$

$\Rightarrow V = 200\pi cm^3$

$\therefore$ Volume of remaining part $(v) = 200\pi cm^3$

57. A tent is in form of a cylinder of diameter 20m and height $2\cdot5m$ surmounted by a cone of equal base and height $7\cdot5m.$ Find capacity of tent and cost of canvas at Rs 100per square meter?

Sol:



Given radius of cylinder $(r) = \frac{20}{2} = 10m$

Height of a cylinder $(h_1) = 2\cdot5m$

Height of cone $(h_2) = 7\cdot5m$

Let 'l' be slant height of cone

$l = \sqrt{r^2 + h_2^2}$

$l = \sqrt{10^2 + 7\cdot5^2}$

$\Rightarrow l = 12\cdot5m$

Volume of cylinder $(V_1) = \pi r^2 h$

$V_1 = \pi(10)^2(2\cdot5)$ ……..(1)

Volume of cone $\left(V_2\right)=\frac{1}{3}\pi r^2 h_2$

$=\frac{1}{3}\pi\left(10\right)^2\left(7\cdot 5\right)m^3$ ……..(2)

Total capacity of tent = (1) + (2)

$V=V_1+V_2$

$V=\pi\left(10\right)^2 2\cdot 5+\frac{1}{3}\pi\left(10\right)^2 7\cdot 5$

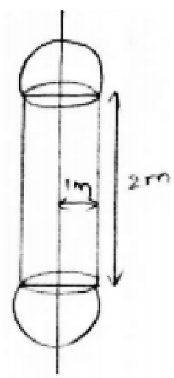
$V=250\pi+250\pi$

$V=500\pi cm^3$

$\therefore$ Total capacity of tent $=500\pi cm^2$

58. A boiler is in the form of a cylinder 2m long with hemispherical ends each of 2m diameter. Find the volume of the boiler?

Sol:



Given height of cylinder $\left(h\right)=2m$

Diameter of hemisphere $\left(d\right)=2m$

Radius $\left(r\right)=1m$

Volume of a cylinder $=\pi r^2 h$

$V_1=\pi\left(1\right)^2\left(2\right)cm^3$ ……..(1)

Volume of hemisphere $=\frac{2}{3}\pi r^3$

Since at ends of cylinder hemisphere are attached

Volumes of 2 hemispheres

$=2\times\frac{2}{3}\pi\left(1\right)^2 cm^2$ ………(2)

Volumes of boiler = (1) + (2)

$V=V_1+V_2$

$V = 2\times\frac{2}{3}\pi(1)^2 + \pi(1)^2(2)$

$V = \frac{220}{21}m^3$

$\therefore$ Volumes of boiler $= \frac{220}{21}m^3$

59. A vessel is a hollow cylinder fitted with a hemispherical bottom of the same base. The depth of cylinder is $\frac{14}{3}m$ and internal surface area of the solid?

Sol:

Given radius of hemisphere $(r) = \frac{3\cdot 5}{2} = 1\cdot 75m$

Height of cylinder $(h) = \frac{14}{3}m$

Volume of cylinder $= \pi r^2 h$

$= \pi(1\cdot 75)^2\left(\frac{14}{3}\right)cm^3$ ……..(1)

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\times\pi(1\cdot 75)^3 cm^3$ ……..(2)

Volume of vessel = (1) + (2)

$V = V_1 + V_2$

$V = \pi r^2 h + \frac{2}{3}\pi r^3$

$V = \pi(1\cdot 75)^2\left(\frac{14}{3}\right) + \frac{2}{3}\pi(1\cdot 75)^2$

$V = 56m^3$

$\therefore$ Volumes of vessel $(v) = 56m^3$

Internal surface area of solid $(s) = 2\pi rh + 2\pi r^2$

S = Surface area of cylinder + surface are of hemisphere

S = $2\pi(1\cdot 75)\left(\frac{14}{3}\right) + 2\pi(1\cdot 75)^2$

$S = 70\cdot 51m^2$

$\therefore$ Internal surface area of solid $(s) = 70\cdot 51m^2$

60. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104cm and radius of each of hemispherical ends is 7cm. find the cost of polishing its surface at the rate of $Rs\ 10\ per\ dm^2$?

Sol:

Given radius of hemispherical ends $= 7cm$

Height of body $(h+2r) = 104cm.$

Curved surface area of cylinder $= 2\pi rh$

$= 2\pi(7)h$ ....(1)

$\Rightarrow h + 2x = 104$

$\Rightarrow h = 104 - 2(r)$

$\Rightarrow h = 90cm$

Substitute 'h' value in (1)

Curved surface area of cylinder $= 2\pi(7)(90)$

$= 3948 \cdot 40cm^2$ ....(2)

Curved surface area of 2 hemisphere $= 2(2\pi r^2)$

$= 2(2 \times \pi \times 7^2)$

$= 615 \cdot 75cm^3$ ....(3)

Total curved surface area = (2) + (3)

$= 3958 \cdot 40 + 615 \cdot 75 = 4574 \cdot 15cm^2 = 45 \cdot 74dm^2$

Cost of polishing for $1dm^2 = Rs10$

Cost of polishing for $45 \cdot 74dm^2 = 45 \cdot 74 \times 10$

$= Rs\ 457 \cdot 4$

61. A cylindrical vessel of diameter 14cm and height 42cm is fixed symmetrically inside a similar vessel of diameter 16cm and height $42cm.$ The total space between two vessels is filled with cork dust for heat insulation purpose. How many cubic cms of cork dust will be required?

Sol:

Given height of cylindrical vessel $(h) = 42cm$

Inner radius of a vessel $(r_1) = \frac{14}{2}cm = 7cm$

Outer radius of a vessel $(r_2) = \frac{16}{2} = 8cm$

Volume of a cylinder $= \pi(r_2^2 - r_1^2)h$

$= \pi(8^2 - 7^2)42$

$= \pi(64-49)42$

$= 15 \times 42 \times \pi$

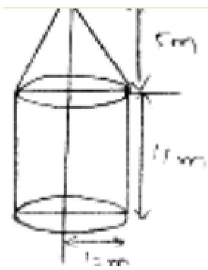
$= 630\pi$

$= 1980 cm^3$

Volume of a vessel $= 1980 cm^2$

62. A cylindrical road solar made of iron is 1m long its internal diameter is 54cm and thickness of the iron sheet used in making roller is 9cm. Find the mass of roller if $1cm^3$ of iron has $7 \cdot 8gm$ mas?

Sol:



Given internal radius of cylindrical road

Roller $(r_1) = \frac{54}{2} = 27cm$

Given thickness of road roller $\left(\frac{1}{b}\right) = 9cm$

Let order radii of cylindrical road roller be R

$\Rightarrow t = R - r$

$\Rightarrow 9 = R - 27$

$\Rightarrow R = 9 + 27 = 36cm$

$R = 36cm$

Given height of cylindrical road roller $(h) = 1m$

$h = 100cm.$

Volume of iron $= \pi h(R^2 - r^2)$

$= \pi(36^2 - 27^2) \times 100$

$= 1780 \cdot 38cm^3$

Volume of iron $= 1780 \cdot 38cm^3$

Mass of $1cm^3$ of iron $= 7 \cdot 8gm$

Mass of $1780 \cdot 38cm^3$ of iron $= 1780 \cdot 38 \times 7 \cdot 8$

$= 1388696 \cdot 4gm$

$= 1388 \cdot 7kg$

$\therefore$ Mass of roller $(m) = 1388 \cdot 7kg$

63. A vessel in from of a hollow hemisphere mounted by a hollow cylinder. The diameter of hemisphere is 14cm and total height of vessel is 13cm. find the inner surface area of vessel?

Sol:

Given radius of hemisphere and cylinder (r)

$= \frac{14}{2} = 7cm$

Given total height of vessel $= 13cm$

$(h + r) = 13cm$

Inner surface area of vessel $= 2\pi r(h + r)$

$= 2 \times \pi \times 7(13)$

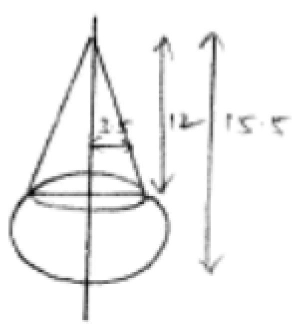
$= 182\pi$

$= 572cm^2$

$\therefore$ Inner surface area of vessel $= 572cm^2$

64. A toy is in the form of a cone of radius $3 \cdot 5cm$ mounted on a hemisphere of same radius. The total height of toy is $15 \cdot 5cm$. Find the total surface area of toy?

Sol:



Given radius of cone $(r) = 3 \cdot 5cm$

Total height of toy $(h) = 15 \cdot 5cm$

Length of cone $(l) = 15 \cdot 5 - 3 \cdot 5$

$= 12cm$

$\therefore$ Length of cone $(l) = 12cm$

Curved surface area of cone $= \pi rl$

$S_1 = \pi(3 \cdot 5)(12)$

$S_1 = 131 \cdot 94cm^2$ ……(1)

Curved surface area of hemisphere $= 2\pi r^2$

$S_2 = 2\pi(3\cdot 5)^2$

$S_2 = 76\cdot 96 cm^2$ ……(2)

$\therefore$ Total surface of toy = (1) + (2)

$S = S_1 + S_2$

$S = 181\cdot 94 + 76\cdot 96$

$S = 208\cdot 90$

$S = 209 cm^2$

$\therefore$ Total surface area of toy $= 209 cm^2$

65. The difference between outside and inside surface areas of cylindrical metallic pipe 14cm long is $44m^2$. If pipe is made of $99cm^3$ of metal. Find outer and inner radii of pipe?

Sol:

Let inner radius of pipe be r_1

Radius of outer cylinder be r_2

Length of cylinder $(h) = 14cm.$

Surface area of hollow cylinder $= 2\pi h(r_2 - r_1)$

Given surface area of cylinder $= 44m^2$

66. A radius circular cylinder bring having diameter 12cm and height 15cm is full ice-cream. The ice-cream is to be filled in cones of height 12cm and diameter 6cm having a hemisphere shape on top find the number of such cones which can be filled with ice-cream?

Sol:

Given radius of cylinder $(r_1) = \frac{12}{2} = 6cm$

Given radius of hemisphere $(r_2) = \frac{6}{2} = 3cm.$

Given height of cylinder $(h) = 15cm..$

Height of cones $(l) = 12cm.$

Volume of cylinder $= \pi r_1^2 h$

$= \pi(6)^2(15) cm^3$ ……(1)

Volume of each cone = volume of cone + volume of hemisphere

$= \frac{1}{3}\pi r_2^2 l + \frac{2}{3}\pi r_2^3$

$$=\frac{1}{3}\pi(3)^2(12)+\frac{2}{3}\pi(3)^3\,cm^3 \qquad \text{........(2)}$$

Let number of cones be 'n'

n(Volume of each cone) = volume of cylinder

$$n\left(\frac{1}{3}\pi(3)^2(12)+\frac{2}{3}\pi(3)^3\right)=\pi(6)^2 15$$

$$\Rightarrow n=\frac{\pi(6)^2 15}{\frac{1}{3}\pi(3)^2(12)+\frac{2}{3}\pi(3)^3}$$

$$\Rightarrow n=\frac{540}{5}=10$$

$$\Rightarrow 2\pi h(r_2-r_1)=44$$

$$\Rightarrow 2\pi(14)(r_2-r_1)=44$$

$$\Rightarrow 28\pi(r_2-r_1)=44$$

$$\Rightarrow (r_2-r_1)=\frac{44}{28\pi}$$

$$\Rightarrow (r_2-r_1)=\frac{1}{2} \qquad \text{..........(1)}$$

Given volume of a hollow cylinder $=99cm^3$

Volume of a hollow cylinder $=\pi h\left(r_2^2-r_1^2\right)$

$$\Rightarrow \pi h\left(r_2^2-r_1^2\right)=99$$

$$\Rightarrow 14\pi\left(r_2^2-r_1^2\right)=99$$

$$\Rightarrow 14\pi(r_1+r_2)(r_2-r_1)=99$$

$$\Rightarrow 14\pi(r_1+r_2)(1)=99$$

$$\Rightarrow 14\pi(r_1+r_2)=99$$

$$\Rightarrow (r_1+r_2)=\frac{9}{2} \qquad \text{..........(2)}$$

Equating (1) and (2) equations we get

$$r_1+r_2=\frac{9}{2}$$

$$-r_1+r_2=\frac{1}{2}$$

$$2r_2=5$$

$$r_2=\frac{5}{2}cm.$$

Substituting r_2 value in (1)

$\Rightarrow r_1 = 2cm$

$\therefore$ Inner radius of pipe $(a) = 2cm$

Radius of outer cylinder $(r_2) = \frac{5}{2} cm.$

67. A solid iron pole having cylindrical portion 110cm high and of base diameter 12cm is surmounted by a cone 9cm high. Find the mass of the pole given that the mass of $1cm^3$ of iron is $8gm$?

Sol:

Given radius of cylindrical part $(r) = \frac{12}{2} = 6cm$

Height of cylinder $(h) = 110cm$

Length of cone $(l) = 9cm$

Volume of cylinder $= \pi r^2 h$

$V_1 = \pi (0)^2 110cm^3$ ....(1)

Volume of cone $= \frac{1}{3}\pi r^2 l$

$V_2 = \frac{1}{3}\pi (6)^2 9 = 108\pi cm^3$ ....(2)

Volume of pole = (1) + (2)

$V = V_1 + V_2$

$\Rightarrow V = \pi (6)^2 110 + 108\pi$

$\Rightarrow V = 12785 \cdot 14 cm^3$

Given mass of $1cm^3$ of iron $= 8gm$

Mass of $12785 \cdot 14cm^3$ of iron $= 12785 \cdot 14 \times 8$

$= 102281 \cdot 12$

$= 102 \cdot 2kg$

$\therefore$ Mass of pole for $12785 \cdot 14cm^3$ of iron is $102 \cdot 2kg$

68. A solid toy is in the form of a hemisphere surmounted by a right circular cone. Height of the cone is 2 cm and the diameter of the base is 4 cm. If a right circular cylinder circumscribes the toy, find how much more space it will cover.

Sol:

Given radius of cone, cylinder and hemisphere $(r) = \frac{4}{2} = 2cm$

Height of cone $(l) = 2cm$

Height of cylinder $(h) = 4cm$

Volume of cylinder $= \pi r^2 h = \pi (2)^2 (4) cm^3$ ………(1)

Volume of cone $= \frac{1}{3}\pi r^2 l$

$= \frac{1}{3}\pi (2)^2 \times 2$

$= \frac{\pi}{3}(4) \times 2cm^3$ ……….(2)

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \pi (2)^3$

$= \frac{2}{3} \times \pi (8) cm^3$ ………..(3)

So remaining volume of cylinder when toy is inserted to it $= \pi r^2 h - \left(\frac{1}{3}\pi r^2 l + \frac{2}{3}\pi r^3\right)$

= (1) – ((2) + (3))

$= \pi (2)^2 (4) - \left(\frac{\pi}{3} \times 8 + \frac{2}{3} \times \pi \times 8\right)$

$= 16\pi - \frac{2}{3}\pi (4+8) = 16\pi - 8\pi = 8\pi cm^3$

$\therefore$ So remaining volume of cylinder when toy is inserted to it $= 8\pi cm^3$

69. A solid consisting of a right circular cone of height 120cm and radius 60cm is placed upright in right circular cylinder full of water such that it touches bottoms. Find the volume of water left in the cylinder. If radius of cylinder is 60cm and its height is $180cm$?

Sol:

Given radius of circular cone $(a) = 60cm$

Height of circular cone $(b) = 120cm.$

Volume of a cone $= \frac{1}{3}\pi r^2 l$

$= \frac{1}{3}\pi (60)^2 (120) cm^3$ …………(1)

Volume of hemisphere $= \frac{2}{3}\pi r^3$

Given radius of hemisphere $= 60cm$

$= \frac{2}{3}\pi(60)^2 \ cm^3$ …………(2)

Given radius of cylinder $= 60cm$

Height of cylinder $(h) = 180cm.$

Volume of cylinder $= \pi r^2 h$

$= \pi(60)^2 \times 180cm^3$ ……………(3)

Volume of water left in cylinder = (3) – ((1) + (2))

$\Rightarrow \frac{1}{3}\pi(60)^3(120) - \left(\frac{2}{3}\pi(60)^3 + \pi(60)^2 \times 180\right)$

$\Rightarrow 113\cdot 1cm^3 = 1\cdot 131m^3$

$\therefore$ Volume of water left in cylinder $= 1\cdot 131m^3$

70. A cylindrical vessel with internal diameter 10cm and height $10\cdot 5cm$ is full of water. A solid cone of base diameter 7cm and height 6cm is completely immersed in water. Find value of water (i) displaced out of the cylinder (ii) left in the cylinder?

Sol:

Given internal radius $(r_1) = \frac{10}{2} = 5cm$

Height of cylindrical vessel $(h) = 10\cdot 5cm$

Outer radius of cylindrical vessel $(l_2) = \frac{7}{2} = 3\cdot 5cm$

Length of cone $(l) = 6cm.$

(i) Volume of water displaced = volume of cone

Volume of cone $= \frac{1}{3}\pi r_2^2 l$

$= \frac{1}{3}\pi \times 3\cdot 5^2 \times 6 = 76\cdot 9cm^3$

$= 77cm^3$

$\therefore$ Volume of water displaced $= 77cm^3$

Volume of cylinder $= \pi r_1^2 h = \pi(5)^2 10\cdot 5$

$= 824\cdot 6$

$= 825cm^2$

(ii) Volume of water left in cylinder = volume of
Cylinder – volume of cone

$= 825 - 77 = 748cm^3$

$\therefore$ Volume of water left in cylinder $= 748cm^3$

71. A hemispherical depression is cut from one face of a cubical wooden block of edge 21cm such that the diameter of hemisphere is equal to the edge of cube determine the volume and total surface area of the remaining block?

Sol:

Given edge of wooden block $(a) = 21cm$

Given diameter of hemisphere = edge of cube

Radius $= \frac{21}{2} = 10 \cdot 5cm$

Volume of remaining block = volume of box – volume of hemisphere

$= a^3 - \frac{2}{3}\pi r^3$

$= (2)^3 - \frac{2}{3}\pi(10 \cdot 5)^3$

$= 6835 \cdot 5cm^3$

Surface area of box $= 6a^2$ ……..(1)

Curved surface area of hemisphere $= 2\pi r^2$ ..……(2)

Area of base of hemisphere $= \pi r^2$ ……..(3)

So remaining surface area of box = (1) – (2) + (3)

$= 6a^2 - \pi r^2 + 2\pi r^2$

$= 6(21)^2 - \pi(10 \cdot 5) + 2\pi(10 \cdot 5)^2$

$= 2992 \cdot 5cm^2$

$\therefore$ Remaining surface area of box $= 2992 \cdot 5cm^2$

Volume of remaining block $= 6835 \cdot 5cm^3$

72. A tag is in the form of a hemisphere surmounted by a right circular cone of same base radius as that of the hemisphere. If the radius of the base of cone is 21cm and its volume is $\frac{2}{3}$ of volume of hemisphere calculate height of cone and surface area of toy?

Sol:

Given radius of cone = radius of hemisphere

Radius $(r) = 21cm$

Given that volume of cone $= \frac{2}{3}$ Volume of hemisphere

$\Rightarrow$ Volume of cone $= \frac{1}{3}\pi r^2 h$

Volume of hemisphere $= \frac{2}{3}\pi r^3$

So $\frac{1}{3}\pi r^2 h = \frac{2}{3}\left(\frac{2}{3}\pi r^3\right)$

$\Rightarrow \frac{1}{3}\pi (21)^2 h = \frac{2}{3}\left(\frac{2}{3}\pi (21)^3\right)$

$\Rightarrow h = \frac{4(21)\pi \times 3}{4\pi (21)}$

$\Rightarrow h = \frac{4}{3} \times 21 = 28cm$

$\therefore$ Height of cone $(h) = 28cm$

Curved surface area of cone $= \pi rl$

$S_1 = \pi (21)(28) cm^2$ ………….(1)

Curved surface area off hemisphere $= 2\pi r^2$

$S_2 = 2 \times \pi (21)^2 cm^2$ …………..(2)

Total surface area $(s) = S_1 + S_2 = (1) + (2)$

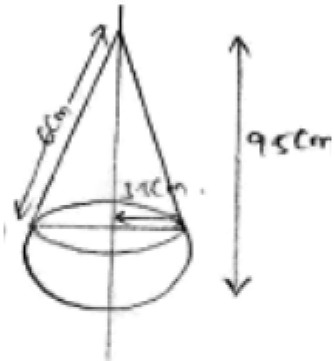
$S = \pi rl + 2\pi r^2$

$S = 5082cm^2$

$\therefore$ Curved surface area of toy $= 5082cm^2$

73. A solid is in the shape of a cone surmounted on hemisphere the radius of each of them is being $3\cdot5cm$ and total height of solid is $9\cdot5cm$. Find volume of the solid?

Sol:



Given radius of hemisphere and cone $=3\cdot5cm$

Given total height of solid $(h)=9\cdot5cm$

Length of cone $(l)=9\cdot5-3\cdot5=6cm$

Volume of a cone $=\frac{1}{3}\pi r^2 l$

$V_1=\frac{1}{3}\pi(3\cdot5)^2\times6\ cm^3$ ....(1)

Volume of hemisphere $=\frac{2}{3}\pi r^3$

$V_2=\frac{2}{3}\pi(3\cdot5)^3\ cm^3$ ....(2)

Volume of solid = (1) + (2)

$V=V_1+V_2$

$V=\frac{1}{3}\pi(3\cdot5)^2\times6+\frac{2}{3}\pi(3\cdot5)^3$

$V=76\cdot96+89\cdot79=166\cdot75cm^3$

$\therefore$ Volume of solid $(v)=166\cdot75cm^3$

Exercise 16.3

1. A bucket has top and bottom diameters of 40 cm and 20 cm respectively. Find the volume of the bucket if its depth is 12 cm. Also, find the cost of tin sheet used for making the bucket at the rate of Rs 1.20 per dm^2. (Use π = 3.14)

Sol:

Given diameter to top of bucket = 40cm

Radius $(r_1) = \frac{40}{2} = 20cm$

Depth of a bucket $(h) = 12cm$

Volume of a bucket $= \frac{1}{3}\pi\left(r_1^2 + r_2^2 + r_1r_2\right)h$

$= \frac{3}{1}\pi\left(20^2 + 10^2 + 20(10)\right)^{12}$

$= 8800cm^3.$

Let 'l' be slant height of bucket

$\Rightarrow l = \sqrt{(r_1 - r_2)^2 + h_2}$

$\Rightarrow l = \sqrt{(20-10)^2 + 12^2}$

$\Rightarrow l = 2\sqrt{61} = 15 \cdot 620cm$

Total surface area of bucket $= \pi(r_1 + r_2) \times l + \pi r_2^2$

$= \pi(20+10) \times 15 \cdot 620 + \pi(10)^2$

$= \frac{1320\sqrt{61} + 2200}{7} cm^2$

$= \frac{1320\sqrt{61} + 2200}{7 \times 100} dm^2 = 17 \cdot 87 dm^2$

Given that cost of tin sheet used for making bucket per $dm^2 = Rs1.20$

So total cost for $17 \cdot 87 dm^2 = 1 \cdot 20 \times 17 \cdot 87$

$= 21 \cdot 40\ Rs.$

$\therefore$ Cost of tin sheet for $17 \cdot 87 dm^2 = Rs2140\,ps$

2. A frustum of a right circular cone has a diameter of base 20cm, of top 12cm and height 3cm. find the area of its whole surface and volume?

Sol:

Given base diameter of cone $(d_1) = 20cm$

Radius $(r_1) = \frac{20}{2} = 10cm$

Top diameter of cone $(d_2) = 12cm$

Radius $(r_2) = \frac{12}{2} = 6cm$

Height of cone $(h) = 3cm$

Volume of frustum right circular cone

$=\frac{1}{3}\pi\left(r_1^2+r_2^2+r_1r_2\right)h$

$=\frac{1}{3}\pi\left(10^2+6^2+(10)(6)\right)3$

$=616cm^3$

Let 'l' be slant height of cone

$\Rightarrow l=\sqrt{\left(r_1-r_2\right)^2+h_2}$

$\Rightarrow l=\sqrt{(10-6)^2+3^2}$

$\Rightarrow l=\sqrt{16+9}=\sqrt{25}cm=5cm$

$\therefore$ Slant height of cone $(l)=5cm$

Total surface area of cone $=\pi\left(r_1+r_2\right)l+\pi r_1^2+\pi r_2^2$

$=\pi(10+6)5+\pi(10)^2+\pi(6)^2$

$=\pi(80+100+36)$

$=\pi(216)=678\cdot 85cm^2$

$\therefore$ Total surface area of cone $=678\cdot 85cm^2$

3. The slant height of the frustum of a cone is 4cm and perimeters of it circular ends are 18cm and 6cm. find curved surface of the frustum?

Sol:

Given slant height of cone $(r)=4cm$

Let radii of top and bottom circles be r_1 and r_2

Given perimeters of its ends as 18cm and 6cm

$\Rightarrow 2\pi r_1=18cm$

$\Rightarrow \pi r_1=9cm$ ……(1)

$\Rightarrow 2\pi r_2=6cm$

$\Rightarrow \pi r_2=3cm$ ……(2)

Curved surface area of frustum cone $=\pi\left(r_1+r_2\right)l$

$=\pi\left(r_1+r_2\right)l$

$=\left(\pi r_1+\pi r_2\right)l$

$=(9+3)4$

$=(12)4=48cm^2$

$\therefore$ Curved surface area of frustum cone $=48cm^2$

4. The perimeters of the ends of a frustum of a right circular cone are 44 cm and 33 cm. If the height of the frustum be 16 cm, find its volume, the slant surface and the total surface.

Sol:

Given perimeters of ends of frustum right circular cone are 44cm an 33cm

Height of frustum cone $= 16cm$

Perimeter $= 2\pi r$

$2\pi r_1 = 44$

$r_1 = 7cm$

$2\pi r_2 = 33$

$r_2 = \frac{21}{4} = 5\cdot 25cm$

Let slant height of frustum right circular cone be l

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$l = \sqrt{(7 - 5\cdot 25)^2 + 16^2 cm}$

$l = 16\cdot 1cm$

$\therefore$ Slant height of frustum cone $= 16\cdot 1cm$

Curved surface area of frustum cone $= \pi(r_1 + r_2)l$

$= \pi(7 + 5\cdot 25)16\cdot 1$

C.S.A of cone $= 619\cdot 65cm^2$

Volume of a cone $= \frac{1}{3}\pi(r_1^2 + r_2^2 + r_1 r_2) \times h$

$= \frac{1}{3}(7^2 + (5\cdot 25)^2 + 7(5\cdot 25) \times 16$

$= 1898\cdot 56cm^3$

$\therefore$ Volume of a cone $= 1898\cdot 56\ cm^3$

Total surface area of frustum cone $= \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$

$= \pi(7 + 5\cdot 25)16\cdot 1 + \pi(7^2 + 5\cdot 25^2)$

$= 860\cdot 27cm^2$

$\therefore$ Total surface area of frustum cone $= 860\cdot 27cm^2$

5. If the radii of circular ends of a conical bucket which is 45cm high be 28cm and 7cm. find the capacity of the bucket?

Sol:

Given height of conical bucket $= 45cm$

Give radii of 2 circular ends of a conical bucket is 28cm and 7cm

$r_1 = 28cm$

$r_2 = 7cm$

Volume of a conical bucket $= \frac{1}{3}\pi\left(r_1^2 + r_2^2 + r_1 r_2\right)h$

$= \frac{1}{3}\pi\left(28^2 + 7^2 + 28(7)\right)45$

$= \frac{1}{3}\pi(1029)45$

$= 15435$

$V = 48510cm^3$

Volume of a conical bucket $= 48510cm^3$

6. The height of a cone is 20cm. A small cone is cut off from the top by a plane parallel to the base. If its volumes be $\frac{1}{25}$ of the volume of the original cone, determine at what height above base the section is made

Sol:

V AB be a cone of height $h_1 = VO_1 = 20cm$

Fronts triangles Vo_1A and VoA_1

$\frac{VO_1}{VO} = \frac{O_1A}{OA_1} \Rightarrow \frac{20}{VO} = \frac{O_1A}{OA_1}$

Volumes of cone $VA_1O = \frac{1}{125}$ times volumes of cone VAB

We have $\frac{1}{3}\pi \times OA_1^2 \times VO = \frac{1}{125} \times \frac{1}{3}\pi \times O_1A_1^2 \times 20$

$\Rightarrow \left(\frac{OA_1}{O_1A}\right)^2 \times VO = \frac{4}{25}$

$\Rightarrow \left(\frac{VO}{20}\right)^2 \times VO = \frac{4}{25}$

$\Rightarrow (VO)^3 = \frac{4\times 400}{25}$

$\Rightarrow VO^3 = 64$

$\Rightarrow VO = 4$

Height at which section is made $= 20 - 4 = 16cm.$

7. If the radii of circular ends of a bucket 24cm high are 5cm and 15cm. find surface area of bucket?

Sol:

Given height of a bucket $(R) = 24cm$

Radius of circular ends of bucket $5cm$ and $15cm$

$r_1 = 5cm$; $r_2 = 15cm$

Let $'l'$ be slant height of bucket

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$\Rightarrow l = \sqrt{(15-5)^2 + 24^2}$

$\Rightarrow l = \sqrt{100+576} = \sqrt{676}$

$l = 26cm$

Curved surface area of bucket $= \pi(r_1 + r_2)l + \pi r_2^2$

$= \pi(5+15)26 + \pi(15)^2$

$= \pi(20)26 + \pi(15)^2$

$= \pi(520+225)$

$= 745\pi cm^2$

$\therefore$ Curved surface area of bucket $= 745\pi cm^2$

8. The radii of circular bases of a frustum of a right circular cone are 12cm and 3cm and height is 12cm. find the total surface area volume of frustum?

Sol:

Let slant height of frustum cone be ‘l’

Given height of frustum cone $12cm$

Radii of a frustum cone are 12cm and 23cm

$r_1 = 12cm$ $r_2 = 3cm$

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$l = \sqrt{(12-3)^2 + 12^2}$

$l = \sqrt{81+144} = 15cm$

$l = 15cm$

Total surface area of cone $= \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$

$= \pi(12+3)15 + \pi(12)^2 + \pi(3)^2$

T.S.A $= 378\pi cm^2$

Volume of cone $= \frac{1}{3}\pi\left(r_1^2 + r_1 r_2 + r_2^2\right) \times h$

$= \frac{1}{3}\pi\left(12^2 + 3^2 + (12)(3)\right)12$

$= 756\pi cm^3$

Volume of frustum cone $= 756\pi cm^3$

9. A tent consists of a frustum of a cone copped by a cone. If radii of ends of frustum be 13m and 7m the height of frustum be 8m and slant height of thee conical cap bee 12m. find canvas required for tent?

Sol:

Given height of frustum $(h) = 8m$

Radii of frustum cone are $13m$ *and* $7m$

$r_1 = 13m \quad r_2 = 7cm$

Let 'l' be slant height of frustum cone

$\Rightarrow l = \sqrt{(r_1 - r_2)^2 + h^2}$

$\Rightarrow l = \sqrt{(13-7)^2 + 8^2} = \sqrt{36+64}$

$\Rightarrow l = 10m$

Curved surface area of friction $(S_1) = \pi(r_1 + r_2) \times l$

$= \pi(13+7) \times 10$

$= 200\pi m^2$

C.S.A of frustum $(S_1) = 200\pi m^2$

Given slant height of conical cap = 12m

Base radius of upper cap cone $= 7m$

Curved surface area of upper cap cone $(S_2) = \pi rl$

$= \pi \times 7 \times 12 = 264m^2$

Total canvas required for tent $(S) = S_1 + S_2$

$S = 200\pi + 264 = 892 \cdot 57m^2$

$\therefore$ Total canvas $= 892 \cdot 57m^2$

10. A reservoir in form of frustum of a right circular contains 44×10^{7} liters off water which fills it completely. The radii of bottom and top of reservoir are 50m and 100m. find depth of water and lateral surface area of reservoir?

Sol:

Let depth of frustum cone be h

Volume of first cone $(V)=\frac{1}{3}\pi\left(r_1^2+r_2^2+r_1r_2\right)h$

$r_1=50m \quad r_2=100m$

$V=\frac{1}{3}\times\frac{22}{7}\times\left(50^2+100^2+50(100)\right)h$

$V=\frac{1}{3}\times\frac{22}{7}\times(2500+1000+5000)h \qquad \ldots(1)$

Volumes of reservoir $=44\times10^{7}$ liters $\quad \ldots.(2)$

Equating (1) and (2)

$\frac{1}{3}\pi(2500)h=44\times10^{2}$

$h=24$

Let 'l' be slant height of cone

$l=\sqrt{(r_1-r_2)^2+h_2}$

$l=\sqrt{(50-100)^2+24^2}$

$l=55\cdot461m$

Lateral surface area of reservoir

$(S)=\pi(r_1+r_2)\times l$

$=\pi(50+100)55\cdot461$

$=1500(55\cdot461)\pi=26145\cdot225m^2$

Lateral surface area of reservoir $=26145\cdot225m^2$

Volume of frustum cone $=\frac{1}{3}\pi\left(r_1^2+r_2^2+r_1r_2\right)h$

$=\frac{1}{3}\pi\left(30^2+18^2+30(18)\right)9$

$=5292\pi cm^3$

Volume $=5292\pi cm^3$

Total surface area of frustum cone =

$=\pi(r_1+r_2)\times l+\pi r_1^2+\pi r_2^2$

$=(30+18)15+\pi(30)^2+(18)^2$

$= \pi\left(48(15)+(30)^2+(18)^2\right)$

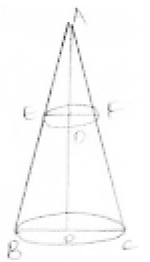
$= \pi(720+900+324)$

$= 1944\pi cm^2$

$\therefore$ Total surface area $= 1944\pi cm^2$

11. A metallic right circular cone 20cm high and whose vertical angle is $90°$ is cut into two parts at the middle point of its axis by a plane parallel to base. If frustum so obtained bee drawn into a wire of diameter $\left(\frac{1}{16}\right)cm.$ find length of the wire?

Sol:



Let ABC be cone. Height of metallic cone $AO = 20cm$

Cone is cut into two parts at the middle point of its axis

Hence height of frustum cone $AD = 10cm$

Since angle A is right angled. So each angles B and $C = 45°$

Angles E and $F = 45°$

Let radii of top and bottom circles of frustum cone bee r_1 and $r_2 cm$

From $\Delta^{le} ADE \Rightarrow \frac{DE}{AD} = \cot 45°$

$\Rightarrow \frac{r_1}{10} = 1$

$\Rightarrow r_1 = 10cm.$

From $\Delta^{le} AOB$

$\Rightarrow \frac{OB}{OA} = \cot 45°$

$\Rightarrow \frac{r_2}{20} = 1$

$\Rightarrow r_2 = 20cm$

12. A bucket is in the form of a frustum of a cone with a capacity of 12308.8 cm3 of water. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of the metal sheet used in its making. (Use π = 3.14).

Sol:

Given radii of top circular ends $(r_1) = 20cm$

Radii of bottom circular end of bucket $(r_2) = 12cm$

Let height of bucket be 'h'

Volume of frustum cone $= \frac{1}{3}\pi\left(r_1^2 + r_2^2 + r_1r_2\right)h$

$= \frac{1}{3}\pi\left(20^2 + 12^2 + 20(12)\right)h$

$= \frac{784}{3}\pi h cm^3$ ……….(1)

Given capacity/volume of bucket $= 123308 \cdot 8cm^3$ ……….(2)

Equating (1) and (2)

$\Rightarrow \frac{784}{3}\pi h = 12308 \cdot 8$

$\Rightarrow h = \frac{12308 \cdot 8 \times 3}{784 \times \pi}$

$\Rightarrow h = 15cm$

$\therefore$ Height of bucket $(h) = 15cm$

Let 'l' be slant height of bucket

$\Rightarrow l^2 = (r_1 - r_2)^2 + h^2$

$\Rightarrow l = \sqrt{(r_1 - r_2)^2 + h^2}$

$\Rightarrow l = \sqrt{(20 + 2)^2 + 15^2} = \sqrt{64 + 225}$

$\Rightarrow l = 17cm$

Length of bucket/ slant height of

Bucket $(l) = 17cm$

Curved surface area of bucket $= \pi(r_1 + r_2)l + \pi r_2^2$

$= \pi(20 + 12)17 + \pi(12)^2$

$= \pi(32)17 + \pi(12)^2$

$= \pi(9248 + 144) = 2160 \cdot 32cm^2$

$\therefore$ Curved surface area $= 2160 \cdot 32cm^2$

13. A bucket made of aluminum sheet is of height 20cm and its upper and lower ends are of radius 25cm an 10cm, find cost of making bucket if the aluminum sheet costs Rs 70 per $100cm^2$

Sol:

Given height of bucket $(h) = 20cm$

Upper radius of bucket $(r_1) = 25cm$

Lower radius of bucket $(r_2) = 10cm$

Let ‘l’ be slant height of bucket

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$l = \sqrt{(25-10)^2 + 20^2} = \sqrt{225+400}$

$l = 25m$

$\therefore$ Slant height of bucket (l) $= 25cm$

Curved surface area of bucket $= \pi(r_1 + r_2)l + \pi r_2^2$

$= \pi(25+10)25 + \pi(10)^2$

$= \pi(35)25 + \pi(100) = 975\pi$

C.S.A $= 3061\cdot 5cm^2$

Curved surface area $= 3061\cdot 5cm^2$

Cost of making bucket per $100cm^2 = Rs70$

Cost of making bucket per $3061\cdot 5cm^2 = \frac{3061\cdot 5}{100} \times 70$

$= Rs\ 2143\cdot 05$

$\therefore$ Total cost for $3061\cdot 5cm^2 = Rs\ 2143\cdot 05\ per$

14. Radii of circular ends of a solid frustum off a cone re 33cm and 27cm and its slant height are 10cm. find its total surface area?

Sol:

Given slant height of frustum cone $= 10cm$

Radii of circular ends of frustum cone are 33 and 27cm

$r_1 = 33cm$; $r_2 = 27cm.$

Total surface area of a solid frustum of cone

$= \pi(r_1 + r_2) \times l + \pi r_1^2 + \pi r_2^2$

$= \pi(33+27) \times 10 + \pi(33)^2 + \pi(27)^2$

$= \pi(60) \times 10 + \pi(33)^2 + \pi(27)^2$

$= \pi(600 + 1089 + 729)$

$= 2418\pi cm^2$

$= 7599 \cdot 42 cm^2$

$\therefore$ Total surface area of frustum cone $= 7599 \cdot 42 cm^2$

15. A bucket made up of a metal sheet is in form of a frustum of cone of height 16cm with diameters of its lower and upper ends as 16cm and 40cm. find thee volume of bucket. Also find cost of bucket if the cost of metal sheet used is Rs 20 per 100 cm^2

Sol:

Given height off frustum cone $= 16cm$

Diameter of lower end of bucket $(d_1) = 16cm$

Lower and radius $(r_1) = \frac{16}{2} = 8cm$

Upper and radius $(r_2) = \frac{40}{2} = 20cm$

Let 'l' be slant height of frustum of cone

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$l = \sqrt{(20-8)^2 + 16^2}$

$l = \sqrt{144 + 256}$

$l = 20cm$

$\therefore$ Slant height of frustum cone $(l) = 20cm.$

Volume of frustum cone $= \frac{1}{3}\pi\left(r_1^2 + r_2^2 + r_1 r_2\right)h$

$= \frac{1}{3}\pi\left(8^2 + 20^2 + 8(20)\right)16$

$= \frac{1}{3}\pi(9984)$

Volume $= 10449 \cdot 92 cm^3$

Curved surface area of frustum cone

$= \pi(r_1 + r_2)l + \pi r_2^2$

$= \pi(20+8)20 + \pi(8)^2$

$= \pi(560 + 64) = 624\pi cm^2$

Cost of metal sheet per $100cm^2 = Rs20$

Cost of metal sheet for $624\pi cm^2 = \frac{624\pi}{100} \times 20$

$= Rs\ 391 \cdot 9$

$\therefore$ Total cost of bucket $= Rs\ 391 \cdot 9$

16. A solid is in the shape of a frustum of a cone. The diameter of two circular ends are $60cm$ and 36cm and height is 9cm. find area of its whole surface and volume?

Sol:

Given height of a frustum cone = 9cm

Lower end radius $(r_1)=\frac{60}{2}cm=30cm$

Upper end radius $(r_2)=\frac{36}{2}cm=18cm$

Let slant height of frustum cone be l

$l=\sqrt{(r_1-r_2)^2+h^2}$

$l=\sqrt{(8-30)^2+9^2}$

$l=\sqrt{144+81}$

$l=15cm$

Volume of frustum cone $=\frac{1}{3}\pi\left(r_1^2+r_2^2+r_1r_2\right)h$

$=\frac{1}{3}\pi\left(30^2+18^2+30(18)\right)9$

$=5292\pi cm^3$

Volume $=5292\pi cm^3$

Total surface area of frustum cone =

$=\pi(r_1+r_2)\times l+\pi r_1^2+\pi r_2^2$

$=\pi(30+18)15+\pi(30)^2+\pi(18)^2$

$=\pi\left(48(15)+(30)^2+(18)^2\right)$

$=\pi(720+900+324)$

$=1944\pi cm^2$

$\therefore$ Total surface area $=1944\pi cm^2$

17. A milk container is made of metal sheet in the shape of frustum of a cone whose volume is $10459\frac{3}{7}cm^3$. The radii of its lower and upper circular ends are 8cm and 20cm. find the cost of metal sheet used in making container at rate of $Rs\ 1\cdot 4\, per\ cm^2$?

Sol:

Given lower end radius of bucket $(r_1)=8cm$

Upper end radius of bucket

Let height of bucket be ‘h’

$V_1 = \frac{1}{3}\pi\left(8^2 + 20^2 + 8(20)\right)h\,cm^3$ ....(1)

Volume of milk container $= 10459\frac{3}{4}cm^3$

$V_2 = \frac{73216}{7}cm^3$ ....(2)

Equating (1) and (2)

$V_1 = V_2$

$\Rightarrow \frac{1}{3}\pi\left(8^2 + 20^2 + 8(20)\right)h = \frac{73216}{7}$

$\Rightarrow h = \frac{10459\cdot 42}{653\cdot 45}$

$\Rightarrow h = 16cm$

$\therefore$ Height of frustum cone $(h) = 16cm$

Let slant height of frustum cone be ‘l’

$l = \sqrt{(r_1 - r_2)^2 + h^2}$

$= \sqrt{(20-8)^2 + 16^2} = \sqrt{144 + 256}$

$l = 20cm$

$\therefore$ Slant height of frustum cone $(l) = 20cm$

Total surface area of frustum cone

$= \pi(r_1 + r_2)l + \pi r_2^2 + \pi r_1^2$

$\Rightarrow \pi(20+8)20\pi(20)^2 + \pi(8)^2$

$= \pi(560 + 400 + 64)$

$= \pi(960 + 64) = 1024\pi = 3216\cdot 99cm^2$

Total surface area $= 3216\cdot 99cm^2$